Recent Advances in Fixed Income Securities Modeling Techniques

Day 1: Equilibrium Models and the Dynamics of Bond Returns

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Introduction

A Canonical Reference Model

- To frame the discussion, I start from the simplest economic model of nominal bond pricing.
  - In the next few days we will explore additional models that expand on this one.
- The basic reference economic model has the following ingredients: (*a*) a model for GDP growth, inflation and expected inflation;(*b*) agents who take optimal actions to maximize their "utility".

1. Real log GDP $y_t = \log(Y_t)$ grows according to the stochastic model

$$dy_t = gdt + \sigma_y dW_{y,t}$$

2. The log CPI $q_t = \log Q_t$ grows according to the stochastic model

$$dq_t = i_t dt + \sigma_q dW_{q,t}$$

$$di_t = (\alpha - \beta i_t)\, dt + \sigma_i dW_{i,t}$$

   - $i_t =$ is the expected inflation rate $i_t = E_t[dq_t]/dt$.
   - Using a simple Continuous Time Kalman Filter, next plots show the behavior of inflation, expected (filtered) inflation, interest rates and GDP growth.

Figure 1: Inflation and Expected Inflation

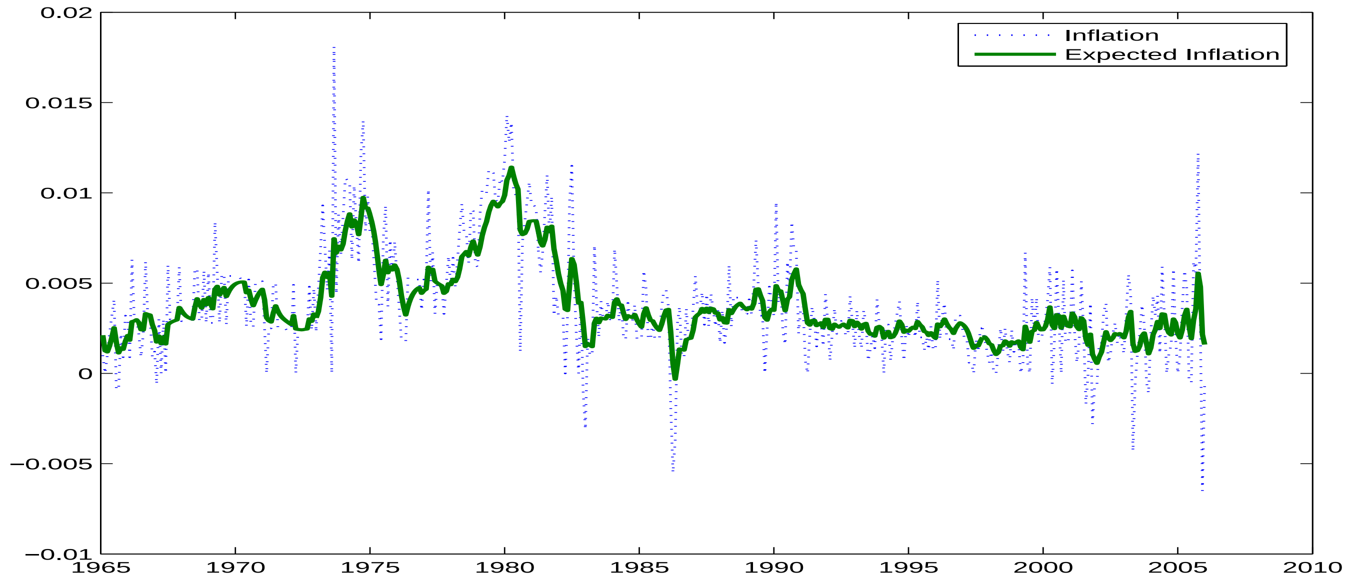


Figure 2: Inflation and 3-month TBill rate

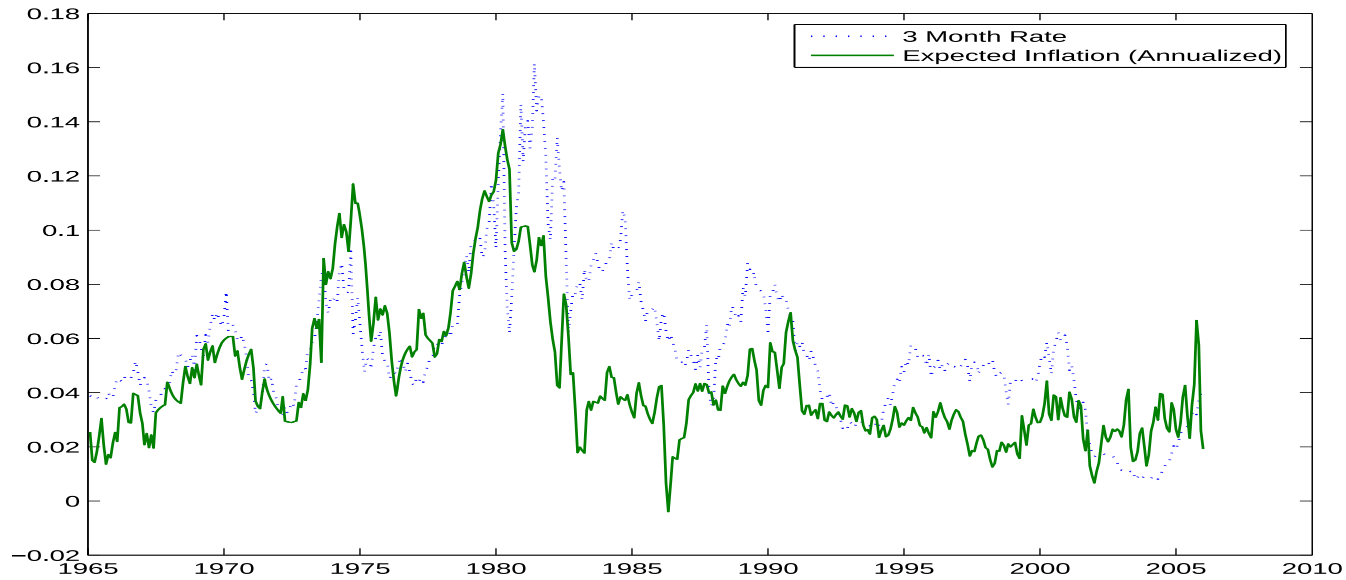
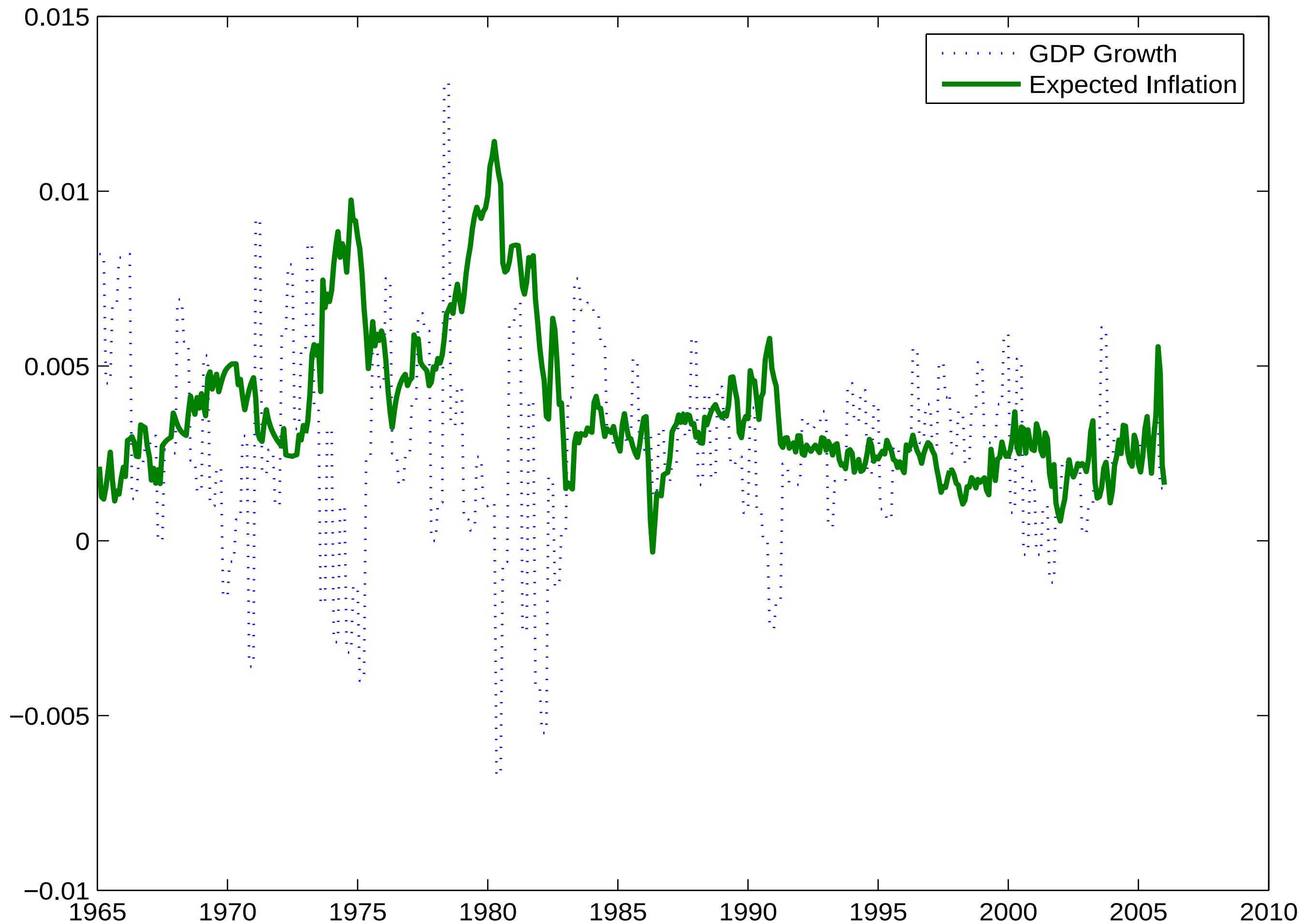


