

# Topics in Dynamic Asset Pricing

Course Presentation.

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## Course Objectives

- This course has two objectives:
  1. Introduce students to the frontier of research in asset pricing: we will cover a number of models and methodologies have been recently developed in the literature to address intriguing empirical regularities.
  2. Teach students how to write coherent research papers: over the eleven weeks I will assign three research ideas that students have to developed into research papers (I provide tips). the TA and I will “referee” such papers providing then feedback on how papers should be written.
- We start by reviewing some (but not all) intriguing empirical regularities.

## A Simple Benchmark Model (Lucas Tree Model)

- Aggregate dividends  $D_t$  are i.i.d.

$$\frac{dD_t}{D_t} = \mu_d dt + \sigma_d dB_t$$

- $P_t$  = price of stock that is a claim on these dividends.  $r_t$  = risk free rate of return.
- A representative agent has infinite life, power utility over consumption, chooses  $C_t$  and asset allocation  $\theta_t$  to

$$\max_{C_t, \theta_t} E_0 \left[ \int_0^\infty e^{-\phi t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right]$$

- Equilibrium:  $C_t = D_t$  and  $\theta_t = 1 \implies \text{SDF} = \lambda_t = e^{-\phi t} C_t^{-\gamma}$

$$P_t = E_t \left[ \int_t^\infty \frac{\lambda_\tau}{\lambda_t} D_\tau d\tau \right] = \frac{D_t}{R - \mu_d}$$

- where  $R$  = discount rate for risky stock

## Implications of Benchmark Model

- A large number of empirical regularities clash with this standard paradigm.

1. **Equity premium puzzle:** Stocks have averaged returns of about 7% over treasuries.

- This number is high compared to the volatility of consumption, of about 1-2%.
- The canonical model implies

$$\text{Expected Excess Return} = \gamma \text{Variance of Consumption Growth}$$

- Even assuming that  $\gamma$  is large, say  $\gamma = 10$ , we have

$$\text{Expected Excess Return} = 10 \times (.02)^2 = 0.4\%$$

- We are an order of magnitude off.

## Implications of Benchmark Model

2. **Volatility Puzzle 1:** Return volatility (about 16 %) is too high compared to the volatility of dividends (about 7%).

- The same classic canonical model has

$$\frac{P_t}{D_t} = \text{Constant}$$

- This implies

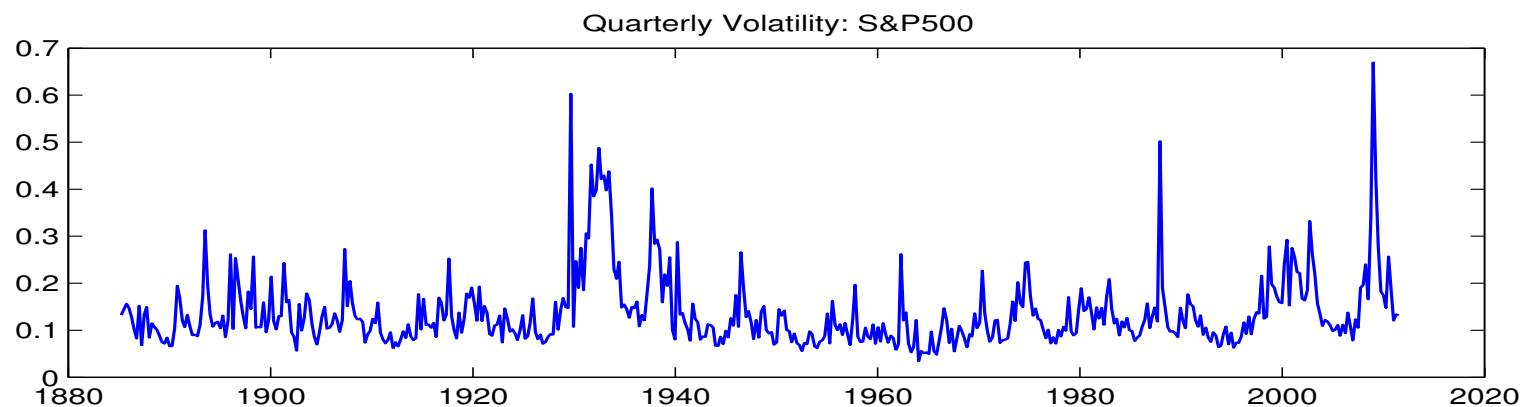
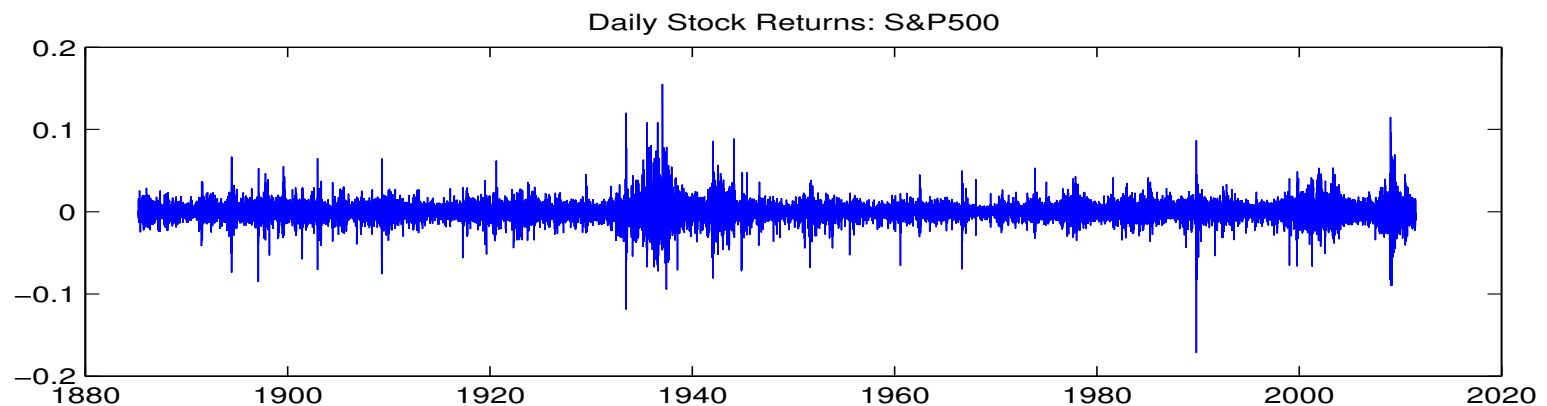
$$\text{Volatility of } \frac{dP_t}{P_t} = \text{Volatility of } \frac{dD_t}{D_t}$$

- Something else must be time varying to make the volatility higher.
- Indeed, the canonical model would imply a constant P/D ratio, which we know it is not.

## Implications of Benchmark Model

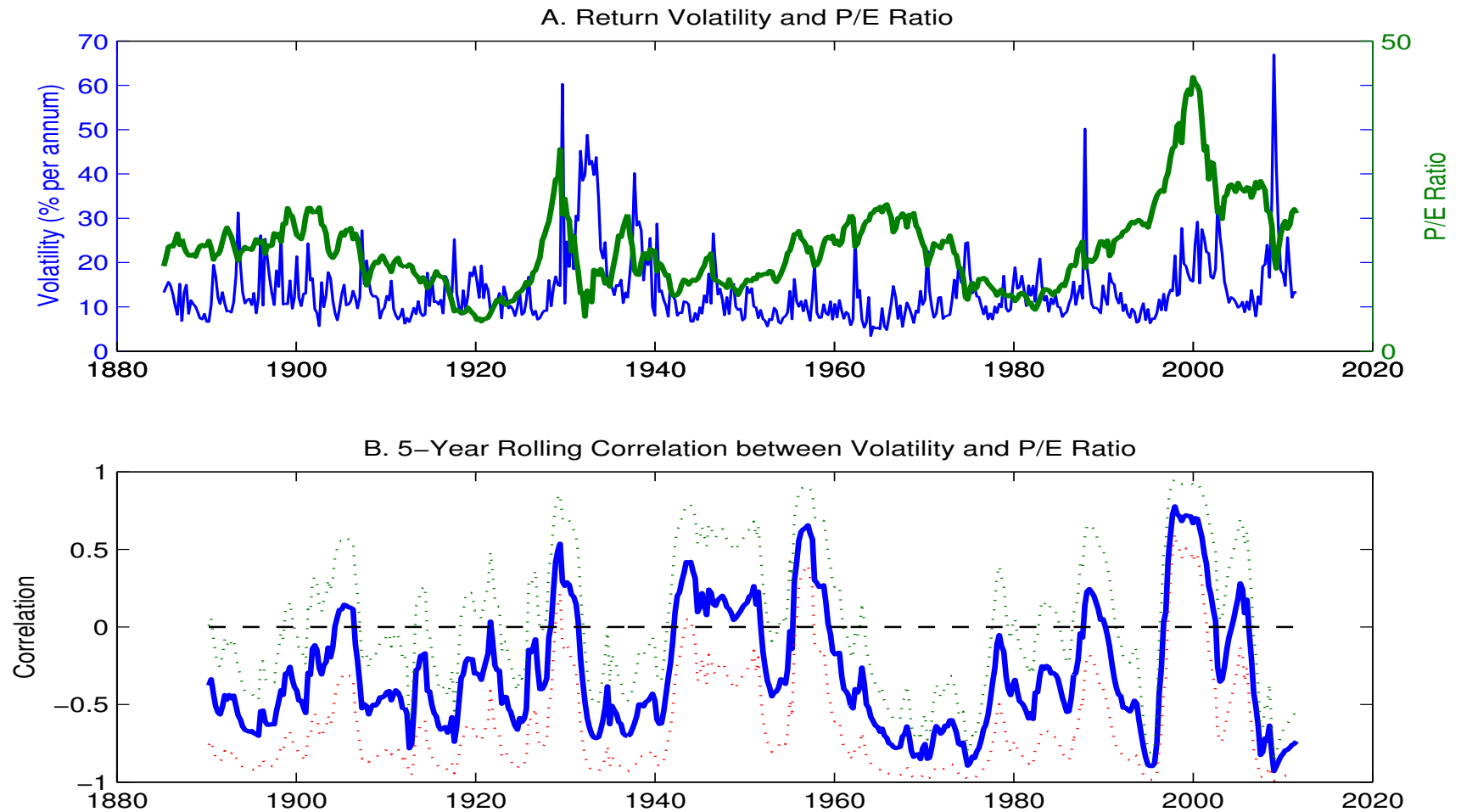
### 3. Volatility Puzzle 2: Return volatility is not only high, but it is time varying.

