Student Solution Manual

to accompany the textbook

Fixed Income Securities:
Valuation, Risk, and Risk Management

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This *student* solution manual provides solutions to selected exercises at the end of each chapter of the book Fixed Income Securities. Solutions to all of the exercises are instead available only to instructors.

Solutions are provided to the following exercises:

Chapter 2 Exercises 2, 4, and 5;

Chapter 3 Exercises 1, 3, 4, 7, 8, and 9;

Chapter 4 Exercises 1, 4, and 7;

Chapter 5 Exercises 2, 5, 7, 9, and 11;

Chapter 6 Exercises 1, 3, 6, 7, and 10;

Chapter 7 Exercises 2 and 5;

Chapter 8 Exercises 1, 2, and 7;

Chapter 9 Exercises 1 and 5;

Chapter 10 Exercises 1 and 3;

Chapter 11 Exercises 3, 5, and 7;

Chapter 12 Exercises 2, 3, and 5;

Chapter 13 Exercises 2, 3, and 4;

Chapter 14 Exercises 2, 4, 5, and 7;

Chapter 15 Exercises 1, 2, 3, 6, 7, and 9;

Chapter 16 Exercises 1, 4, and 7;

Chapter 17 Exercises 1, 3, and 6;

Chapter 18 Exercises 3, and 5;

Chapter 19 Exercises 1, and 4;

Chapter 20 Exercises 1, 4, and 5;

Chapter 21 Exercises 2, 4, 6, and 8;

Chapter 22 Exercises 1, 2, and 3.

Solutions to Chapter 2

Exercise 2. Compute the quoted price P of the T-bill as:

$$P = 100 \times \left[1 - \frac{n}{360} \times d\right], \tag{1}$$

using the discount rate given, d. The simple (bond equivalent) yield measures your annualized return as:

$$BEY = \frac{100 - P}{P} \times \frac{365}{n}. \tag{2}$$

Let $\tau = \frac{n}{365}$ be the time to maturity expressed as fraction of a year, and let T denote the maturity date of a given T-bill. The continuously compounded yield follows as:

$$r(t, T) = -\frac{1}{\tau} \ln \frac{P}{100}. \tag{3}$$

Finally, to obtain the semi-annually compounded yield for the 1-year T-bill, use:

$$r_2(0, 1) = 2 \times \left(\frac{1}{(P/100)^{1/2}} - 1 \right) \tag{4}$$

	n	$T-t$	Discount, d	Price, P	BEY	Semi-annual yield	Date
a.	28	0.083	3.48%	99.7293	3.5379%	3.53%	12/12/2005
b.	28	0.083	0.13%	99.9899	0.13%	0.13%	11/6/2008
c.	90	0.25	4.93%	98.7675	5.06%	5.03%	7/10/2006
d.	90	0.25	4.76%	98.8100	4.88%	4.86%	5/8/2007
e.	90	0.25	0.48%	99.8800	0.49%	0.49%	11/4/2008
f.	180	0.5	4.72%	97.6400	4.90%	4.84%	4/21/2006
g.	180	0.5	4.75%	97.6250	4.93%	4.87%	6/6/2007
h.	180	0.5	0.89%	99.5550	0.91%	0.90%	11/11/2008
i.	360	1	1.73%	98.2700	1.78%	1.77%	9/30/2008
j.	360	1	1.19%	98.8100	1.22%	1.21%	11/5/2008

For bond i. and j. the continuously compounded yields are 1.75% and 1.20% respectively.

Exercise 4. Using Table 2.4, obtain the discount factor $Z(t,T)$ for each maturity $T-t$ from 0.25 to 7.5 years:

$$Z(t,T) = \frac{1}{\left(1+\frac{r_2(t,T)}{2}\right)^{2(T-t)}}. \tag{5}$$

Use Z to price each bond:

a. $P_z(0,5) = 100 \times Z(0,5) = 72.80$

b. $P_{c=15\%,n=2}(0,7) = \frac{15}{2} \times \sum_{i=1}^{14} Z(0,i/2) + 100 \times Z(0,7) = 151.23$

c. $P_{c=7\%,n=4}(0,4) = \frac{7}{4} \times \sum_{i=1}^{16} Z(0,i/4) + 100 \times Z(0,4) = 101.28$

d. $P_{c=9\%,n=2}(0,3.25) = \frac{9}{2} \times \sum_{i=1}^{7} Z(0,i/2-0.25) + 100 \times Z(0,3.25) = 108.55$

e. 100 (see Fact 2.11)

f. $P_{FR,n=1,s=0} = Z(0,0.5) \times 100 \times (1+\frac{6.8\%}{1}) = 103.44$, where we assume that $r_1(0) = 6.8\%$

g. $P_{FR,n=4,s=0.35\%}(0,5.5) = 100 + \frac{0.35}{4} \sum_{i=1}^{22} Z(0,i/4) = 101.6$

h. $P_{FR,n=2,s=0.40\%} = Z(0,0.25) \times 100 \times (1+\frac{6.4\%}{2}) + \frac{0.40}{2} \sum_{i=1}^{15} Z(0,i/2 - 0.25) = 104$, where we assume that $r_2(0) = 6.4\%$

Exercise 5.

a. When coupon c is equal to the yield to maturity y the bond trades at par; when coupon is below (above) the yield to maturity the bond trades above (below) par. Obtain bond prices given yield and the coupon using:

$$P_c(0,T) = \sum_{i=1}^{20} \frac{c/2 \times 100}{(1+y/2)^i} + \frac{100}{(1+y/2)^{20}} \tag{6}$$

It follows:

c	y	P
5%	6%	107.79
6%	6%	100
7%	6%	92.89

b. Figure **??** plots bond prices implied by different yields to maturity.

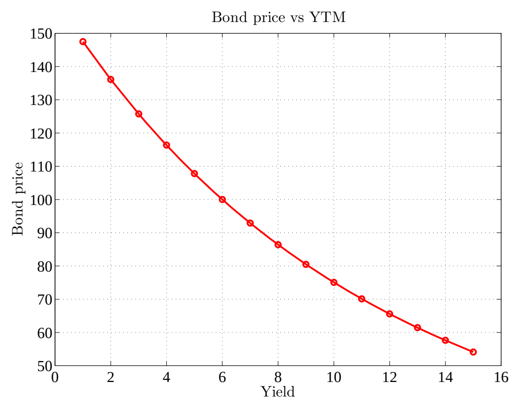

Figure 1: Bond price as function of yield to maturity

Solutions to Chapter 3

Exercise 1.

a. 3; equal to the maturity of the zero bond

b. 2.9542; the duration of the coupon bond is the weighted average of the coupon payment times

c. 0.9850

d. 0.5; equal to the time left to the next coupon payment

e. 0.5111; obtain the price P_{FR} of the floating rate bond (see Chapter 2, equation (2.39)). In analogy to a coupon bond, the duration is computed as:

$$D_{FR} = \frac{100}{P_{FR}} \times 0.5 + \frac{0.5s \times \sum_{t=0.5}^{3} Z(0,t) \times t}{P_{FR}}. \qquad (7)$$

f. 0.2855; proceed as in point e. above but recognize that the valuation is outside the reset date. Assume that the coupon applying to the next reset date has been set at $r_2(0) = 6.4\%$.

Exercise 3. Obtain yield to maturity y for each security. Compute modified and Macaulay duration accodring to equation (3.19) and (3.20) in the book.

	Yield	Duration	Modified	Macaulay
a.	6.95%	3	3	2.8993
b.	6.28%	2.9542	2.9974	2.9061
c.	6.66%	0.9850	0.9850	0.9689
d.	0.00%	0.5	0.5	0.5
e.	6.82%	0.5111	0.5111	0.4943
f.	6.76%	0.2855	0.2855	0.2761

Exercise 4. Compute the duration of each asset and use the fact that the dollar duration is the bond price times its duration.

	Price	Duration	$ Duration
a.	$89.56	4.55	$407.88
b.	$67.63	-7.00	($473.39)
c.	$79.46	3.50	$277.74
d.	$100.00	0.5	$50.00
e.	$100.00	-0.25	($25.00)
f.	$102.70	-0.2763	($28.38)

Exercise 7.

a. $10 mn

b. Compute the dollar duration of the cash flows in each bond, and then the dollar duration of the portfolio:

Security	$ (mn)	Price	N	$\$D$	$\$D \times N$
6yr IF @ 20% - fl quart	20.00	146.48	0.137	1,140.28	155.69
4yr fl 45bps semi	20.00	101.62	0.197	53.54	10.54
5yr zero	(30.00)	76.41	-0.393	382.052	-150.00
Portfolio	**$10.00 mn**				**16.23**

Note: negative values denote short positions.

Exercise 8.

a. The price of the 3yr @ 5% semi bond is $97.82. You want the duration of the hedged portfolio to be zero. You need to short 0.058 units of the 3-year bond, i.e. the short position is -$5.69.

b. The total value of the portfolio is: $4.31 mn.

Exercise 9. Compute the new value of the portfolio assuming the term structure of interest rates as of May 15, 1994.

	Original	Now	Δ value
Unhedged port.	\$10.00	\$8.97	(\$1.03)
Hedge	(\$5.69)	(\$5.44)	\$0.25
Total	\$4.31	\$3.53	(\$0.78)

a. \$8.97 mn

b. \$3.53 mn

c. The immunization covered part of the loss. The change in the value of the portfolio is both due to (*i*) the passage of time (coupon) and (*ii*) the increase in interest rates.

Solutions to Chapter 4

Exercise 1.

a. 16

b. 4.7 (see Fact 4.3)

c. 3.85

d. 0.28 = 0.25 (due to floater with zero spread) + 0.03 (due to the spread)

e. 0.14 = 0.06 (due to floater with zero spread) + 0.08 (due to the spread).
   Assume $r_2(0) = 6.40\%$.

Exercise 4. Follow the steps in Example 4.3 using a 2-year bond instead of a 10-year bond.

Duration Hedging				
Spot Curve Shift	$P_c(0,10)$	$P_z(0,2)$	Position	Change in Portfolio Value
Initial Values	103.58	91.39	-4.5507	
$dr = .1\%$	102.75	91.21	-4.5507	0.0030
$dr = 1\%$	95.63	89.58	-4.5507	0.2880
$dr = 2\%$	88.38	87.81	-4.5507	1.1087
$dr = -.1\%$	104.41	91.58	-4.5507	0.0030
$dr = -1\%$	112.29	93.24	-4.5507	0.3113
$dr = -2\%$	121.84	95.12	-4.5507	1.2957

The hedge performs better in that for any scenario the change in the value of the portfolio is positive.

Exercise 7. You need factor sensitivities for yields with maturities from 3 months to 4.25 years with a 0.25-year spacing. To obtain the 0.25-year grid of sensitivities, interpolate linearly the β's between available maturities. To compute factor durations, use Fact 4.5.

	Security	Price	Level D	Slope D	Curvature D
a.	4 yr zero	81.45	4.10	0.20	0.99
b.	2.5 yr @ 3% semi	96.24	2.50	-0.61	0.77
c.	3.25 yr float 0 bps	100.71	0.25	-0.06	-0.08
d.	4.25 yr float 35 bps	102.12	0.29	-0.07	-0.07

Solutions to Chapter 5

Exercise 2. Use the facts that:

$$F(0, T-\Delta, T) = e^{-f(0,T-\Delta,T)\Delta} \quad (8)$$

$$Z(0, T_i) = Z(0, T_{i-1}) \times F(0, T_{i-1}, T_i). \quad (9)$$

(t, T)	$f(t, T-\Delta, T)$	$F(t, T-\Delta, T)$	yield	$Z(t, T)$
0.25	3.53%	0.9912	3.53%	0.9912
0.50	3.58%	0.9911	3.55%	0.9824
0.75	4.19%	0.9896	3.77%	0.9721
1.00	3.99%	0.9901	3.82%	0.9625
1.25	4.54%	0.9887	3.97%	0.9516
1.50	5.00%	0.9876	4.14%	0.9398
1.75	4.76%	0.9882	4.23%	0.9287
2.00	5.88%	0.9854	4.43%	0.9151
2.25	5.30%	0.9868	4.53%	0.9031
2.50	4.92%	0.9878	4.57%	0.8921
2.75	6.09%	0.9849	4.71%	0.8786
3.00	5.29%	0.9869	4.76%	0.8670
3.25	6.48%	0.9839	4.89%	0.8531
3.50	6.20%	0.9846	4.98%	0.8400
3.75	6.34%	0.9843	5.07%	0.8268
4.00	6.00%	0.9851	5.13%	0.8145
4.25	5.99%	0.9851	5.18%	0.8024
4.50	6.58%	0.9837	5.26%	0.7893
4.75	6.26%	0.9845	5.31%	0.7770
5.00	6.69%	0.9834	5.38%	0.7641
5.25	6.12%	0.9848	5.42%	0.7525
5.50	5.70%	0.9859	5.43%	0.7419
5.75	6.81%	0.9831	5.49%	0.7294
6.00	6.50%	0.9839	5.53%	0.7176
6.25	6.59%	0.9837	5.57%	0.7059
6.50	7.06%	0.9825	5.63%	0.6935
6.75	6.87%	0.9830	5.68%	0.6817
7.00	6.37%	0.9842	5.70%	0.6709

Exercise 6.

a. $P^{fwd} = 100 \times F(0, 0.5, 2) = \$90.3210$

b. $M = \frac{\$50\,mn}{P^{fwd}} = 0.554\,mn$

Exercise 7.

a. Payoff $= M \times (P(0.5, 2) - P^{fwd}) = \$0.64\,mn$

b. You make money on the long forward.

Exercise 9. Use fact 5.11 to obtain the swap rate $c(t, T)$:

(t, T)	yield	$Z^L(t, T)$	$c(t, T), n = 2$
0.50	2.76%	0.9863	2.77%
1.00	2.95%	0.9710	2.96%
1.50	2.98%	0.9564	2.99%
2.00	3.20%	0.9383	3.20%

Exercise 11. On that day, the net spread was strongly negative:

$$\begin{aligned} SS &= c - ytm = -0.33\% \\ LRS &= LIBOR - repo = 1.71\% \\ SS - LRS &= -2.04\% \end{aligned}$$

You could envision the following strategy to exploit the large negative spread:

1. Buy Treasury through a repo transaction:	get coupon, pay repo
2. Enter the floating-for-fixed swap:	pay fixed, get LIBOR

Solutions to Chapter 6

Exercise 1. From semi-annually compounded yields obtain (*i*) the discount factors, and (*ii*) the forward discount factors. Use Fact 5.9 in Chapter 5 to compute the forward price of a coupon bond.

a. 98.85

b. 0

c. Compute the time-t value of the forward as:

$$V(t) = Z(t,T) \times \left[P_c^{fwd}(t,T,T^*) - K\right],\ K = P_c^{fwd}(0,T,T^*) \tag{10}$$

d. See Table below

e. Obtain the daily overnight discount factor $Z(o/n)$, then:

$$Total\ P/L(t_i) = P/L(t_i) + \frac{Total\ P/L(t_{i-1})}{Z_{o/n}(t_{i-1})} \tag{11}$$

	2/15/94	2/16/94	2/17/94	2/18/94	2/22/94	2/23/94
	t_0	t_1	t_2	t_3	t_4	t_5
P_c^{fwd}	98.85	98.73	98.23	97.77	98.13	97.84
$V(t)$	0	-0.1156	-0.5691	-0.9913	-0.6623	-0.9206
P/L		-0.1262	-0.4964	-0.4616	0.3588	-0.2846
r o/n		3.54%	3.78%	3.90%	4.44%	2.88%
$Z(o/n)$		0.9999	0.9999	0.9998	0.9998	0.9999
Tot. P/L		-0.1262	-0.6227	-1.0844	-0.7257	-1.0105

Exercise 3.

a. Yes, the rates did converge: Libor equals the futures rate at expiry.

b. Total profit from futures: $0.475 = 0.25 \times (97.2912 - 95.3900)$

c. Total profit from forward: $0.547 = 100 \times (0.9933 - 0.9878)$

d.,e.,f. See Figure **??**.

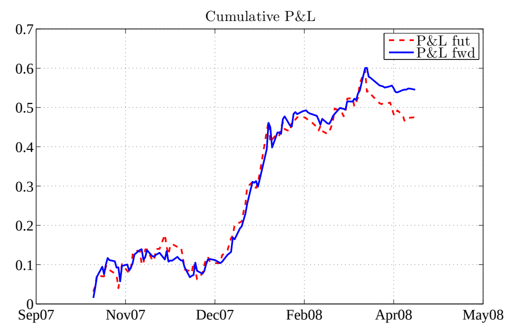

Figure 2: P&L from the futures and forward contracts

Exercise 6.

a.i. The empirical probability that the firm has enough cash for the lawsuit is 79% (see Figure **??**, a.i).

a.ii The firm would have enough cash in 54% of scenarios (see Figure **??**, a.ii).

α	0.00404
β	0.88152
s.e.	0.00853
$r(0)$	5.2088%

b. Ex post the company will not have enough cash: $98.78 \times (1 + 2.7088\%/4) = 99.4489$.

