Habits and Leverage

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Motivation

- Much discussion in the academic literature and in policy circles about leverage and its impact on the real economy and on financial markets

- Various related themes, such as:
  - Excess credit supply may lead to financial crisis
  - The excessive growth of household debt and the causal relation between households' deleveraging and their low future consumption growth
  - Leverage cycle: Leverage is high when prices are high and volatility is low
  - Active deleveraging of financial institutions generate "fire sales" of risky financial assets, which further crash asset prices
  - The leverage ratio of financial institutions is a risk factor
  - Balance sheet recessions
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- But leverage is endogenous....

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  - Heterogeneous time varying risk-bearing capacity $\Longrightarrow$ leverage dynamics

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- Model's predictions consistent with empirical evidence
- Model aggregates to representative agent models with external habit

$\Longrightarrow$ It can be calibrated to yield reasonable asset pricing quantities.

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PROPORTIONAL TO UTILITY OF REPRESENTATIVE AGENT

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- Endowments w_i are also heterogeneous, with $\int w_i di = 1$

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- Technical restrictions:
  - $Y_t > \lambda \geq 1$ for all t: $\sigma_D(Y_t) \to 0$ as $Y_t \to \lambda$. Otherwise $\sigma_D(Y_t)$ general.
  - Endowments satisfy

$$w_i > \frac{a_i(\overline{Y} - \lambda) + \lambda - 1}{\overline{Y}}$$

Optimal Risk Sharing

- No consumption externalities $\Longrightarrow$ solve planner's problem
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- Less risk averse agents provide insurance to more risk averse agents

Representative Agent and State Price Density

- Our model aggregates to Menzly, Santos, and Veronesi (2004):
- As in Campbell and Cochrane (1999), define

$$\textit{Surplus consumption ratio} = S_t = \frac{D_t - \int X_{it} di}{D_t} = \frac{1}{Y_t} \tag{1}$$

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- **Proposition**. The equilibrium state price density

$$M_t = e^{-\rho t} D_t^{-1} S_t^{-1} \tag{2}$$

  – which follows

$$dM_t/M_t \; = \; -r_t dt - \sigma_{M,t} dZ_t \quad \textbf{with} \quad \sigma_{M,t} = (1+v)\sigma_D(S_t)$$

- We use S_t as state variable for notational convenience.

Competitive Equilibrium – 2

- **Proposition.** The competitive equilibrium has:

(Stock price) $$P_t = \left(\frac{\rho + k\overline{Y} S_t}{\rho \left(\rho + k \right)} \right) D_t$$

(Risk-free rate) $$r_t = \rho + \mu_D - (1+v)\sigma_D(S_t)^2 + k\left(1 - \overline{Y} S_t\right)$$

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- Stock and bond holdings depend on $w_i - a_i$ and the function $H(S_t)$.
- Stock price and risk-free rate are independent of distribution of w_i and a_i.

  $\Longrightarrow$ Prices and quantities have no causal relation with each other.

Implications: Leverage, Consumption, and Business Cycle

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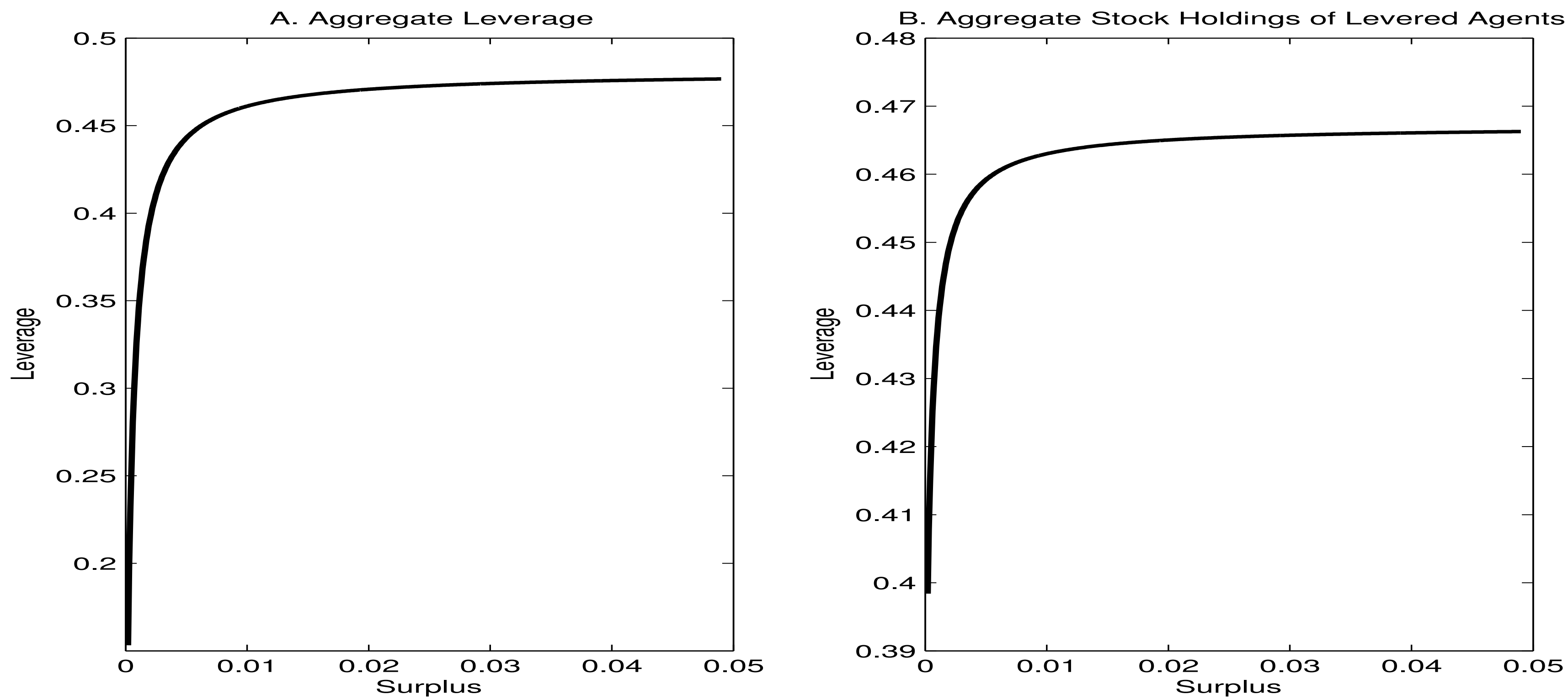