- Historically, (annualized) market return volatility fluctuated wildly, ranging between 60 - 70 % in the 30s (and 2008-2009) to less than 5% in the middle of the 1960s.



## Implications of Benchmark Model

### 4. Volatility Puzzle 3: Volatility and price/earning ratios are sometimes positively correlated.



## Implications of Benchmark Model

5. **Risk Free Rate Puzzle:** The usual canonical model implies that the interest rate is given by

$$r = \phi + \gamma\mu_c - \frac{1}{2}\gamma(\gamma + 1)\sigma_c^2$$

- If  $\gamma = 10$  for instance, using  $\mu_c = 2\%$ ,  $\sigma_c = 1\%$  and  $\phi = 2\%$  we find  $r = 21\%$
- The problem is  $\gamma$  that is too high: If we set  $\gamma = 2$  we obtain  $r = 6\%$ .
- Note the tension between equity premium puzzle (need  $\gamma$  high) and risk free rate puzzle (need  $\gamma$  low).



## Implications of Benchmark Model

6. **Predictability:** Stock returns are predictable by, say, the dividend price ratio, earnings price ratio, etc.

- Predictability regression

$$\text{Cumulated Returns } (t \rightarrow t + \tau) = \alpha + \beta x_t + \epsilon_{t,t+\tau}$$

where  $x_t$  is a predictor observable at time  $t$ .

**Table 1: Return Predictability – CRSP Sample: 1927 - 2010**

Predictor	Horizon (Quarters)	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$
Log Div Yield	1	0.10	0.02	1.72	1.53	1.1%
Log Earn Yield (1 y)	1	0.09	0.03	2.32	1.99	1.1%
Log Earn Yield (10 y)	1	0.13	0.04	2.80	2.55	2.2%
Term Spread	1	0.01	0.46	0.74	1.05	0.3%
Return Variance	1	0.02	-0.15	2.47	-0.24	0.0%
Credit Spread	1	0.01	0.62	0.40	0.37	0.2%
Book / Market	1	-0.02	0.06	-1.27	2.03	2.4%
Log Payout yield	1	0.15	0.06	2.44	2.28	1.7%
Log Div Yield	4	0.42	0.11	2.56	2.29	5.2%
Log Earn Yield (1 y)	4	0.35	0.11	3.08	2.59	3.9%
Log Earn Yield (10 y)	4	0.52	0.17	3.57	3.09	9.2%
Term Spread	4	0.02	2.12	0.65	1.94	1.6%
Return Variance	4	0.05	0.04	2.51	0.03	0.0%
Credit Spread	4	0.03	1.78	0.87	0.54	0.4%
Book / Market	4	-0.09	0.23	-1.46	2.93	8.0%
Log Payout yield	4	0.73	0.32	3.89	3.47	10.2%
Log Div Yield	12	1.12	0.29	4.37	3.75	14.2%
Log Earn Yield (1 y)	12	1.00	0.32	3.20	2.71	11.6%
Log Earn Yield (10 y)	12	1.31	0.42	3.25	2.78	22.1%
Term Spread	12	0.02	8.53	0.18	2.16	9.3%
Return Variance	12	0.15	-0.37	2.43	-0.09	0.0%
Credit Spread	12	0.11	4.13	1.00	0.73	0.7%
Book / Market	12	-0.17	0.53	-1.01	2.33	15.7%
Log Payout yield	12	1.76	0.75	3.21	2.77	22.6%

Note: t-statistics computed using Newey West standard errors

**Table 2: Return Predictability – “cay” Sample: 1952 - 2010**

Predictor	Horizon (Quarters)	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$
Log Div Yield	1	0.10	0.02	2.02	1.75	1.5%
Log Earn Yield (10 y)	1	0.07	0.02	1.64	1.34	0.9%
cay	1	0.01	0.87	2.42	3.88	4.3%
Term Spread	1	0.00	0.66	0.34	1.61	1.3%
Book / Market	1	0.00	0.02	0.16	0.86	0.4%
Investment/Capital	1	0.15	-3.88	2.98	-2.69	3.0%
Log Payout yield	1	0.09	0.04	1.57	1.36	0.9%
Log Div Yield	4	0.43	0.11	2.33	1.99	6.6%
Log Earn Yield (10 y)	4	0.30	0.09	2.05	1.65	4.2%
cay	4	0.05	3.61	2.92	3.97	16.6%
Term Spread	4	0.02	2.29	0.58	2.21	3.6%
Book / Market	4	0.00	0.10	-0.03	1.15	2.1%
Investment/Capital	4	0.47	-11.78	2.57	-2.19	6.1%
Log Payout yield	4	0.47	0.19	2.41	2.10	5.5%
Log Div Yield	12	1.06	0.26	3.30	2.94	16.1%
Log Earn Yield (10 y)	12	0.73	0.20	2.32	1.91	9.9%
cay	12	0.14	8.59	4.12	6.70	38.1%
Term Spread	12	0.07	4.99	1.30	3.28	6.6%
Book / Market	12	0.07	0.15	0.57	0.78	1.9%
Investment/Capital	12	1.28	-31.38	4.29	-3.80	16.7%
Log Payout yield	12	1.12	0.44	3.63	3.15	12.0%

Note: t-statistics computed using Newey West standard errors

## Implications of Benchmark Model

- This result raises a number of issues, such as:
  - (a) Why are stock return predictable?
  - (b) Why the regression coefficients (and significance) depend on the time interval used?
  - (c) What are the implication for an investor who is allocating his wealth between stocks and bonds to maximize his life time utility?
  - (d) Why stock return volatility **does not** predict future excess returns? After all, the canonical model has

$$\text{Expected Excess Return} = \gamma \text{Variance of Stock Return}$$

- Using more sophisticated models for volatility, some studies find a significantly positive relation, but some others find a significant negative relation. There is still a considerable debate.

## Implications of Benchmark Model

7. **Cross-sectional Predictability Puzzle:** Some type of stocks yield an average return that is not consistent with the canonical model.

- The canonical model implies that expected excess returns of asset  $i$  is given by:

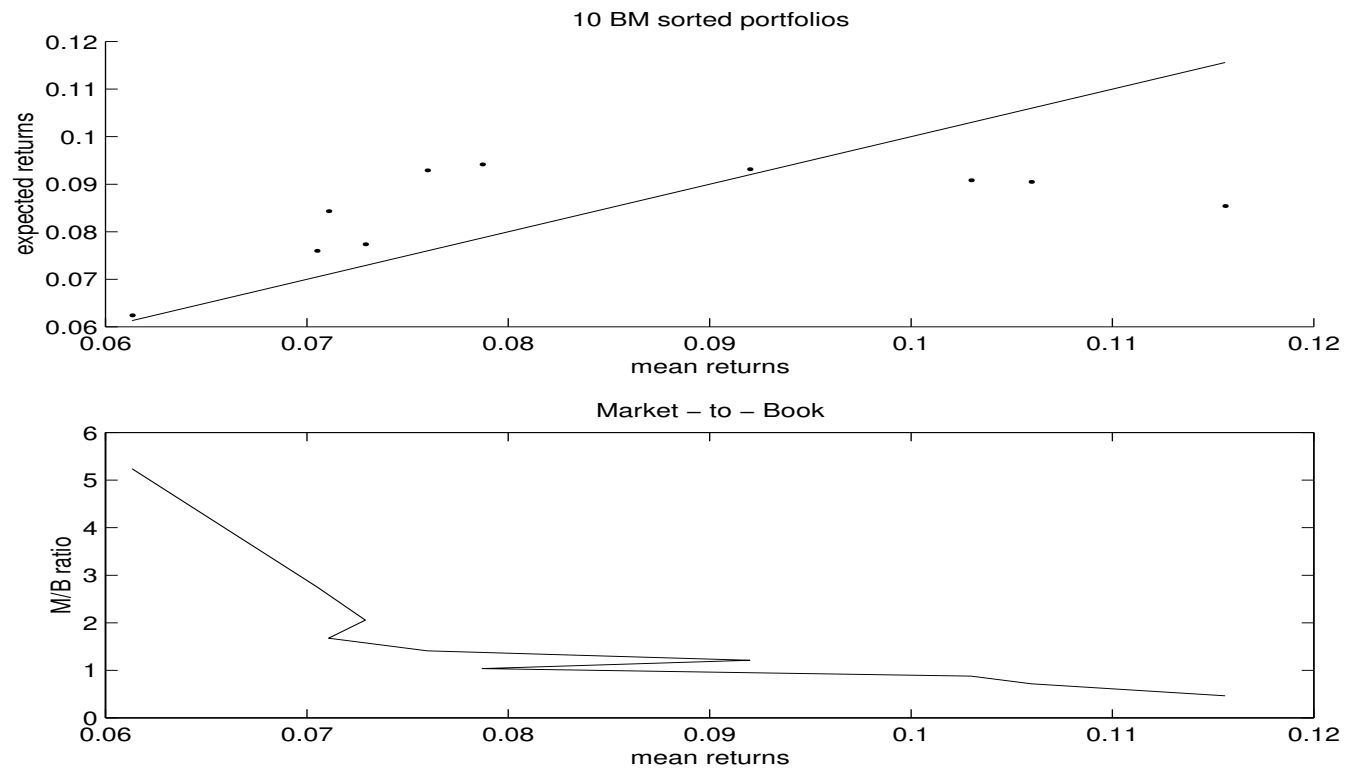
$$\begin{aligned} E [\text{Excess Return}_t^i] &= \gamma \text{Cov} (\text{Return}^i, \text{Consumption Growth}) \\ &= \beta^i E [\text{Excess Return of Mkt Portfolio}] \end{aligned}$$