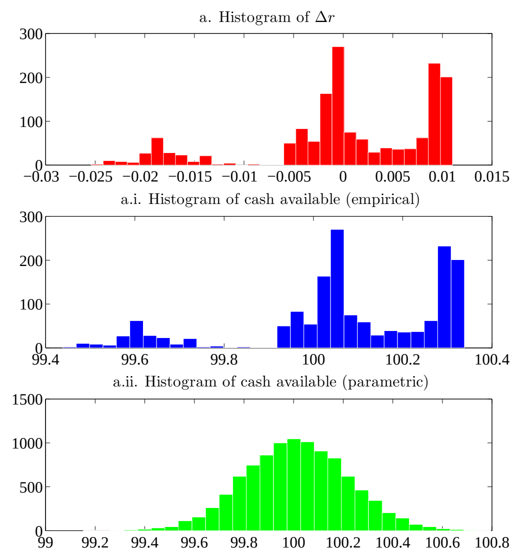

Figure 3: Distribution of possible cash flows for the firm

Exercise 7.

	Price	P&L	Total	3mo Libor	After 90 days	Final ($ mn)
a.	$98.86	0.475	99.336	2.7088%	100.009	0.009
b.	$98.86	-0.475	98.385	6.5112%	99.987	-0.013

Under scenario in b. (from Table 6.12), the firm is short on the cash they need, receiving 99.987 out of 100 required.

Exercise 10.

a. The securities are not correctly priced, as the put-call parity is violated:

Put	$0.1044
Call	$0.2934
$P^{fwd}(0, 0.5, 0.75)$	98.96
K	99.12
$Z(0, 0.5) \times (P^{fwd} - K)$	-0.1590
Call from P/C parity	($0.0546)

b. Strategy long call, short put, short forward gives a positive cash-flow of \$0.3480 at no risk.

Solutions to Chapter 7

Exercise 2.

a. Using Equation 7.28, minimize squared distance between the observed and the model-based prices (computed from equation 7.27) to obtain the parameters in the table below. Using the parameters compute real yields and the discount factors (see Figure **??**).

Parameter	Value
θ_0	6278.301
θ_1	-6278.23
θ_2	-6289.19
θ_3	-0.18757
κ_1	27056.49
κ_2	32.19053

b. Obtain the analytical expression for the short interest rate by taking the limit of $r(0,T)$ implied by the Nelson-Siegel model:

$$\lim_{T\to 0} r(0,T) = \theta_0 + \theta_1 = 7.58\% \tag{12}$$

Use the following facts:

$$\lim_{T\to 0} \frac{1-e^{-T/\kappa_1}}{T/\kappa_1} = 1 \tag{13}$$

$$\lim_{T\to 0} = e^{-T/\kappa_1} = 1. \tag{14}$$

c.,d. Using the NS yield curve, the TIPS with maturity 3.372 years (maturing 4/14/2012) is priced at \$86.23. This is \$2.755 lower relative to the traded price.

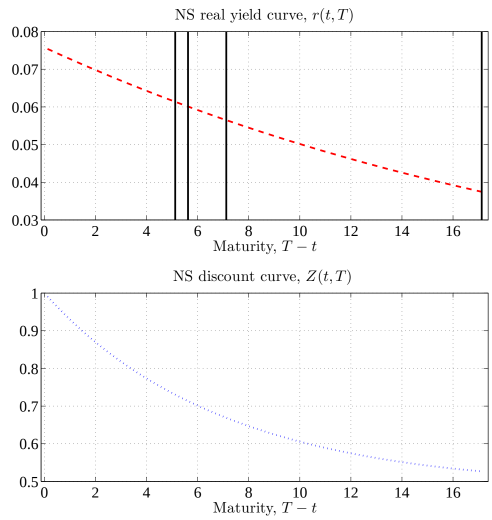

Figure 4: NS yields and discount factors. Vertical lines indicate the maturities of bonds used in estimation.

Exercise 5.

b. If there is no inflation, the security will pay real coupon $\times 1.00031$, as the index ratio will not change. If there is a deflation, and should the index ratio for the security go below 1, it will be automatically reset to 1. In this case, the security will pay an amount equal to the real coupon.

c. The bond is more valuable to investors. Compared to the other securities, the bond offers a deflation floor, and this attractive feature will be priced in.

d. The price of the security using the NS real curve is 89.57. (You have to compute dirty prices of bonds first.)

e. The squared pricing error for this security is

$$(\hat{P}^{NS} - Dirty\ \hat{P})^2 = (89.57 - 100.7)^2 = 123.92, \tag{15}$$

and is much larger than on all other securities.

f. The model-implied clean price is: $(89.57 - 0.85) \times 1.00031 = 88.74$. The squared error on this price is: $(88.74 - 99.88)^2 = 123.99$.

g. Yes.

Solutions to Chapter 8

Exercise 1. The results are reported in the following Table.

$T-t$	$Yield$	$Z(t,T)$	$T-t$	$Yield$	$Z(t,T)$	$T-t$	$Yield$	$Z(t,T)$
0.017	5.336	0.999	0.997	5.379	0.948	7.206	5.596	0.668
0.036	5.336	0.998	1.083	5.383	0.943	7.456	5.603	0.659
0.039	5.336	0.998	1.167	5.386	0.939	7.956	5.617	0.640
0.047	5.337	0.997	1.206	5.388	0.937	8.456	5.630	0.621
0.056	5.337	0.997	1.242	5.389	0.935	8.706	5.636	0.612
0.075	5.338	0.996	1.333	5.393	0.931	9.206	5.648	0.595
0.083	5.339	0.996	1.414	5.397	0.927	9.706	5.660	0.577
0.092	5.339	0.995	1.456	5.398	0.924	14.206	5.744	0.442
0.111	5.340	0.994	1.500	5.400	0.922	14.706	5.752	0.429
0.131	5.341	0.993	1.581	5.403	0.918	14.956	5.755	0.423
0.150	5.342	0.992	1.667	5.407	0.914	15.206	5.759	0.417
0.167	5.342	0.991	1.706	5.409	0.912	15.456	5.762	0.410
0.186	5.343	0.990	1.750	5.410	0.910	15.956	5.768	0.398
0.206	5.344	0.989	1.831	5.414	0.906	16.456	5.774	0.387
0.225	5.345	0.988	1.917	5.417	0.901	16.706	5.777	0.381
0.242	5.346	0.987	1.956	5.419	0.899	17.456	5.785	0.364
0.250	5.346	0.987	1.997	5.420	0.897	17.956	5.790	0.354
0.269	5.347	0.986	2.083	5.424	0.893	18.206	5.793	0.348
0.289	5.348	0.985	2.167	5.427	0.889	18.706	5.797	0.338
0.308	5.349	0.984	2.206	5.429	0.887	19.206	5.801	0.328
0.328	5.350	0.983	2.242	5.430	0.885	19.456	5.803	0.323
0.333	5.350	0.982	2.333	5.434	0.881	19.706	5.805	0.319
0.344	5.350	0.982	2.414	5.437	0.877	20.206	5.809	0.309
0.364	5.351	0.981	2.456	5.439	0.875	20.456	5.810	0.305
0.383	5.352	0.980	2.500	5.440	0.873	20.706	5.812	0.300
0.403	5.353	0.979	2.581	5.443	0.869	20.956	5.814	0.296
0.414	5.353	0.978	2.706	5.448	0.863	21.706	5.818	0.283
0.422	5.354	0.978	2.956	5.458	0.851	21.956	5.819	0.279
0.442	5.355	0.977	3.206	5.467	0.839	22.206	5.820	0.275
0.456	5.355	0.976	3.456	5.476	0.828	22.706	5.822	0.267
0.461	5.356	0.976	3.706	5.485	0.816	23.956	5.827	0.248
0.481	5.356	0.975	3.956	5.494	0.805	24.206	5.827	0.244
0.500	5.357	0.974	4.206	5.503	0.793	24.706	5.828	0.237
0.517	5.358	0.973	4.456	5.512	0.782	25.206	5.829	0.230
0.581	5.361	0.969	4.706	5.520	0.771	25.706	5.830	0.223
0.667	5.365	0.965	4.956	5.528	0.760	25.956	5.830	0.220
0.706	5.366	0.963	5.206	5.536	0.750	26.206	5.831	0.217
0.747	5.368	0.961	5.456	5.544	0.739	26.706	5.831	0.211
0.750	5.368	0.961	5.622	5.550	0.732	26.956	5.831	0.208
0.831	5.372	0.956	5.872	5.557	0.722	27.706	5.831	0.199
0.917	5.375	0.952	6.206	5.568	0.708	27.956	5.830	0.196
0.956	5.377	0.950	6.456	5.575	0.698	28.206	5.830	0.193
0.994	5.379	0.948	6.706	5.582	0.688	28.706	5.830	0.188

Exercise 2. Follow the lines of Example 8.2 and Table 6.3. Plug in the NS parameters to compute the discount curve.

a. The price is \$316.39 mn, and is above the \$300 mn par value of the security.

b. Under the (unrealistic) assumption of constant PSA, you can apply the definition of duration in Chapter 3, and compute it in a standard way (see Section 3.2.3). The duration is 6.47.

c.,d. Compute the prices of the pass through under the different scenarios taking into account the parallel shift in the curve and the change in the PSA. Use definitions 8.1 and 8.2 to obtain the effective duration and convexity, respectively.

Prices under the three scenarios:	
P(dr = 0bps, PSA=150%)	316.39
P(dr = +50bps, PSA=120%)	306.68
P(dr = −50bps, PSA=200%)	323.89

Effective Duration	5.44
Effective Convexity	-278.44

The effective duration is lower than the one obtained under the assumption that the change in rates does not affect the PSA. Standard duration overstates the sensitivity of the MBS price to changes in interest rates.

In contrast to the Treasury bonds, the convexity of an MBS is negative (i.e. the value profile is concave with respect to interest rate changes). Therefore, convexity presents a source of risk to investors. This risk is associated with the prepayment option that homeowners have. Effectively, an MBS investor is short an American call option to the homeowners.

Exercise 7. Results are reported in the following table

Tranche	A	B	C	E	G	H
Face	127.50	51.00	25.50	68.00	59.50	93.50
	a. 10/1/1993, PSA = 450%					
Price	119.76	36.06	11.38	57.83	51.09	72.48
Duration	1.99	4.81	7.53	2.82	2.71	2.86
	b. 4/4/1994, PSA = 450%					
Price	108.52	34.45	9.55	53.76	47.80	70.37
Duration	1.67	4.30	6.93	2.43	2.31	2.23
	c. 4/4/1994, PSA = 200%					
Price	108.52	34.45	9.55	53.76	38.55	23.55
Duration	1.67	4.30	6.93	2.43	2.70	7.71

(b) i. The interest rates have increased (in a nonparallel fashion). The behavior of the G+H portfolio is shown in the following Table.

				a.	b.ii.	c.iv.
PSA				450%	450%	200%
Date				10/1/93	4/4/94	4/4/94
	Tranche	Inv.	Weight	Price	Price	Price
	G	50	0.98	51.09	47.80	38.55
	H	50	0.69	72.48	70.37	23.55
Port.		100		100	95.32	53.98
ΔPort.					-4.68	-46.02

(c) i. Yes, the term structure has increased, so the PSA has declined (lower prepayment speed).

ii. No. Since all tranches are PO, the portfolio is a bet on decreasing interest rates (it wins when interest rates decline). Between October and April interest rates moved up, therefore the portfolio is losing money. The computed duration confirms this intuition.

Solutions to Chapter 9

Exercise 1.

a. The expected return is equal to 2.5%.

b. The forward rate (continously compounded) is equal to 3.0954%. This is higher than the expected rate computed in Part (a). If we observe high forward rates it may be because of two posibilities: either market participants expect higher future interest rates; or they are strongly averse to risk, and thus the price of long term bonds is low today.

c. The market price of risk equals: λ = -0.1980. The high (negative) market price of risk, means that market participants have high risk aversion, which may explain the price of long term bonds today.

d. The risk neutral probability equals: $p* = 0.7000$. The interpretation is the same as in Part (c).

Exercise 5.

a. The value of r_0 is: 5%

b. The following three pairs of values for $(r_{1,u}, r_{1,d})$ are consistent with the two bond prices: (7.010, 3.005), (7.000, 3.015) and (6.000, 4.000). In fact the relationship between both interest rate scenarios is linear and can be summarized by the following equation:

$$r_{1,d} = -0.9852 \times r_{1,u} + 9.9115 \tag{16}$$

Note that this is consistent with the idea of risk neutrality. In order to compensate a decrease (increase) in the up state, the down side must increase (decrease); given that the risk neutral probabilities are fixed.

c. The option gives an information on actual prices, so we can infer the market price of risk and therefore can pin down the actual values for $(r_{1,u},\, r_{1,d})$. These values should be consistent in order to value the option at the given price. The values for $(r_{1,u},\, r_{1,d})$ are: (7.000, 3.015).

d. If you didn't know $p*$ you would also need the value of the interest rate in the up and down states or the prices of the bonds in these states.

Solutions to Chapter 10

Exercise 1.

a. The bond evolves in the following way:

i = 0	1	2
90.0000	93.2394	100.000
	97.0446	100.000
		100.000

b. The market price of risk is: λ = -0.3709

c. The anwers are the following:

   i. The market price of risk is: λ = -0.3709. Which is the same as the one computed from the bond, this is because the market price of risk is the same across securities since it measures the willingness of agents to hold risky assets. In other words it is a measure on the economic agents and not on the instruments themselves.

   ii. The price of the option is 0.2238.

   iii. The result holds the same.

d. The price of the option is 0.0245.

e. The option has the following structure over time:

i = 0	1	2
0.0245	0.0000	0.0000
	0.2234	0.0000
		2.0199

The positions on the short-term bond (N_1) and on the three-period bond (N_2) vary over time, in order to replicate the option structure, in the following way:

N_1		N_2	
i=0	1	i=0	1
-0.0244	0.0000	0.0287	0.0000
	-0.6632		0.6972

Exercise 3. The following results vary greatly with the dataset used. The results presented use values from September 1, 2009. Which are the following:

LIBOR 3-month

I	1999	5.009%	I	2003	1.288%	I	2007	5.348%
II	1999	5.355%	II	2003	1.116%	II	2007	5.359%
III	1999	6.083%	III	2003	1.160%	III	2007	5.621%
IV	1999	6.005%	IV	2003	1.157%	IV	2007	5.131%
I	2000	6.289%	I	2004	1.111%	I	2008	3.058%
II	2000	6.778%	II	2004	1.604%	II	2008	2.681%
III	2000	6.816%	III	2004	2.005%	III	2008	2.811%
IV	2000	6.403%	IV	2004	2.558%	IV	2008	2.217%
I	2001	4.877%	I	2005	3.100%	I	2009	1.264%
II	2001	3.791%	II	2005	3.505%	II	2009	0.656%
III	2001	2.597%	III	2005	4.006%	III	2009	0.348%
IV	2001	1.883%	IV	2005	4.530%			
I	2002	2.031%	I	2006	4.990%			
II	2002	1.860%	II	2006	5.509%			
III	2002	1.806%	III	2006	5.373%			
IV	2002	1.383%	IV	2006	5.360%			

Swaps and Eurodollar Futures

Maturity	Swap rate	Maturity	Eurodollar Futures
1 year	0.61%	3 months	99.535
2 year	1.29%	6 months	99.305
3 year	1.91%	9 months	98.945
4 year	2.37%	12 months	98.555
5 year	2.72%		
7 year	3.20%		

a. The current 3-month interest rate is 0.3475%, when converted to continous compounding we get: 0.3473%. The regression gives the following parameters (t-stats in parenthesis): $\alpha =$ -0.0003 (-0.1497) and $\beta = 0.97763$ (19.7565). The predicted value for next quarter is $m_{t+i} = 0.3396\%$.

b. We estimate $\sigma = 0.006055$. The tree for the first 3 ($i = 3$) periods looks like this:

$i = 0$	1	2	3
0.35%	0.74%	1.12%	1.50%
	0.13%	0.52%	0.90%
		-0.09%	0.29%
			-0.31%

c. The zero coupon yield curve can be summarized with the following discounts:

$Z(0,T)$	Value
$Z(0,0.25)$	99.9132
$Z(0,0.50)$	99.7533
$Z(0,0.75)$	99.5200
$Z(0,1.00)$	99.3922
$Z(0,1.25)$	99.0297
$Z(0,1.50)$	99.0640
$Z(0,1.75)$	98.0851
$Z(0,2.00)$	97.4451
$Z(0,2.25)$	96.7885
$Z(0,2.50)$	97.2324
$Z(0,2.75)$	94.8043
$Z(0,3.00)$	94.3874

These values were computed with the Eurodollar futures and swap data.

d. With the emprical σ you get probabilites outside of the [0,1] range. If you increase σ enough you can go into the range. In this specific case, volatility had to be increased by 10 percentage points, to get the first eight periods to be within the range.

e. When the probabilities are working as they should, we get that the expected risk neutral rate is higher than the one predicted by the regression.