Figure 3: Expected Inflation and GDP growth



- Any security that whose price is negatively correlated with inflation is risky
  * It yields a low return when GDP is low (bad times) and high return when GDP is high (good times)
  * Systematic risk is high, and thus a premium is required.

Market Participants' Preferences

3. Agents are identical, and have utility function defined over real consumption C_t given by

$$U\left(C_t, t\right) = e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma}$$

   - The more they consume, the more they are happy.
   - γ is the coefficient of risk aversion; ρ is the subjective intertemporal discount.

- In equilibrium, assume that all of the GDP has to be consumed (endowment economy), so that the equilibrium condition is $C_t = Y_t$

Market Participants First Order Conditions

- Agents have the choice between:

  1. Spend \$1 today (t) and buy $\frac{1}{Q_t}$ amount of the consumption good, which will procure him/her an additional amount of utility
  $$U'(C_t,t)\frac{1}{Q_t}$$
  2. Save the dollar, and buy $\frac{1}{Z(t,T)}$ zero coupon bonds with maturity T. At that time, the agent can buy $\frac{1}{Z(t,T)}\frac{1}{Q_T}$ of the consumption good, for an additional amount of utility
  $$\frac{1}{Z(t,T)}\frac{1}{Q_T}U'(C_T,T)$$

- *In equilibrium*, agents should be indifferent between the first and the second choice. However, we do not know at time t what $\frac{1}{Z(t,T)}\frac{1}{Q_T}U'(C_T,T)$ will be, so the equilibrium condition must be
$$U'(C_t,t)\frac{1}{Q_t}=E_t\left[\frac{1}{Z(t,T)}\frac{1}{Q_T}U'(C_T,T)\right]$$

Deriving Nominal Bonds

- We can re-arrange, to find that the value of a (nominal) zero coupon bond is given by

$$Z(t,T) = E_t\left[\frac{Q_t}{Q_T}\frac{U'(C_T,T)}{U'(C_t,t)}\right]$$

  – Substitute

$$U'(C_t,t) = C_t^{-\gamma}e^{-\rho t} \text{ and } U'(C_T,T) = C_T^{-\gamma}e^{-\rho T}$$

  – and obtain

$$Z(t,T) = E_t\left[e^{-\rho(T-t)}\frac{Q_t C_t^{\gamma}}{Q_T C_T^{\gamma}}\right]$$

  – Our assumption $C_t = Y_t$ implies

$$\begin{aligned} Z(t,T) &= E_t\left[e^{-\rho(T-t)}\frac{Q_t Y_t^{\gamma}}{Q_T Y_T^{\gamma}}\right] \\ &= E_t\left[e^{-\rho(T-t)-(q_T-q_t)-\gamma(y_T-y_t)}\right] \end{aligned}$$

The Bond Pricing Formula

- The solution is

$$Z\left(i_t, t; T\right) = e^{A_0(\tau) - A_\beta(\tau) i_t}$$

- where $\tau = T - t$,

$$\begin{aligned} A_\beta\left(\tau\right) &= \frac{1 - e^{-\beta\tau}}{\beta} \\ A_0\left(\tau\right) &= -c\tau - \left(\alpha - \gamma\sigma_i\sigma_y\rho_{iy}\right)\frac{1}{\beta}\left(\tau - A_\beta\left(\tau\right)\right) \\ &\quad + \sigma_i\sigma_q\rho_{iq}\frac{1}{\beta}\left(\tau - A_\beta\left(\tau\right)\right) + \frac{\sigma_i^2}{2\beta^2}\left(\tau + A_{2\beta}\left(\tau\right) - 2A_\beta\left(\tau\right)\right) \end{aligned}$$

- and

$$c = \left(\rho + \gamma g - \frac{1}{2}\gamma^2\sigma_y^2\right) - \gamma\sigma_y\sigma_q\rho_{qy} - \frac{1}{2}\sigma_q^2$$

- The constant c is related to
  - the "real" interest rate (first parenthesis)
  - the inflation risk premium (second term)
  - A convexity term (the third term)