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- The risk premium for any asset with return $dR_{it} = (dP_{it} + D_{it})/P_{it}$ is

$$E_t[dR_{it} - r_t dt] = \underbrace{Cov_t\left(\frac{dD_t}{D_t}, dR_{it}\right)}_{\text{Consumption CAPM}} + \underbrace{\frac{q'(\ell_t)}{q(\ell_t)} Cov_t\left(d\ell_t, dR_{it}\right)}_{\text{Leverage risk premium}}$$

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- Two potential measures of leverage:

Debt/Output Ratio: $\ell_t = Q_{it}^{D/O}(S_t) = -\frac{N_{it}^0 B_t}{D_t} = v\,(w_i - a_i)\,H\,(S_t)$

Debt/Equity Ratio: $\ell_t = Q_{it}^{D/W}(S_t) = -\frac{N_{it}^0 B_t}{W_{it}} = \dfrac{\sigma_{Wi}(S_t)}{\sigma_P(S)} - 1$

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- **Result:** The price of leverage risk is

(a) $\lambda_t^{D/O} = \frac{q^{D/O\,\prime}(\ell_t)}{q^{D/O}(\ell_t)} > 0$ if $\ell_t =$ Debt/Output Ratio ("book leverage").

(b) $\lambda_t^{D/W} = \frac{q^{D/W\,\prime}(\ell_t)}{q^{D/W}(\ell_t)} < 0$ if $\ell_t =$ Debt/Equity Ratio ("market leverage").

- In bad times:
  - agents deleverage $\Longrightarrow$ debt/output $\downarrow \Longrightarrow$ book leverage risk price > 0.
  - high discounts $\Longrightarrow$ debt/equity $\uparrow \Longrightarrow$ market leverage risk price < 0.

Quantitative Predictions

- Previous results independent of the functional form of $\sigma_D(Y_t)$.
- Assume now a specific functional form to make model comparable to MSV and obtain reasonable asset pricing implications:

$$\sigma_D(Y_t) = \sigma^{max}(1 - \lambda Y_t^{-1})$$

- $\Longrightarrow$ Economic uncertainty increases in bad times, but bounded between $[0, \sigma^{max}]$
- $\Longrightarrow$ Obtain same process for Y_t as in MSV $\Longrightarrow$ Use their same parameters.
  - Additional parameter σ^{max} chosen to fit average consumption volatility
- All asset pricing results are similar (or stronger) than MSV.

Calibrate to Household Consumption Systematic Volatility

- $a_i \sim U(\underline{a}, \overline{a})$ and Pareto weights $\phi_i = LogN\left(-\frac{1}{2}\sigma_\psi^2, \sigma_\psi^2\right)$

Table 2. Cross Sectional Parameters and Household Consumption Moments

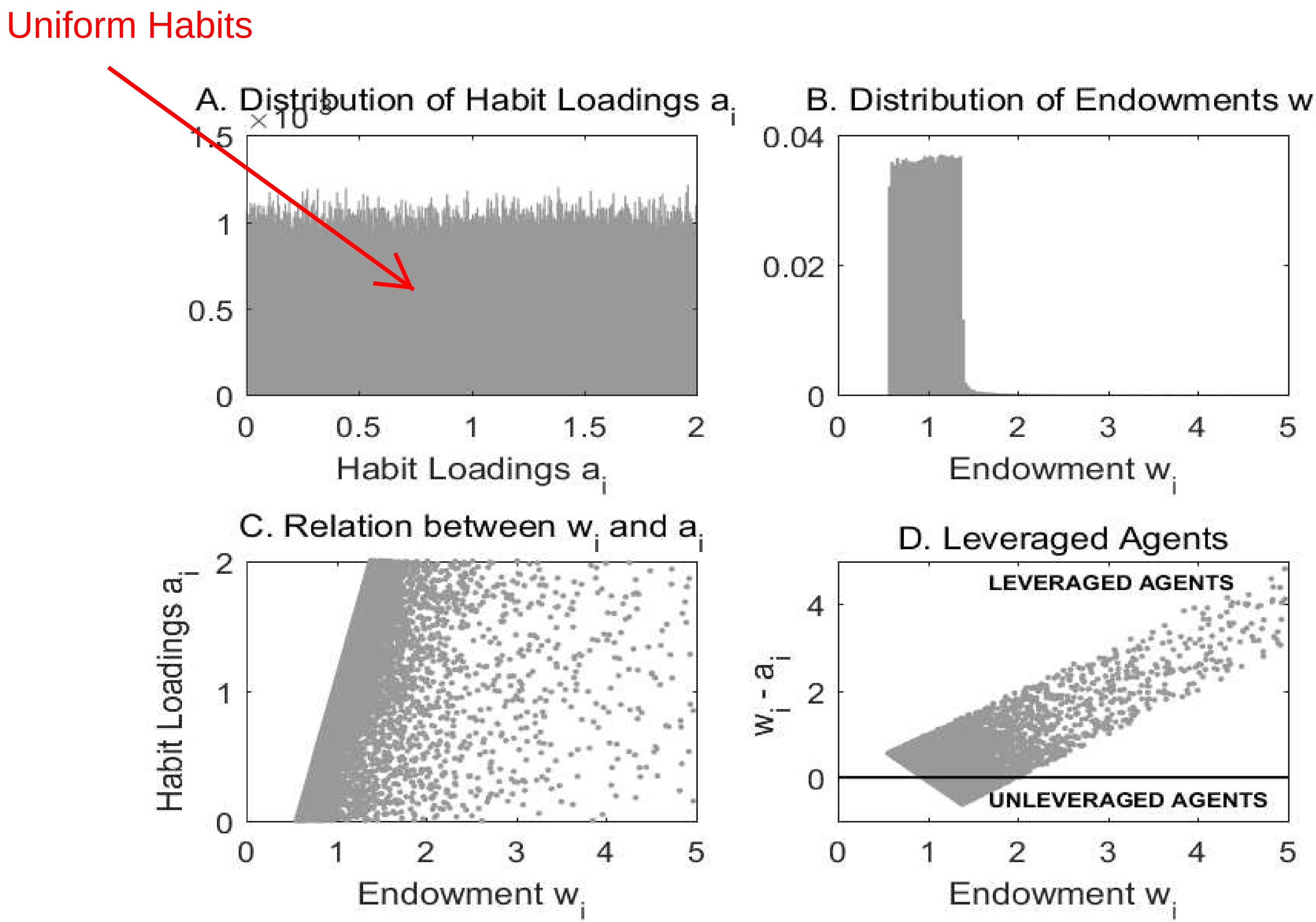
Panel A. Households Quarterly Consumption Moments. Data