This makes sense in order to make economic agents risk neutral, they must be compensated with enough upside. Recall the discussion at the end of chapter 9 (section 9.4.4).

Solutions to Chapter 11

Exercise 3.

a. The price of the zero coupon bonds is:

$Z(0,T)$	Value
$Z(0,1)$	0.9608
$Z(0,2)$	0.9141
$Z(0,3)$	0.8659

b. The swap rate $c(3)$ is: 4.89%.

c. For the option described in the exercise:

i. The value of the option is: 0.1532.

ii. In order to hedge this security:

1. Choose two securities to hedge this one, for example the 1-year zero coupon bond and the 2-year zero coupon bond.
2. Compute the values for N_1 and N_2 which give the amount of each security that we need to buy. In this case the values are N_1 = 8.1358 and N_2 = -8.3835.
3. Verify that this strategy actually gives the value of the bond:

$$[N_1 \times Z(0,1)] + [[N_2 \times Z(0,2)] = 0.1532$$

You can also verify that this replicates the up and down states, by using the inputs for $i = 1$:

* Up State - When interest rates go from 4% to 7% we have that:

$$[N_1 \times Z_{up}(1,1)] + [[N_2 \times Z_{up}(1,2)] = 0.3190$$

* Down State - When interest rates go from 4% to 3% we have that:

$$[N_1 \times Z_{dn}(1,1)] + [[N_2 \times Z_{dn}(1,2)] = 0.0000$$

Which matches exactly the payoffs of the option.

d. For the Procter & Gamble leveraged swap we have:

i. The value of the security is: -27.5495.

ii. Intuitively the value of $\bar{r}$ that makes the swap value zero should be higher than $c(3)$. P&G is paying more now, through the states in which it has to pay the spread, which means that in order to be compensated it should expect a higher rate from the fixed rate that Bankers Trust pays.

iii. Using MS Excel Solver we find the value of this rate to be: 14.94%.

Exercise 5.

a. The zeros from the LIBOR curve are the following:

t	$P_z(0,t)$	t	$P_z(0,t)$	t	$P_z(0,t)$
0.25	99.2904	2.75	92.0454	5.25	81.4258
0.50	98.6053	3.00	90.9898	5.50	80.3754
0.75	98.1544	3.25	89.9522	5.75	79.3266
1.00	97.6061	3.50	88.8979	6.00	78.2843
1.25	96.9954	3.75	87.8326	6.25	77.2529
1.50	96.3399	4.00	86.7628	6.50	76.2365
1.75	95.6373	4.25	85.6875	6.75	75.2401
2.00	94.8950	4.50	84.6072	7.00	74.2689
2.25	94.0613	4.75	83.5326	7.25	73.3133
2.50	93.0947	5.00	82.4744	7.50	72.3608

b. The Ho-Lee model in the first 2 years ($i = 8$):

$i = 0$	1	2	3	4	5	6	7	8
$t = 0$	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
2.85%	2.82%	2.96%	3.42%	4.08%	4.67%	5.28%	5.87%	6.67%
	2.04%	2.17%	2.63%	3.30%	3.89%	4.50%	5.08%	5.89%
		1.39%	1.85%	2.51%	3.11%	3.72%	4.30%	5.11%
			1.07%	1.73%	2.33%	2.93%	3.52%	4.32%
				0.95%	1.54%	2.15%	2.73%	3.54%
					0.76%	1.37%	1.95%	2.76%
						0.59%	1.17%	1.98%
							0.39%	1.19%
								0.41%

c. The following table compares the risk neutral expected future interest rate with the forward rate.

	$\sigma = 0.0078$		$\sigma = 0.05$		$\sigma = 0.001$	
t	$E^*[r_t]$	$f(t\text{-}1,t)$	$E^*[r_t]$	$f(t\text{-}1,t)$	$E^*[r_t]$	$f(t\text{-}1,t)$
0.00	2.849%	2.849%	2.849%	2.849%	2.849%	2.849%
0.25	2.430%	2.430%	2.438%	2.430%	2.430%	2.430%
0.50	2.174%	2.173%	2.204%	2.173%	2.173%	2.173%
0.75	2.243%	2.241%	2.311%	2.241%	2.241%	2.241%
1.00	2.514%	2.511%	2.636%	2.511%	2.511%	2.511%
1.25	2.717%	2.712%	2.908%	2.712%	2.712%	2.712%
1.50	2.935%	2.928%	3.209%	2.928%	2.928%	2.928%
1.75	3.126%	3.117%	3.499%	3.117%	3.117%	3.117%
2.00	3.542%	3.530%	4.029%	3.530%	3.530%	3.530%
2.25	4.147%	4.132%	4.764%	4.132%	4.132%	4.132%
2.50	4.553%	4.534%	5.315%	4.534%	4.534%	4.534%
2.75	4.637%	4.614%	5.558%	4.614%	4.614%	4.614%

Note that as volatility increases, so does the difference between the rates.

d. The value of the 1-year cap is 0.2374, the value of the 2-year cap is 0.6003, and the value of the 3-year cap is 1.3805. In general the model underestimates the price of the securities.

e. The value of the swap is zero, as expected.

f. The value of the swaption is: 1.9956.

Exercise 7.

a. The following table reports the first values of the LIBOR curve, in discount factors:

t	$P_z(0,t)$
0.50	99.6053
0.75	99.4966
1.00	99.3555
1.25	99.1319
1.50	98.8526
1.75	98.5171
2.00	98.1323
2.25	97.6366
2.50	96.9852

b. The implied volatilities are:

t	$\sigma_t^{implied}$
0.50	0.1775
0.75	0.2288
1.00	0.3460
1.25	0.4298
1.50	0.4616
1.75	0.4804
2.00	0.4801
2.25	0.4788
2.50	0.4752

c. The forward volatilities are:

t	$\sigma_t^{forward}$
0.50	0.2925
0.75	0.2975
1.00	0.3054
1.25	0.2597
1.50	0.2054
1.75	0.1926
2.00	0.1813
2.25	0.1826
2.50	0.1659

d. The price of the corridor note is 80.218.

Solutions to Chapter 12

Exercise 2.

a. The value of the American swaption is 0.2101.

b. The value of the callable bond is 99.74.

c. In order to *only* hedge the prepayment risk you hedge the underlying option, intead of the callable bond itself. The hedging strategy is the following:

$i =$	0	1
Short-term bond (N1)	0.0077	0.0012
American swaption (N2)	-1.6144	-0.1252

Note that there is no hedging strategy for node (1,1) since the option is retired at that point.

d. To hedge the interest rate risk (which includes prepayment risk) of the callable bond the investor can use the following strategy:

$i =$	0	1
Short-term bond (N1)	1.0525	1.0525
American swaption (N2)	-1.8073	-0.9418

As in the previous part, there is no hedging strategy for node (1,1).

Exercise 3.

(a) The results on the mortgage are as follows:

i. The value of the coupon is $C = 11,132.65$. The stream of scheduled interest payments, principal payments, and the remaining principal is:

i	t	Interest paid	Principal payments	Remaining principal
0	0.0	0.0	0.0	100,000
1	0.5	2,000	9,133	90,867
2	1.0	1,817	9,315	81,552
3	1.5	1,631	9,502	72,050
4	2.0	1,441	9,692	62,359
5	2.5	1,247	9,885	52,473
6	3.0	1,049	10,083	42,390
7	3.5	848	10,285	32,105
8	4.0	642	10,491	21,615
9	4.5	432	10,700	10,914
10	5.0	218	10,914	0

ii. The interest rate tree at semi-annual frequency, for the first seven periods, is:

$i = 0$	1	2	3	4	5	6	7
1.74%	2.87%	4.69%	6.39%	8.72%	10.67%	12.17%	13.72%
	2.17%	3.53%	4.82%	6.58%	8.04%	9.17%	10.34%
		2.66%	3.63%	4.96%	6.06%	6.91%	7.79%
			2.73%	3.73%	4.57%	5.21%	5.87%
				2.81%	3.44%	3.92%	4.43%
					2.60%	2.96%	3.34%
						2.23%	2.51%
							1.90%

iii. The value of the mortgage without the prepayment option is:

$i = 0$	1	2	3	4	5	6	7
100,359	88,791	77,636	67,110	57,045	47,527	38,283	29,049
	91,416	80,252	69,578	59,266	49,384	39,716	30,054
		82,305	71,519	61,015	50,845	40,840	30,839
			73,031	62,377	51,981	41,713	31,447
				63,430	52,859	42,387	31,915
					53,533	42,903	32,273
						43,297	32,546
							32,753

iv. The value of the American option implicit in the mortgage is:

$i = 0$	1	2	3	4	5	6	7
288	33	5	0	0	0	0	0
	548	61	10	0	0	0	0
		753	114	21	1	0	0
			981	210	42	2	0
				1071	386	84	4
					1060	513	168
						907	441
							648

v. The option-adjusted value of the mortgage is: 100,071.

A. The prepayment option is going to be exercised when the value of the mortgage exceeds the outstanding principal. This occurs in the following nodes:

$i =$	0	1	2	3	4	5	6	7
0	H	H	H	H	H	H	H	H
1		X	H	H	H	H	H	H
2			R	H	H	H	H	H
3				R	H	H	H	H
4					R	X	H	H
5						R	R	X
6							R	R
7								R

where H stands for Hold, X for Exercise, and R for Retired.

B. Yes. Any path leading to nodes (7,5) or (9,6).

C. It is fairly priced in the sense that it takes into account the prepayment option, assuming that agents act optimally. But this may not be the case, people may forget to exercise at the right time or they may be other factors affecting their decision that are not taken into account in the model.

D. If the homeowner doesn't refinance then the value of the option is higher than before. Refinancing is done for the homeowner's benefit, if he 'forgets' to do so, it is the holder of the security that gets an extra amount of cash.

(b) The results for the mortgage backed securities are as follows:

i. The value of the pass-through is 99,334. The spot rate duration is: 4.4084.

ii. The value of the PO strip is 93,989 and the value of the IO strip is 5,345.

A. The sum of the IO and PO strips equals the value of the pass-trough security.

Exercise 5.

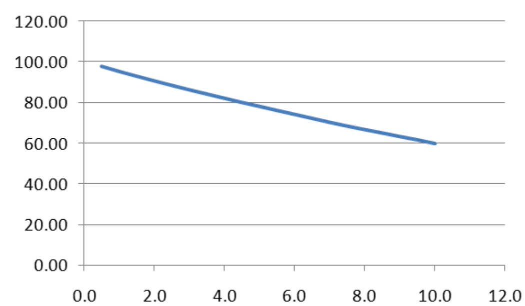

Figure 5: Price of zeros (discounts) for different maturities

a. The following table reports all zero coupon bonds using the bootstrap method. See Figure **??**

T	$P_z(0,T)$
0.5	97.58
1.0	95.10
1.5	92.76
2.0	90.49
2.5	88.26
3.0	86.08
3.5	84.01
4.0	81.96
4.5	79.89
5.0	77.97
5.5	76.01
6.0	74.07
6.5	72.11
7.0	70.17
7.5	68.32
8.0	66.60
8.5	64.95
9.0	63.22
9.5	61.59
10.0	59.77

b. The following shows the values of the BDT tree for the first 6 periods, using $\sigma = 0.2018$.

i= 0	1	2	3	4	5	6
4.91%	5.87%	6.51%	7.40%	8.52%	9.74%	10.92%
	4.41%	4.89%	5.56%	6.40%	7.32%	8.21%
		3.68%	4.18%	4.81%	5.50%	6.17%
			3.14%	3.62%	4.14%	4.64%
				2.72%	3.11%	3.49%
					2.34%	2.62%
						1.97%

c. On the Option Adjusted Spread (OAS):

i. The price of the callable note is 98.818.

ii. The OAS is: 26 bps.

iii. In order to make the OAS equal to zero, $\sigma = 0.15$.

d. The price of the callable note is 42.312. The main reason is that volatilty has a bigger impact in the Ho-Lee model, which in turn raises the value of the option and reduces the value of the callable security.

e. The duration of the security is: 3.9973.

f. Convexity:

   i. The dollar spot rate duration at the required nodes is: $D^{\$}_{i+1,j} = 443.2313$ and $D^{\$}_{i+1,j+1} = 261.7521$.

   ii. Spot rate convexity is then: -126.025.

j. If instead of June, 2009 the lockout period finishes June, 2008 (i.e. instead of lasting two years it lasts one year), the convexity goes to -222.62.

Solutions to Chapter 13

Exercise 2.

a. The first 6 steps of the monthly Ho-Lee tree look like the following:

$i = 0$	1	2	3	4	5	6
5.00%	5.29%	5.58%	5.87%	6.16%	6.44%	6.73%
	4.71%	5.00%	5.29%	5.58%	5.87%	6.16%
		4.42%	4.71%	5.00%	5.29%	5.58%
			4.13%	4.42%	4.71%	5.00%
				3.85%	4.13%	4.42%
					3.56%	3.85%
						3.27%

b. The price of the interest rate barrier option would be 0.1316. The problem with the backward methodology is that we cannot distinguish the states that go down and out, since this is path dependant.

c. Through the BDT model we get a price of 0.0131, which is significantly lower than the previous value. This is because the BDT tends to be more concentrated around the center than the Ho-Lee model. This means that positive results are less extreme than in the Ho-Lee model. Thus the lower value.

Exercise 3.

a. The trigger rates are:

i	r_{-i}
0.5	2.17%
1.0	2.66%
1.5	2.73%
2.0	2.81%
2.5	3.44%
3.0	2.96%
3.5	3.34%
4.0	2.97%
4.5	3.64%

b. The value of the mortgage is: 100,477, which is still far from the value of the security obtained in the previous section: 100,071 (still outside of the confidence intervals). This may be because only 1,000 simulations were used.

c. The value of the 3.5% passthrough security is: 99,154.

d. Adding some probability to the model we get:

   i. Adding a 50 PSA we get that the price of the mortgage is now: 99,301.

   ii. If 80% of homeowners forget to exercise at the optimal time we get that the price of the passthrough is now: 99,294.

e. This is good news for the bank issuing the mortgage since now it gets more money. As seen in the previous question, when homeowners don't exercise optimally it means that mortgages increase in value.

f. The price of the IO strips is 9,259 with spot rate duration -3.81. The price of the PO strips is 89,895 with spot rate duration -5.95. Note that here only optimal exercise is considered.

Exercise 4.

a. The results are the same as in chapter 11, exercise 5.

b. See chapter 11, exercise 5.

c. After around 600 simulations the average error over 40 periods goes below 10 cents on the value of a zero coupon bond with face value 100.

d. The price from the tree model for a 1-year cap is 0.2401. Through the simulation approach (1000 simulations) we get 0.2381, with a a standard error of 0.0051. The price from the tree is well within the confidence intervals. Sigma used to price a 1-year cap is 0.00795.

e. The price of the Asian cap is 0.2249 with a standard error of 0.0119. Sigma for pricing a 2 year bond is 0.012354.

f. Spot rate duration for the option is -250.557.

Solutions to Chapter 14

Exercise 2. Results for both cases should be the same as figures in the book. Changes may appear from different values in the random number generator.

Exercise 4.

a. Consider the base case with the following parameters: $\gamma = 1$, $\bar{r} = 5.40\%$, $\sigma = 1\%$, $r(0) = 0.17\%$ and $dt = 1/252$; we have:

  i. For γ: As this parameter increases, the 'speed' by which the rate converges to the long-term rate, $\bar{r}$, increases. Inversely as this parameter decreases, the rate converges slower to $\bar{r}$ (see Figure **??**).

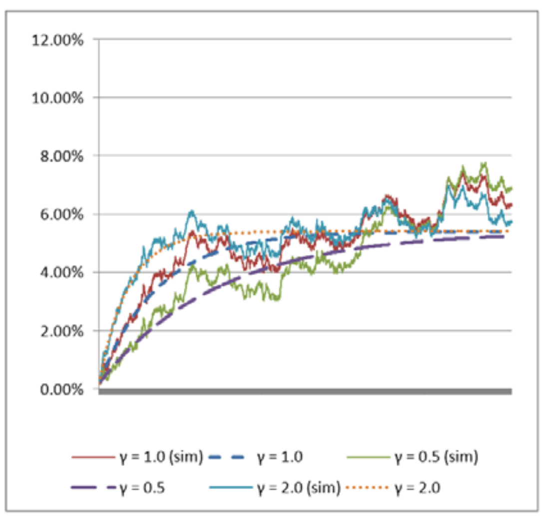

Figure 6: Vasicek simulations and expected value for different values of γ

ii. For σ: As this parameter increases the volatility of the process goes up. Inversely as it goes down so does the volatility of the process (see Figure ??).