- where

$$\beta^i = \frac{\text{Cov} (\text{Return}^i, \text{Return Mkt Portfolio})}{\text{Var} (\text{Return Mkt Portfolio})}$$

- Portfolios of stocks that are sorted by Book-to-Market Ratio or by Size and Book to Market do not satisfy this relation.
- For instance, using Book-to-Market sorted portfolios, we obtain the following

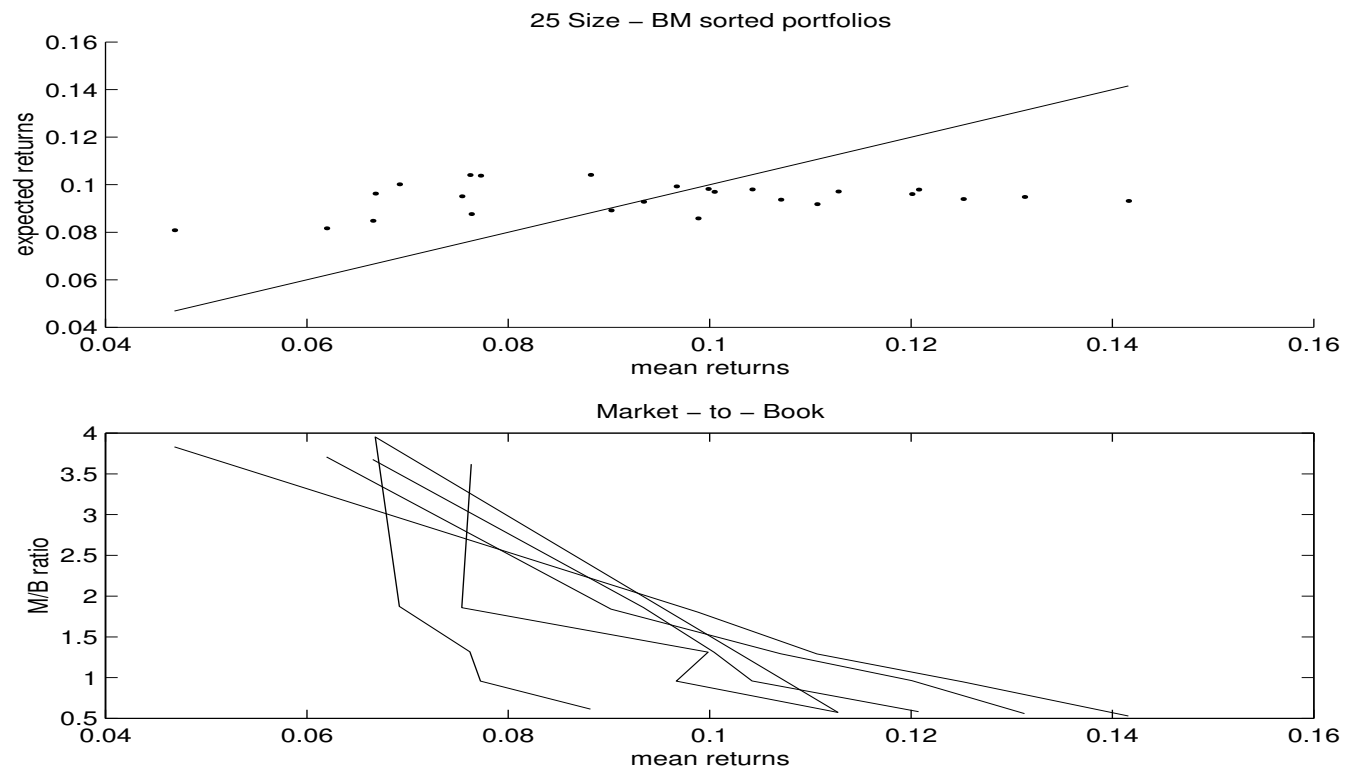
## Implications of Benchmark Model



- The top panel shows the the average return on B/M sorted portfolio on the x-axis, and the one implied by the CAPM ( =  $\beta \times \text{Average Return of Market Portfolio}$ ) on the y-axis
- They should line up, but they don't

## Implications of Benchmark Model

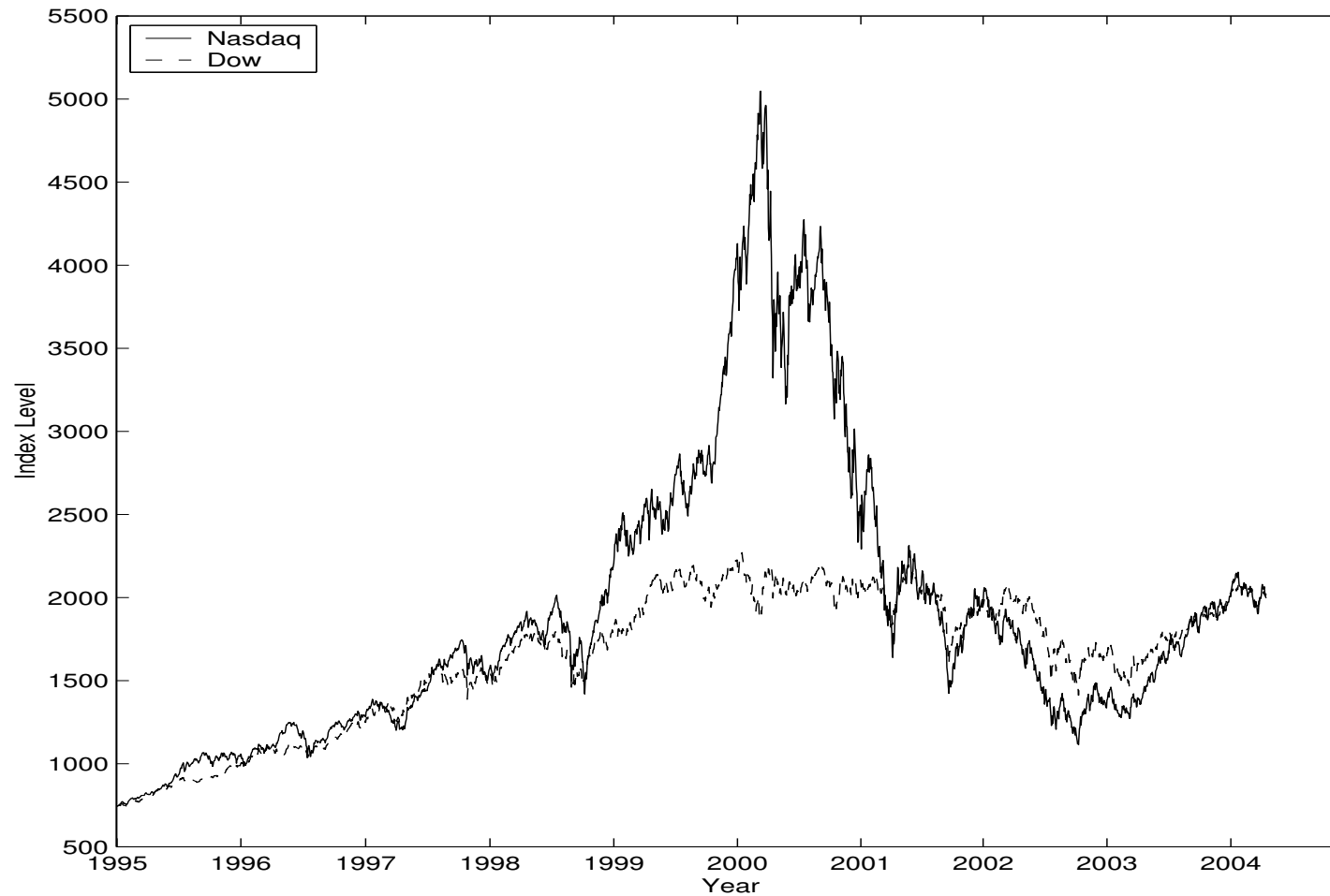
- It is even worse if one uses Size and Book-to-Market portfolios (the so-called FF 25 portfolios)



- Adding to this, momentum portfolios (sorted by past winners and losers) show similar and perhaps more striking pattern.

