Dynamic Asset Pricing Models: Recent Developments

Day 3. Habits, Long Run Risk and Cross-sectional Predictability

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Overview

I. Santos and Veronesi (2006): Habit Preferences and the Cross-Section of Stock Returns

- Discuss the empirical evidence on the value premium

II. Bansal and Yaron (2005): Recursive Preferences and Long Run Risk

III. Bansal, Dittmar and Lundbland (2005): Cash Flow risk and the Cross-Section of Stock Returns

Motivation

- The value premium:

  Stocks with high book-to-market ratios, value stocks, have yielded higher average returns than stocks with low book-to-market ratios, growth stocks.

- The value premium puzzle: The CAPM fails to price value sorted portfolios.

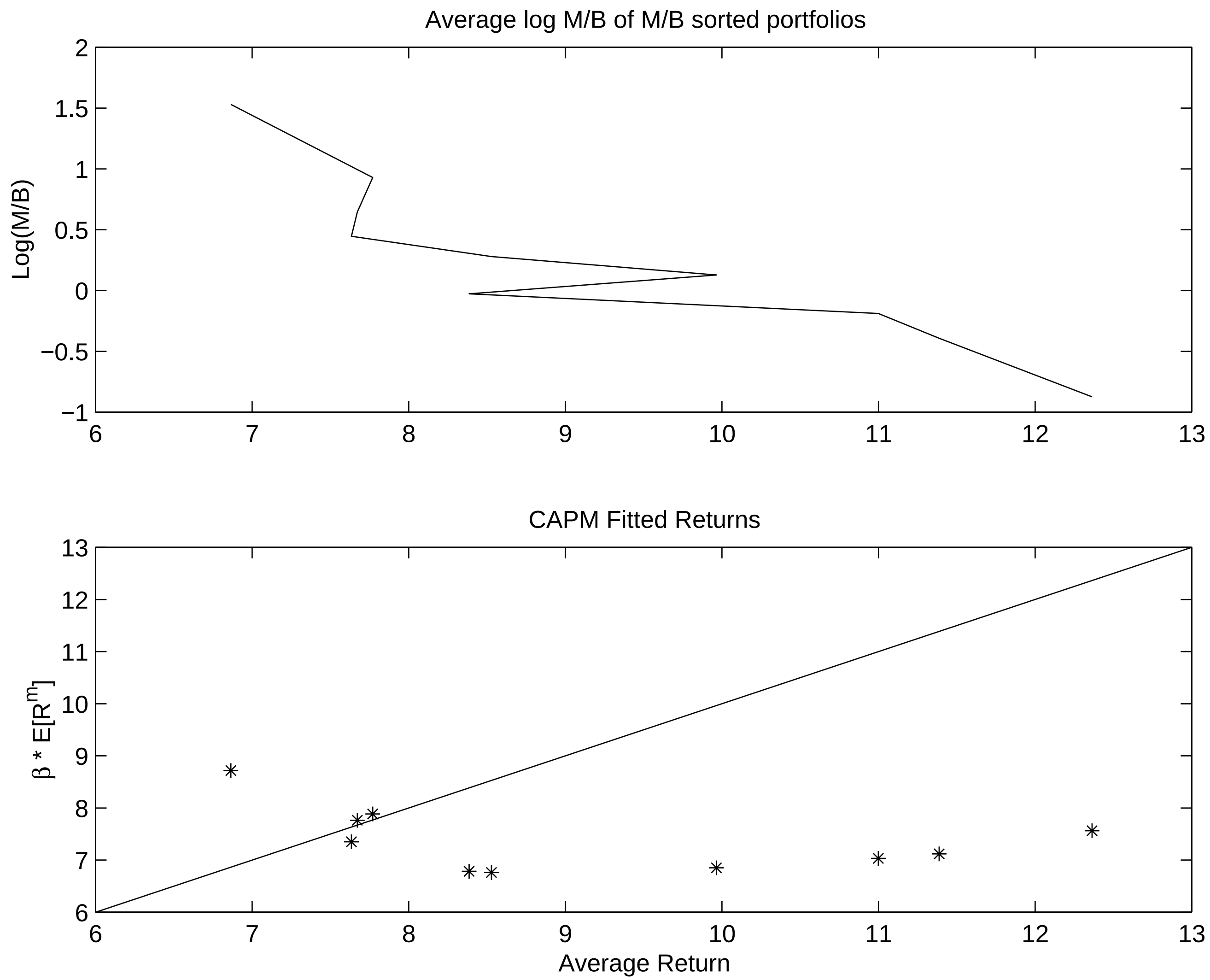

Average log M/B of M/B sorted portfolios
Log(M/B)
2
1.5
1
0.5
0
−0.5
−1
6
7
8
9
10
11
12
13
CAPM Fitted Returns
β * E[R^m]
13
12
11
10
9
8
7
6
Average Return