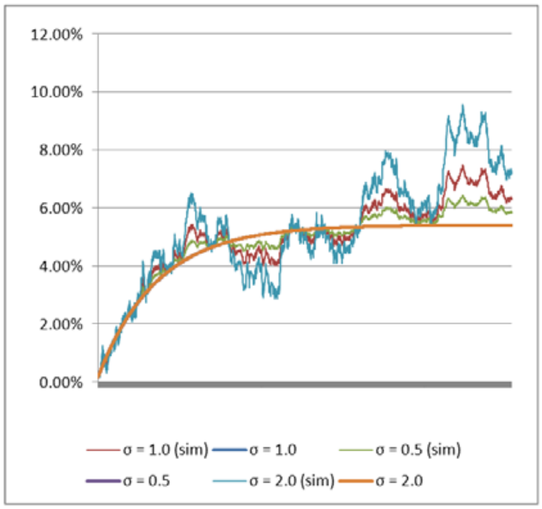

Figure 7: Vasicek simulations and expected value for different values of σ

b. The effects of having different values for r(0) can be seen in Figure ??. We choose values that are higher, lower and the same as $\bar{r}$

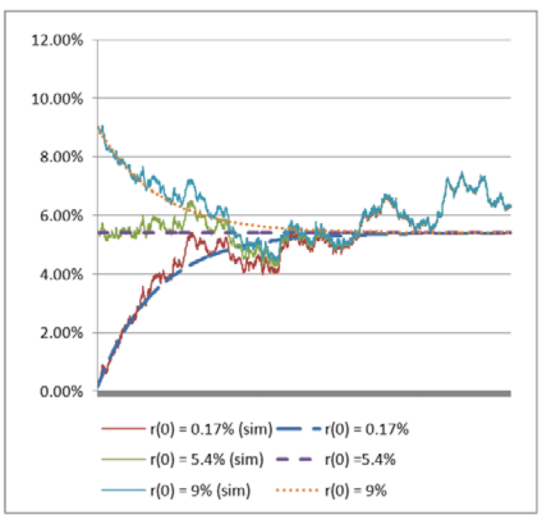

Figure 8: Vasicek simulations and expected value for different values of r_0

Exercise 5.

a. We have that: $\beta = -\gamma dt$ and $\alpha = \gamma \bar{r} dt$. For σ we can simply estimate the standard error of the regression and annualize it by dividing the standard error by $\sqrt{dt}$. Using daily data on Treasury interest rates from 1/1/2008 to 3/5/2010 we obtain the following regression estimates: $\alpha = 0.004466$; $\beta = -0.01439$; and $\sigma_{SE} = 0.113332$. This leads to the following parameters: $\gamma = 3.625184$; $\bar{r} = 0.310414$; and $\sigma = 1.799097$. In this particular case, this leads to a highly volatile process, which easily gives negative rates. The reason for this shortcoming is twofold: on the one hand we are using a period of particular interest rate volatility to estimate the model (the range of rates during this 2 year period was between 3.37% and 0.00%); on the other hand, r_0 is 0.11% which is already very close to zero.

b. Figure **??** plots the forecast with a simulation which shows the problems with the estimated model mentioned above.

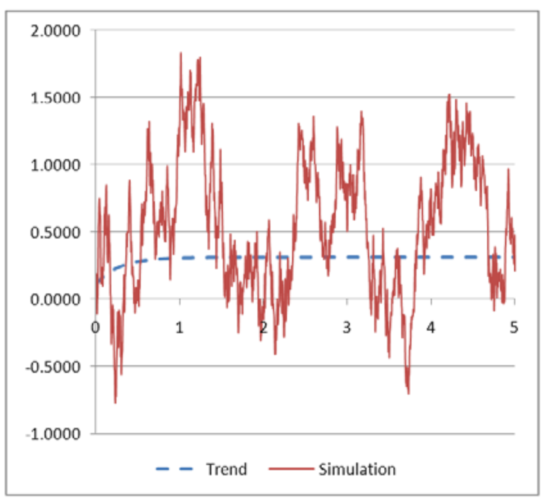

Figure 9: Vasicek model expected and simulated rates

c. Figure **??** shows the histograms for the forecasted values.

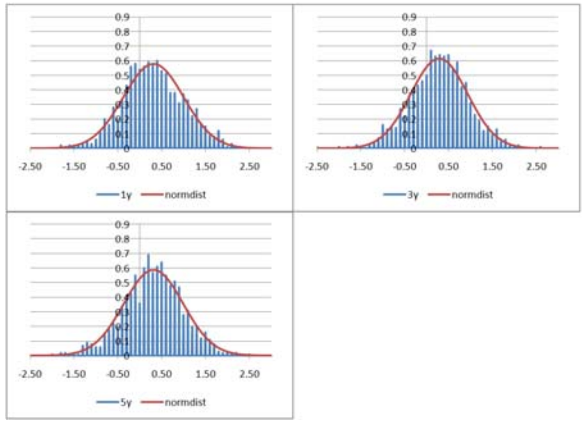

Figure 10: Histogram of rates at 1 year, 3 years and 5 years

Exercise 7. Given the process:

$$dr_t = \gamma(\bar{r} - r_t)dt + \sigma dX_t$$

We have that $m(r_t, t) = \gamma(\bar{r} - r_t)dt$; and $s(r_t, t) = \sigma$. Using Ito's lemma:

$$dP_t = \left[\left(\frac{\partial F}{\partial t}\right) + \left(\frac{\partial F}{\partial r}\right) m(r_t, t) + \frac{1}{2}\left(\frac{\partial^2 F}{\partial r^2}\right) s(r_t, t)^2 \right] dt + \left(\frac{\partial F}{\partial r}\right) s(r_t, t) dX_t$$

a. For $F(r) = A + Br$:

$$\frac{\partial F}{\partial t} = 0; \frac{\partial F}{\partial r} = B; \frac{\partial^2 F}{\partial r^2} = 0$$

So:

$$dP_t = B\gamma(\bar{r} - r_t)dt + B\sigma dX_t = Bdr_t$$

b. For $F(r) = e^{A-Br}$

$$\frac{\partial F}{\partial t} = 0; \frac{\partial F}{\partial r} = -Be^{A-Br}; \frac{\partial^2 F}{\partial r^2} = B^2 e^{A-Br}$$

So:

$$dP_t = \left[-BP\gamma(\bar{r} - r_t) + \frac{1}{2}B^2 P\sigma^2 \right] dt + BP\sigma dX_t$$

$$dP_t = P\left[\frac{1}{2}B^2\sigma^2 dt - Bdr_t\right]$$

Solutions to Chapter 15

Exercise 1.

a. In order to value the zero coupons we use the risk neutral parameters.

τ	Z(t,T)	τ	Z(t,T)
0.25	0.9944	5.25	0.7827
0.50	0.9877	5.50	0.7713
0.75	0.9802	5.75	0.7599
1.00	0.9718	6.00	0.7487
1.25	0.9627	6.25	0.7376
1.50	0.9531	6.50	0.7266
1.75	0.9429	6.75	0.7157
2.00	0.9324	7.00	0.7049
2.25	0.9216	7.25	0.6943
2.50	0.9105	7.50	0.6838
2.75	0.8992	7.75	0.6734
3.00	0.8877	8.00	0.6632
3.25	0.8761	8.25	0.6531
3.50	0.8644	8.50	0.6432
3.75	0.8527	8.75	0.6333
4.00	0.8409	9.00	0.6237
4.25	0.8292	9.25	0.6141
4.50	0.8175	9.50	0.6047
4.75	0.8058	9.75	0.5954
5.00	0.7942	10.00	0.5863

b. Figure **??** presents the yield curve for the bonds.

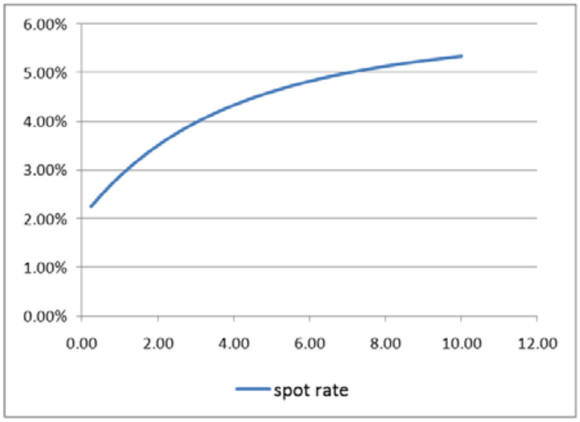

Figure 11: Spot rate of bonds up to 10 years

c. Spot rate duration is:

τ	B(t;T)	τ	B(t;T)
0.25	0.2360	5.25	1.9624
0.50	0.4461	5.50	1.9829
0.75	0.6331	5.75	2.0011
1.00	0.7996	6.00	2.0174
1.25	0.9478	6.25	2.0319
1.50	1.0797	6.50	2.0447
1.75	1.1972	6.75	2.0562
2.00	1.3017	7.00	2.0664
2.25	1.3948	7.25	2.0755
2.50	1.4776	7.50	2.0836
2.75	1.5514	7.75	2.0908
3.00	1.6170	8.00	2.0972
3.25	1.6754	8.25	2.1029
3.50	1.7275	8.50	2.1080
3.75	1.7738	8.75	2.1125
4.00	1.8150	9.00	2.1165
4.25	1.8517	9.25	2.1201
4.50	1.8844	9.50	2.1233
4.75	1.9134	9.75	2.1261
5.00	1.9393	10.00	2.1287

Exercise 2.

a. Figure **??** presents the effect on the term structure of interest rates due to changes in γ^*.

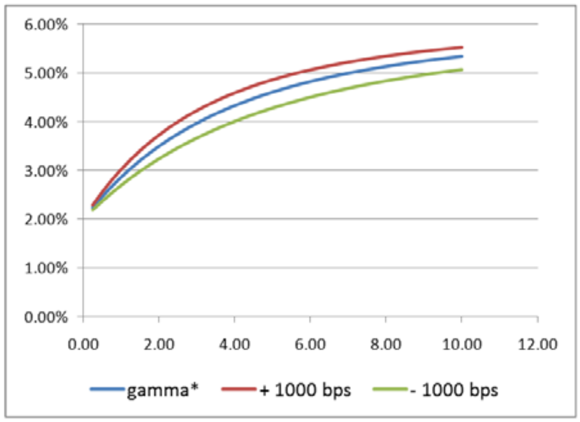

Figure 12: Term structure of interest rates for three choices of γ^* (Vasicek)

b. Figure **??** presents the effect on the term structure of interest rates due to changes in $\bar{r}^*$.

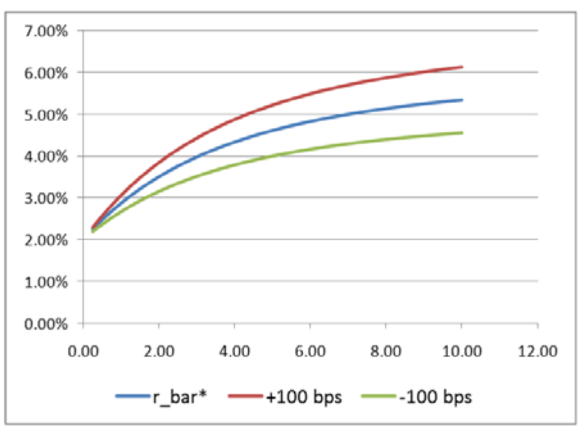

Figure 13: Term structure of interest rates for three choices of $\bar{r}^*$ (Vasicek)

c. Figure **??** presents the effect on the term structure of interest rates due to changes in σ.

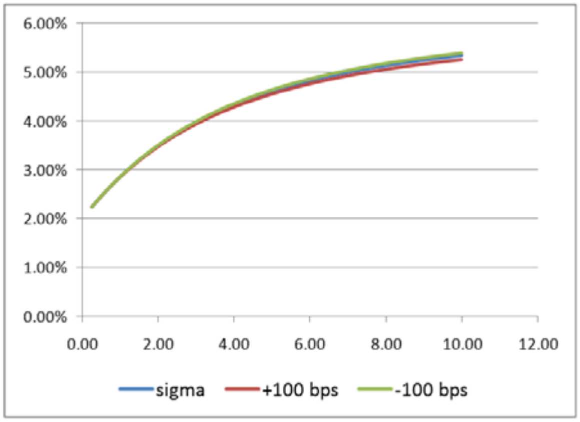

Figure 14: Term structure of interest rates for three choices of σ (Vasicek)

Exercise 3.

a. Figure **??** presents the effect on the term structure of interest rates due to changes in γ^*.

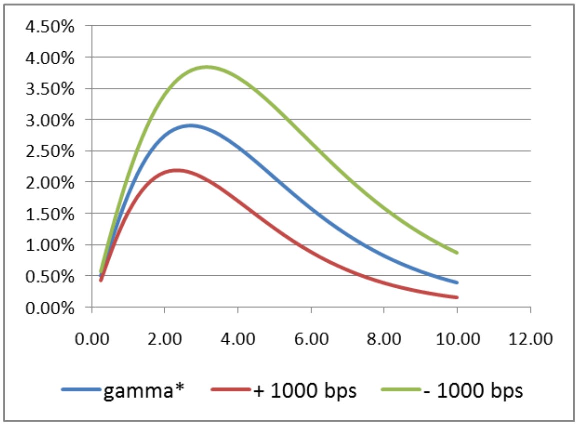

Figure 15: Term structure of interest rates for three choices of γ^* (CIR)

b. Figure **??** presents the effect on the term structure of interest rates due to changes in $\bar{r}^*$.

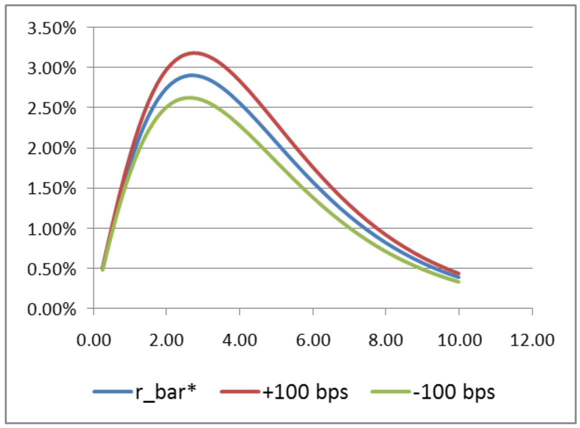

Figure 16: Term structure of interest rates for three choices of $\bar{r}^*$ (CIR)

c. Figure **??** presents the effect on the term structure of interest rates due to changes in σ.

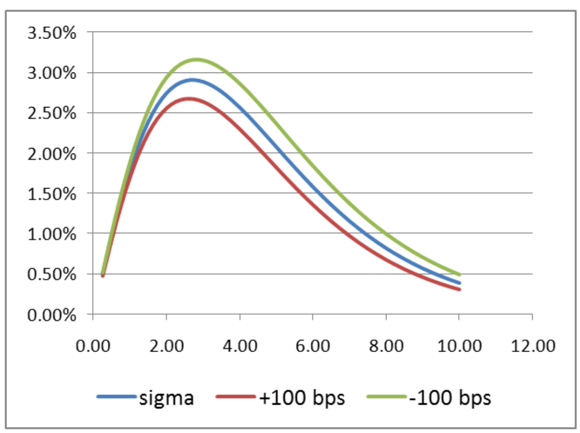

Figure 17: Term structure of interest rates for three choices of σ (CIR)

Exercise 6. Short-term bonds have higher yield volatility than long-term bonds (see Figure **??**). Return volatility and yield volatility are not the same since yields are a sort of "average" measure in order to make comparable different maturities. Returns will be affected by the yield volatility times the time to maturity. Return volatility is always increasing in relation to time to maturity.