	Growth Rate (%)				Volatility (%)		
	Mean	Median	Std. Dev.		Mean	Median	Std. Dev.
Arithmetic	6.04	-0.63	40.13	Total	36.53	27.10	42.35
Logarithmic	-0.59	-0.66	35.78	Systematic	8.94	6.61	10.42

Panel B. Households Quarterly Consumption Moments. Model

$U[\underline{a}, \overline{a}]$, σ_ϕ	Arithmetic Growth Rate (%)			Volatility (%)		
	Mean	Median	Std. Dev.	Mean	Median	Std. Dev.
$U[0, 2]$, 3	0.73	0.52	4.31	6.58	4.08	8.37
$U[1, 1]$, 3	0.52	0.52	0.67	1.41	0.91	3.38
$U[0, 2]$, 0	0.73	0.52	4.23	6.58	3.91	8.48
$U[1, 1]$, 0	0.52	0.52	0.00	1.19	1.19	0.00

The Cross-Section of Agents' Behavior: Who Levers?



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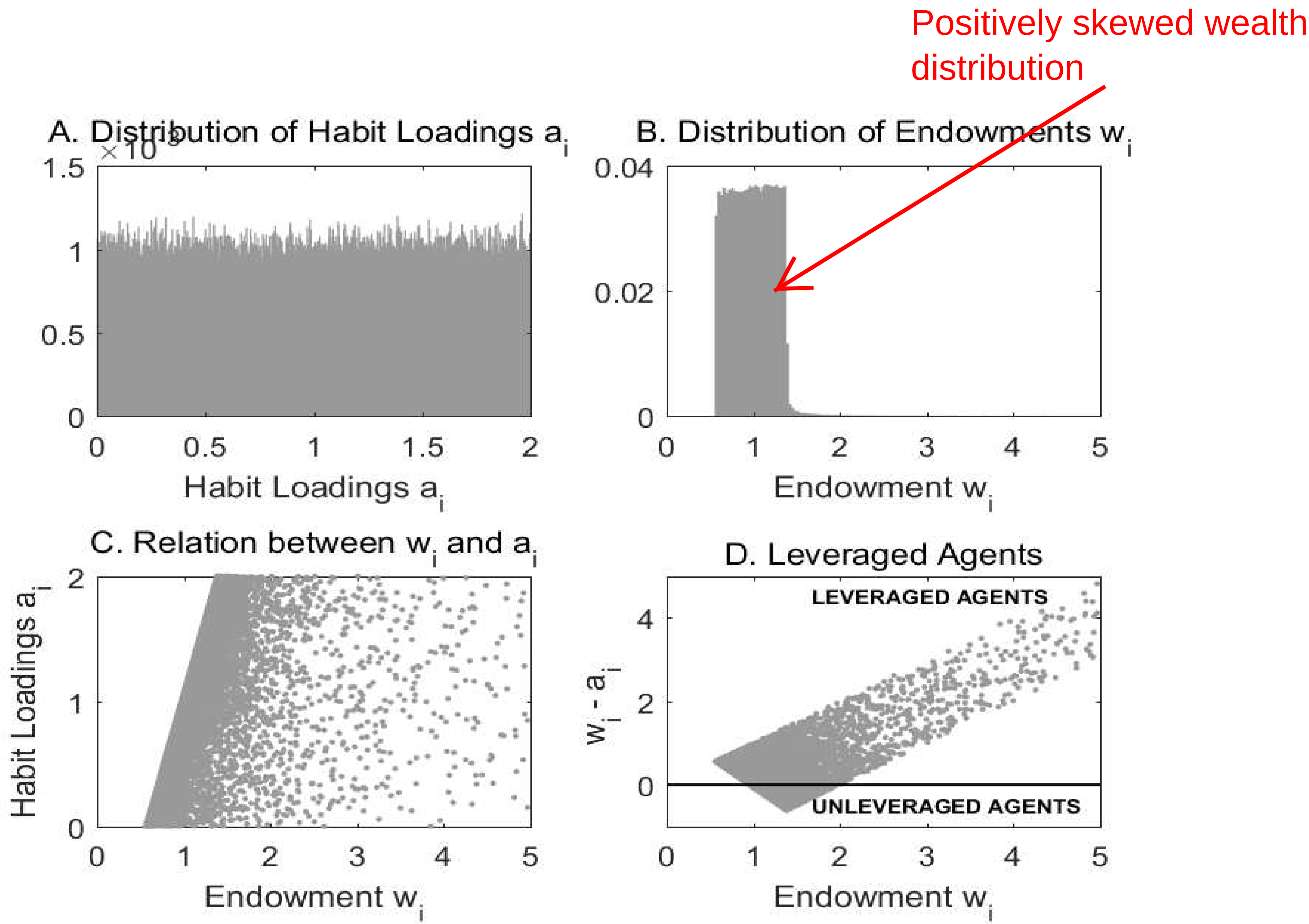