## Implications of Benchmark Model

### 8. Tech “Bubble”: Typical to talk about technology bubbles (e.g. late 1990s)





## Implications of Benchmark Model

- Was it a bubble?
- Why do stock prices tend to go up and then down around technological revolutions?
- Examples:
  - the early 1980s (biotechnology, PC)
  - the early 1960s (electronics)
  - the 1920s (electricity, automobiles)
  - the early 1900s (radio)

## Implications of Benchmark Model

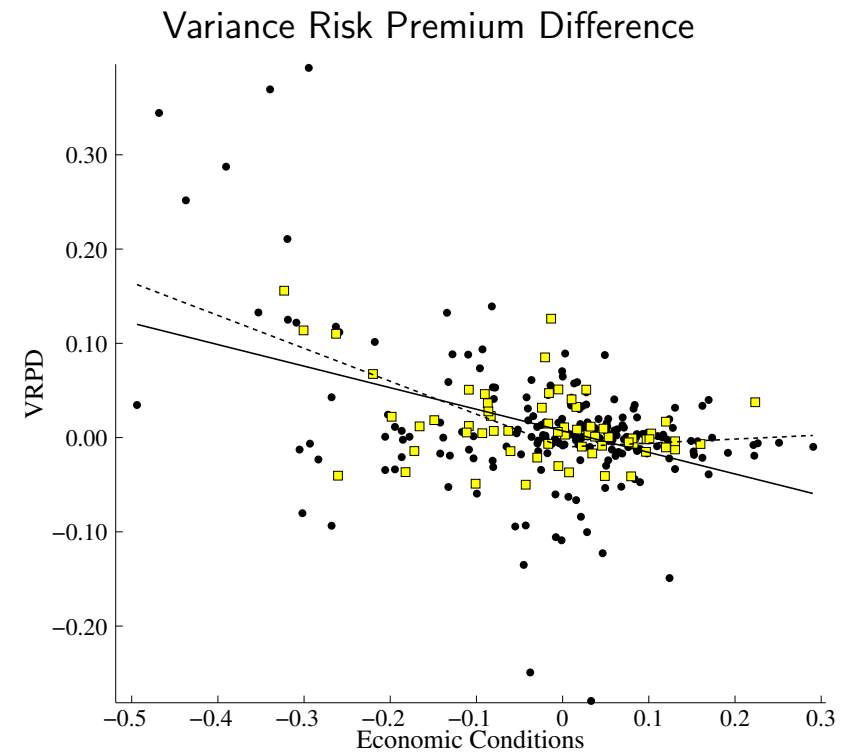
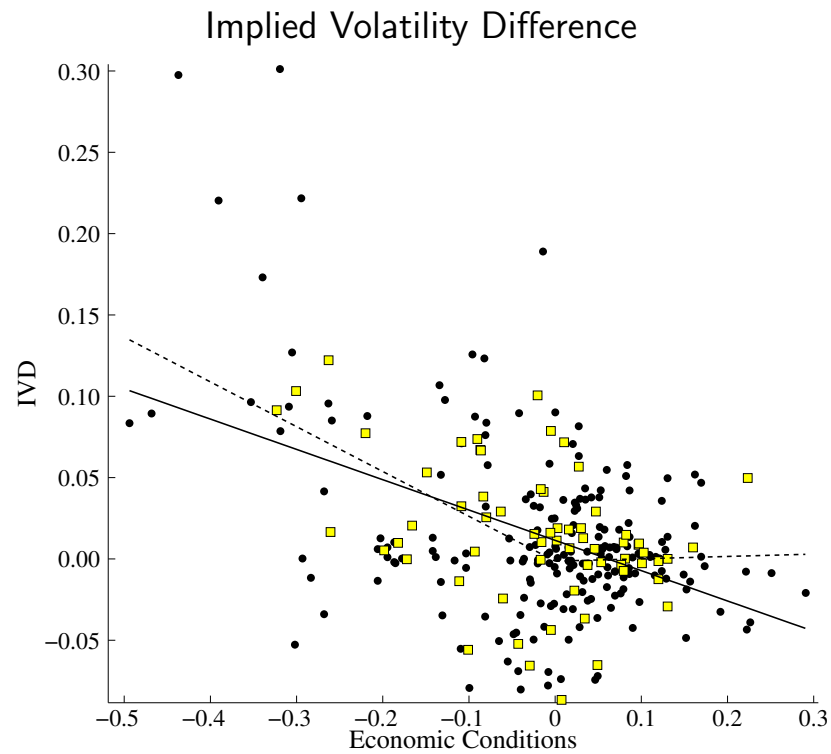
### 9. Presidential Cycle. Why are average excess returns higher during democratic presidencies?

	Sample: 1927 - 2009		
	Rep	Dem	t-diff
Average Excess Returns (%/year)	0.79	10.37	2.30
Average Real Div Growth (%/year)	4.17	5.93	1.29
Average P/D Ratio	32.00	28.95	1.4 (logs)
Average Volatility (%/year)	15.48	14.39	1.67
Median Excess Return (%/year)	7.75	16.11	-
Median Nominal Dividend Growth (%/year)	7.00	7.92	-
Median P/D Ratio	26.83	23.62	-
Median Volatility (%/year)	12.08	11.66	-

See also: Santa Clara and Valkanov "Political Cycles and the Stock Market" *Journal of Finance*, 2003

## Implications of Benchmark Model

### 10. Political events and Risk Premia. Why put options are especially expensive around elections or global summits?



## Benchmark Portfolio Allocation Model

- Consider now the model above for stock returns with same preferences, but now we do not impose market clearing ( $\theta = 1$ ).
- In this case, the utility maximization problem of an investor with investment horizon  $T$  is

$$J(W_0, 0) = \max_{\{(C_t), (\theta_t)\}} E_0 \left[ \int_0^T e^{-\phi t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right]$$

- subject to the budget constraint

$$dW_t = \{W_t(\theta_t(\mu - r) + r) - C_t\} dt + W_t\theta_t\sigma d\mathbf{B}_t$$

- The solution to this program yields an investment in stocks equal to

$$\text{Fraction of Wealth Invested in Stocks} = \theta_t = \frac{\text{Excess Return on the Stock Market}}{\gamma \text{Variance of Stock Returns}}$$

## Implications of Benchmark Portfolio Allocation Model

1. **Portfolio Allocation Puzzle 1:** The typical stockholders holds too little in stocks compared to what a canonical model would require.

- Using unconditional averages, Excess Stock Return = 7% and Volatility of Returns = .16 %, we obtain

Table: Portfolio Allocation

	Risk Aversion				
	2	4	6	8	10
Investment	136%	68%	45%	34 %	27 %

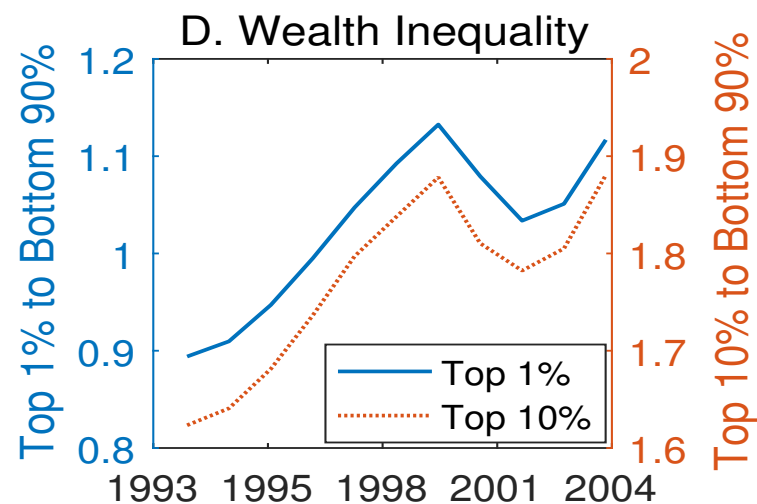
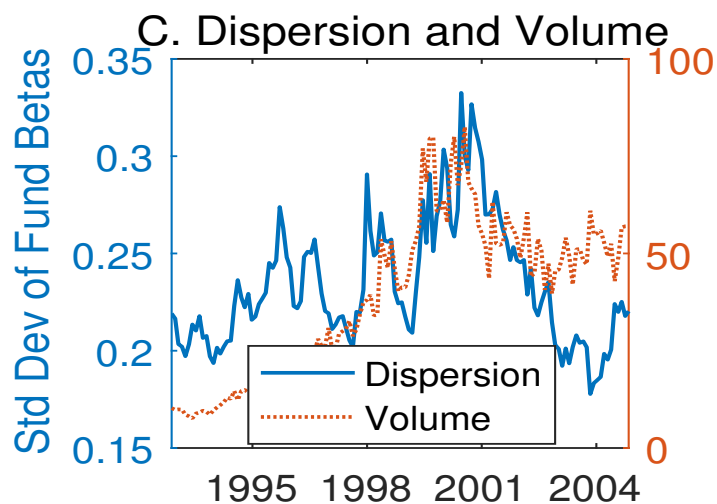
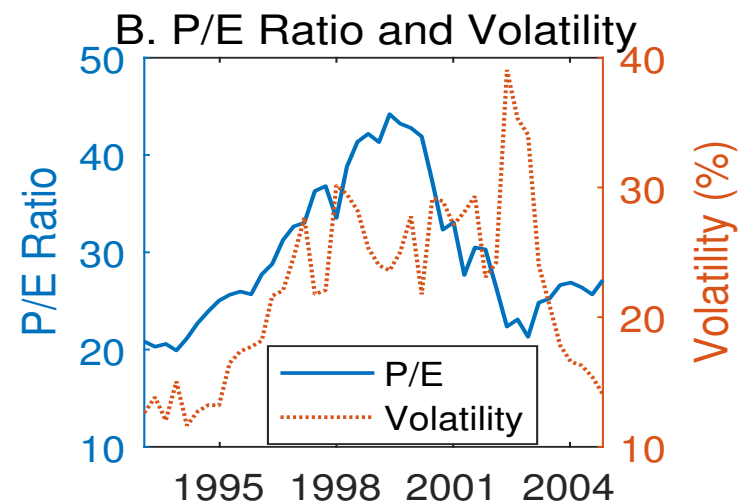
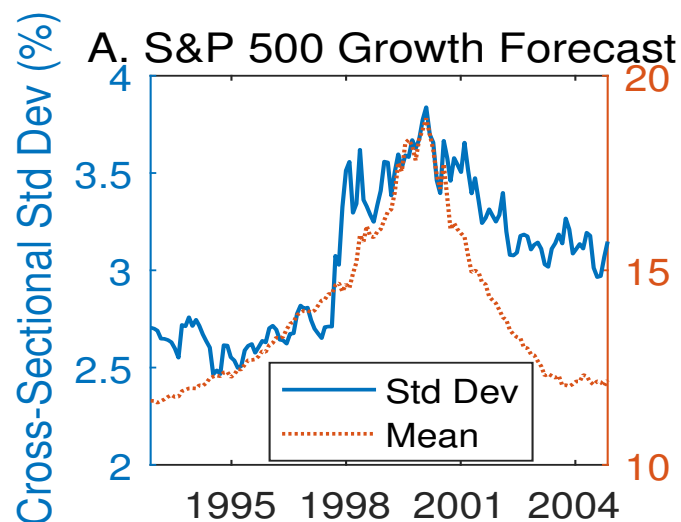
- In contrast, depending on estimates, typical household holds between 6 % to 20 % in equity. Conditional on participating to the stock market, these number increase to about 40% of financial assets.