Bond Yields

- The yield to maturity of the zero coupon bond is

$$y(t;T) = -\frac{\log(Z(i_t, t;T))}{\tau} = -\frac{A_0(\tau)}{\tau} + \frac{A_\beta(\tau)}{\tau} i_t$$

- and the instantaneous rate is

$$r_t = \lim_{T-t\to 0} y(t;T) = -\lim_{\tau \to 0} \frac{A_0(\tau) - A_1(\tau) i_t}{\tau} = c + i_t$$

- Implications:

  1. The instantaneous nominal rate r_t is given by the constant real rate + inflation risk premium + expected inflation.
  2. The whole yield curve depends on the current expected inflation $i_t = E[dq_t]/dt$.
  3. The nominal rate follows a Vasicek model

$$\begin{aligned} dr_t &= di_t = (\alpha - \beta i_t)\, dt + \sigma_i dW_{i,t} \\ &= (\widetilde{\alpha} - \beta r_t)\, dt + \sigma_i dW_{i,t} \end{aligned}$$

  where $\widetilde{\alpha} = \alpha + c\beta$

Bond Yields

4. The long run unconditional average nominal rate $E[r]$ is then

$$\overline{r} = E[r] = \frac{\widetilde{\alpha}}{\beta} = \frac{\alpha}{\beta} + c$$

   – i.e. $\overline{r} =$ long run expected inflation $E[i] = \alpha/\beta$ plus the constant real rate c.

5. The long end of the yield curve can be computed from $y(t;\tau)$. Thus, as $T \to \infty$ we have

$$y\left(i_t, t; T\right) \;\to\; y_\infty = \overline{r} - \frac{1}{\beta}\left(\gamma\sigma_i\sigma_y\rho_{iy} + \sigma_i\sigma_q\rho_{iq}\right) - \frac{\sigma_i^2}{2\beta^2}$$

   – Long term yield equals the long run unconditional average nominal rate $\overline{r}$, minus an adjustment for risk $\gamma\sigma_i\sigma_y\rho_{iy}/\beta$ minus a convexity term.[1]

     * Note that since $\rho_{iy} < 0$ (typically), $\gamma\sigma_i\sigma_y\rho_{iy}/\beta < 0$. Higher risk or risk aversion, the higher the long end of the yield curve.

[1] The convexity terms enter because there is a convex relation between inflation i_t and the bond price $Z(i, t; T)$.

The Slope of the Term Structure

6. The Term Spread (Slope) is

$$y_\infty - r_t = \left(\frac{\alpha}{\beta} - i_t\right) - \frac{1}{\beta}\left(\gamma\sigma_i\sigma_y\rho_{iy} + \sigma_i\sigma_q\rho_{iq}\right) - \frac{\sigma_i^2}{2\beta^2}$$

   – The first term is the difference between long term expected inflation and current inflation.
   – The second term is a risk adjustment (and convexity adjustment).

– For instance, the long end of the current yield curve (that is, as of 13 October 2005) is lower than in the past.

– According to this model, this is due to either

   1. lower expected long term inflation (α/β)
   2. lower risk aversion of market participants γ.
   3. lower risk $\sigma_y\sigma_i\rho_{yi}$

The Term Structure of Volatility

- The volatility of bond yields changes ($\sigma(dy)$) is then obtained from an application of Ito's Lemma

$$\sigma_y(t;T) = \frac{1-e^{-\beta\tau}}{\beta\tau}\sigma_i$$

- The standard deviation of bond yields ($\sigma(\ y(t;T)\ )$) is instead

$$\sigma(\ y(t;T)\ ) = \left(\frac{1-e^{-\beta\tau}}{\beta\tau}\sigma_i\right)\sqrt{\frac{1-e^{-2\beta(T-t)}}{2\beta}}$$

A (Simple) Calibration

- Using data on inflation and GDP growth, we obtain the following parameters for the processes

α	β	g	σ_y	σ_q	σ_i	ρ_{yq}	ρ_{yi}	ρ_{iq}
.0160	0.3805	0.02*	0.02*	0.0106	0.0073	-.1409	-.2894	0.8360

* The estimates of GDP growth were $g = .0321$ and $\sigma_y = 0.0098$, which made it hard to generate sensible yield functions. The parameters assumed are closer to consumption growth

- I use the utility parameters $\rho = .1$ and $\gamma = 104$. This implies a real rate $c = .02$.
- Figure 4 plots three yield curves, assuming current expected inflation is $i_0 = 0.01, 0.0420, 0.0720$. The corresponding nominal interest rates are $r_0 = 0.0299, 0.0619, 0.0919$.
- Figure 5 plots the term structure of volatility.