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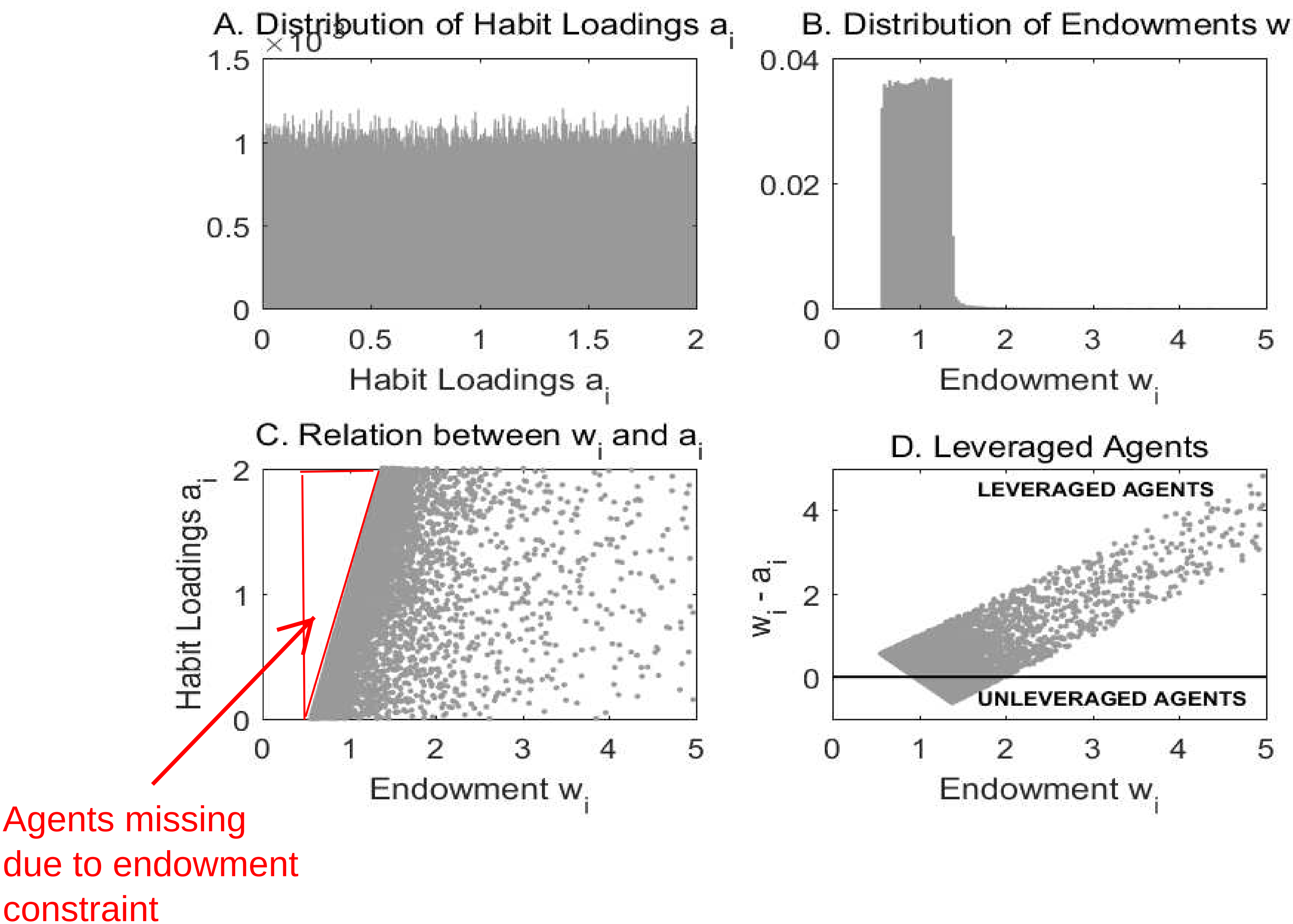
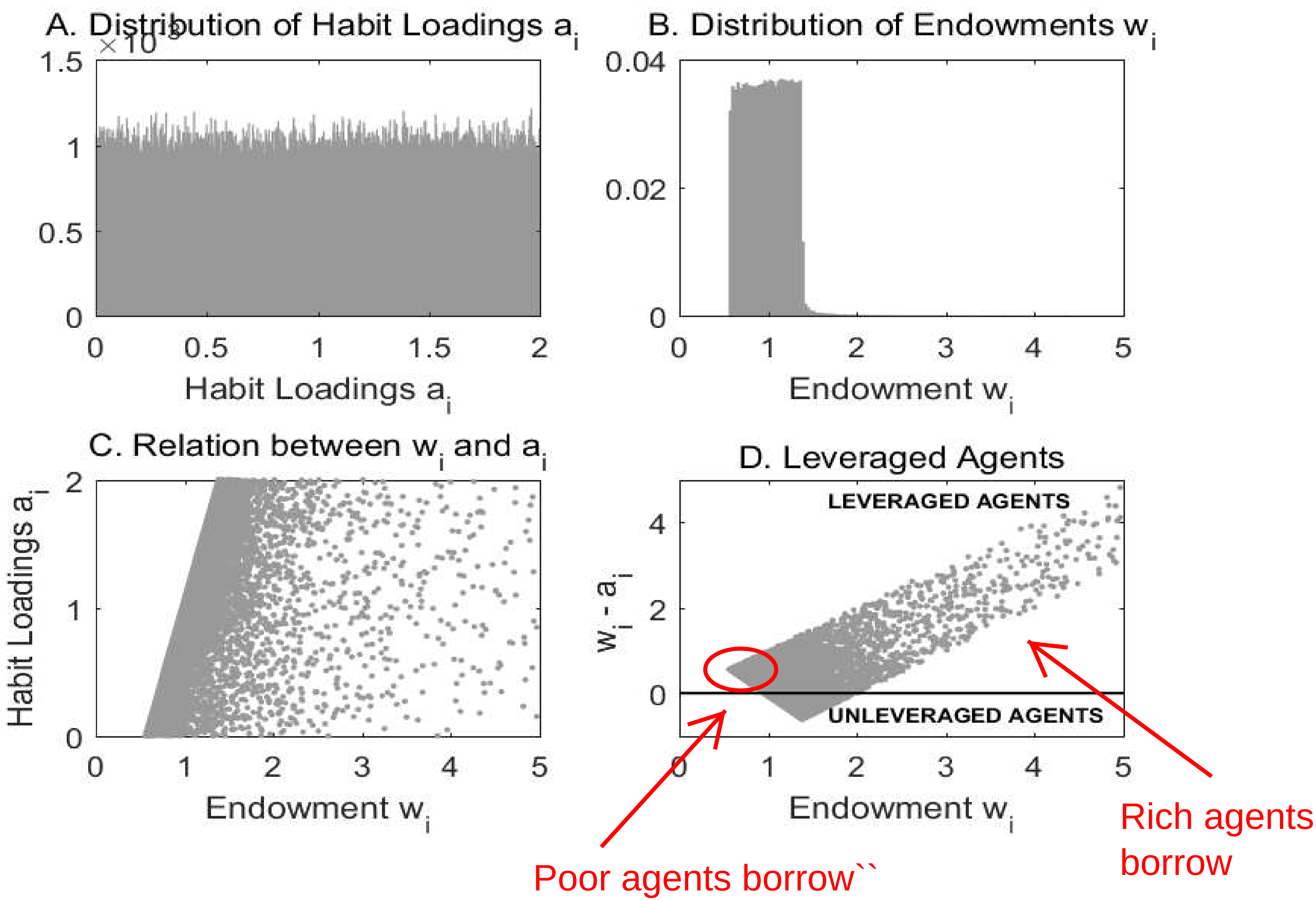
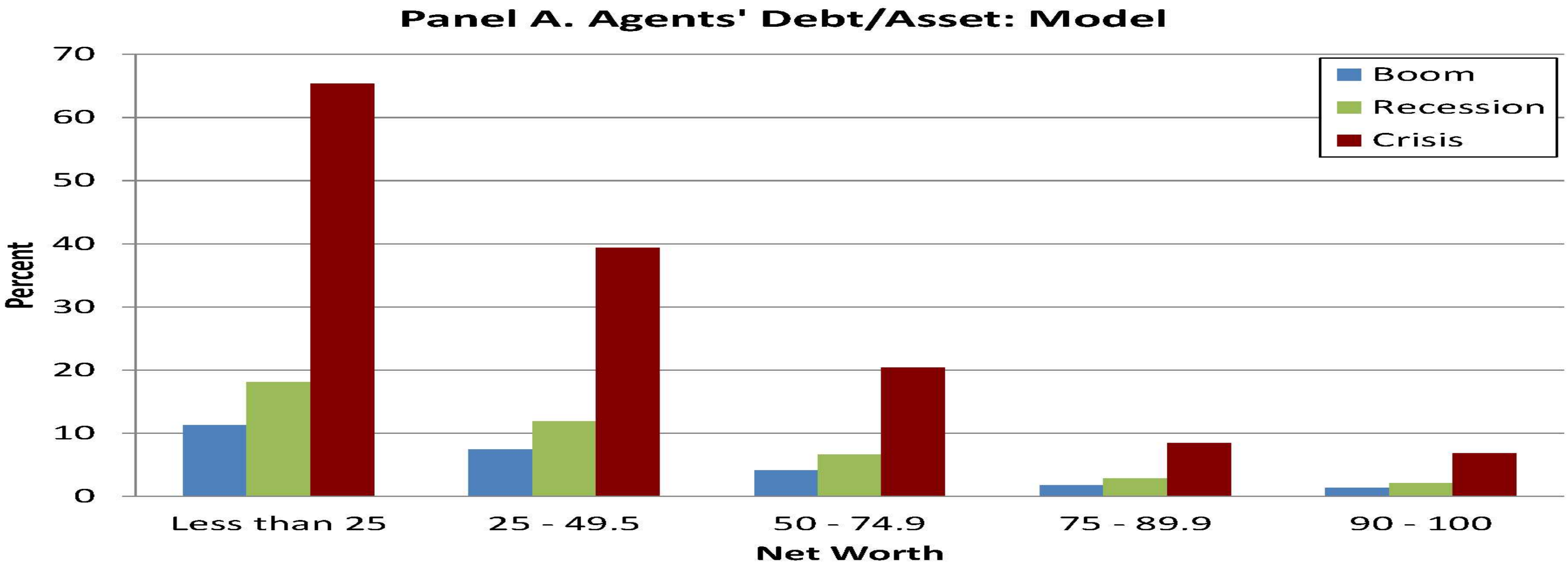