Table I (cont.)
Basic moments

Panel C: The value premium

	Growth									Value
Portf.	1	2	3	4	5	6	7	8	9	10
$\overline{R}$	6.86%	7.77%	7.67%	7.63%	8.53%	9.96%	8.39%	11.00%	11.39%	12.36%
$\overline{ME/BE}$	5.05	2.68	2.00	1.63	1.38	1.18	1.01	.86	.70	.45
$\overline{P/D}$	43.47	31.38	26.87	24.65	22.65	21.62	20.64	19.95	20.00	21.77
SR	.352	.450	.452	.461	.555	.640	.522	.657	.644	.600
CAPM β	1.13	1.02	1.01	.95	.88	.89	.88	.91	.92	.98

Notice:

I. The value premium

II. The value premium puzzle

III. The Sharpe ratio is decreasing in the ME/BE and P/D.

- Alternatives:
  - Rational
    * Multifactor models: Fama and French (1993)
    * Conditional CAPM: Lettau and Ludvigson (2001)
    * Cash flow risk: Campbell and Vuoltenaaho (2003), Bansal, Dittmar, and Lundblad (2005), Parker and Julliard (2005).
    * Long-run risks: Hansen, Heaton and Li (2005).
    * Composition effect: Santos and Veronesi (2005), Lettau and Wachter (2005).
  - Behavioral
    * Rosenberg, Reid and Lanstein (1985), DeBondt and Thaler (1987), Lakonishok, Shleifer, and Vishny (1994).

- These explanations are typically detached from the literature that focuses on the properties of the market portfolio:
  - The equity premium (puzzle), the volatility of returns, and the predictability of stock returns.
- In this paper we show that:

  I. The time series behavior of the market portfolio imposes general equilibrium restrictions on the behavior of the cross-section of average returns of price sorted portfolios

  II. These restrictions generate tight implications for the cash-flow characteristics of value and growth stocks.

  III. Moreover, we show that these implications extend to the dynamics of the value premium.

  IV. The model allow us to assess all these effects and implications quantitatively.
  - Standard in the equity premium literature, not so in the cross sectional one.

- Sketch of the model and strategy

  - The model has two ingredients

    - Stochastic discount factor: Habit persistence a la Campbell and Cochrane (1999).

    - A model of cash-flows a la Santos and Veronesi (2005) and Menzly, Santos, and Veronesi (2004).

  - The first ingredient is related to *discount effects*: How "risk averse" is the representative agent?

  - The second ingredient is related to individual *cash-flow effects*:

    - Duration: High or low expected dividend growth and

    - Cross sectional differences in cash-flow risk: Covariance of cash-flow growth with consumption growth.

  - We are going to calibrate the discount effects to get reasonable properties for the market portfolio and then see how much do we need in terms of cash-flow risk to generate reasonable properties for the cross section.

- Results:

  I. Value stocks are (endogenously) those with high cash-flow risk:

  - Empirical evidence: Cohen, Polk and Vuolteenaho (2003), Bansal, Dittmar, and Lundblad (2005), Hansen, Heaton and Li (2005).

  II. Value stocks are *particularly* risky in "bad times:" Time variation in risk attitudes interact with the cross sectional variation in cash-flow risk to generate fluctuations in the value premium.

  - Empirical evidence: The conditional asset pricing literature (Lettau and Ludvigson (2001)).

  III. Interpretation of asset pricing models in light of the present paper:

  (A) CAPM: The value premium and puzzle obtain.

  (B) The Fama and French (1993) model performs well because

  - the loadings on HML capture cross sectional differences in cash-flow risk and
  - it captures the component of the value premium that is related to time series variation in the premium on HML.

  (C) Conditional CAPM models capture the time series variation of the value premium.

  - All these models capture different aspects of the cash-flow effects (and their interaction with discount effects).

IV. Magnitudes:

- In the absence of cash-flow risk only discount risk effects matter and in this case a "growth premium" obtains.
- Thus cash-flow risk is needed to generate the value premium.
- We want to assess the "amount" of cross-sectional variation in cash-flow risk needed to generate the value premium.
- We find that, *in the context of our model,* the amount of cash-flow risk needed to generate the value premium is "large."

The Model

- **Preferences**
  - A representative agent with preferences

$$E\left[\int_0^\infty u\left(C_t, X_t, t\right) dt\right] \qquad \text{with} \qquad u\left(C_t, X_t, t\right) = \begin{cases} e^{-\rho t}\frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 1 \\ e^{-\rho t}\log\left(C_t - X_t\right) & \text{if } \gamma = 1 \end{cases}$$

  - Habit is given by

$$X_t = \lambda \int_{-\infty}^t e^{-\lambda(t-\tau)} C_\tau d\tau \qquad \Rightarrow \qquad dX_t = \lambda\left(C_t - X_t\right) dt$$

Define

$$G_t = \left(\frac{C_t}{C_t - X_t}\right)^\gamma \qquad \Rightarrow \qquad dG_t = \left[\mu_G\left(G_t\right) - \sigma_G\left(G_t\right)\mu_{c,1}\left(\mathbf{s}_t\right)\right] dt - \sigma_G\left(G_t\right)\sigma_c dB_t^1$$