Figure 4: Yield curves

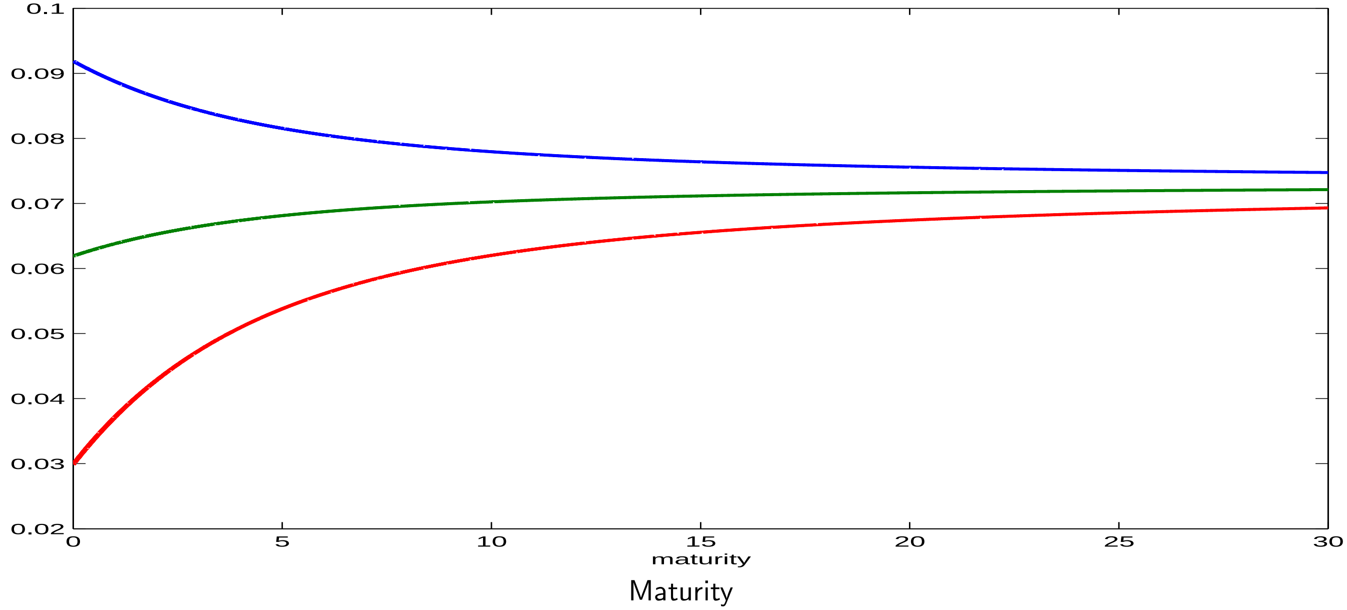
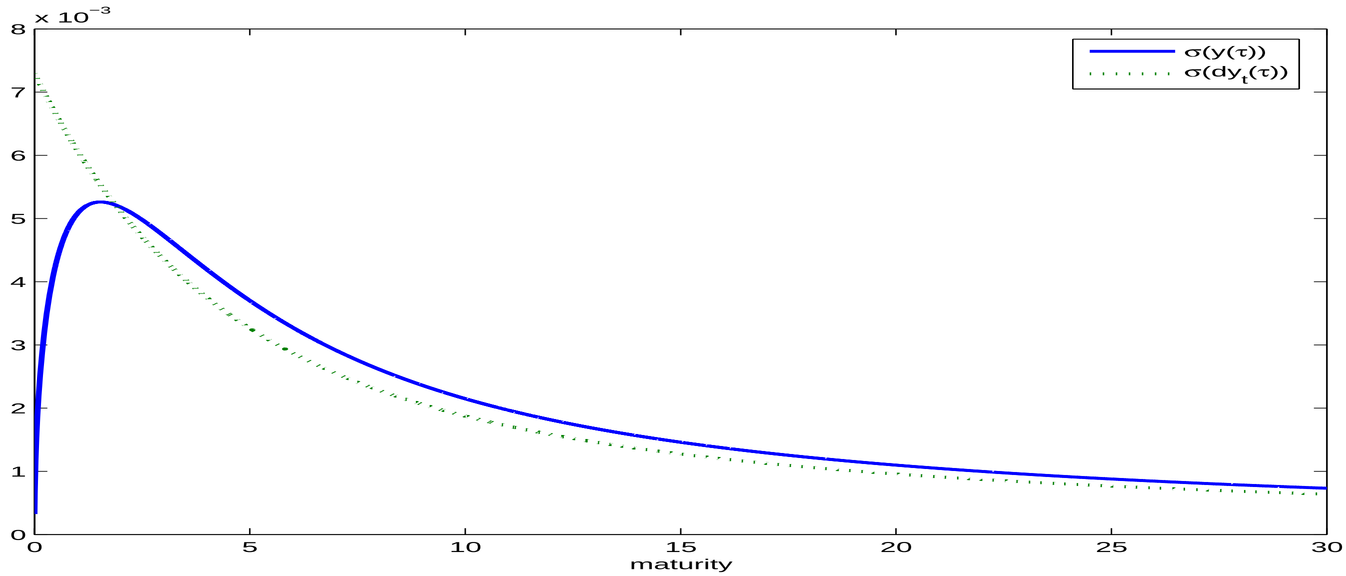


Figure 5: Term Structure of Volatility



Expected Bond Return and Risk Premium

- Consider an investor who buys a long term bond for $Z(r,t;T)$.
  - What is the expected return over a short period dt?
- From the bond pricing formula, the process for bond returns is

$$\frac{dZ}{Z} = (r_t + \mu_Z) + \sigma_Z dW_{i,t}$$

- The volatility component is given by

$$\sigma_Z = \frac{1}{Z}\frac{\partial Z}{\partial i}\sigma_i = -A_\beta(\tau)\,\sigma_i$$

- The risk premium is given by

$$E\left[\frac{dZ}{Z}\right]/dt - r_t = \mu_Z = \sigma_Z \lambda$$

  where $\lambda = \gamma\sigma_y\rho_{iy} + \sigma_q\rho_{iq}$ is **Market Price of (inflation) Risk**
  - Caveat: Note that the diffusion term in bond process is negative $\sigma_Z < 0$
    * This is simply because $dW_{i,t}$ is the shock to expected inflation
    * Positive shock to expected inflation increases the nominal interest rate and thus depress the bond price
    * i.e. the negative sign defines the negative *correlation* between interest rate shocks and bond prices

The Market Price of Risk

- The market price of risk is given by

$$\lambda = \frac{E_t\left[dZ/Z - r_t dt\right]}{\sigma_Z} = \text{(negative of) Sharpe Ratio}$$

$$= \gamma \sigma_y \rho_{iy} + \sigma_q \rho_{iq}$$

  - The market price of risk λ depends on
    - the coefficient of risk aversion γ
    - the volatility of GDP (σ_y)
    - the correlation between expected inflation and GDP growth.
  - Higher risk aversion γ increases the market price of risk λ.
  - Note that $\rho_{iy} < 0$ (typically), as high expected inflation is correlated with low GDP growth.

Implications of the Canonical Equilibrium Model

1. The model implies that level, slope and curvature are perfectly correlated.
   - $\Longrightarrow$ Need more than one factor.
2. The model requires a large risk aversion to produce reasonable yield curves and a reasonable market price of risk λ
   - For instance, with $\gamma = 104$, we obtain $\lambda = -0.5931$
   - Risk free rate puzzle kicks in:
     - For "reasonable" γ, the interest rate is too high.
     - Lowering γ to $\gamma \approx 0.5$ generates also reasonable yield curves, but they are not upward sloping in average. Moreover, the market price of risk is too low.
3. The model implies expected holding return should be constant

$$E\left[\frac{dZ}{Z}\right]/dt - r_t = \sigma_Z \lambda$$

   - $\Longrightarrow$ no predictability of bond returns.
4. The model implies constant volatility of both returns and yields.