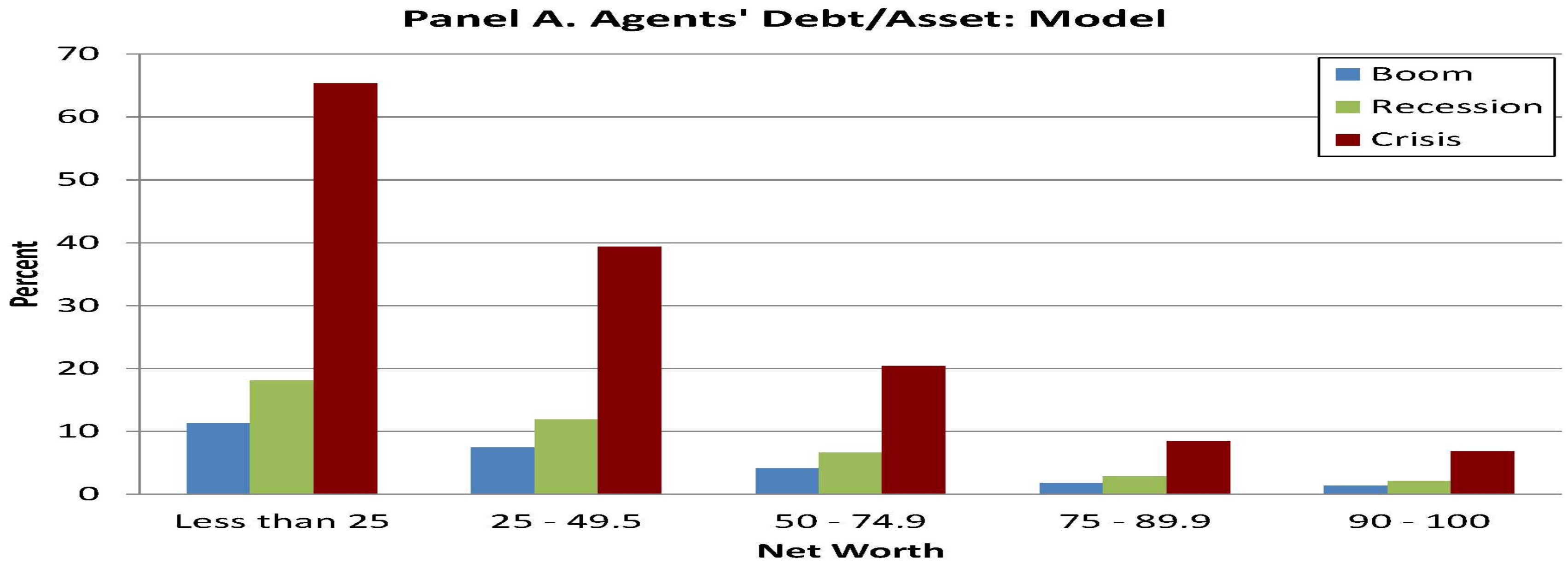
The Cross-Section of Agents' Behavior: Who Levers?