  - We simply assume that

$$\mu_G\left(G_t\right) = k\left(\overline{G} - G_t\right) \qquad \text{and} \qquad \sigma_G\left(G_t\right) = \alpha\left(G_t - \lambda\right)$$

Thus

$$\uparrow dB_t^1 \quad \Rightarrow \quad \downarrow dG_t \quad \Rightarrow \quad \uparrow S_t = \frac{C_t - X_t}{C_t}$$

- **Endowment: Cash flows**
  - We make two assumptions:
    - **Assumption 1**

$$\frac{dC_t}{C_t} = \mu_c(\mathbf{s}_t)\,dt + \boldsymbol{\sigma}_c'\,d\mathbf{B}_t$$

where

$$\mu_c(\mathbf{s}_t) = \overline{\mu}_c + \mathbf{s}_t'\,\boldsymbol{\theta}_{CF} \qquad \text{and} \qquad \boldsymbol{\sigma}_c = (\sigma_c, 0, ..., 0)'$$

    - **Assumption 2**

$$ds_t^i = \phi\left(\overline{s}^i - s_t^i\right)dt + s_t^i\boldsymbol{\sigma}^i(\mathbf{s}_t)\cdot d\mathbf{B}_t \qquad \text{and} \qquad \boldsymbol{\sigma}^i(\mathbf{s}_t) = \boldsymbol{\nu}_i' - \sum_{j=1}^n s_t^j\boldsymbol{\nu}_j'$$

      - Each asset represents a certain long run value of the overall economy, $\overline{s}^i$
      - No firm will take over the economy.
      - The choice of the volatility ensures that the shares are positive and add up to one.
      - Dividends are then

$$D_t^i = s_t^i C_t$$

– Dividends: By Ito's Lemma:

$$\frac{dD_t^i}{D_t^i} = \mu_{D,t}^i dt + \boldsymbol{\sigma}_D^i (\mathbf{s}_t) \, d\mathbf{B}_t$$

where

$$\mu_{D,t}^i = \overline{\mu}_c + \theta_{CF}^i + \phi \left(\frac{\overline{s}^i}{s_t^i} - 1 \right) \qquad \text{and} \qquad \boldsymbol{\sigma}_D^i (\mathbf{s}_t) = \boldsymbol{\sigma}_c + \boldsymbol{\sigma}^i (\mathbf{s}_t)$$

In these formulas,

$$\theta_{CF}^i = \boldsymbol{\nu}_i' \cdot \boldsymbol{\sigma}_c$$

– Cash-flow risk: The covariance between dividend an consumption growth:

$$\sigma_{CF,t}^i \equiv Cov_t \left(\frac{dD_t^i}{D_t^i}, \frac{dC_t}{C_t} \right) = \boldsymbol{\sigma}_c \boldsymbol{\sigma}_c' + \theta_{CF}^i - \mathbf{s}_t' \, \boldsymbol{\theta}_{CF}$$

We can impose

$$\sum_{j=1}^n \overline{s}^j \theta_{CF}^j = 0 \qquad \Rightarrow \qquad \overline{\sigma}_{CF}^i = E\left[\sigma_{CF,t}^i\right] = \boldsymbol{\sigma}_c \boldsymbol{\sigma}_c' + \theta_{CF}^i$$

Equilibrium Asset Prices and Returns

I. Strategy

- The stochastic discount factor

$$m_t = e^{-\rho t} \left(C_t - X_t \right)^{-\gamma} = e^{-\rho t} C_t^{-\gamma} G_t \qquad \Rightarrow \qquad \frac{dm_t}{m_t} = -r_t^f dt + \boldsymbol{\sigma}_m' d\mathbf{B}_t$$

- The first, and only non-zero entry of $\boldsymbol{\sigma}_m'$

$$\sigma_{m,t}^1 = -\left[\gamma + \alpha \left(1 - \lambda S_t^{\gamma} \right) \right] \sigma_c.$$

- We have to solve for

$$m_t P_t^i = E_t \left[\int_t^{\infty} m_\tau s_\tau^i C_\tau d\tau \right] \qquad \text{and} \qquad E_t \left[dR_t^i \right] = -cov \left(\frac{dm_t}{m_t}, dR^i \right) = -\boldsymbol{\sigma}_m' \boldsymbol{\sigma}_R^i$$

II. General Results

(A) The total wealth portfolio

1. Prices

$$\frac{P_t^{TW}}{C_t} = \alpha_0^{TW}(\mathbf{s}_t) + \alpha_1^{TW}(\mathbf{s}_t) S_t^{\gamma} \qquad \text{where} \qquad S_t = \frac{C_t - X_t}{C_t}$$

Intuition:

$$\text{For a given } \mathbf{s}_t \quad \uparrow S_t \quad \Rightarrow \quad \downarrow \frac{\gamma}{S_t} \quad \Rightarrow \quad \uparrow \frac{P_t^{TW}}{C_t}$$