Figure 6: Monthly Volatility of Yields

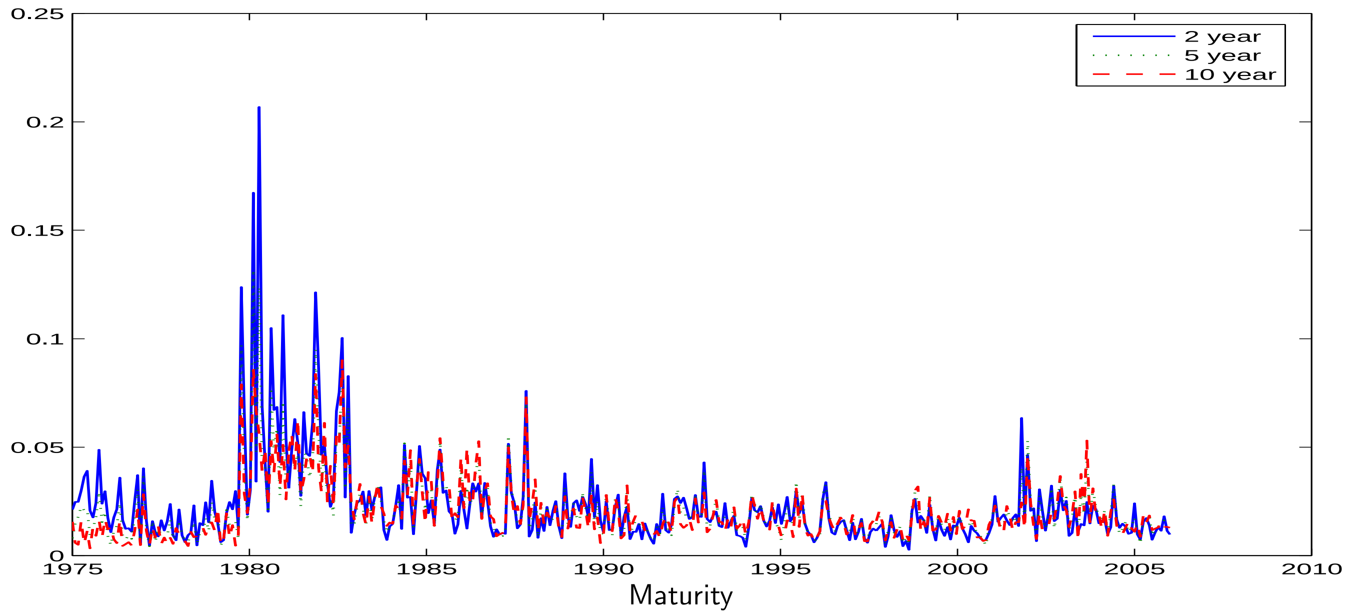
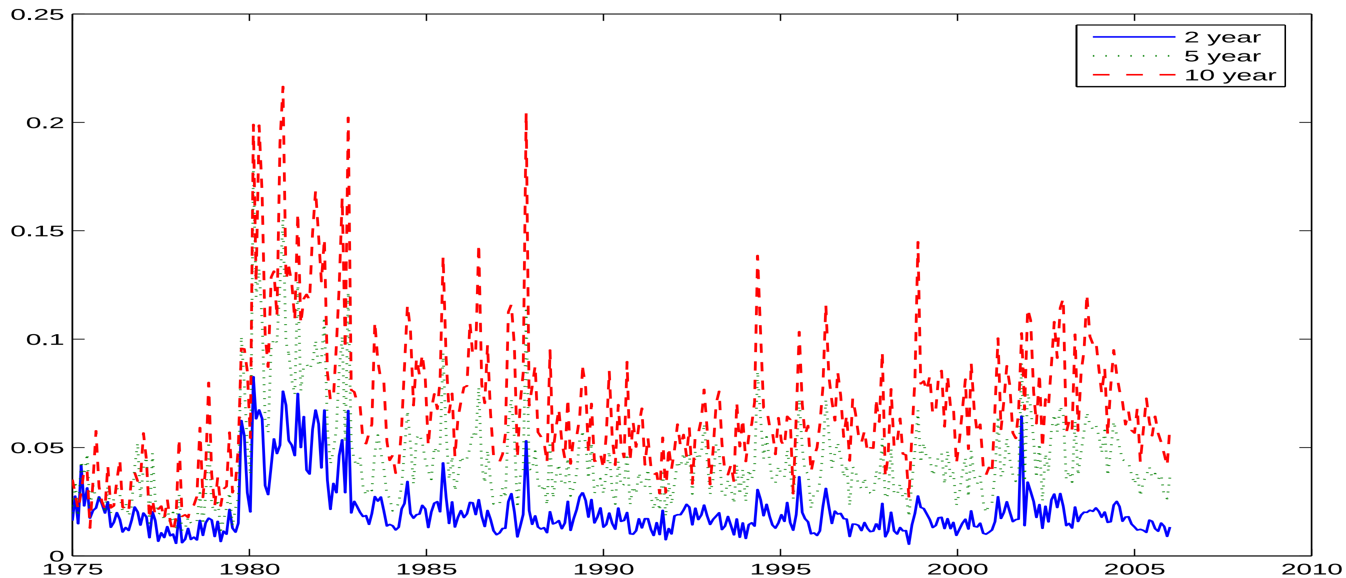


Figure 7: Monthly Volatility of Bond Returns



Bond Predictability. Fama Bliss (1987)

- Fama and Bliss classic paper show that bond return are predictable by the forward spread.

$$rx^n_{t,t+1} = \alpha + \beta \left(f^{(n)}_t - y(t,1) \right) + \epsilon_t$$

- where $n =$ horizon (in years), and

$$\text{holding period excess log return:} \quad rx^n_{t,t+1} = \log \left(\frac{Z(t+1,n-1)}{Z(t,n)} \right) - y(t,1)$$

$$\text{forward rate:} \quad f^{(n)}_t = \log \left(\frac{Z(t,n-1)}{Z(t,n)} \right)$$

TABLE 2—FAMA-BLISS EXCESS RETURN REGRESSIONS

Maturity n	β	Small T	R^2	$\chi^2(1)$	p-val	EH p-val
2	0.99	(0.33)	0.16	18.4	⟨0.00⟩	⟨0.01⟩
3	1.35	(0.41)	0.17	19.2	⟨0.00⟩	⟨0.01⟩
4	1.61	(0.48)	0.18	16.4	⟨0.00⟩	⟨0.01⟩
5	1.27	(0.64)	0.09	5.7	⟨0.02⟩	⟨0.13⟩

Notes: The regressions are $rx^{(n)}_{t+1} = \alpha + \beta(f^{(n)}_t - y^{(1)}_t) + \varepsilon^{(n)}_{t+1}$. Standard errors are in parentheses "( )", probability values in angled brackets "⟨ ⟩". The 5-percent and 1-percent critical values for a $\chi^2(1)$ are 3.8 and 6.6.

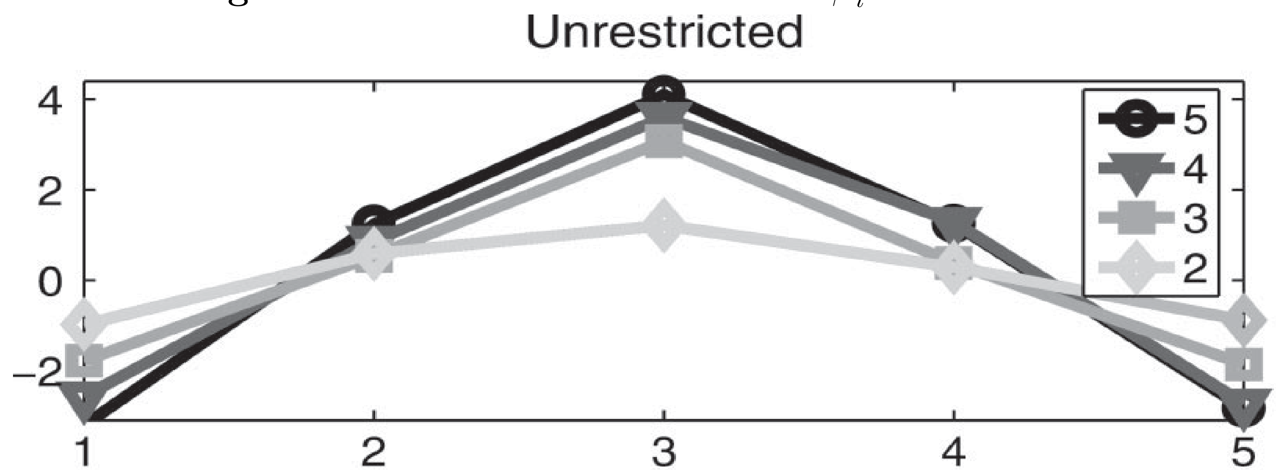
Source: Cochrane and Piazzesi (2005, AER)

Bond Predictability: Cochrane and Piazzesi (2005)

- Cochrane and Piazzesi shows that a particular combination of forward rates predicts well excess bond returns.
- Consider the regression

$$rx_{t+1}^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} y(t,1) + \beta_2^{(n)} f_t^{(2)} + \ldots + \beta_5^{(n)} f_t^{(5)} + \epsilon_{t+1}^{(n)}$$

Figure 8: Cochrane and Piazzesi: β_i coefficients'



Source: Cochrane and Piazzesi (2005, AER)

- The coefficients have a "tent shape"
  - Suggest that there is a particular combination of forwards that contain most of the information in forward curve.