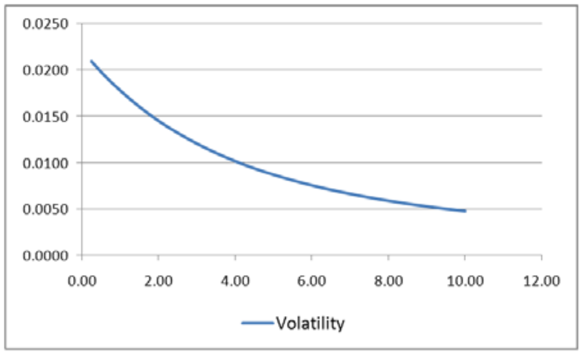

Figure 18: Yield volatility with respect to τ

Exercise 7. The Fundamental Pricing Equation states:

$$\frac{\partial Z}{\partial t} + \frac{\partial Z}{\partial r} m^*(r,t) + \frac{1}{2}\frac{\partial^2 Z}{\partial r^2}\sigma^2 = rZ$$

where:

$$m^*(r,t) = \gamma^*(\bar{r}^* - r)$$

The Vasicek formula for pricing a zero coupon bond:

$$Z(r,t;T) = e^{A(t;T) - B(t;T)r}$$

where:

$$B(t;T) = \frac{1}{\gamma^*}\left(1 - e^{-\gamma^*(T-t)}\right)$$

$$A(t;T) = (B(t;T) - (T-t))\left(\bar{r}^* - \frac{\sigma^2}{2(\gamma^*)^2}\right) - \frac{\sigma^2 B(t;T)^2}{4\gamma^*}$$

For the Left Hand Side of the Fundamental Pricing Equation we have:

$$\frac{\partial Z}{\partial t} = [A'(t;T) - B'(t;T)r]\, Z(r,t;T)$$

$$\frac{\partial Z}{\partial r} = -B(t;T)Z(r,t;T)$$

$$\frac{\partial^2 Z}{\partial r^2} = B(t;T)^2 Z(r,t;T)$$

Where:

$$B'(t;T) = -e^{-\gamma^*(T-t)}$$
$$A'(t;T) = (1 + B'(t;T))\left(\bar{r}^* - \frac{\sigma^2}{2(\gamma^*)^2}\right) - \frac{\sigma^2}{2\gamma^*}B'(t;T)B(t;T)$$

Note that:

$$B(t;T) = \frac{1}{\gamma^*}(1 + B'(t;T))$$

then:

$$\gamma^* B(t;T) = 1 + B'(t;T)$$

and

$$\gamma^* B(t;T) - B'(t;T) = 1$$

So we can present $A'(t;T)$ as:

$$A'(t;T) = \gamma^* B(t;T)\left(\bar{r}^* - \frac{\sigma^2}{2(\gamma^*)^2}\right) - \frac{\sigma^2}{2\gamma^*}B'(t;T)B(t;T)$$
$$= B(t;T)\left[\gamma^*\bar{r}^* - \frac{\sigma^2}{2\gamma^*} - \frac{\sigma^2}{2\gamma^*}B'(t;T)\right] = B(t;T)\left[\gamma^*\bar{r}^* - \frac{\sigma^2}{2\gamma^*}(1 + B'(t;T))\right]$$
$$= B(t;T)\left[\gamma^*\bar{r}^* - \frac{\sigma^2}{2\gamma^*}\gamma^* B(t;T)\right] = B(t;T)\left[\gamma^*\bar{r}^* - \frac{\sigma^2}{2}B(t;T)\right]$$

So:

$$A'(t;T) = \gamma^*\bar{r}^* B(t;T) - \frac{\sigma^2}{2}B(t;T)^2$$

Which means that:

$$\frac{\partial Z}{\partial t} = \left[B(t;T)\gamma^*\bar{r}^* - \frac{\sigma^2}{2}B(t;T)^2 - B'(t;T)r\right]Z(r,t;T)$$

So we can present the Left Hand Side of the Fundamental Pricing Equation as:

$$Z(r,t;T)\left[B(t;T)\gamma^*\bar{r}^* - \frac{\sigma^2}{2}B(t;T)^2 - B'(t;T)r - B(t;T)\gamma^*(\bar{r}^* - r) + \frac{\sigma^2}{2}B(t;T)^2\right]$$
$$= Z(r,t;T)\left[B(t;T)\gamma^* r - B'(t;T)r\right] = Z(r,t;T)r\left[B(t;T)\gamma^* - B'(t;T)\right]$$

Recall that: $B(t;T)\gamma^* - B'(t;T) = 1$, so:

$$LHS = Z(r,t;T)r$$

Exercise 9.

a. Parameter Estimation:

   i. The risk neutral parameters are: $\gamma^* = 0.4841$ and $\bar{r}^* = 7.279\%$. Note that σ is obtained through the regression estimate.

   ii. The risk natural parameters are: $\gamma = 0.1766$, $\bar{r} = 5.97\%$ and $\sigma = 0.017431$.

   iii. The risk neutral model has a quicker convergence speed and a higher long-term rate than the risk natural model. This makes sense because we need to compensate economic agents in order for them to become risk neutral (we consider them to be risk averse).

b. Sensitivity to interest rates:

   i. Because of the assumption $r < 15\%$, we can say that:

$$P^{IF} = P_{zero} + P_{fixed} - P_{float}$$

In other words we price the inverse floater as a portfolio of different bonds, so $\frac{\partial P^{IF}}{\partial r}$ can be presented as:

$$\frac{\partial P^{IF}}{\partial r} = \frac{\partial P_{zero}}{\partial r} + \frac{\partial P_{fixed}}{\partial r} - \frac{\partial P_{float}}{\partial r}$$

For the Vasicek model, the price of a zero coupon bond is given by:

$$Z(r, t; T) = e^{A(t;T) - B(t;T)r}$$

which means that:

$$\frac{\partial Z(r, t; T)}{\partial r} = -B(t; T)Z(r, t; T)$$

We know all parameters for this so:

$$\frac{\partial P_{zero}}{\partial r} = \frac{\partial Z(r, t; 3)}{\partial r} = -1.3608$$

The fixed coupon bond is composed by several zeros:

$$\frac{\partial P_{fixed}}{\partial r} = \sum_{T=1}^{3} c\frac{\partial Z(r,t;T)}{\partial r} + \frac{\partial Z(r,t;3)}{\partial r} = -1.8547$$

From the $Z(r,0;1)$ we can obtain the annually compounded 1-year rate $r_1(0,1) = 3.99\%$ which we can use to obtain the value of a floating rate bond:

$$P_{Float} = (1 + r_1(0,1))Z(0,r;1)$$

So:

$$\frac{\partial P_{Float}}{\partial r} = (1 + r_1(0,1))\frac{\partial Z(0,r;1)}{\partial r} = -0.7927$$

Thus:

$$\frac{\partial P^{IF}}{\partial r} = -1.3608 - 1.8547 + 0.7927 = -2.4227$$

ii. For convexity we have the same idea as in the previous exercise but instead we use:

$$\frac{\partial^2 Z(r,t;T)}{\partial r^2} = B(t;T)^2 Z(r,t;T)$$

So:

$$\frac{\partial^2 P_{zero}}{\partial r^2} = \frac{\partial^2 Z(r,t;3)}{\partial r^2} = 2.1530$$

$$\frac{\partial^2 P_{fixed}}{\partial r^2} = \sum_{T=1}^{3} c\frac{\partial^2 Z(r,t;T)}{\partial r^2} + \frac{\partial^2 Z(r,t;3)}{\partial r^2} = 2.7914$$

$$\frac{\partial^2 P_{Float}}{\partial r^2} = (1 + r_1(0,1))\frac{\partial^2 Z(0,r;1)}{\partial r^2} = 0.6284$$

Thus:

$$\frac{\partial^2 P^{IF}}{\partial r^2} = 2.1530 + 2.7914 - 0.6284 = 4.3160$$

iii. For the whole portfolio we simply weigh the sensitivities of interest rate of both assets, so we get:

$$\frac{\partial \Pi}{\partial r} = -40.1588$$

$$\frac{\partial^2 \Pi}{\partial r^2} = 69.3465$$

c. The Fundamental Pricing Equation:

i. The equation that $\Pi(r, t; T)$ must satisfy is:

$$r\Pi = \frac{\partial \Pi}{\partial t} + \frac{\partial \Pi}{\partial r}\eta^* (\bar{r} - r) + \frac{1}{2}\frac{\partial^2 \Pi}{\partial r^2}\sigma^2$$

ii. In order to obtain $\frac{\partial \Pi}{\partial t}$ we can rearrange the previous equation:

$$\frac{\partial \Pi}{\partial t} = r\Pi - \frac{\partial \Pi}{\partial r}\eta^* (\bar{r} - r) - \frac{1}{2}\frac{\partial^2 \Pi}{\partial r^2}\sigma^2$$

From previous exercises we know both $\frac{\partial \Pi}{\partial r} = -40.1588$ and $\frac{\partial^2 \Pi}{\partial r^2} = 69.3465$, so:

$$\frac{\partial \Pi}{\partial t} = 1.4368$$

iii. In one day, the capital gain that can be contributed to the passage of time is:

$$0.0751 \times \frac{1}{252} = 0.0057$$

This is the partial derivative of the value of the portfolio against time. Thus, we can interpret this as the following: each day, the value of the portfolio will rise by .0057 billion when interest rate does not change. Since this portfolio is not "Delta Hedged", the Theta-Gamma relation does not hold. However, holding Delta constant, we can still see the impact that a low (or even negative) Gamma will tend to lead to high Theta.

d. ii. The following historgram plots the possible values for the portfolio in one year.

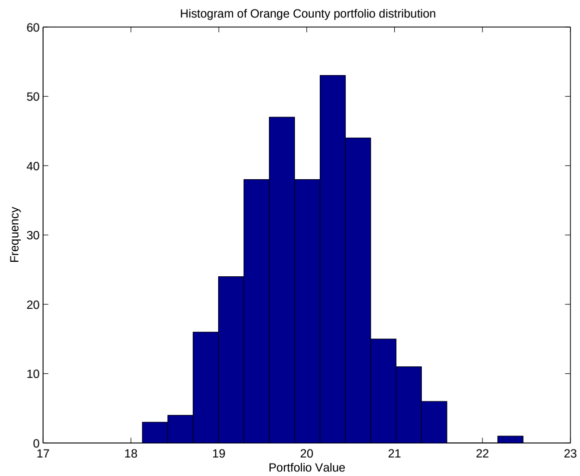

Figure 19: Historgram of possible Orange County portfolio values in one year

iii. The following table summarizes the results:

VaR	Initial	1-year	Loss
@ 5%	20.5	18.81	-1.68
@ 1%	20.5	18.36	-2.13

As seen in the table the results experienced by Oragne County's portfolio where completely forseeable.

Solutions to Chapter 16

Exercise 1. We follow these steps:

1. Parameter Estimation: We use the data given in Table 15.4 to fit the Vasicek model, for which we obtain $\gamma^* = 0.0290$, $\bar{r}^* = 2.09\%$ and $\sigma = 1.78\%$. The graph shows the yield curve and its fitted values according to the Vasicek model. For σ one possiblity is to estimate the value from historical data or simply include it as a free parameter in the optimization process (see Figure **??**).

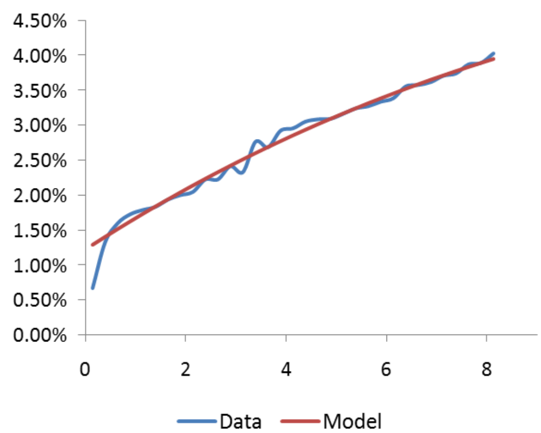

Figure 20: Yield curve and Vasicek model (September 25, 2008)

2. Relative Pricing Error Discovery: We look for the bond where there is the highest difference between the model and the data prices.

τ	Z^{Data}	Z^{Model}	Diff.
0.139	99.908	99.821	
0.389	99.488	99.457	
0.639	98.989	99.038	
0.889	98.483	98.569	
1.139	97.992	98.050	
1.389	97.497	97.485	
1.639	96.889	96.875	
1.889	96.304	96.224	
2.139	95.732	95.533	
2.389	94.840	94.805	
2.639	94.314	94.042	
2.889	93.284	93.247	
3.139	92.967	92.421	0.546
3.389	91.082	91.567	
3.639	90.715	90.688	
3.889	89.284	89.784	
4.139	88.508	88.858	
4.389	87.488	87.913	
4.639	86.694	86.950	
4.889	85.998	85.970	0.028
5.139	85.024	84.977	
5.389	84.009	83.971	
5.639	83.182	82.954	
5.889	82.195	81.928	
6.139	81.267	80.895	
6.389	79.716	79.855	
6.639	78.908	78.811	
6.889	77.982	77.764	
7.139	76.782	76.714	
7.389	75.895	75.664	
7.639	74.447	74.614	
7.889	73.603	73.566	
8.139	72.096	72.520	-0.424

Two bonds appear to be particularly promising $\tau = 3.139$ and $\tau = 8.139$, since they have the biggest gap between prices. We will use $\tau = 3.139$, which seems to be overpriced.

3. Set-Up Trading Strategy:

   i. Since $Z(3.139)$ seems to be overpriced we short sell it, making a profit of \$92.9670 per bond.

   ii. Using the bond maturing at $\tau = 4.889$ we buy a synthetic position:

$$\hat{Z}(3.139) = \Delta \times Z(4.889) + C_0 = \$92.4213$$

   where:

$$\Delta = \frac{B(0, 3.139) \times Z^{Model}(r, 0; 3.139)}{B(0, 4.889) \times Z(4.889)} = 0.7074$$

$$C_0 = Z^{Model}(r, 0; 3.139) \times 100 - \Delta \times Z(4.889) \times 100 = 31.5866$$

   In other words, for each $Z(3.139)$ bond we shorted, we buy 0.7074 of a $Z(4.889)$ bond and take a \$31.5866 cash position. This gives a \$0.5457 profit per bond.

Exercise 4.

a. The price of the option is: Call (x 100) = \$13.23.

b. In order to hedge the call we pick the zero coupon bond maturing at the same time. Two methodologies were proposed to obtain the hedge ratio for the replicating portfolio:

   i. Using equation (16.31) we obtain: $\Delta = 0.6608$ and $C_0 = -0.4797$.

   ii. Through the numerical approximation we obtain: $\Delta = 0.6608$ and $C_0 = -0.4797$. Which is almost identical to the number obtained with the previous methodology.

c. Figure **??** shows that the strategy does a good job replicating the option.

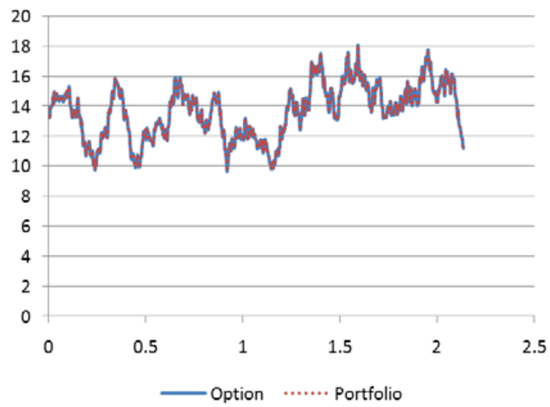

Figure 21: Value of the asset and of the replicating portfolio

Exercise 7. In order to perform the trade we proceed as follows:

1. Parameter Estimation:

   i. Risk Natural Parameters. Recall that the Cox-Ingersoll-Ross (CIR) process for the short rate is defined by:

$$dr = (\eta - \gamma r_t)\, dt + \sqrt{\alpha r_t} dX$$

As in the Vasicek case we would estimate risk neutral parameters through calibrating the model to the data (using the Non-Linear Least Squares methodology), yet the volatility parameter α we would derive from the actual data. Additionally, if we want to test the model against simulated values of interest rates we will eventually need the risk natural parameters as well. For this reason, we start by obtaining the risk natural parameters and the volatility parameter, before calibrating the model to the risk neutral parameters. As a reminder, recall that in the case of the Vasicek model we had:

$$dr_t = \gamma (\bar{r} - r_t)\, dt + \sigma dX_t$$

and we estimated the risk natural parameters through a regression of the form (using data on the overnight repo rate from May 21, 1991 - February 17, 2004):

$$r_{t+dt} = \alpha + \beta_1 r_t + \sigma_{S.E.}\epsilon$$

where: $\alpha = \gamma\bar{r}dt$; $\beta_1 = (1 - \gamma dt)$, $\sigma = \frac{\sigma_{S.E.}}{\sqrt{dt}}$ and $\epsilon \quad N(0,1)$. In the case of the CIR model this regression cannot be used because there is a $\sqrt{r_t}$ in the variance term. In order to eliminate it we must divide both sides of the equation by $\sqrt{r_t}$:

$$\frac{dr_t}{\sqrt{r_t}} = \frac{\gamma(\bar{r} - r_t)dt}{\sqrt{r_t}} + \sqrt{\alpha dt}\epsilon$$

Decomposing dr_t and rearranging a few terms, we have:

$$\frac{r_{t+dt}}{\sqrt{r_t}} = \gamma\bar{r}dt\frac{1}{\sqrt{r_t}} + (1 - \gamma dt)\frac{r_t}{\sqrt{r_t}} + \sqrt{\alpha dt}\epsilon$$

Which can be summarized as the following regression:

$$\frac{r_{t+dt}}{\sqrt{r_t}} = \beta_1\frac{1}{\sqrt{r_t}} + \beta_2\frac{r_t}{\sqrt{r_t}} + \sigma_{S.E.}\epsilon$$

where: $\beta_1 = \gamma\bar{r}dt$, $\beta_2 = (1 - \gamma dt)$ and $\sigma_{S.E.} = \sqrt{\alpha dt}$.

Running a simple regression gives the following results (p-values in parenthesis), as well as the following parameters for the CIR model.