2. Returns

- The expected excess return on the total wealth portfolio

$$E_t\left[dR_t^{TW}\right] = \begin{cases} \left(\gamma + \alpha\left(1 - \lambda S_t^{\gamma}\right)\right) \frac{S_t^{\gamma}\alpha\left(1-\lambda S_t^{\gamma}\right)}{f_1^{TW}(\mathbf{s}_t)+S_t^{\gamma}}\sigma_c^2 & \text{Related to discount effects} \\ + & \\ \left(\gamma + \alpha\left(1 - \lambda S_t^{\gamma}\right)\right) \Sigma_{j=1}^{n} w_{jt}^{TW} \sigma_{CF,t}^{j} & \text{Related to changes in } E_t\left(dc_t\right) \end{cases}$$

(B) **Individual securities**

1. <u>Prices</u>

$$\frac{P_t^i}{D_t^i} = \alpha_0^i + \alpha_1^i S_t^\gamma + \alpha_2^i(\mathbf{s}_t)\left(\frac{\overline{s}^i}{s_t^i}\right) + \alpha_3^i(\mathbf{s}_t) S_t^\gamma \left(\frac{\overline{s}^i}{s_t^i}\right)$$

For a given distribution of shares $\mathbf{s}_t$

a. Expected dividend growth:

$$\uparrow \frac{\overline{s}^i}{s_t^i} \quad \Rightarrow \quad \uparrow E_t\left[\frac{dD_t^i}{D_t^i}\right] \quad \Rightarrow \quad \uparrow \frac{P_t^i}{D_t^i}$$

b. Aggregate discount effects:

$$\uparrow S_t \quad \Rightarrow \quad \downarrow \frac{\gamma}{S_t} \quad \Rightarrow \quad \uparrow \frac{P_t^i}{D_t^i}$$

c. A duration effect

An increase in S_t has a stronger impact on prices the higher the expected dividend growth.

2. Returns

- *Expected excess returns*

  – The expected excess returns

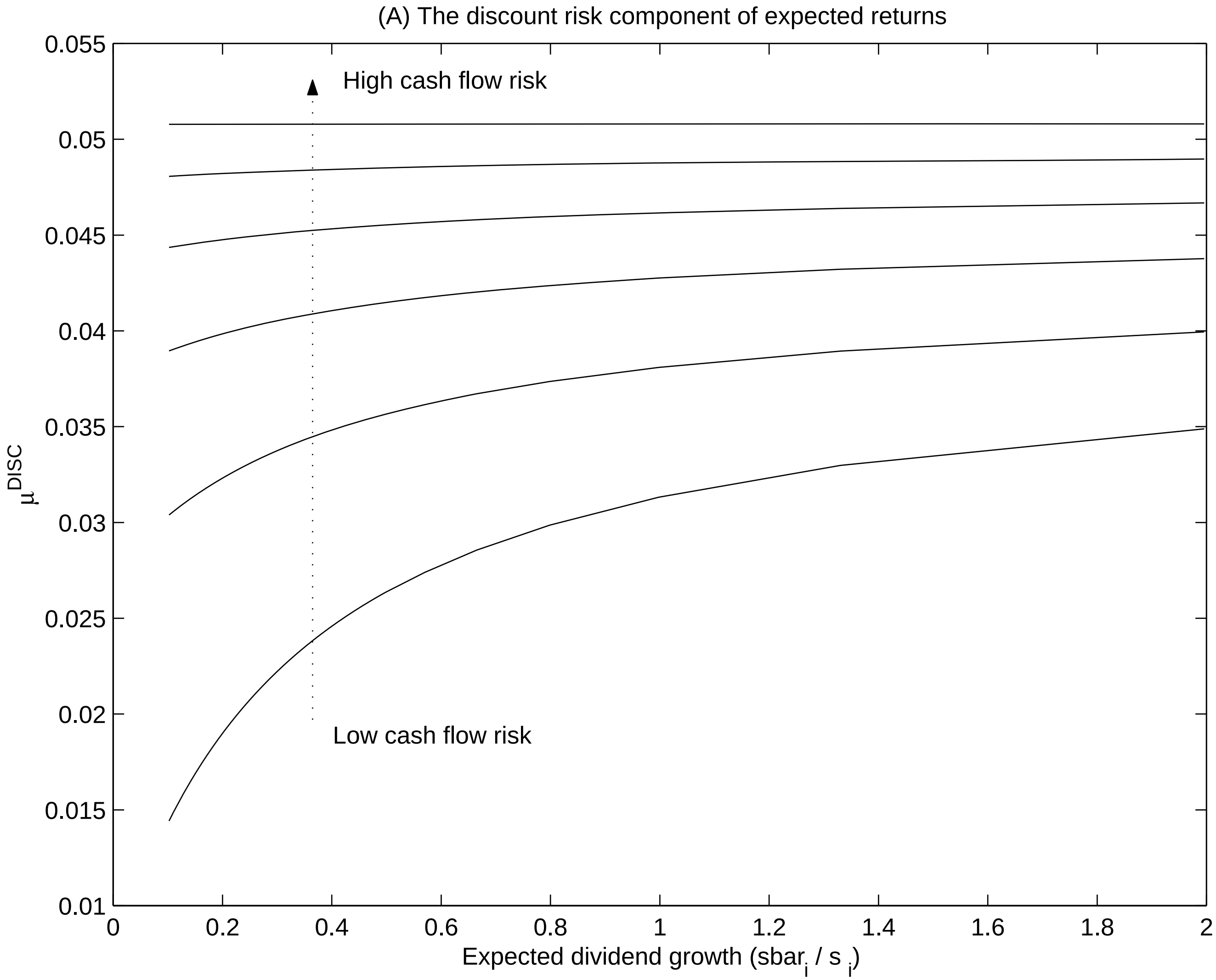
$$E_t\left[dR_t^i\right] = \mu_{i,t}^{DISC} + \mu_{i,t}^{CF}.$$