Bond Predictability: Cochrane and Piazzesi (2003)

- Define $\mathbf{f}_t = [y(t,1), f_t^{(2)}, .., f_t^{(n)}]$ and compute $\boldsymbol{\gamma}$ from the regression

$$\frac{1}{4}\sum_{i=2}^{5} rx_{t+1}^{(n)} = \boldsymbol{\gamma}\mathbf{f}_t + \overline{\epsilon}_{t+1}$$

- Define the predictor $\boldsymbol{\gamma}\mathbf{f}_t$, and use it to predict future excess returns.

TABLE 1—ESTIMATES OF THE SINGLE-FACTOR MODEL

A. Estimates of the return-forecasting factor, $\overline{rx}_{t+1} = \boldsymbol{\gamma}^\top \mathbf{f}_t + \bar{\varepsilon}_{t+1}$

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	R^2	$\chi^2(5)$
OLS estimates	−3.24	−2.14	0.81	3.00	0.80	−2.08	0.35	
				Asymptotic (Large T) distributions				
HH, 12 lags	(1.45)	(0.36)	(0.74)	(0.50)	(0.45)	(0.34)		811.3
NW, 18 lags	(1.31)	(0.34)	(0.69)	(0.55)	(0.46)	(0.41)		105.5
Simplified HH	(1.80)	(0.59)	(1.04)	(0.78)	(0.62)	(0.55)		42.4
No overlap	(1.83)	(0.84)	(1.69)	(1.69)	(1.21)	(1.06)		22.6
				Small-sample (Small T) distributions				
12 lag VAR	(1.72)	(0.60)	(1.00)	(0.80)	(0.60)	(0.58)	[0.22, 0.56]	40.2
Cointegrated VAR	(1.88)	(0.63)	(1.05)	(0.80)	(0.60)	(0.58)	[0.18, 0.51]	38.1
Exp. Hypo.							[0.00, 0.17]	

B. Individual-bond regressions

Restricted, $rx_{t+1}^{(n)} = b_n(\boldsymbol{\gamma}^\top \mathbf{f}_t) + \varepsilon_{t+1}^{(n)}$						Unrestricted, $rx_{t+1}^{(n)} = \boldsymbol{\beta}_n \mathbf{f}_t + \varepsilon_{t+1}^{(n)}$			
n	b_n	Large T	Small T	R^2	Small T	R^2	EH	Level R^2	$\chi^2(5)$
2	0.47	(0.03)	(0.02)	0.31	[0.18, 0.52]	0.32	[0, 0.17]	0.36	121.8
3	0.87	(0.02)	(0.02)	0.34	[0.21, 0.54]	0.34	[0, 0.17]	0.36	113.8
4	1.24	(0.01)	(0.02)	0.37	[0.24, 0.57]	0.37	[0, 0.17]	0.39	115.7
5	1.43	(0.04)	(0.03)	0.34	[0.21, 0.55]	0.35	[0, 0.17]	0.36	88.2

Notes: The 10-percent, 5-percent and 1-percent critical values for a $\chi^2(5)$ are 9.2, 11.1, and 15.1 respectively. All p-values are less than 0.005. Standard errors in parentheses "( )", 95-percent confidence intervals for R^2 in square brackets "[ ]". Monthly observations of annual returns, 1964–2003.

Source: Cochrane and Piazzesi (2005, AER)

Bond Predictability: Level, Slope and Curvature

- "Slope" from Principal Component Analysis forecast excess returns.

TABLE 4—EXCESS RETURN FORECASTS USING YIELD FACTORS AND INDIVIDUAL YIELDS

		NW, 18		Simple S		Small T		5 percent
Right-hand variables	R^2	χ^2	p-value	χ^2	p-value	χ^2	p-value	crit. value
Slope	0.22	60.6	⟨0.00⟩	22.6	⟨0.00⟩	24.9	⟨0.00⟩	9.5
Level, slope	0.24	37.0	⟨0.00⟩	20.5	⟨0.00⟩	18.6	⟨0.00⟩	7.8
Level, slope, curve	0.26	31.9	⟨0.00⟩	17.3	⟨0.00⟩	16.7	⟨0.00⟩	6.0
$y^{(5)} - y^{(1)}$	0.15	85.5	⟨0.00⟩	30.2	⟨0.00⟩	33.2	⟨0.00⟩	9.5
$y^{(1)}, y^{(5)}$	0.22	45.7	⟨0.00⟩	24.6	⟨0.00⟩	22.2	⟨0.00⟩	7.8
$y^{(1)}, y^{(4)}, y^{(5)}$	0.33	9.1	⟨0.01⟩	4.6	⟨0.10⟩	4.9	⟨0.09⟩	6.0

Notes: The χ^2 test is $c = 0$ in regressions $\overline{rx}_{t+1} = a + bx_t + cz_t + \bar{\varepsilon}_{t+1}$ where x_t are the indicated right-hand variables and z_t are yields such that $\{x_t, z_t\}$ span all five yields.

Source: Cochrane and Piazzesi (2005, AER)

Bond Predictability: Diebold and Li (2006) Level, Slope and Curvature

- Diebold and Li (2006) use Nelson and Siegel model to predict future interest rates.
- Nelson and Siegel model posits a functional form of the term structure given by

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}\right) + \beta_{3,t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right)$$