## Implications of Benchmark Portfolio Allocation Model

2. **Portfolio Allocation Puzzle 2:** The canonical model with constant investment opportunity set implies that the portfolio allocation should not depend on the age of investor.
  - This is in contrast with the behavior of investors: Investors increase their holdings in equity for the first 1/2 of their life cycle, and decrease it afterwards.
3. **Portfolio Allocation Puzzle 3:** Many investors do not participate in the stock market, while the canonical model would imply always some participation to the market (at worse, short the market).
4. **Portfolio Allocation Puzzle 4:** Many investors invest in own company stocks, especially in their retirement plan. Diversification arguments clearly points at “shorting” the stock, if anything.

## Benchmark Model: No Trading and Wealth Inequality

- Representative agents models don't have implications about trading or cross-sectional differences in wealth inequality, which are important features of the data.



## Benchmark Model: No Trading and Wealth Inequality

- What forces determine trading and wealth inequality dynamics?
  - Asymmetric information
  - Differences of beliefs
  - Heterogeneity in preferences
- Can we reconcile standard representative agent models with such trading dynamics?
  - Aggregation becomes difficult. But progress in recent times.



## Nominal Long Term Bonds in Benchmark Model

- I now introduce an exogenous inflation process, and obtain nominal long term bond prices.
- The log dividend (consumption)  $c = \log(C)$  and log CPI  $q_t = \log Q_t$  grow according to the joint stochastic model

$$dc_t = gdt + \sigma_c dW_{c,t}$$

$$dq_t = i_t dt + \sigma_q dW_{q,t}$$

$$di_t = (\alpha - \beta i_t) dt + \sigma_i dW_{i,t}$$

–  $i_t$  = is the expected inflation rate  $i_t = E_t[dq_t]/dt$ .

- The First Order Condition is (recall  $\lambda_t = e^{-\phi t} C_t^{-\gamma}$ )

$$Z(i_t, t; T) = E \left[ \frac{\lambda_T Q_t}{\lambda_t Q_T} \right]$$

- yielding

$$Z(i_t, t; T) = e^{A_0(\tau) - A_\beta(\tau) i_t}$$

- where  $A_\beta(\tau)$  and  $A_0(\tau)$  are two function of time to maturity  $\tau = T - t$

## Implications of Benchmark Model

1. The instantaneous nominal rate  $r_t$  is given by the constant real rate + inflation risk premium + expected inflation

$$r_t = \lim_{T-t \rightarrow 0} y(t; T) = - \lim_{\tau \rightarrow 0} \frac{A_0(\tau) - A_1(\tau) i_t}{\tau} = c + i_t$$

- where

$$c = \left( \rho + \gamma g - \frac{1}{2} \gamma^2 \sigma_c^2 \right) - \gamma \sigma_c \sigma_q \rho_{qc} - \frac{1}{2} \sigma_q^2$$

2. The whole yield curve depends on the current expected inflation  $i_t = E[ dq_t ] / dt$ .

$$y(t; T) = - \frac{\log(Z(i_t, t; T))}{\tau} = - \frac{A_0(\tau)}{\tau} + \frac{A_\beta(\tau)}{\tau} i_t$$

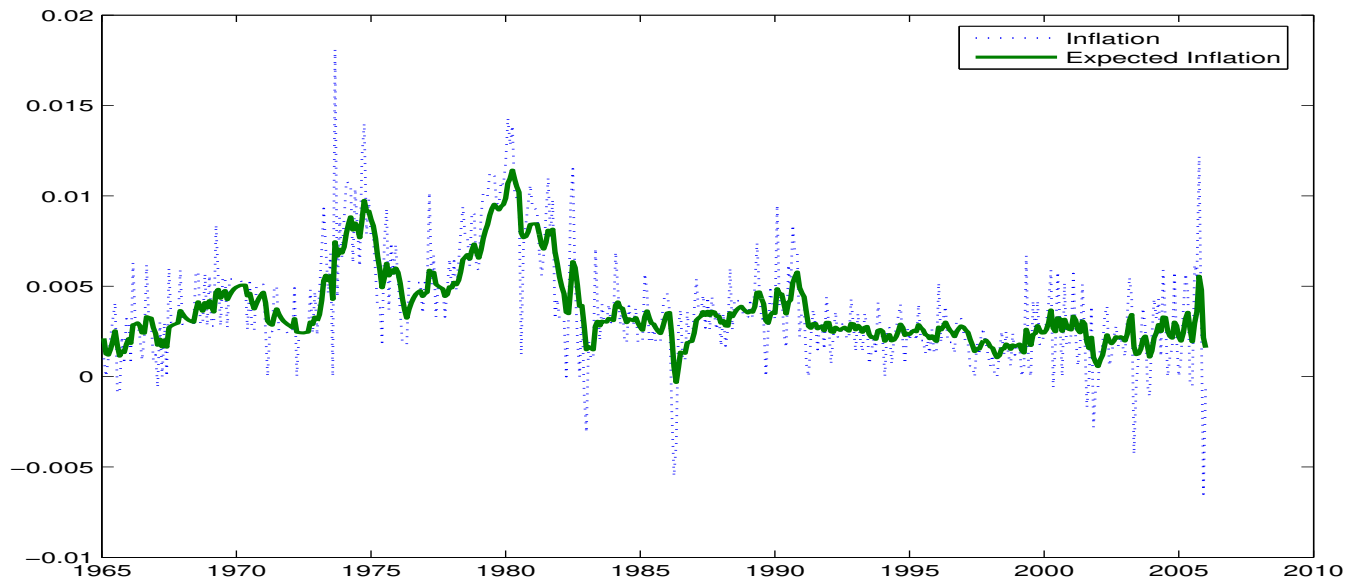
- In particular, all of the yields are perfectly correlated.

3. The Term Spread (Slope) is

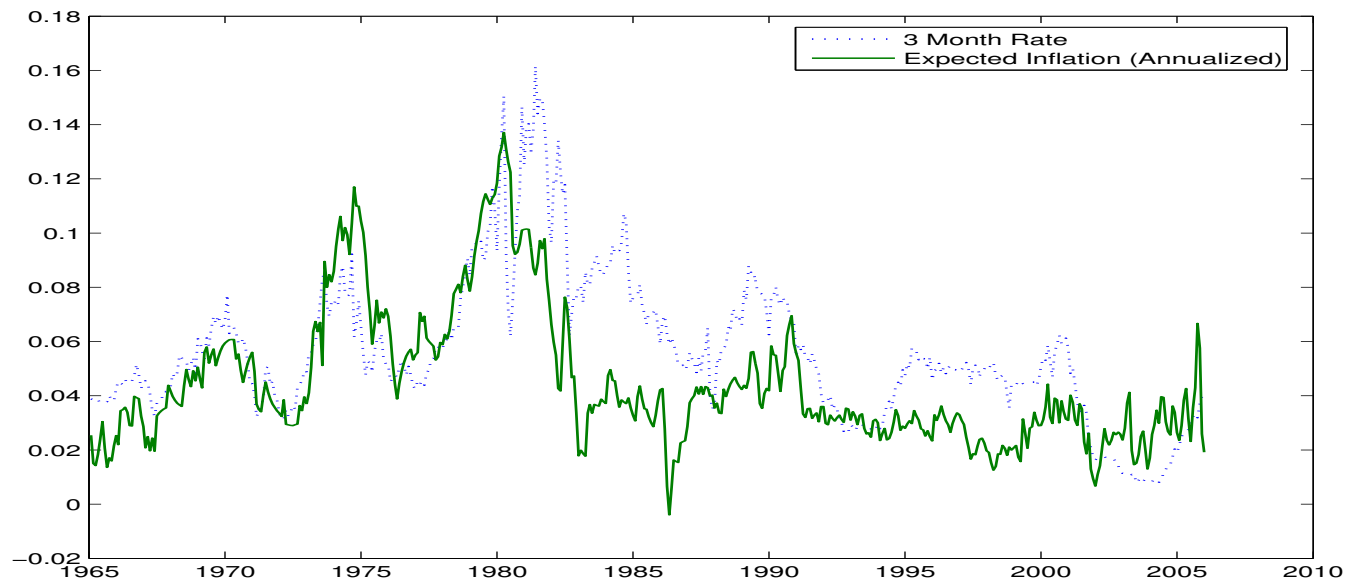
$$y_\infty - r_t = \left( \frac{\alpha}{\beta} - i_t \right) - \frac{1}{\beta} \left( \gamma \sigma_i \sigma_c \rho_{ic} + \sigma_i \sigma_q \rho_{iq} \right) - \frac{\sigma_i^2}{2\beta^2}$$

- Note that since  $\rho_{ic} < 0$  (typically),  $\gamma \sigma_i \sigma_c \rho_{ic} / \beta < 0$ . Higher risk or risk aversion, the higher the long end of the yield curve.