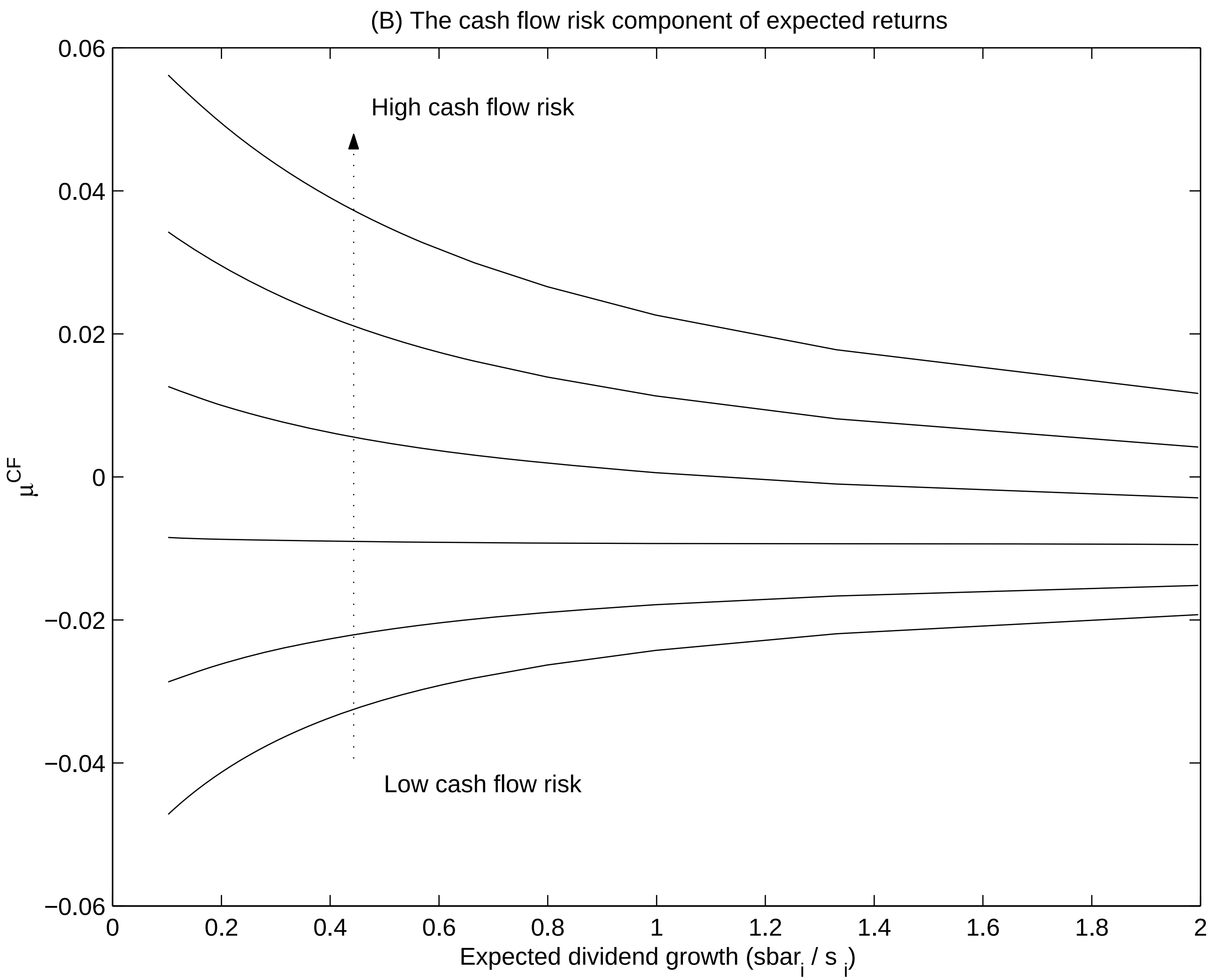
  * The discount component:

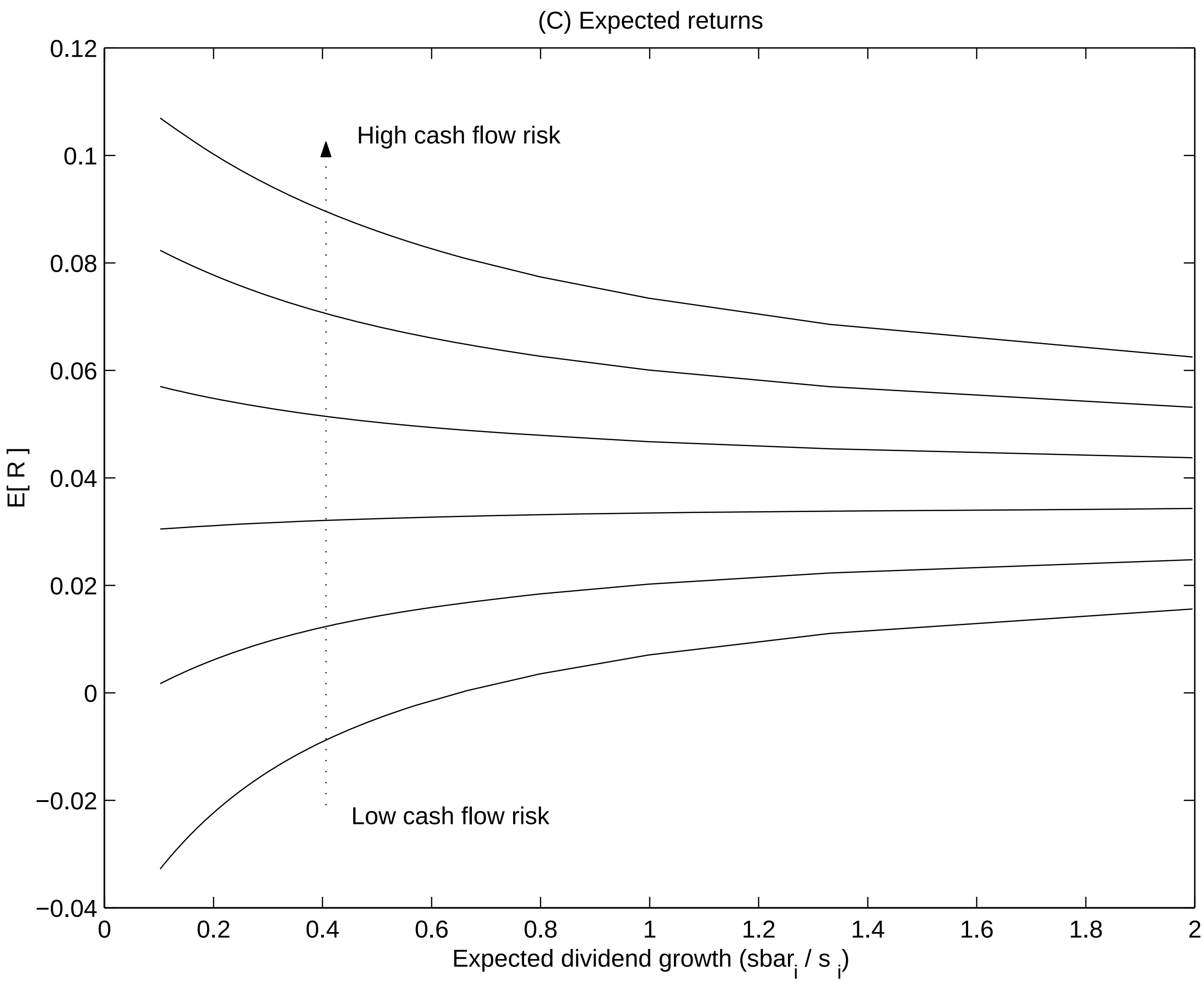
$$\mu_{i,t}^{DISC} = \left(\gamma + \alpha\left(1 - \lambda S_t^{\gamma}\right)\right)\left(\frac{\alpha S_t^{\gamma}\left(1 - \lambda S_t^{\gamma}\right)}{f_1^i\left(\frac{\overline{s}^i}{s_t^i}, \mathbf{s}_t\right) + S_t^{\gamma}}\right)\sigma_c^2$$

  * The cash-flow component:

$$\mu_{i,t}^{CF} = \left(\gamma + \alpha\left(1 - \lambda S_t^{\gamma}\right)\right)\left[\left(\frac{1}{1 + f_2^i\left(S_t, \mathbf{s}_t\right)\left(\frac{\overline{s}^i}{s_t^i}\right)} + \eta_{it}^i\right)\sigma_{CF,t}^i + \sum_{j \neq i}\eta_{jt}^i\sigma_{CF,t}^j\right]$$


(A) The discount risk component of expected returns
High cash flow risk
Low cash flow risk
μ^DISC
Expected dividend growth (sbar_i / s_i)
0.055
0.05
0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


(B) The cash flow risk component of expected returns
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
High cash flow risk
Low cash flow risk
μ^CF
Expected dividend growth (sbar_i / s_i)