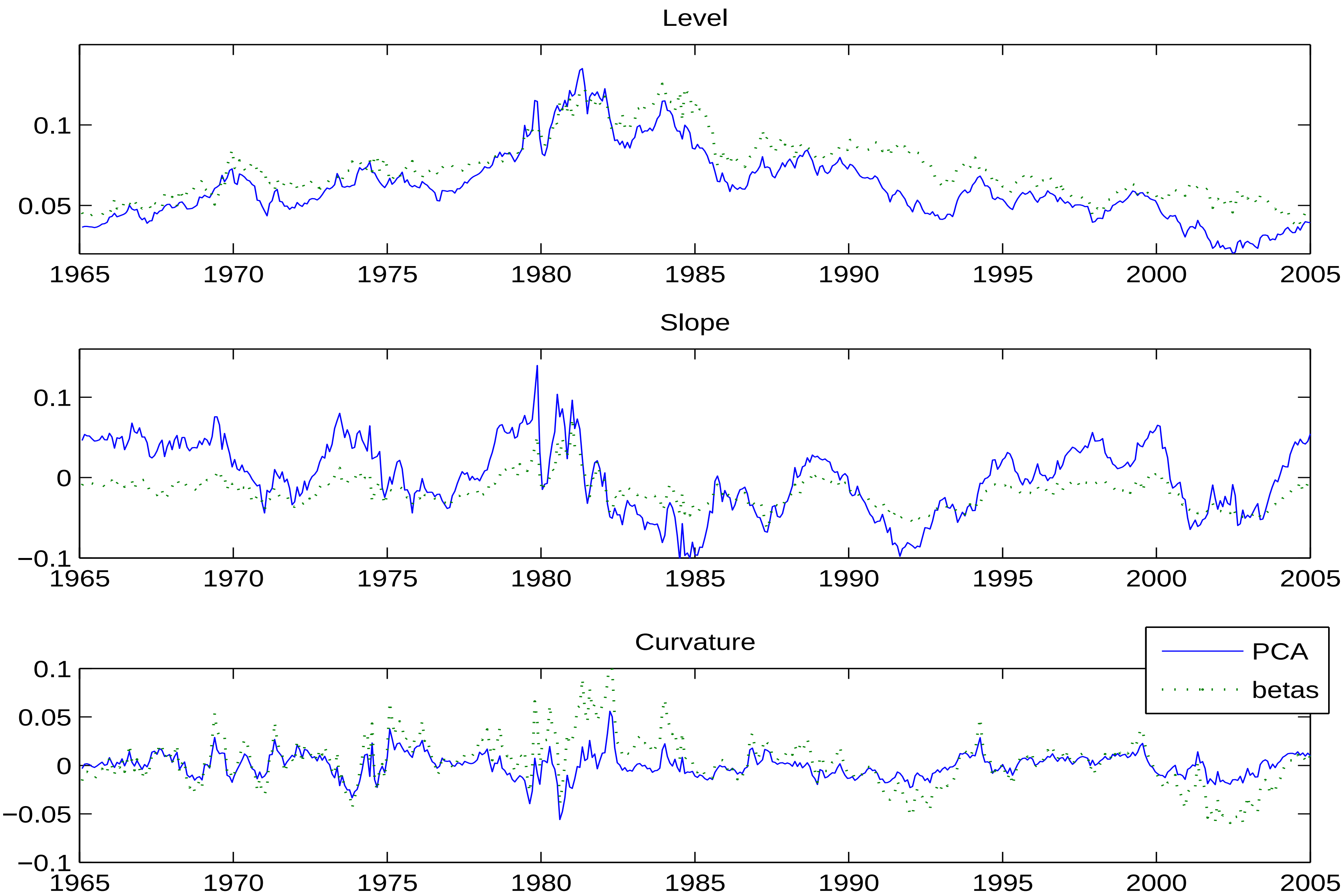
- The coefficient betas, obtained by fitting the model monthly, are correlated with level, slope and curvature.
- Diebold and Li (2006) show that their model predict well interest rates. What about excess returns?

Table 1: Predictability of Annual Excess Return from NS betas

const.	$\beta_{1,t}$ (Level)	$\beta_{2,t}$ (Slope)	$\beta_{3,t}$ (Curvature)	t(const.)	t($\beta_{1,t}$)	t($\beta_{2,t}$)	t($\beta_{3,t}$)	R^2
-0.0221	0.3103	-0.2346	0.0866	-1.9305	2.2049	-1.9107	1.2429	0.1431
-0.0391	0.5029	-0.5456	0.1732	-1.8292	1.8870	-2.4059	1.4167	0.1466
-0.0601	0.7311	-0.8924	0.2237	-2.0541	1.9611	-2.8480	1.3962	0.1776
-0.0753	0.8609	-1.1915	0.3064	-2.1360	1.9089	-3.1772	1.6354	0.1922

Sample: 1968 - 2005

Figure 9: Nelson and Siegel Coefficients and PCA Factors



CP Factor versus NS Betas

- The results in Diebold and Li (2006) original paper seem to suggest that the NS betas dominate the Cochrane Piazzesi factors
  - Important difference between predicting interest rates versus predicting premia
    - E.g. The earlier toy model implies predictable yields but constant expected returns
  - The sample was very different.

Table 2: Predictability of Annual Excess Return from NS betas: DL Sample.

const.	$\beta_{1,t}$ (Level)	$\beta_{2,t}$ (Slope)	$\beta_{3,t}$ (Curvature)	t(const.)	t($\beta_{1,t}$)	t($\beta_{2,t}$)	t($\beta_{3,t}$)	R^2
-0.0396	0.6676	0.1212	-0.0266	-4.1925	4.8373	0.7970	-0.1729	0.4120
-0.0798	1.2820	0.1400	0.0297	-4.0777	4.6835	0.4637	0.0971	0.4175
-0.1189	1.8442	0.0371	0.1070	-3.9957	4.4916	0.0833	0.2427	0.4325
-0.1490	2.2132	-0.1329	0.2567	-4.0392	4.4241	-0.2349	0.4668	0.4317

const.	$\beta_{1,t}$ (Level)	$\beta_{2,t}$ (Slope)	$\beta_{3,t}$ (Curvature)	CP Factor	t(const.)	t($\beta_{1,t}$)	t($\beta_{2,t}$)	t($\beta_{3,t}$)	t(CP Factor)	R^2
-0.0215	0.4103	0.3223	-0.0797	0.4066	-1.6834	2.2260	1.9508	-0.5753	2.2654	0.4920
-0.0430	0.7589	0.5487	-0.0782	0.8264	-1.7866	2.2681	1.7188	-0.2849	2.5398	0.5035
-0.0647	1.0725	0.6401	-0.0522	1.2192	-1.8623	2.2659	1.3817	-0.1331	2.6758	0.5190
-0.0896	1.3663	0.5288	0.0820	1.3380	-2.0311	2.3346	0.8791	0.1643	2.3972	0.4984

Diebold Li Sample: 1985 - 2000

Bond Predictability: Volatility

Table 3: Annual Excess Return Predictability and Volatility

maturity	cons.	Term Spread	t(const)	t(TermSpread)	R^2
2	0.0016	0.4532	0.3533	2.0769	0.0771
3	-0.0008	1.0105	-0.0980	2.5080	0.1083
4	-0.0042	1.6180	-0.3860	2.9304	0.1447
5	-0.0090	2.1003	-0.6962	3.1624	0.1622

maturity	cons.	Term Spread	Yield Vol.	t(const)	t(TermSpread)	t(Yield Vol)	R^2
2	-0.0000	0.4655	1.0913	-0.0052	2.1600	0.3868	0.0801
3	-0.0035	1.0310	1.8211	-0.3663	2.5876	0.3553	0.1106
4	-0.0067	1.6368	1.6760	-0.5499	3.0105	0.2480	0.1457
5	-0.0118	2.1215	1.8873	-0.8291	3.2454	0.2380	0.1631
maturity	cons.	Term Spread	Bond Vol.	t(const)	t(TermSpread)	t(Bond Vol)	R^2
2	-0.0000	0.4655	1.0913	-0.0052	2.1600	0.3868	0.0801
3	-0.0035	1.0310	1.8211	-0.3663	2.5876	0.3553	0.1106
4	-0.0067	1.6368	1.6760	-0.5499	3.0105	0.2480	0.1457
5	-0.0118	2.1215	1.8873	-0.8291	3.2454	0.2380	0.1631

Sample: 1968 - 2005

Conclusions from Evidence

1. Bond excess returns are strongly predictable
2. Predictability is correlated with the slope of the term structure, but not fully explained by it
   - Cochrane and Piazzesi (2005) factor is not explained by the first 3 PCA factors. It is related to some combination of the 4th and 5th factor.
3. Volatility of bond yields and returns is strongly time varying
4. However, the volatility does not explain variation in bond premia

- Are these patterns of predictability only special to US?