α	0.000542	dt	1/252
	(0.9224)		
β_1	0.000150	γ	1.6367
	(0.7290)	$\bar{r}$	0.0231
β_2	0.993505	α	0.0265
	(0.0000)		
$\sigma_{S.E.}$	0.010258		

Note that neither the intercept nor β_1 are significant. The reason is that the regression is including the intercept, which is not present in the model above. We must run the regression without intercept in order to have a better fit with our model. The results of doing so are the following:

α	0	dt	1/252
	(N/A)		
β_1	0.000192	γ	1.2373
	(0.0026)	$\bar{r}$	0.0391
β_2	0.995090	α	0.0265
	(0.0000)		
$\sigma_{S.E.}$	0.010256		

ii. Risk Neutral Parameters. In order to price the bonds we need to find the risk neutral parameters (γ^* and $\bar{r}^*$). To do so we use the Non-Linear Least Squares approach to calibrate the parameters to the data given in Table 16.4 (we follow the same steps as in section 16.8). The minimization procedure yields $\bar{r}^* = 16.53\%$ and $\gamma = 0.0622$. Figure **??** presents the spot rate from the CIR model.

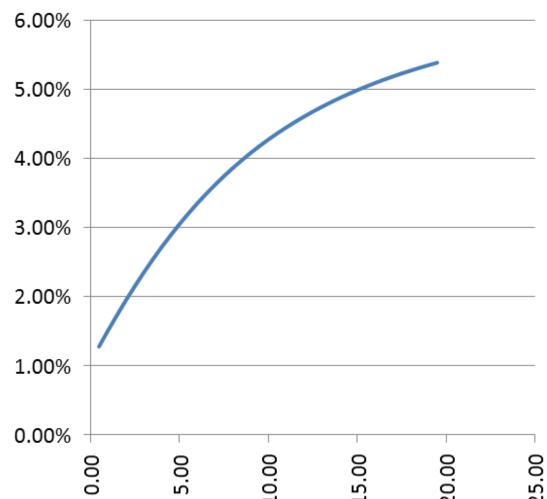

Figure 22: Spot rate

2. Relative Pricing Error Discovery. Figure **??** presents pricing errors from the CIR model. Note that these are very similar to the ones given by using the Vasicek model (see Figure 16.7 in the book), so hedging the 1.5-year bond seems like a good idea. In this case, though, it seems that the 6-year bond is a better hedging instrument than the 7.5-year bond.

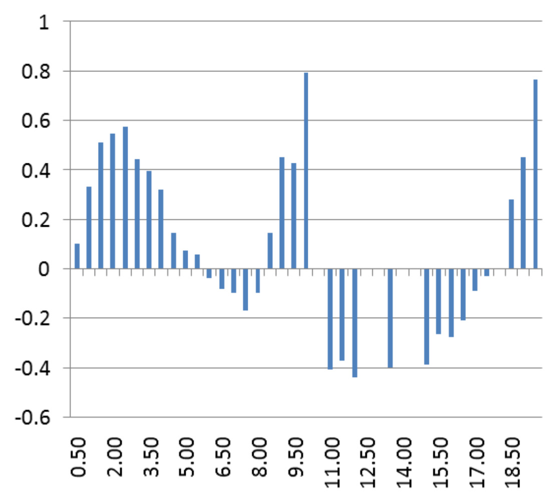

Figure 23: Pricing Errors

3. Set Up a Trading Strategy. We decide to sell short the 1.5-coupon bond and take a long position on the 6-year bond and the cash acount (in order to replicate the shorted bond). At time zero, we have:

$$\Delta_0 = \frac{\partial P_{1.5-yr}(r,0)/\partial r}{\partial P_{6.0-yr}(r,0)/\partial r} = \frac{-147.41}{-456.45} = 0.3229$$

and:

$$C_0 = -P^{CIR}_{1.5-yr} + \Delta_0 \times P_{6.0-yr} = -107.0385 + 0.3229 \times 117.5625 = -69.0725$$

4. Simulations. We then simulate a path on interest rates to check how well the model works (see Figure **??**). Note that this model is not as good as the Vasicek model in replicating the value of the bond.

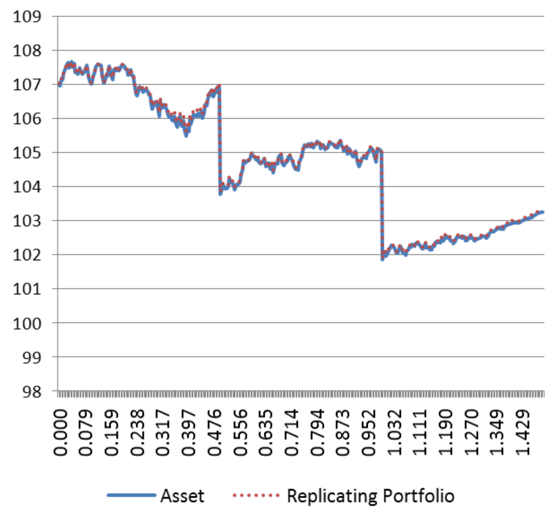

Figure 24: Asset value and replicating portfolio

Solutions to Chapter 17

Exercise 1. Figure **??** presents the result of fitting the Vasicek model with the parameters given in this chapter. Additionally we present four variations:

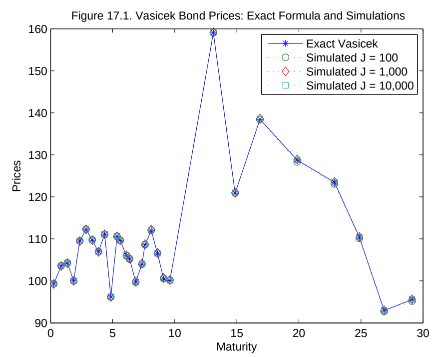

Figure 25: Vasicek Bond Prices: Exact Formula and Simulation

i. Figure **??** presents the result when: $\gamma^* + 300$ bps.

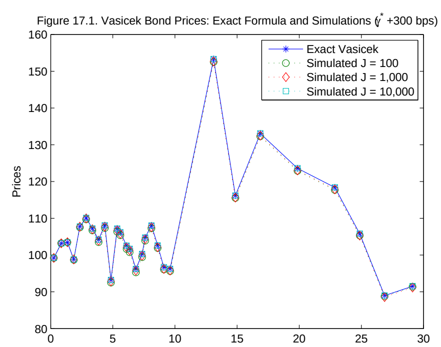

Figure 26: Vasicek Bond Prices: Exact Formula and Simulations ($\gamma^* + 300$ bps)

ii. Figure **??** presents the result when: γ^* - 300 bps.

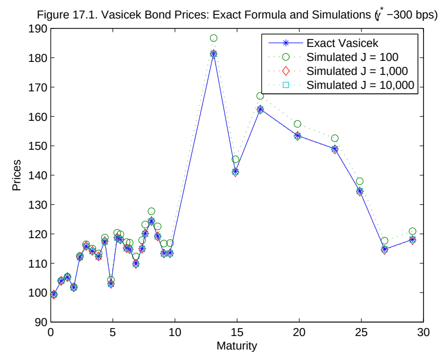

Figure 27: Vasicek Bond Prices: Exact Formula and Simulations (γ^* - 300 bps)

iii. Figure **??** presents the result when: σ + 200 bps.

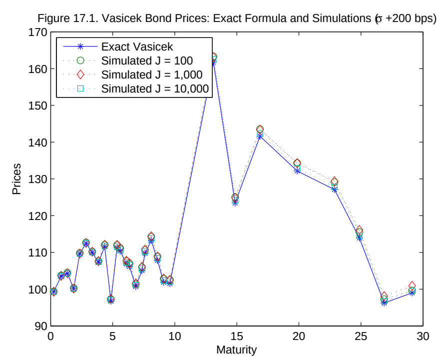

Figure 28: Vasicek Bond Prices: Exact Formula and Simulations (σ + 200 bps)

iv. Figure **??** presents the result when: σ - 200 bps.

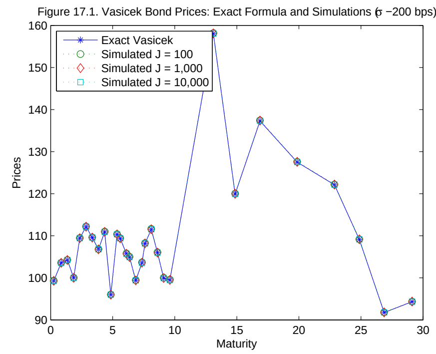

Figure 29: Vasicek Bond Prices: Exact Formula and Simulations (σ - 200 bps)

All results tend to converge to the analytical formula do to the Feynman-Kac theorem. Note that decreasing γ^* and increasing σ requieres more simulations for the results to converge, this is intuitive since γ refers to the "speed" by which the process converges to the long-term mean ($\bar{r}$) and σ refers to the volatility of the process. Lower convergence speed and higher volatility will require more simulations for the methodologies to converge.

Exercise 3. Figure **??** presents the result of fitting the CIR model with the parameters given in this chapter.

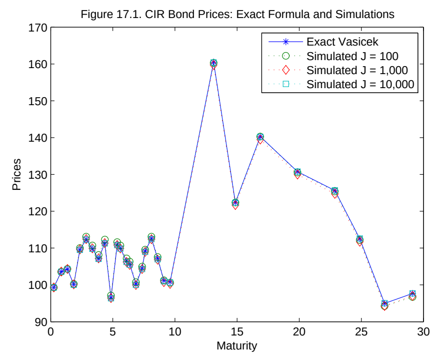

Figure 30: CIR Bond Prices: Exact Formula and Simulations

Additionally we present four variations:

i. Figure **??** presents the result when: γ^* + 3000 bps.

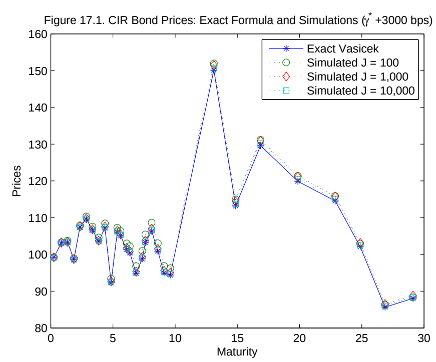

Figure 31: CIR Bond Prices: Exact Formula and Simulations (γ^* + 3000 bps)

ii. Figure **??** presents the result when: γ^* - 3000 bps.

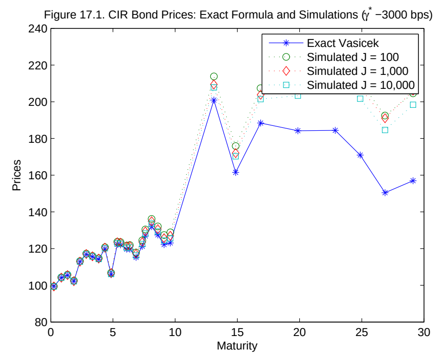

Figure 32: CIR Bond Prices: Exact Formula and Simulations (γ^* - 3000 bps)

iii. Figure **??** presents the result when: α + 300 bps.

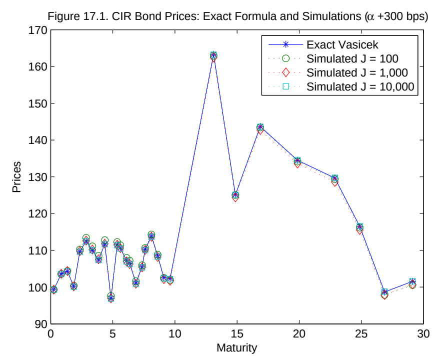

Figure 33: CIR Bond Prices: Exact Formula and Simulations (α + 300 bps)

iv. Figure ?? presents the result when: α - 300 bps.

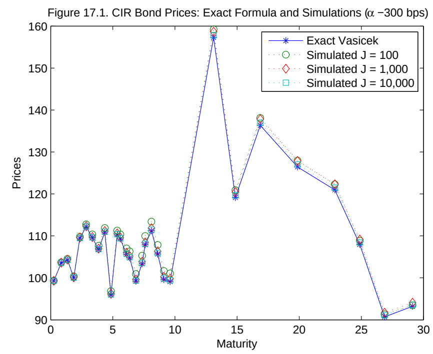

Figure 34: CIR Bond Prices: Exact Formula and Simulations (α - 300 bps)

Exercise 6.

a. Fitting the Vasicek model we obtain the following parameters: $\gamma^* = 0.7738$, $\bar{r}^* = 0.0503$ and σ = 8.3280e-005. Figures ?? and ?? present the bootstrapped discount curve and yield curve, respectively.

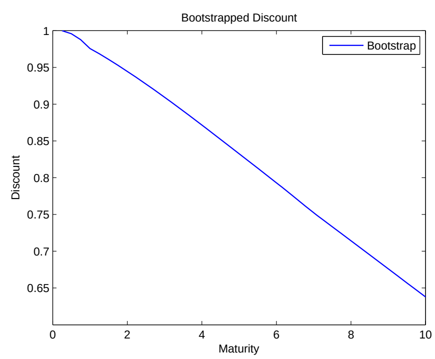

Figure 35: Bootstrapped Discount Curve

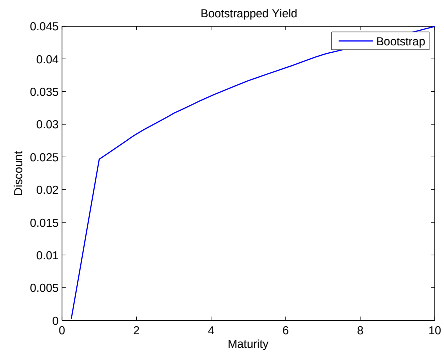

Figure 36: Bootstrapped Yield Curve

b. Figures ?? and ?? present the comparisson of the fitted Vasicek model to the bootstrapped discount curve and yield curve, respectively.

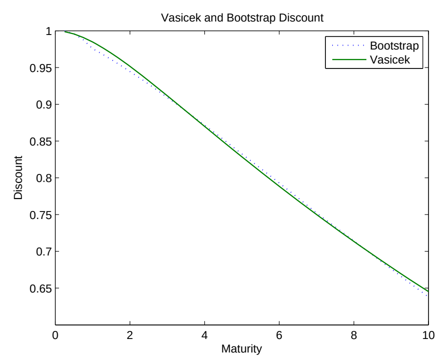

Figure 37: Vasicek and Bootstrap Discount Curve

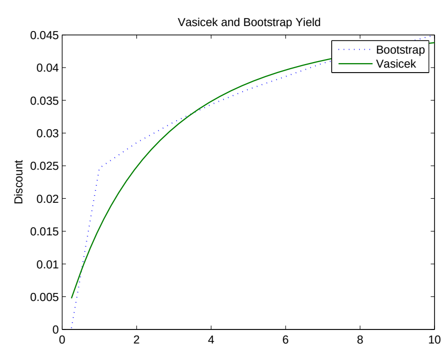

Figure 38: Vasicek and Bootstrap Yield Curve

c. The price of the Corridor Note is 1.1196.

d. These notes are essentially a bet on where the 6 month rate will stay within the next 10 years. In particular, these notes could be used as a hedge for positions which payoff if the 6 month LIBOR becomes either unusually high or low. The floating nature of the payoff for the last 5 years of the life of the note could also be used as a hedge for 3 month LIBOR swaps.

e. We have that Δ = -9.3965 and Γ = -0.0017116.

Solutions to Chapter 18

Exercise 3. To perform the risk analysis we have the following steps:

i. Simulate n times the short rate up to six months using the risk natural parameters.

ii. We have a cash flow occuring six months from now so we need to compute it. So we have to count the days on which the 6-month rate has been within the boundaries for the period between the previous coupon (3-months after the beginning of the contract) and the current coupon (6-month coupon after the beginning of the contract). Since this has already happened we use the rates obtained from the risk natural parameters (approximately the last half of the rates obtained in i., except the last rate). We get n cash flows occuring in 6-months, one for each risk natural process simulated in i.

iii. The last rate obtained in i. is the expected short rate in 6-months. We now use it with the risk neutral parameters to price the corridor note with 6-months less maturity. This is particularly cumbersome (i.e. takes a long time) because, in order to price, for each simulated rate in i. we have to make additional simulations until final maturity (9.5 years times 360 days), but with the risk neutral parameters. Additionally there is a lot of details to be aware of on obtaing the cash flow of the Corridor Note.

iv. We add the corresponding first cash flow (from ii.) with the value of the remaining discounted cash flows (from iii.). These is a set of random posible values of the Corridor Note in 6-months (if we want the ex-coupon value we don't include ii.).

v. We subtract the original value of the Corridor Note (obtaining the Profit and Loss values for the Corridor Note), from which we can obtain Value-at-Risk and expected shortfall

Figure **??** presents the histogram for these values.