(C) Expected returns
High cash flow risk
Low cash flow risk
E[R]
Expected dividend growth (sbar_i / s_i)
0.12
0.1
0.08
0.06
0.04
0.02
0
−0.02
−0.04
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

- The source of the value premium

  a. Discount effects only: A "growth premium" obtains:

  – $\theta^i_{CF} = 0$ for all i, and whatever cross-sectional differences are driven by $\overline{s}^i / s^i_t$.

$$\uparrow \frac{P^i_t}{D^i_t} \quad \Rightarrow \quad \uparrow \overline{s}^i / s^i_t \qquad \text{but} \qquad \uparrow \overline{s}^i / s^i_t \quad \Rightarrow \quad \uparrow E\left[dR^i_t\right]$$

  – Thus

$$\textbf{Growth premium:} \quad \uparrow \frac{P^i_t}{D^i_t} \quad \Rightarrow \quad \uparrow E\left[dR^i_t\right]$$

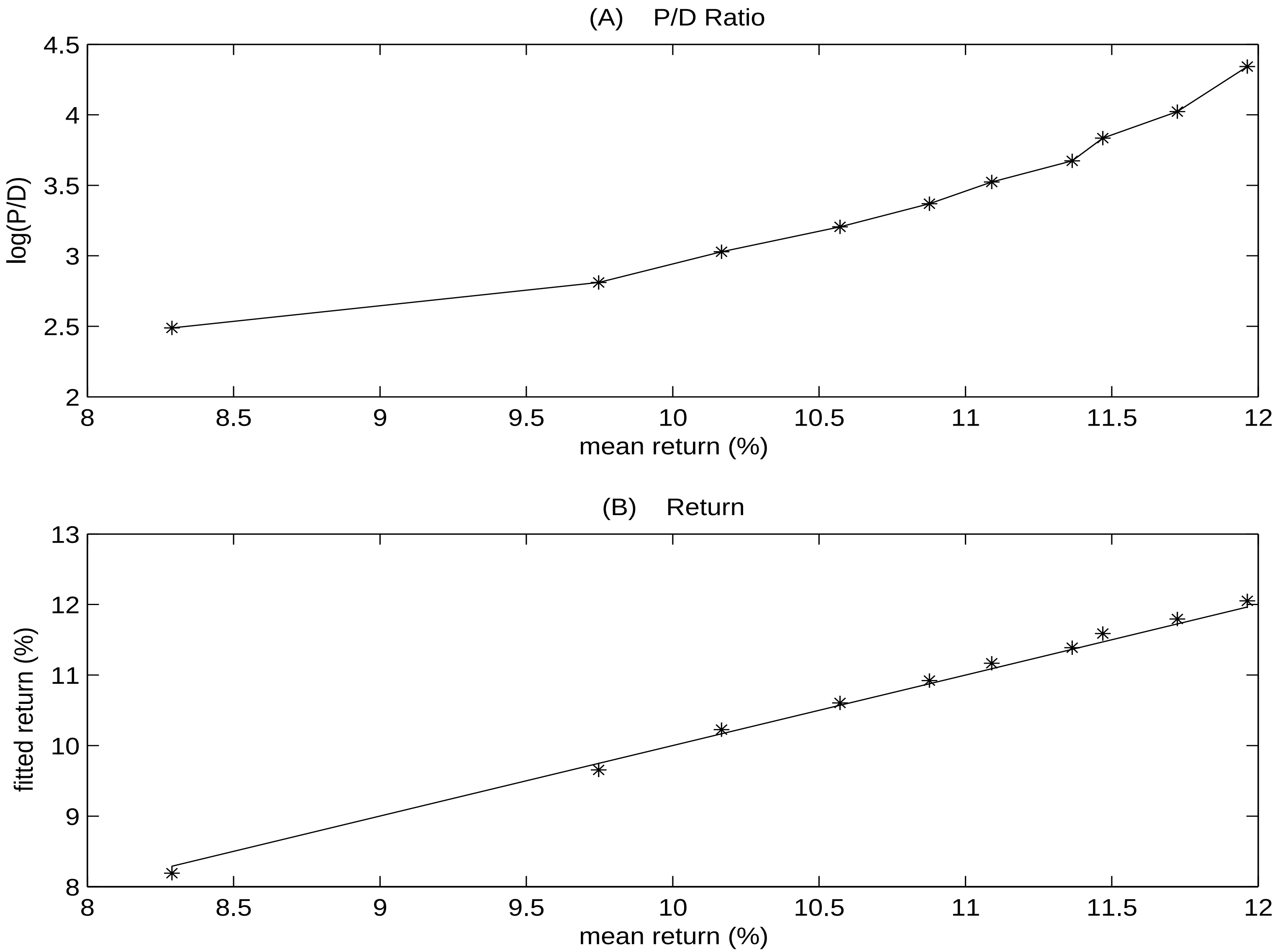


Figure: Simulations with no cross-sectional differences in CF risk.
Methodology: (a) Simulate prices, (b) sort portfolios by P/D, (c) take averages.