The International Evidence: UK 1979 - 2007

Table 2: FAMA-BLISS REGRESSION COEFFICIENTS

						Residual Autos (Yearly Lag)				
maturity (n)	a	s(a)	b	s(b)	R^2	1	2	3	4	5
Regression of holding period excess returns on forward spot spread:										
$hx(n, n-1:t+1) - r(1:t) = a + b * [f(n, n-1:t) - r(1:t)] + u(t+n)$										
2	0.27	0.26	0.61	0.26	0.09	-0.29	-0.04	-0.08	-0.22	0.27
3	0.51	0.45	0.77	0.35	0.07	-0.39	-0.02	-0.06	-0.18	0.30
4	0.76	0.61	0.79	0.43	0.04	-0.40	-0.02	-0.06	-0.16	0.29
5	1.05	0.76	0.71	0.48	0.03	-0.37	-0.02	-0.07	-0.16	0.26
Regression of changes in yields on forward spot spread:										
$r(1:t+n) - r(1:t) = a + b * [f(n, n-1:t) - r(1:t)] + u(t+n)$										
2	-0.27	0.26	0.39	0.26	0.03	-0.15	-0.09	-0.13	-0.27	0.22
3	-0.71	0.45	0.61	0.20	0.11	0.32	-0.25	-0.30	-0.20	0.02
4	-1.20	0.52	0.93	0.30	0.25	0.46	0.00	-0.30	-0.31	-0.18
5	-1.61	0.48	1.09	0.32	0.36	0.51	0.19	-0.23	-0.48	-0.28

Table 10: RESTRICTED REGRESSION:

$$\mathrm{hx}_{t+1}^{(n)} = \mathrm{b}_n * (\boldsymbol{\gamma}^{\boldsymbol{T}} \boldsymbol{f_t}) + \epsilon_{t+1}^n$$

n	b_n	$s(b_n)$	R^2	χ^2	p-val
2	0.42	0.09	0.22	21.70	0.00
3	0.85	0.17	0.26	26.23	0.00
4	1.22	0.24	0.26	26.12	0.00
5	1.51	0.30	0.24	25.29	0.00

The International Evidence: Germany 1972 - 2007

Table 2: FAMA-BLISS REGRESSION COEFFICIENTS

						Residual Autos (Yearly Lag)				
maturity (n)	a	s(a)	b	s(b)	R^2	1	2	3	4	5
Regression of holding period excess returns on forward spot spread:										
$hx(n, n-1:t+1) - r(1:t) = a + b * [f(n, n-1:t) - r(1:t)] + u(t+n)$										
2	0.42	0.34	0.39	0.37	0.03	0.08	0.06	-0.23	-0.49	-0.19
3	0.65	0.71	0.58	0.49	0.04	-0.02	0.07	-0.25	-0.43	-0.08
4	0.72	1.04	0.74	0.56	0.05	-0.07	0.05	-0.27	-0.37	0.01
5	0.74	1.34	0.86	0.64	0.06	-0.09	0.02	-0.29	-0.32	0.06
Regression of changes in yields on forward spot spread:										
$r(1:t+n) - r(1:t) = a + b * [f(n, n-1:t) - r(1:t)] + u(t+n)$										
2	-0.42	0.33	0.61	0.37	0.08	0.15	0.01	-0.25	-0.48	-0.24
3	-1.18	0.46	0.89	0.24	0.16	0.55	0.02	-0.38	-0.59	-0.43
4	-2.04	0.43	1.27	0.11	0.31	0.63	0.07	-0.40	-0.56	-0.49
5	-2.81	0.32	1.60	0.15	0.50	0.57	0.13	-0.28	-0.39	-0.39

Table 10: RESTRICTED REGRESSION:

$$\mathrm{hx}_{t+1}^{(n)} = \mathrm{b}_n * (\boldsymbol{\gamma}^{\boldsymbol{T}} \boldsymbol{f_t}) + \epsilon_{t+1}^n$$

n	b_n	$s(b_n)$	R^2	χ^2	p-val
2	0.45	0.12	0.13	14.25	0.00
3	0.87	0.20	0.15	17.95	0.00
4	1.20	0.27	0.16	19.84	0.00
5	1.48	0.33	0.16	20.33	0.00

The International Evidence: Japan 1989 - 2007

Table 2: FAMA-BLISS REGRESSION COEFFICIENTS

						Residual Autos (Yearly Lag)				
maturity (n)	a	s(a)	b	s(b)	R^2	1	2	3	4	5
Regression of holding period excess returns on forward spot spread:										
$hx(n, n-1 : t+1) - r(1 : t) = a + b * [f(n, n-1 : t) - r(1 : t)] + u(t+n)$										
2	0.42	0.25	0.14	0.59	0.00	-0.02	-0.00	0.02	0.24	-0.55
3	0.65	1.03	0.81	1.23	0.05	-0.17	-0.05	-0.01	0.40	-0.40
4	0.48	1.74	1.30	1.18	0.12	-0.28	0.04	-0.13	0.37	-0.32
5	-1.47	1.93	2.66	0.93	0.28	-0.31	-0.08	-0.19	0.24	-0.20
Regression of changes in yields on forward spot spread:										
$r(1 : t+n) - r(1 : t) = a + b * [f(n, n-1 : t) - r(1 : t)] + u(t+n)$										
2	-0.42	0.25	0.86	0.59	0.17	0.03	-0.05	0.05	0.21	-0.56
3	-1.17	0.70	0.78	0.80	0.10	0.38	0.12	0.49	0.23	-0.12
4	-1.93	1.03	0.72	0.44	0.14	0.70	0.69	0.46	0.60	0.37
5	-2.31	1.52	0.52	0.39	0.05	0.80	0.58	0.69	0.66	0.49

Table 9: RESTRICTED REGRESSION:

$hx_{t+1}^{(n)} = b_n * (\boldsymbol{\gamma}^{\boldsymbol{T}} \boldsymbol{f_t}) + \epsilon_{t+1}^{n}$

n	b_n	$s(b_n)$	R^2	χ^2	p-val
2	0.33	0.03	0.43	92.41	0.00
3	0.76	0.07	0.45	120.67	0.00
4	1.22	0.07	0.48	266.70	0.00
5	1.68	0.11	0.49	236.93	0.00