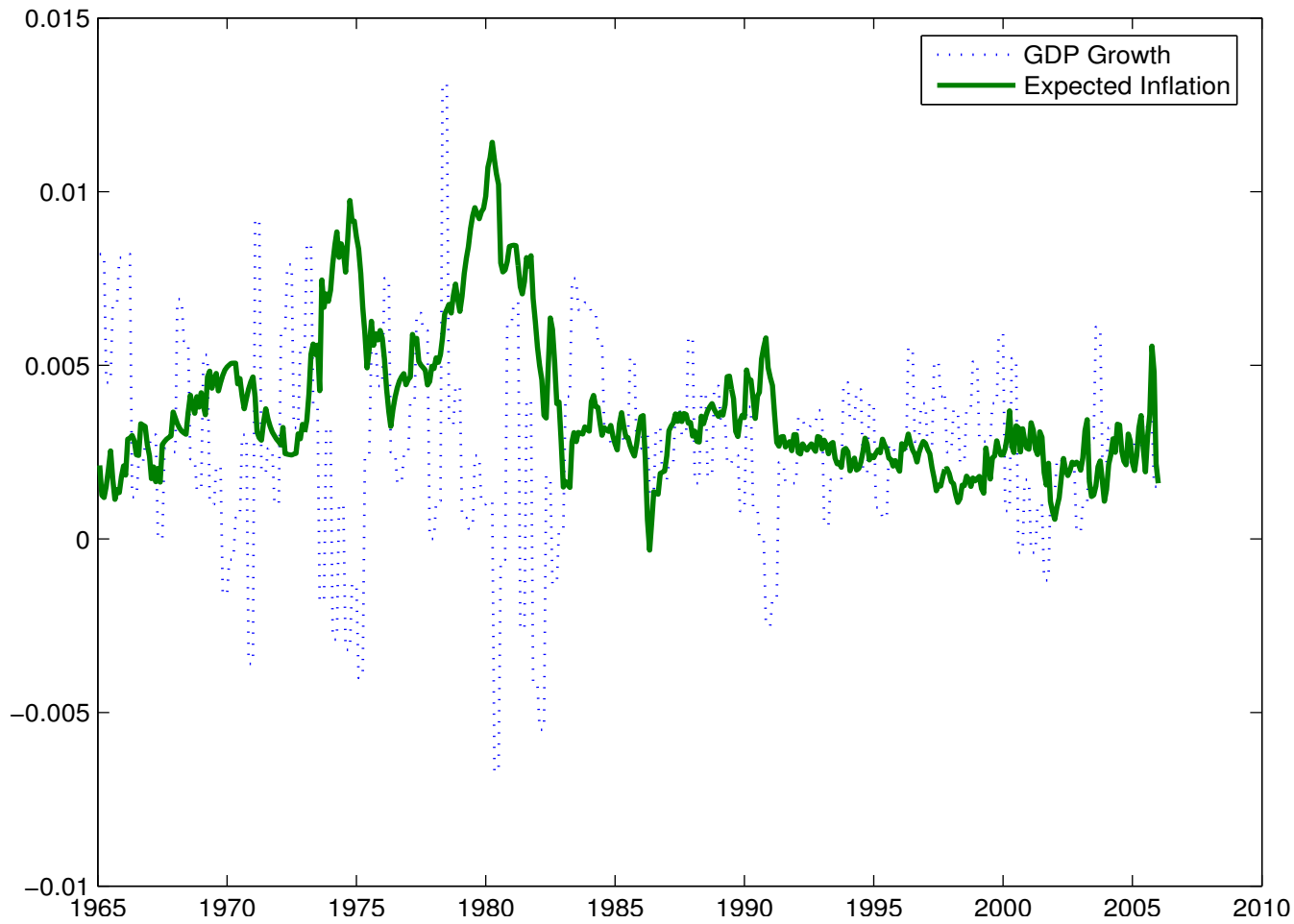
## Inflation and Expected Inflation



## Inflation and 3-month TBill rate



### Expected Inflation and GDP growth



## Implications of Benchmark Model

4. The model requires a large risk aversion to produce reasonable yield curves and a reasonable market price of risk  $\lambda$

- Using data on inflation and GDP growth ( $= C$ ), we obtain the following parameters for the processes

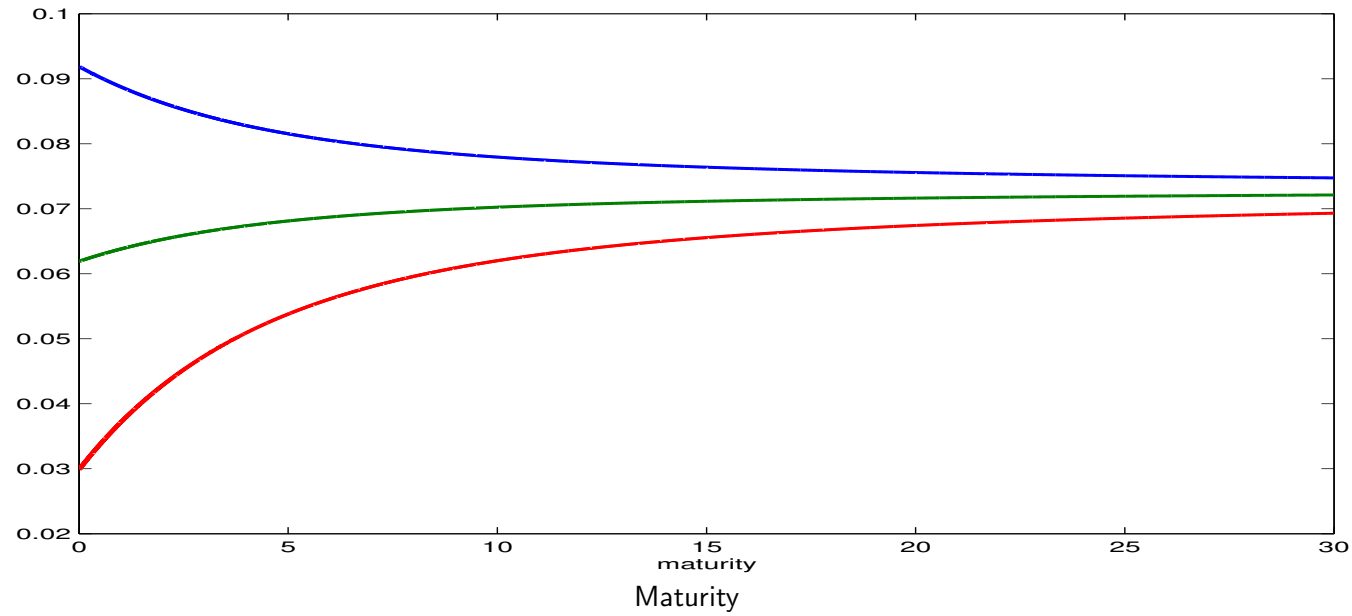
$\alpha$	$\beta$	$g$	$\sigma_y$	$\sigma_q$	$\sigma_i$	$\rho_{yq}$	$\rho_{yi}$	$\rho_{iq}$
.0160	0.3805	0.02*	0.02*	0.0106	0.0073	-.1409	-.2894	0.8360

\* The estimates of GDP growth were  $g = .0321$  and  $\sigma_y = 0.0098$ , which made it hard

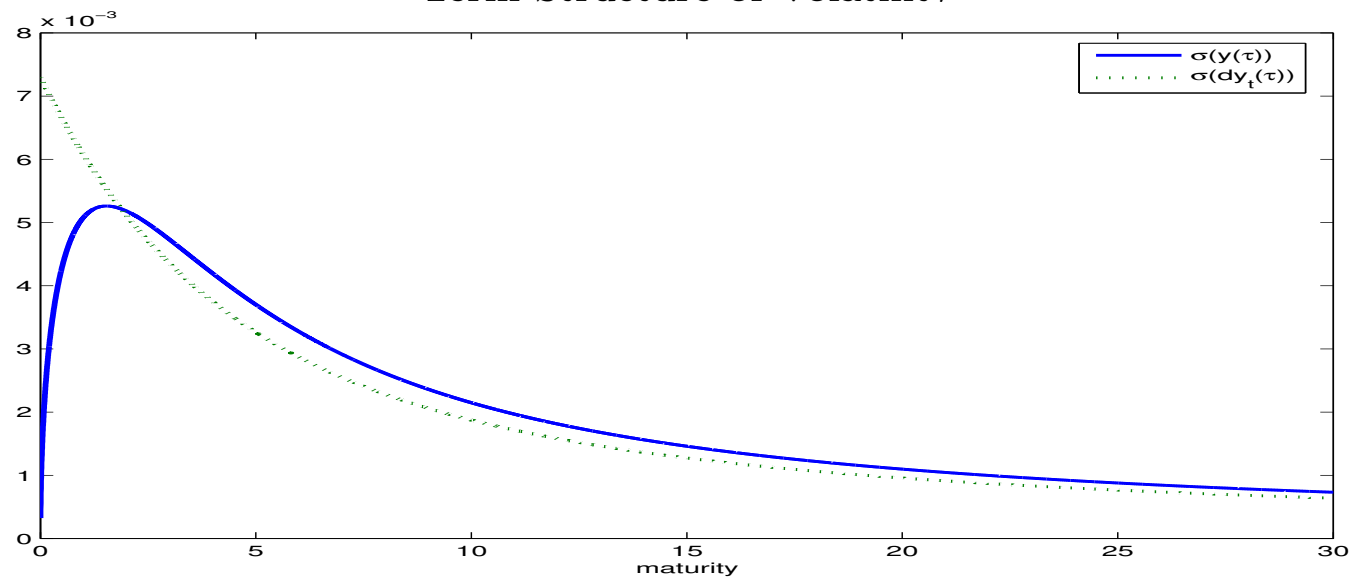
to generate sensible yield functions. The parameters assumed are closer to consumption growth