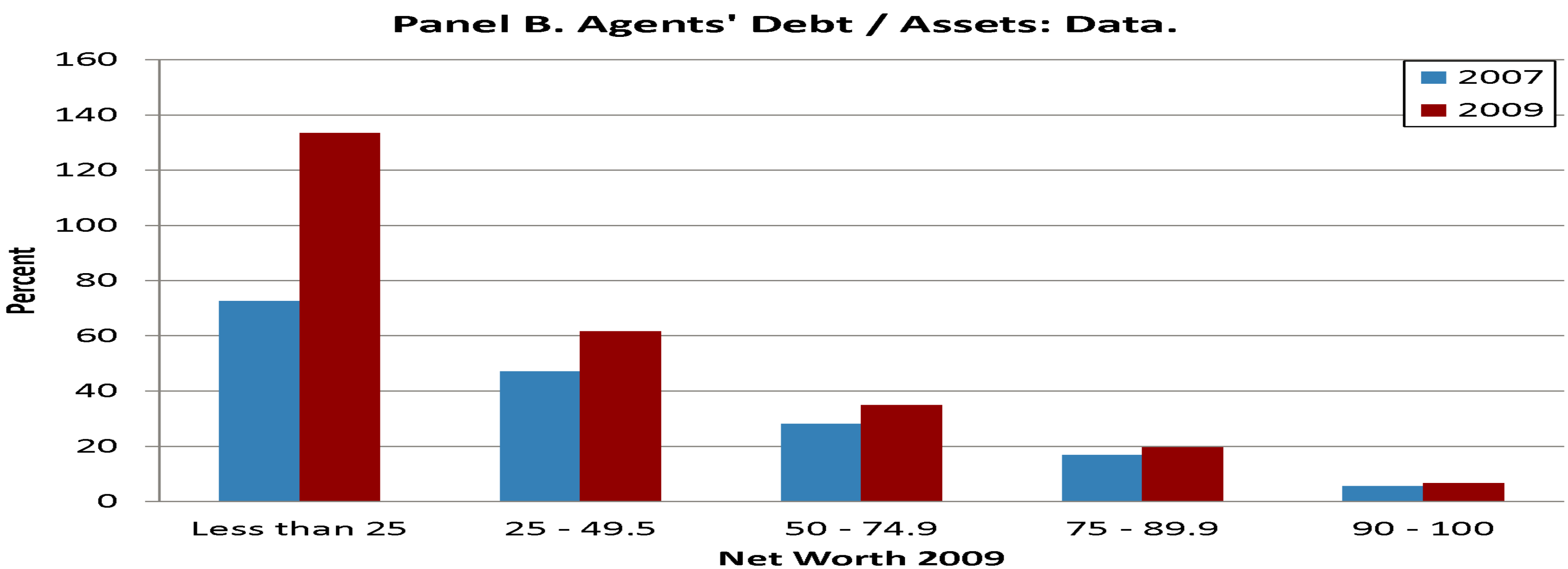


Household Leverage in Good and Bad Times



Household Leverage in Good and Bad Times





“Fire Sales” in a Simulation Run

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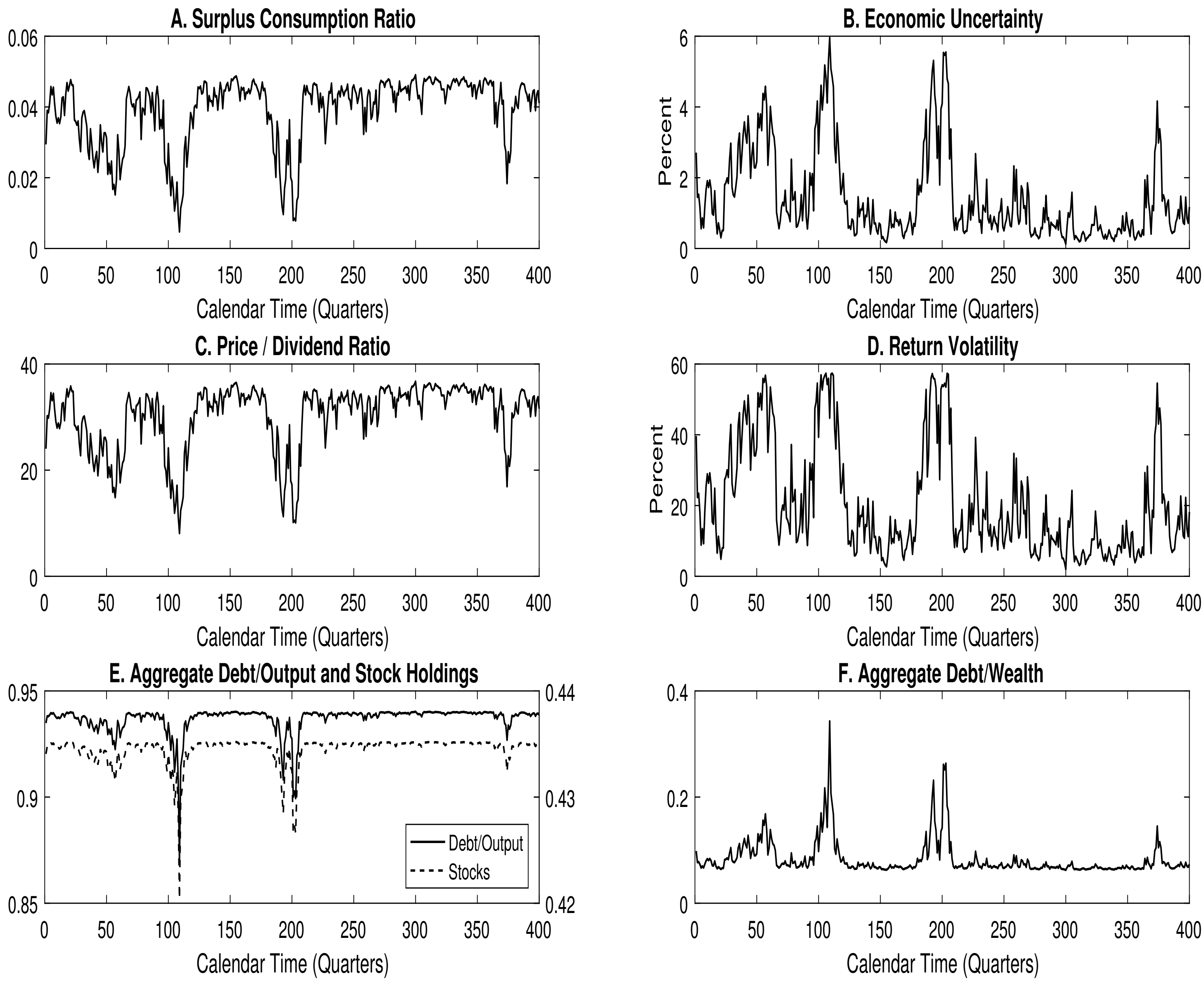
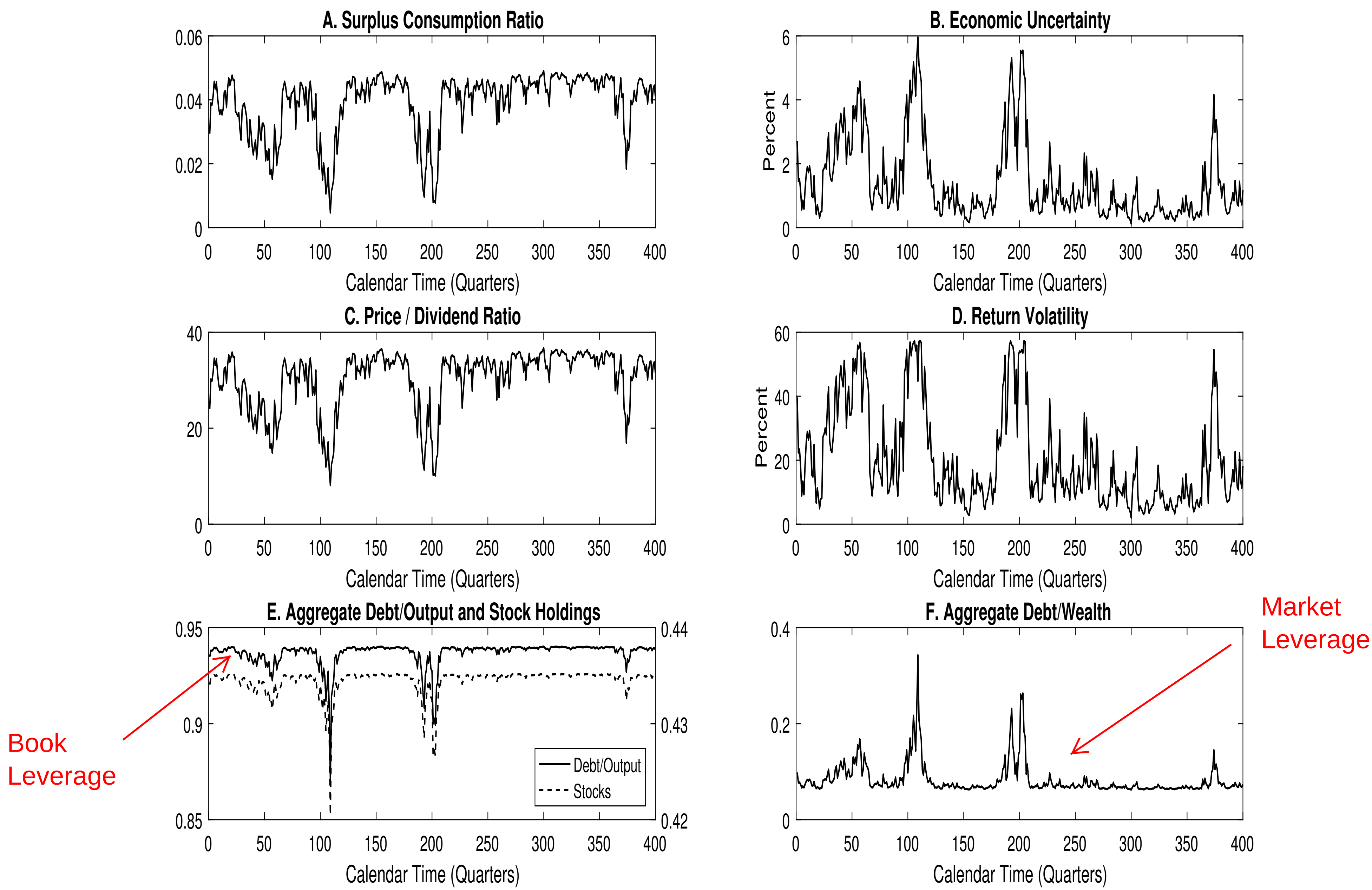


"Fire Sales" in a Simulation Run



Intermediary Asset Pricing: Book and Market Leverage

Table 3: The Market Price of Leverage Risk

Panel A - Data			
α	3.19	0.76	1.07
	(3.05)	(0.62)	(0.97)
Market Return	-0.89	0.97	0.82
	(-0.72)	(0.69)	(0.61)
Market Leverage		-0.22	
		(-2.13)	
Book Leverage			0.63
			(3.07)
R^2 (%)	6.54	50.77	53.35
Panel B - Model			
α	0.02	0.10	0.08
Market Return	2.05	1.96	1.98
Market Leverage		-0.04	
Book Leverage			0.03

Different signs in the data ....

... and in the model

Intermediary Asset Pricing: Book and Market Leverage

Table 4: The Predictability of Aggregate Stock Returns

	1 year	2 year	3 year	4 year	5 year
Panel A. Predictability with Book Leverage. Data					
Coef ($\times 100$)	-1.78	-1.79	-2.17	-3.13	-9.89
	(-0.83)	(-0.72)	(-0.89)	(-1.03)	(-3.29)
R^2	0.01	0.01	0.01	0.01	0.07
Panel B. Predictability with Market Leverage. Data					
Coef ($\times 100$)	3.66	6.21	8.56	10.03	13.06
	(1.57)	(1.50)	(2.18)	(2.51)	(3.84)
R^2	0.04	0.07	0.10	0.12	0.19
Panel C. Predictability with Book Leverage. Model					
Coef ($\times 100$)	-3.57	-7.28	-10.49	-12.62	-14.13
	(-3.45)	(-3.09)	(-3.18)	(-3.34)	(-3.52)
R^2	0.02	0.05	0.08	0.10	0.12
Panel D. Predictability with Market Leverage. Model					
Coef ($\times 100$)	5.86	10.91	14.69	17.35	19.36
	(8.08)	(7.69)	(7.55)	(7.54)	(7.74)
R^2	0.06	0.12	0.17	0.20	0.22

Different signs in the data...