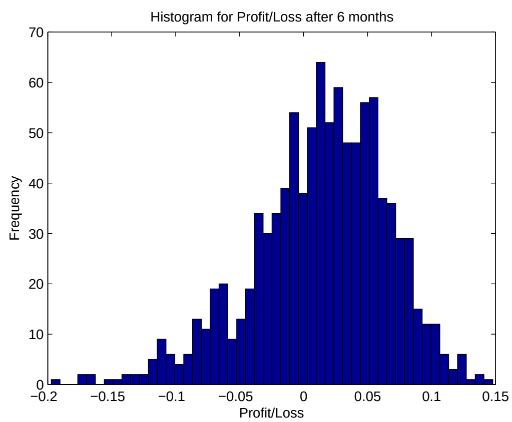

Figure 39: Histogram for Profit/Loss after 6 months

For a 6-month horizon we have that the Value at Risk is (par = \$100):

95% Value-at-Risk	-8.4706
99% Value-at-Risk	-13.4893

The expected shortfall is:

95% Expected Shortfall	-11.3965
99% Expected Shortfall	-15.8254

Exercise 5. The exercise uses the following basic parameters (see Table 18.1): $\bar{i} = 4.20\%$, $\gamma = 0.3805$, $g = 0.02$, $\sigma_y = 0.02$, $\sigma_q = 0.0106$, $\sigma_i = 0.0073$, $\rho_{yq} = -0.1409$, $\rho_{yi} = -0.2894$, $h = 104$, and $\rho_{iq} = 0.8360$. Additionally, we consider the case when $i_0 = 4.20\%$.

a. For variations in risk aversion (h) we pick 103.7, 104 and 104.3. As seen in Figures **??** and **??**, as risk aversion increases the yield curve becomes steeper and the spread increases. Also the values for λ are -0.5835, -0.5852 and -0.5870, respectively. As explained in this chapter, as risk aversion increases so does the market price of risk (being the driving force behind the changes in the yield curve and the spread).

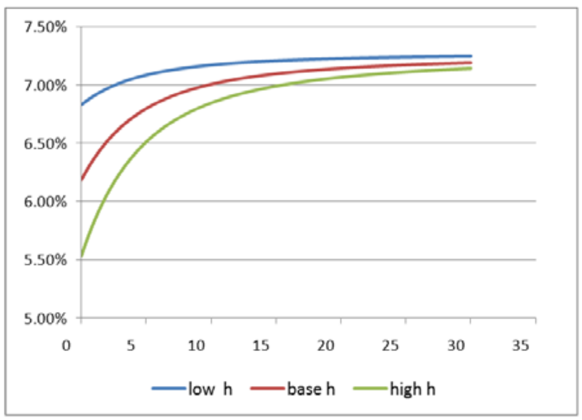

Figure 40: Yield Curve for Variations of h

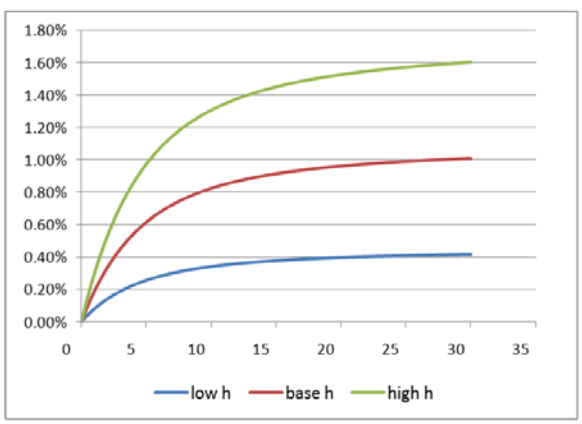

Figure 41: Term Spread for Variations of h

For variations in 'impatience' (ρ) we pick 0.095, 0.100 and 0.105. As seen in Figures **??** and **??**, as impatience increases the higher rates are required (since agents will borrow more to consume), also the spread increases. The values for λ is -0.5852 for all cases, because impatience does not affect risk aversion, volatility of GDP growth or the correlation between GDP growth and expected infaltion.

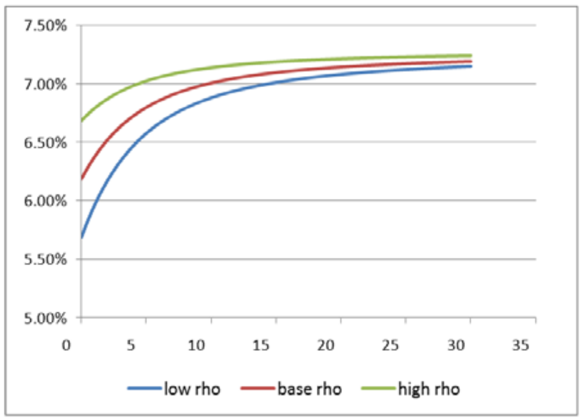

Figure 42: Yield Curve for Variations of ρ

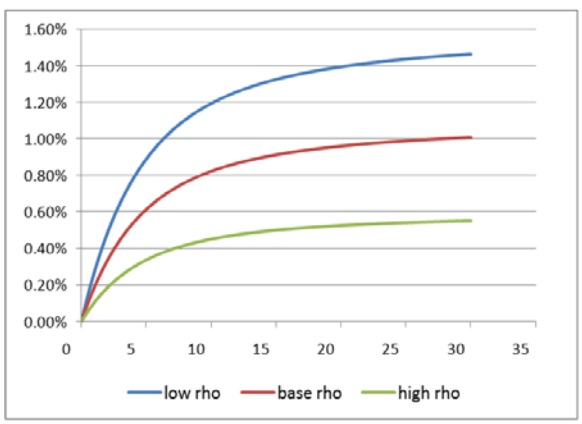

Figure 43: Term Spread for Variations of ρ

b. For variations in the correlation between GDP growth and expected inflation (ρ_{yi}) we pick -0.1, -0.2894 and -0.5. As seen in Figure **??**, as this correlation increases so does the level of interest rates demanded. Also the values for λ are -0.1913, -0.5852 and -1.0233, respectively. These results reflect the fact that this is a risk factor (investors loose money when growth is low and inflation is high), so investors will require additional compensation.

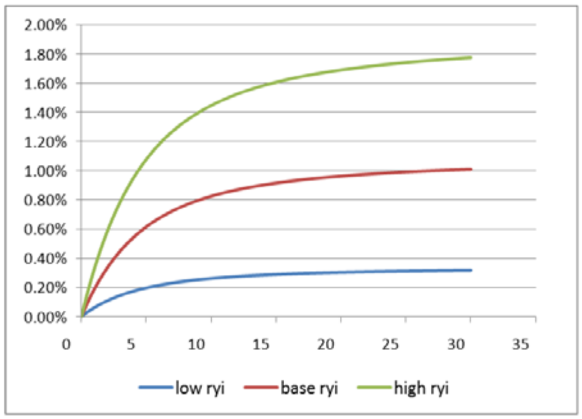

Figure 44: Yield Curve for Variations of ρ_{yi}

For variations in the volatility of GDP (σ_y) we pick 0.01999, 0.02 and 0.02001. As volatility increases so do the level of interest rates demanded. Also the values for λ are -0.5849, -0.5852 and -0.5855, respectively. These results reflect the fact that this is a risk factor (high volatility will make investments more risky), so investors will require additional compensation (see Figure **??**).

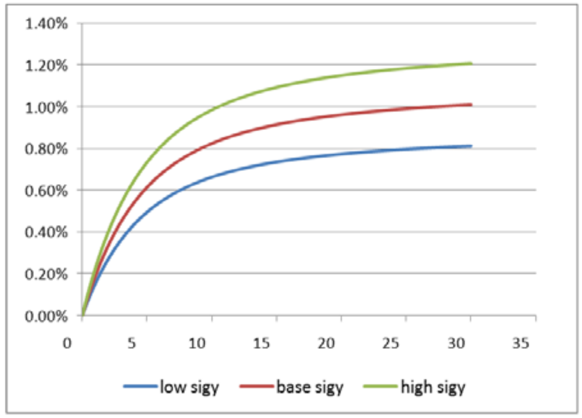

Figure 45: Yield Curve for Variations of σ_y

Solutions to Chapter 19

Exercise 1.

a. In order to obtain the LIBOR yield curve from the swap rates presented:

   i. Obtain the quarterly disocunts from the swap rates.

   ii. From the discounts obtain the continously compounded spot rates, at a quarterly frequency.

   iii. Fit a 10th order polynomial to the data to complete the gaps in the curve (see Figure **??**). The parameters from the polynomial are:

parameter	value
α	2.56e-02
β_1	3.84e-02
β_2	-1.61e-01
β_3	2.69e-01
β_4	-2.47e-01
β_5	1.38e-01
β_6	-4.91e-02
β_7	1.11e-02
β_8	-1.55e-03
β_9	1.22e-04
β_{10}	-4.13e-06

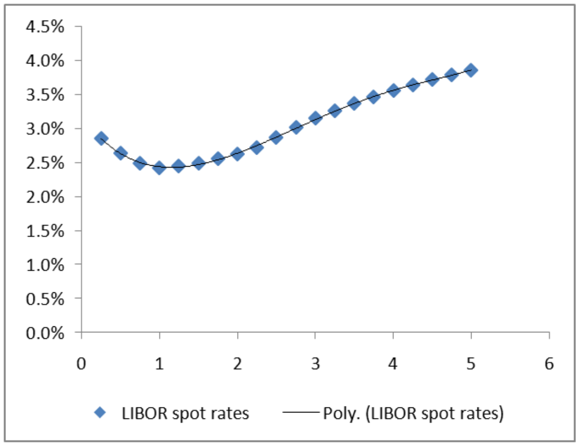

Figure 46: LIBOR yield curve

Note that, as explained in this chapter, when the forward rate is above the yield curve; the yield curve is uppward sloping. Same happens in the opposite case (see Figure **??**).

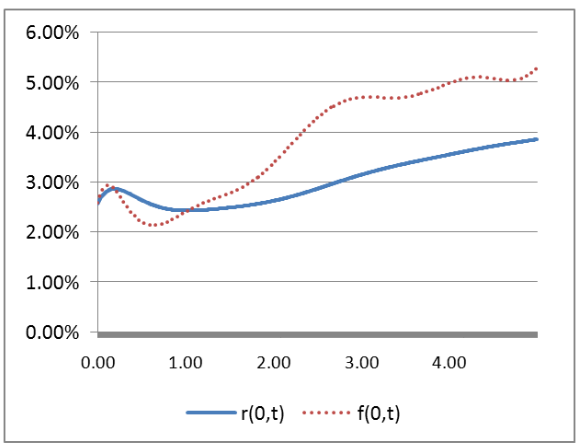

Figure 47: LIBOR yield curve and forward curve

b. The value of σ is 1.00%. The fitted values of the Ho-Lee model match exactly the yield curve.

c. The following chart presents the value of θ_t and the forward curve (See Figure **??**). Note that θ_t is very close to the slope of the forward curve.

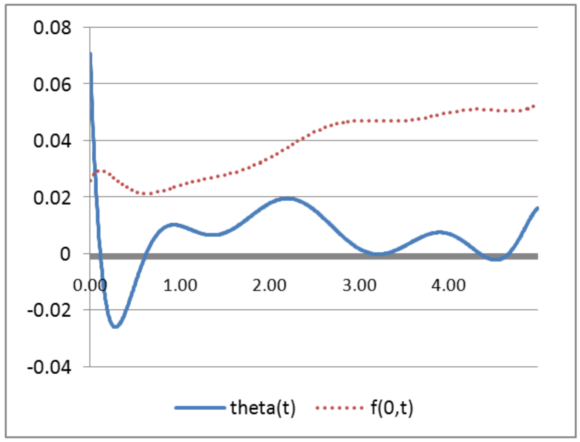

Figure 48: LIBOR yield curve

Exercise 4.

a. The value of the swaption through the Ho-Lee model is: \$0.6522.

b. The value of the swaption through the Hull-White model is: \$0.6483.

Exercise 5. Using a Ho-Lee tree the value of the American swaption is: \$0.8637, although this might be slightly overestimated since the price of the European swaption via the Ho-Lee tree is: \$0.7246. This difference comes mainly because the tree used is "cut to rough" in the sense that the time-step is: $dt = 1/4$. As dt becomes smaller the result would converge to the one obtained previously.

Solutions to Chapter 20

Exercise 1.

a. Figure **??** presents the results for the LIBOR curve. Note that the swap curve was interpolated using a polynomial of 6th order. This is not the ideal interpolation method (splines or cubic Hermite interpolation are better), but it is done in order to present the result in Excel.

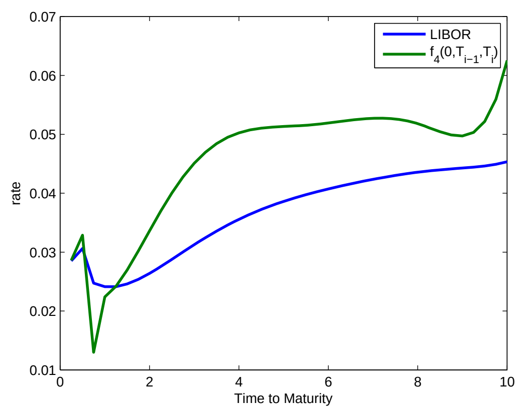

Figure 49: LIBOR yield curve and forward curve

b. The dollar value of a 1-year cap ($\times$ 100) is: \$0.3314 while the dollar value of a 2-year cap ($\times$ 100) is: \$1.0273.

Exercise 4. The flat volatilities corresponding to Table 20.7 are:

τ	Volatility
1Y	118.77%
2Y	75.79%
3Y	49.56%
4Y	35.85%
5Y	29.85%
7Y	24.65%
10Y	21.97%

Exercise 5.

a. The BDT can be calibrated through two methodologies:

   i. Leaving σ_i free so that it is calibrated along with $r_{i,1}$.

   ii. Using the forward volatilities computed in the Black model as σ_i, only $r_{i,1}$ is free.

   Figure ?? presents the caps through each different methodology.

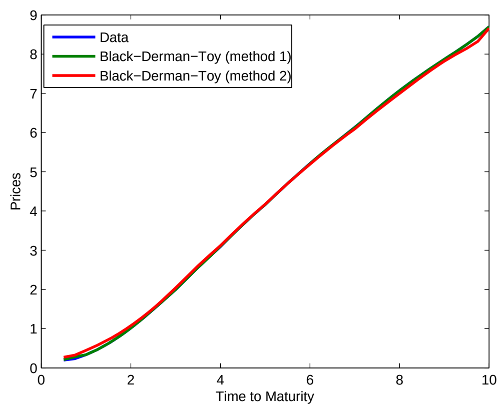

Figure 50: Cap prices under both BDT methodologies

b. Figure ?? presents the errors of each methodology with respect to the data. Note that using the forward volatilities as σ_i is not as far from the results using the first methodology.

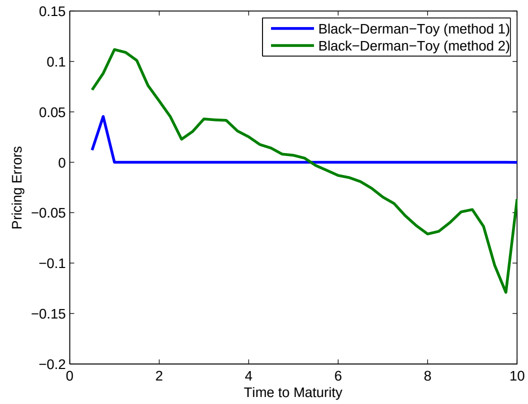

Figure 51: Pricing errors under both BDT methodologies

Solutions to Chapter 21

Exercise 2. Applying the analytical formulas discussed in section 21.7 we have that the value under the Unnatural Lag is: 0.5100. If we applied the convexity adjustment as an approximation we would have: 0.5083, which is lower than the actual value but higher than what we would have with the Natural Lag: 0.4873.

Exercise 4. Figure **??** presents the comparisson between both types of volatilities (it is identical to Figure 21.2 from the book).

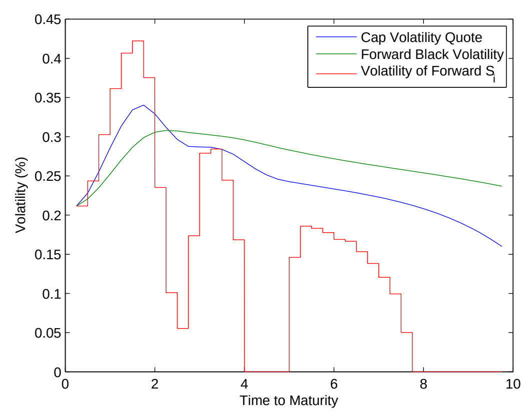

Figure 52: Forward Volatility and Volatility of Forwards

Exercise 6. The value of the Constant Maturity Swap is: 3.3055.