- Using utility parameters  $\rho = .1$  and  $\gamma = 104$  we get a real rate  $c = .02$ .  $\xi = -0.5931$
- Risk free rate puzzle kicks in:
  - For “reasonable”  $\gamma$ , the interest rate is too high.
  - Lowering  $\gamma$  to  $\gamma \approx 0.5$  generates also reasonable yield curves, but they are not upward sloping in average. Moreover, the market price of risk is too low.

## Yield curves



## Term Structure of Volatility



## Implications of Benchmark Model

5. The volatility of bond yields changes ( $\sigma(dy)$ ) is constant over time but depends on maturity:

$$\sigma_y(t; T) = \frac{1 - e^{-\beta\tau}}{\beta\tau} \sigma_i$$

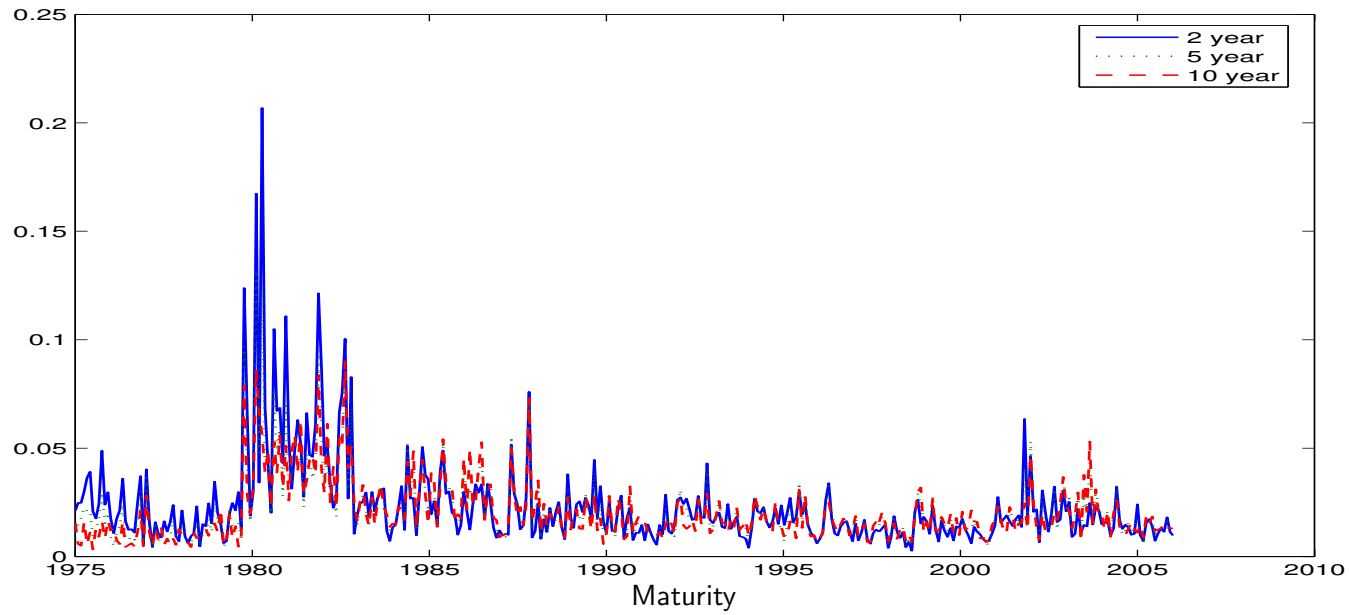
6. The bond risk premium is also constant, and given by

$$E \left[ \frac{dZ}{Z} \right] / dt - r_t = \sigma_Z \xi$$

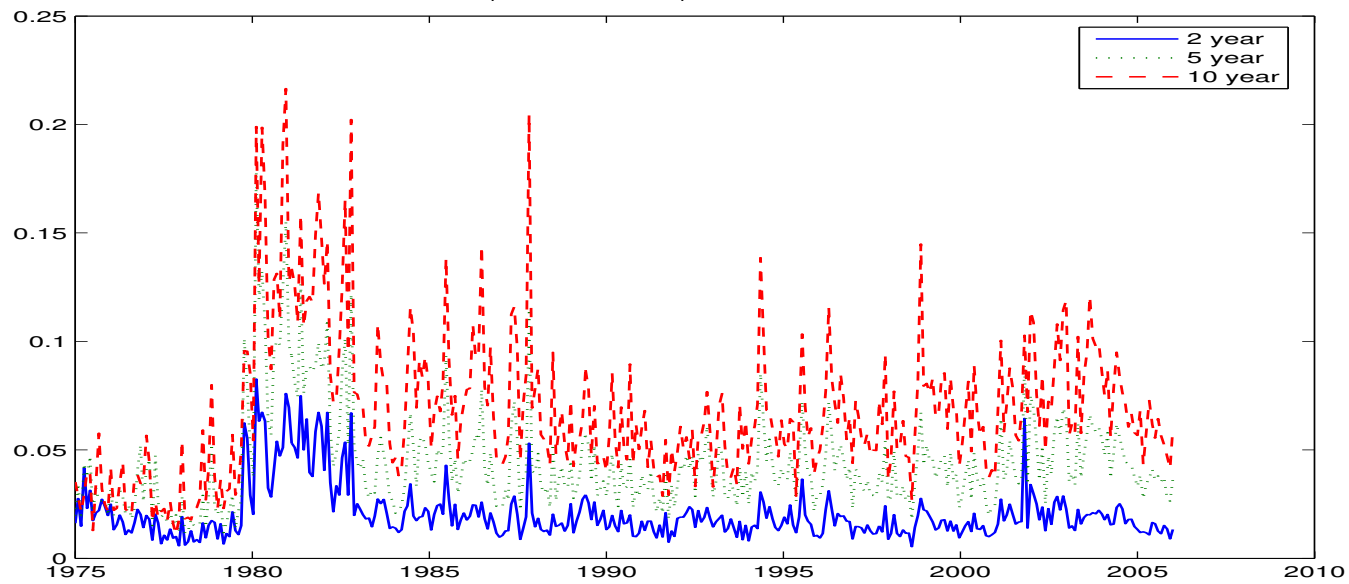
where

- $\sigma_Z = \text{vol of } dZ/Z = -A_\beta(\tau)\sigma_i$
- $\xi = \gamma\sigma_c\rho_{ic} + \sigma_q\rho_{iq}$  is **Market Price of (inflation) Risk**
  - No time varying risk premium and no predictability