... and in the model

Conclusions

- A frictionless dynamic general equilibrium model with heterogeneous agents and external habits seem consistent with many stylized facts.
- Risk sharing motives generate endogenous leverage dynamics
- Our model predicts:
  1. Aggregate debt $\Uparrow$ in good times when prices $\Uparrow$ and volatility $\Downarrow$
  2. Poorer agents borrow more than richer agents
  3. Leveraged agents enjoy a "consumption boom" in good times, followed by a consumption slump
  4. Crisis time $\Longrightarrow$ leveraged agents delever by "fire-selling" stocks, but their debt/wealth ratio $\Uparrow$ due to strong discount effects.
  5. Intermediaries leverage is a priced risk factor.
  6. Wealth dispersion $\Uparrow$ in good times
- Leverage dynamics is due to the differential impact of aggregate shocks on agents' risk aversion.

Table 1. Parameters and Moments

Panel A. Parameters (MSV)

ρ	k	$\overline{Y}$	λ	$\overline{v}$	μ	σ^{max}
0.0416	0.1567	34	20	1.1194	0.0218	0.0641

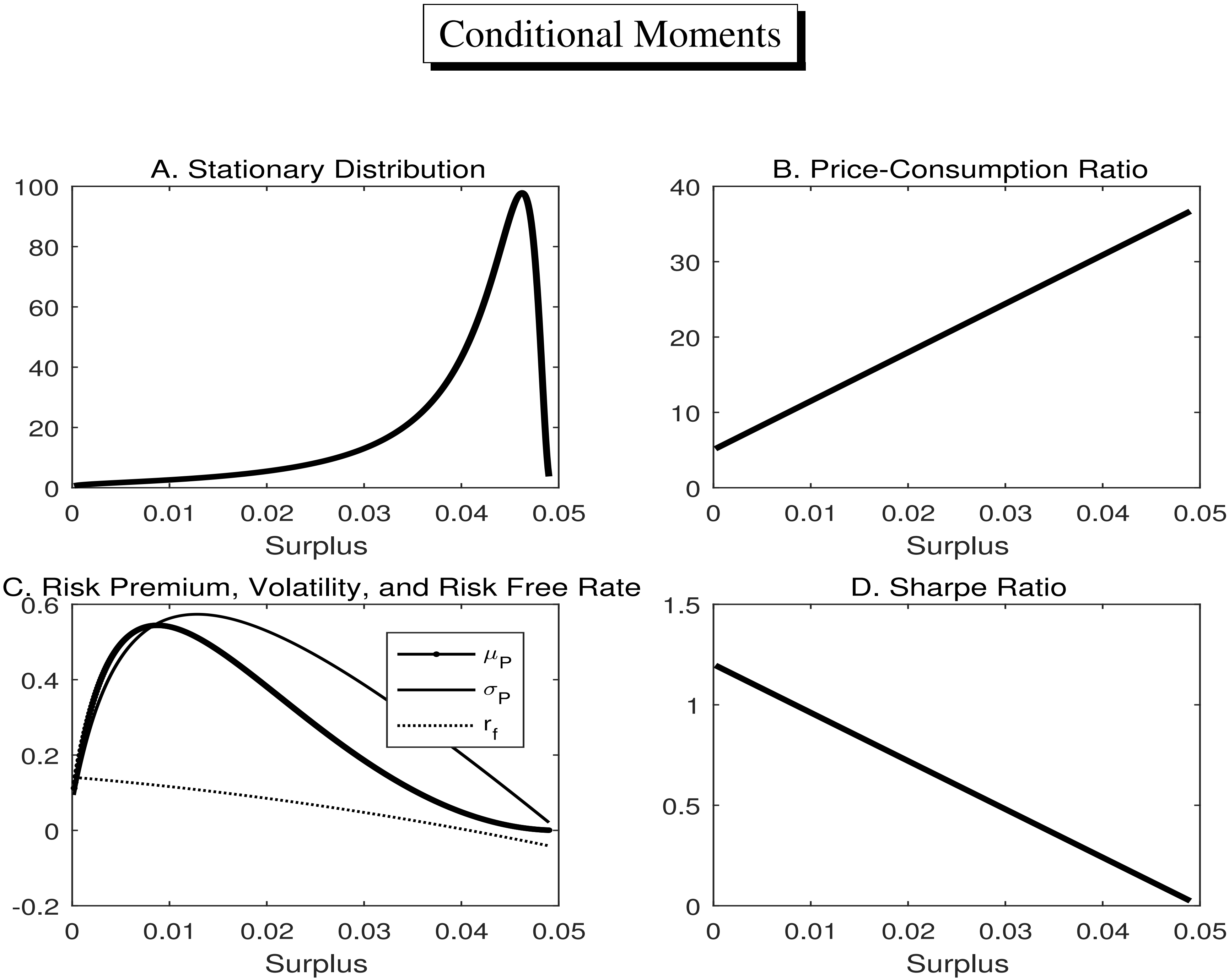
Panel B. Moments (1952 – 2014)

	$E[R]$	$Std(R)$	$E[r_f]$	$Std(r_f)$	$E[P/D]$	$Std[P/D]$	SR	$E[\sigma_t]$	Std(σ_t)
Data	7.13%	16.55%	1.00%	1.00%	38	15	43%	1.41%	0.52%
Model	8.19%	25.08%	0.54%	3.77 %	30.30	5.80	32.64%	1.43%	1.18%

Panel C. P/D Predictability R^2

	1 year	2 year	3 year	4 year	5 year
Data	5.12%	8.25%	9.22%	9.59%	12.45%
Model	14.18%	23.67%	30.01%	33.81%	35.92

- Model matches asset pricing moments well.


Conditional Moments
A. Stationary Distribution
100
80
60
40
20
0
0
0.01
0.02
0.03
0.04
0.05
Surplus
B. Price-Consumption Ratio
40
30
20
10
0
0
0.01
0.02
0.03
0.04
0.05
Surplus
C. Risk Premium, Volatility, and Risk Free Rate
0.6
0.4
0.2
0
-0.2
μ_P
σ_P
r_f
0
0.01
0.02
0.03
0.04
0.05
Surplus
D. Sharpe Ratio
1.5
1
0.5
0
0
0.01
0.02
0.03
0.04
0.05
Surplus