Exercise 8. The convexity adjustment states that:

$$f(0,T,T+\tau) = f^{fut}(0,T,T+\tau) - \int_0^T \frac{\sigma_Z(t,T+\tau)^2 - \sigma_Z(t,T)^2}{2\tau}\,dt$$

Recall that: $x^2 - y^2 = (x+y)(x-y)$, so:

$$\sigma_Z(t,T+\tau)^2 - \sigma_Z(t,T)^2 = \left(\sigma_Z(t,T+\tau) + \sigma_Z(t,T)\right)\left(\sigma_Z(t,T+\tau) - \sigma_Z(t,T)\right)$$

For the Hull-White model we have:

$$\sigma_Z(t,T) = -B(t,T)\sigma = -\frac{1-e^{-\gamma^*(T-t)}}{\gamma^*}\sigma$$

$$\sigma_Z(t,T+\tau) = -B(t,T+\tau)\sigma = -\frac{1-e^{-\gamma^*(T+\tau-t)}}{\gamma^*}\sigma$$

also keep in mind that:

$$\sigma_f(t,T) = \sigma e^{-\gamma^*(T-t)}$$

$$\sigma_f(t,T+\tau) = \sigma e^{-\gamma^*(T+\tau-t)}$$

So:

$$\sigma_Z(t,T+\tau) + \sigma_Z(t,T) = \frac{-2\sigma + \sigma_f(t,T+\tau) + \sigma_f(t,T)}{\gamma^*}$$

$$\sigma_Z(t,T+\tau) + \sigma_Z(t,T) = \frac{\sigma_f(t,T+\tau) - \sigma_f(t,T)}{\gamma^*}$$

We have then that:

$$\sigma_Z(t,T+\tau)^2 - \sigma_Z(t,T)^2 = \frac{1}{\left(\gamma^2\right)^2}\left[-2\sigma\sigma_f(t,T+\tau) + 2\sigma\sigma_f(t,T) + \sigma_f(t,T+\tau)^2 - \sigma_f(t,T)^2\right]$$

The integral in the adjustment can be presented as:

$$\int_0^T \frac{\sigma_Z(t,T+\tau)^2 - \sigma_Z(t,T)^2}{2\tau}\,dt =$$

$$\int_0^T \frac{1}{2\tau\left(\gamma^2\right)^2}\left[-2\sigma\sigma_f(t,T+\tau) + 2\sigma\sigma_f(t,T) + \sigma_f(t,T+\tau)^2 - \sigma_f(t,T)^2\right]\,dt =$$

$$\frac{1}{2\tau\left(\gamma^2\right)^2}\left[\underbrace{-2\sigma\int_0^T \sigma_f(t,T+\tau)\,dt}_{A} + \underbrace{2\sigma\int_0^T \sigma_f(t,T)\,dt}_{B} + \underbrace{\int_0^T \sigma_f(t,T+\tau)^2\,dt}_{C} - \underbrace{\int_0^T \sigma_f(t,T)^2\,dt}_{D}\right]$$

- For A:

$$-2\sigma \int_0^T \sigma_f(t, T+\tau)\, dt = -2\sigma \int_0^T \sigma e^{-\gamma^*(T+\tau-t)}\, dt = -2\sigma^2 \frac{e^{-\gamma^*(T+\tau-t)}}{\gamma^*}\Bigg|_0^T =$$

$$-\frac{2\sigma^2}{\gamma^*} e^{\gamma^*\tau} + \frac{2\sigma^2}{\gamma^*} e^{\gamma^*(T+\tau)} + \underbrace{\left(\frac{2\sigma^2}{\gamma^*} - \frac{2\sigma^2}{\gamma^*}\right)}_{=0} = \frac{2\sigma^2}{\gamma^*}\left(1 - e^{\gamma^*\tau}\right) - \frac{2\sigma^2}{\gamma^*}\left(1 - e^{\gamma^*(T+\tau)}\right)$$

Recall the definition of σ_Z, so that:

$$-2\sigma \int_0^T \sigma_f(t, T+\tau)\, dt = -2\sigma\sigma_Z(0,\tau) + 2\sigma\sigma_Z(0, T+\tau)$$

- For B:

$$2\sigma \int_0^T \sigma_f(t, T)\, dt = 2\sigma \int_0^T \sigma e^{-\gamma^*(T-t)}\, dt = 2\sigma^2 \frac{e^{-\gamma^*(T-t)}}{\gamma^*}\Bigg|_0^T =$$

$$\frac{2\sigma^2}{\gamma^*} - \frac{2\sigma^2}{\gamma^*} e^{\gamma^* T} = \frac{2\sigma^2}{\gamma^*}\left(1 - e^{\gamma^* T}\right) = 2\sigma\sigma_Z(0, T)$$

- For C:

$$\int_0^T \sigma_f(t, T+\tau)^2\, dt = \sigma^2 \int_0^T e^{-2\gamma^*(T+\tau-t)}\, dt = \frac{\sigma^2}{2\gamma^*} e^{-2\gamma^*(T+\tau-t)}\Bigg|_0^T =$$

$$\frac{\sigma^2}{2\gamma^*} e^{-2\gamma^*\tau} - \frac{\sigma^2}{2\gamma^*} e^{-2\gamma^*(T+\tau)} + \underbrace{\left(\frac{\sigma^2}{2\gamma^*}\right) - \left(\frac{\sigma^2}{2\gamma^*}\right)}_{=0} =$$

$$-\frac{\sigma^2}{2\gamma^*}(1 - e^{-2\gamma^*\tau}) + \frac{\sigma^2}{2\gamma^*}(1 - e^{-2\gamma^*(T+\tau)}) = \frac{\sigma}{2}\sigma_Z(0, 2\tau) - \frac{\sigma}{2}\sigma_Z(0, 2(T+\tau))$$

- For D:

$$-\int_0^T \sigma_f(t, T)^2\, dt = -\sigma^2 \int_0^T e^{-2\gamma^*(T-t)}\, dt = -\frac{\sigma^2}{2\gamma^*} e^{-2\gamma^*(T-t)}\Bigg|_0^T =$$

$$-\frac{\sigma^2}{2\gamma^*} + \frac{\sigma^2}{2\gamma^*} e^{-2\gamma^* T} = -\frac{\sigma^2}{2\gamma^*}\left(1 - e^{-2\gamma^* T}\right) = \frac{\sigma}{2}\sigma_Z(0, 2T)$$

So putting everything together we have that:

$$\int_0^T \frac{\sigma_Z(t, T+\tau)^2 - \sigma_Z(t, T)^2}{2\tau}\, dt =$$

$$\frac{\sigma}{2\tau\left(\gamma^*\right)^2}\left[2[\sigma_Z(0,T+\tau)-\sigma_Z(0,T)-\sigma_Z(0,\tau)]-\frac{1}{2}[\sigma_Z(0,2(T+\tau))-\sigma_Z(0,2T)-\sigma_Z(0,2\tau)]\right]$$

So we have that the convexity adjustment for the Hull-White model is:

$$f(0,T,T+\tau)=f^{fut}(0,T,T+\tau)$$

$$-\frac{\sigma}{2\tau\left(\gamma^*\right)^2}\left[2[\sigma_Z(0,T+\tau)-\sigma_Z(0,T)-\sigma_Z(0,\tau)]-\frac{1}{2}[\sigma_Z(0,2(T+\tau))-\sigma_Z(0,2T)-\sigma_Z(0,2\tau)]\right]$$

Solutions to Chapter 22

Exercise 1. Figure ?? and Figure ?? present the yield curve and the discount prices under both the Vasicek and the 2-Factor Vasicek models, respectively. Apparently there is no gain by using the 2-Factor model model. Yet this is mostly due to the time framed selected to present these observations (Figure 22.1 expands the chart over 30 years, where there is a larger divergence in values from the models). Additionally, when we take into account volatility we note that the 2-Factor model does a much better job. Figure ?? presents the yield volatility for both of these models and, also, the 2-Factor model with correlated factors. Note that both 2-Factor models significantly outperform the single factor Vasicek model. Figure ?? presents the correlation and how well each model captures this. The 2-Factor model with correlated factors does a much better job in capturing this.

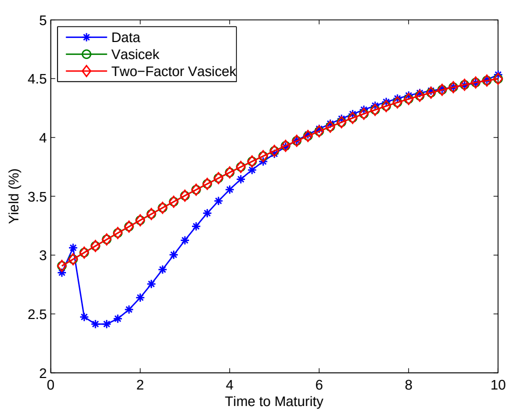

Figure 53: Yield curve under Single Factor and 2-Factor Vasicek

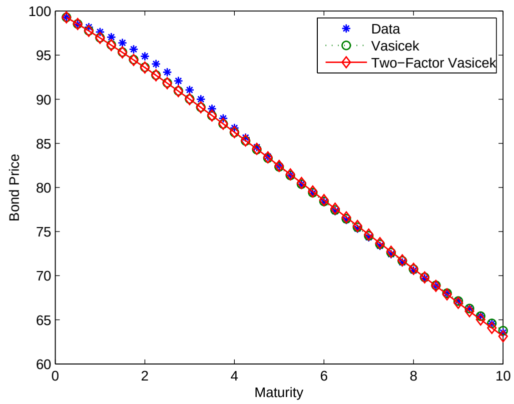

Figure 54: Discounts under Single Factor and 2-Factor Vasicek

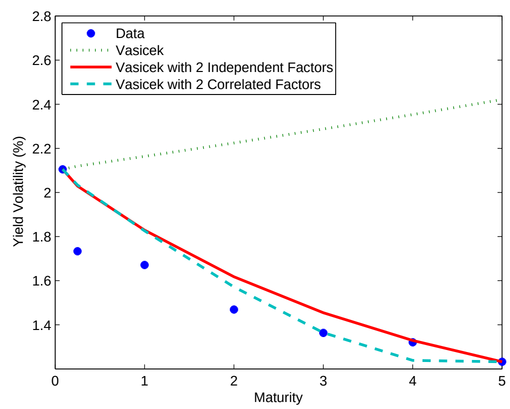

Figure 55: Yield volatility under Single Factor, 2-Factor and 2-Factor with Correlated Factors Vasicek

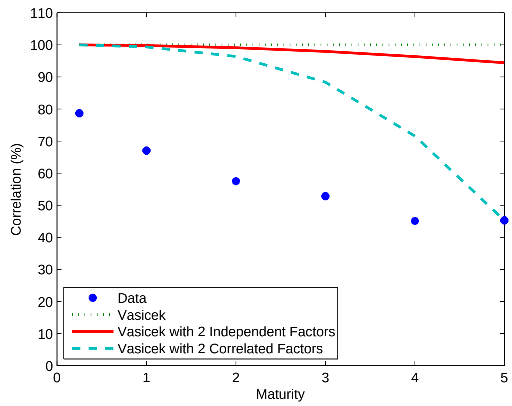

Figure 56: Correlation under Single Factor, 2-Factor and 2-Factor with Correlated Factors Vasicek

Exercise 2.

a. Figure **??** presents the cap prices under the Hull-White models (Single Factor and 2-Factor), as well as the Ho-Lee model. It is very difficult to tell which one does a better job pricing the caps from this chart. Figure **??** shows the pricing errors. Here we can see clearly that the 2-Factor Hull-White model does a better job than both the Ho-Lee and the Single Factor Hull-White models.

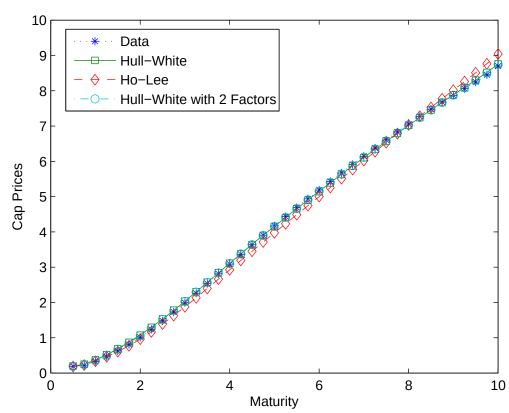

Figure 57: Cap Prices for Ho-Lee, Single Factor Hull-White and 2-Factor Hull-White models

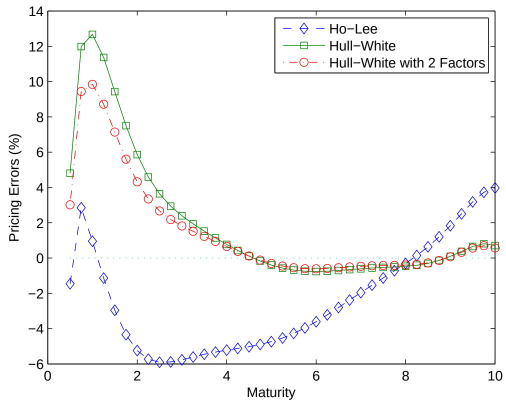

Figure 58: Pricing Errors for Ho-Lee, Single Factor Hull-White and 2-Factor Hull-White models

b. Figure **??** presents volatility estimates for all three models. Both Hull-White models are very close to the volatility implied by the data.

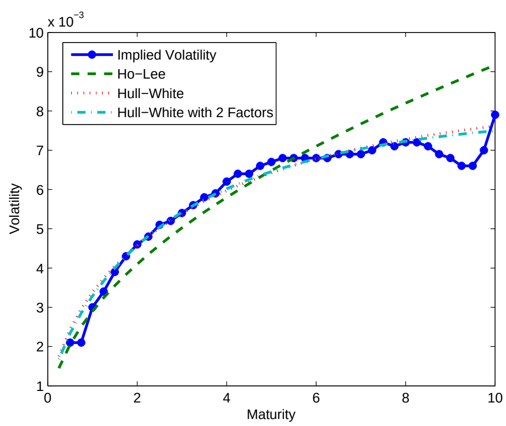

Figure 59: Volatility estimates for Ho-Lee, Single Factor Hull-White, 2-Factor Hull-White models

c. Unsing the 2-Factor Hull-White we price the Eurpean swaptions quoted in Table 22.3 (or 20.6). As it can be seen this swaption implied price is much closer to the data than the cap implied price (see Figures **??** and **??**). Also included is the volatility surface of these swaptions (Figure **??**).

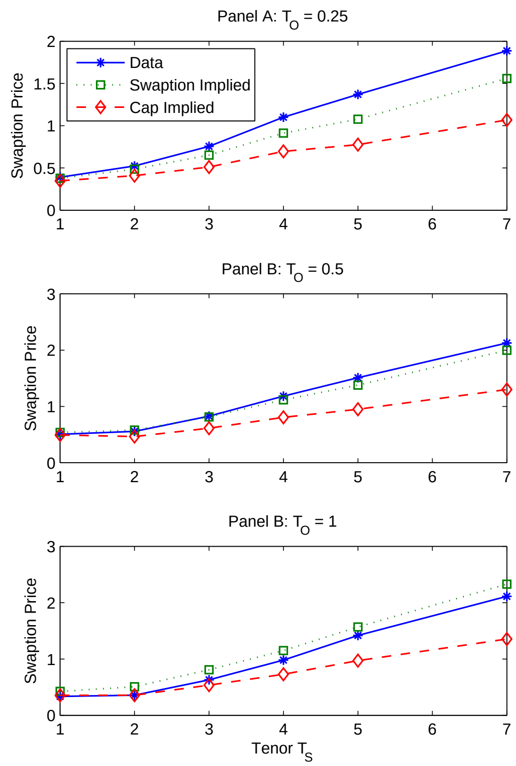

Figure 60: Prices for swaptions

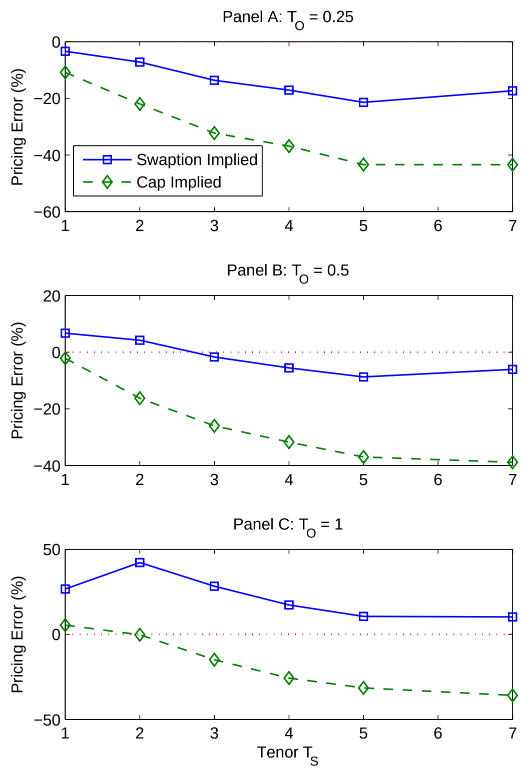

Figure 61: Pricing Errors for swaptions

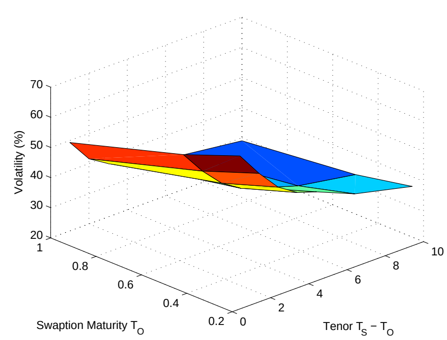

Figure 62: Volatility surface of swaptions

Exercise 3. Price of the caplet is 0.1253.