### Monthly Volatility of Yields



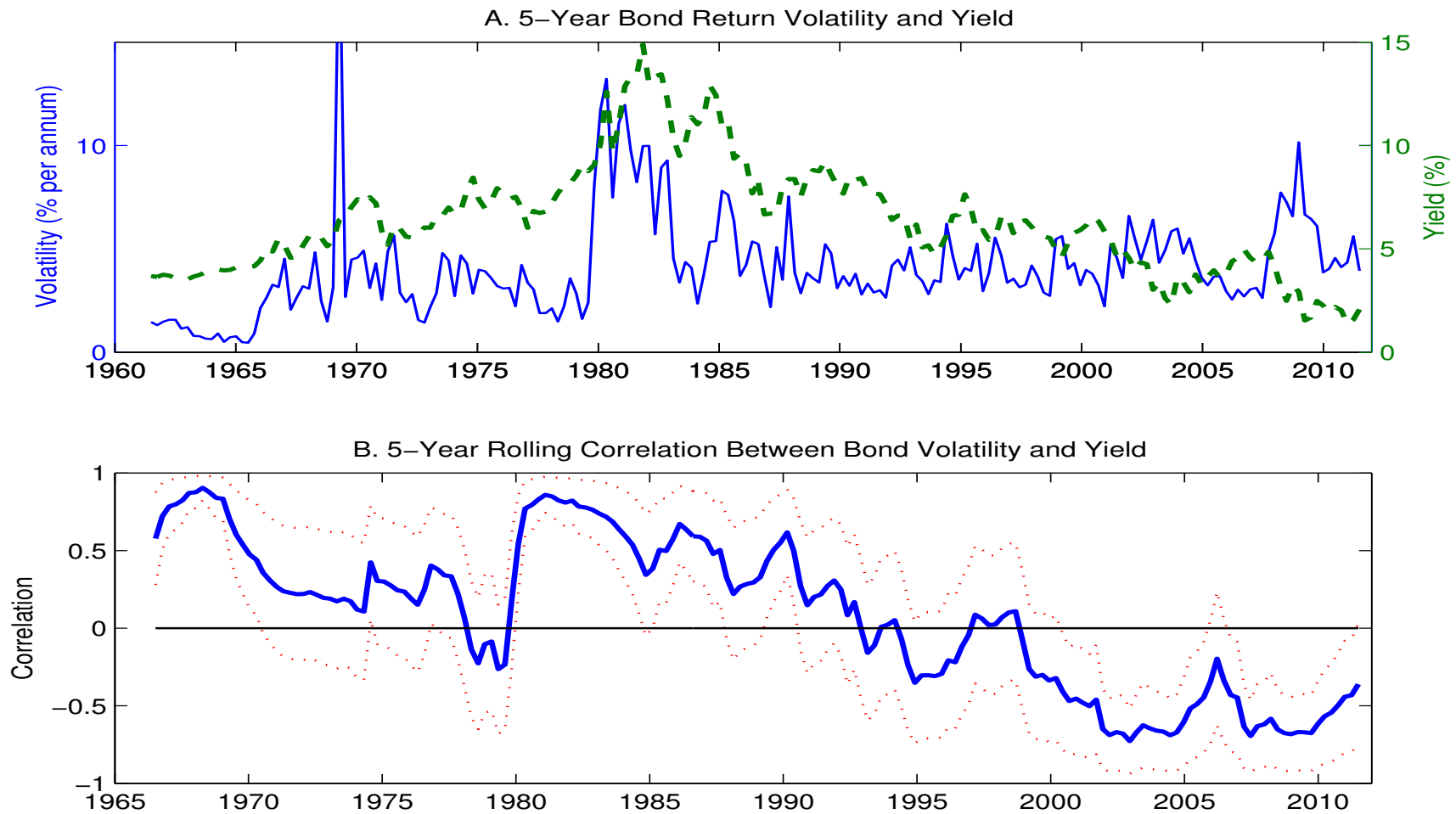
### Monthly Volatility of Bond Returns





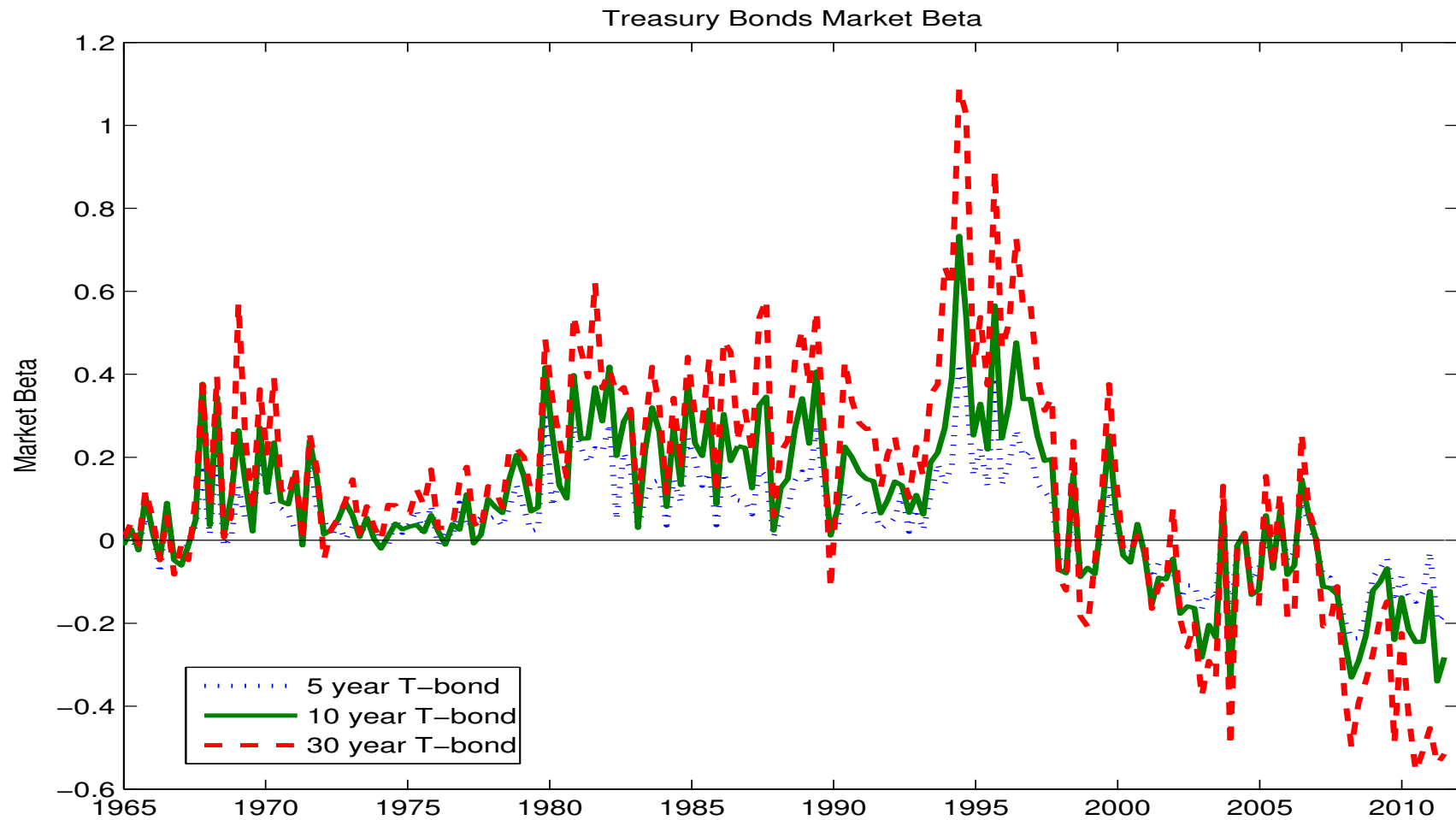
## Bond Return Volatility and Yields

- Bond return volatility and yields are positively or negatively correlated



## The Treasury Bond Market Betas

- Bond return become negative beta assets in the last two decades



## Bond Predictability. Fama Bliss (1987)

- Fama and Bliss classic paper show that bond return are predictable by the forward spread.

$$\text{holding period excess log return} = \alpha + \beta \left( f_t^{(n)} - y(t, 1) \right) + \epsilon_t$$

- where  $n =$  horizon (in years)

### Fama Bliss Regressions: 1960 - 2010

$n$	$\alpha$	$\beta$	$t(\alpha)$	$t(\beta)$	$R^2$
2	0.0018	0.7850	0.6020	2.8129	10.94%
3	-0.0002	1.2246	-0.0446	3.5671	17.50 %
4	-0.0031	1.5325	-0.4309	3.4255	17.19 %
5	0.0014	1.0862	0.1316	1.8760	6.97%

- However, evidence from Euro, UK, Japan is much less clearcut. What's different there?

## Bond Predictability. Cochrane and Piazzesi (2003)

- Cochrane and Piazzesi (2003) show that there is a single combination of forwards that explain bond excess returns.
  - What is an economic model that generates that effect?
  - Intriguingly, Cochrane Piazzesi factor works also outside US, while Fama Bliss regressions do not. What is the factor capturing?

## This Course Covers (subject to change, though)

- Foundations: Complete markets, state price densities, consumption/portfolio allocation, the martingale method.
- (Some) portfolio allocation models with
  - Time varying investment opportunities
  - Incomplete information (learning)
- Habit formation and time varying risk preferences
  - Time-series and cross-sectional predictability
- Heterogeneous preferences, beliefs, and trading
  - Heterogeneous risk aversion
  - Heterogeneous habits
  - Heterogeneous beliefs
- Incomplete information, learning and stock and bond returns
  - Valuation with uncertainty in long term growth.
- Politics and asset prices
  - Political news and returns
- Market incompleteness and constraints

## Requirements

- Big Homework Assignments (30%):
  - I will assign three research ideas / projects during the terms.
  - Your assignments will be to develop such research ideas into coherent papers. This will involve (a) solving a model; (b) obtain predictions; (c) compare predictions with the data.
  - The paper must have the form of a paper, with an introduction, body of the paper, data analysis, conclusion, appendix.
  - The TA and I will be the referees: Give you feedback to improve writing.
  - You can work in groups, but with a limit of 3 per group.
- Weekly Mini Homework Assignments (10%):
  - Every class, I will write on the board some assignments related to missing steps in the notes.
  - You must turn in a sheet with the missing steps. This is individual work (no group).
- In-class Exam (30%)
  - There will be a “midterm” on Tuesday February 18. Essentially 1 1/2 hour on the material covered in class.
- Term Paper (25%)
  - A paper on the topics covered in class, due by the beginning on Spring quarter.
- Class participation (5%).

## Crazy Schedule

	Tuesdays	Fridays	Comment
Week 1	Jan 7. 3:30-6:30 pm	Jan 10. 8:30-11:30 am	
Week 2	Jan 14. No class		
Week 3	Jan 21. 3:30-6:30 pm		
Week 4	Jan 28. 3:30-6:30 pm	Jan 31. 8:30-11:30 am	
Week 5	Feb 3. No class		Paper #1
Week 6	Feb 11. 3:30-6:30 pm		
Week 7	Feb 18. 3:30-6:30 pm (midterm)	Feb 21. 8:30-11:30 am	
Week 8	Feb 25. No class		Paper #2
Week 9	March 3. 3:30-6:30 pm		
Week 10	March 10. No class		Paper #3
Week 11	March 17 - 3:30-6:30 pm	March 20 - 8:30-11:30 am	(Exam week)