b. Discount effects + cash-flow effects: a "value premium" may obtain:

– Differences in ${\theta^i}_{CF}$ and $\overline{s}^i/s_t^i$ drive cross-sectional differences.

$$\uparrow \frac{P_t^i}{D_t^i} \quad \Rightarrow \quad \begin{cases} \uparrow \overline{s}^i/s_t^i \;\Rightarrow\; \uparrow \mu_{i,t}^{DISC} & \text{Discount risk effect} \\ \uparrow \overline{s}^i/s_t^i \;\Rightarrow\; \downarrow \mu_{i,t}^{CF} & \text{Cash-flow risk effect - 1} \\ \downarrow \theta_{CF}^i \;\Rightarrow\; \downarrow E_t\left[dR_t^i\right] & \text{Cash-flow risk effect - 2} \end{cases}$$

– Thus, if cash-flow risk effects are *sufficiently strong*

$$\textbf{Value premium:} \quad \uparrow \frac{P_t^i}{D_t^i} \quad \Rightarrow \quad \downarrow E\left[dR_t^i\right]$$

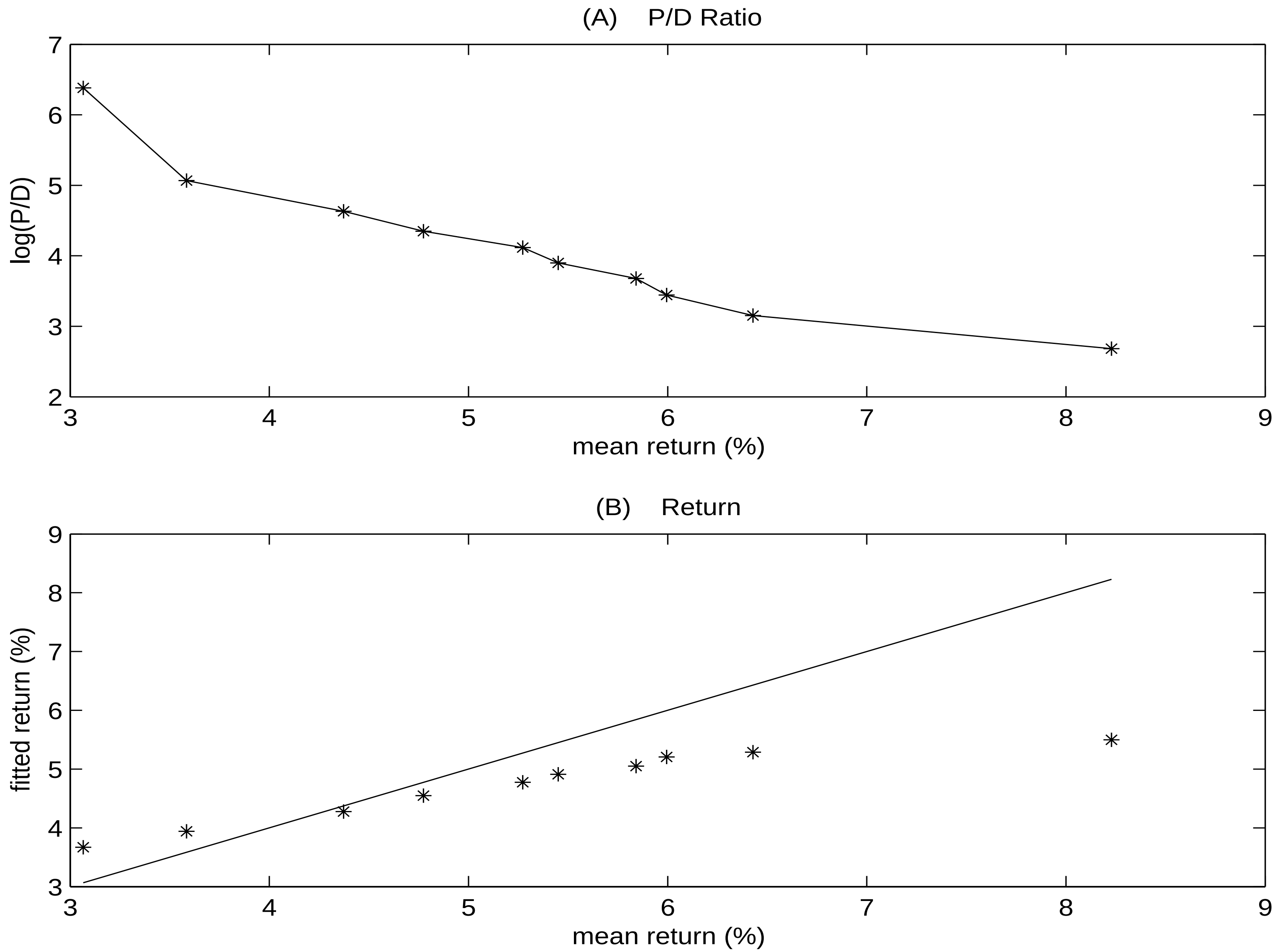
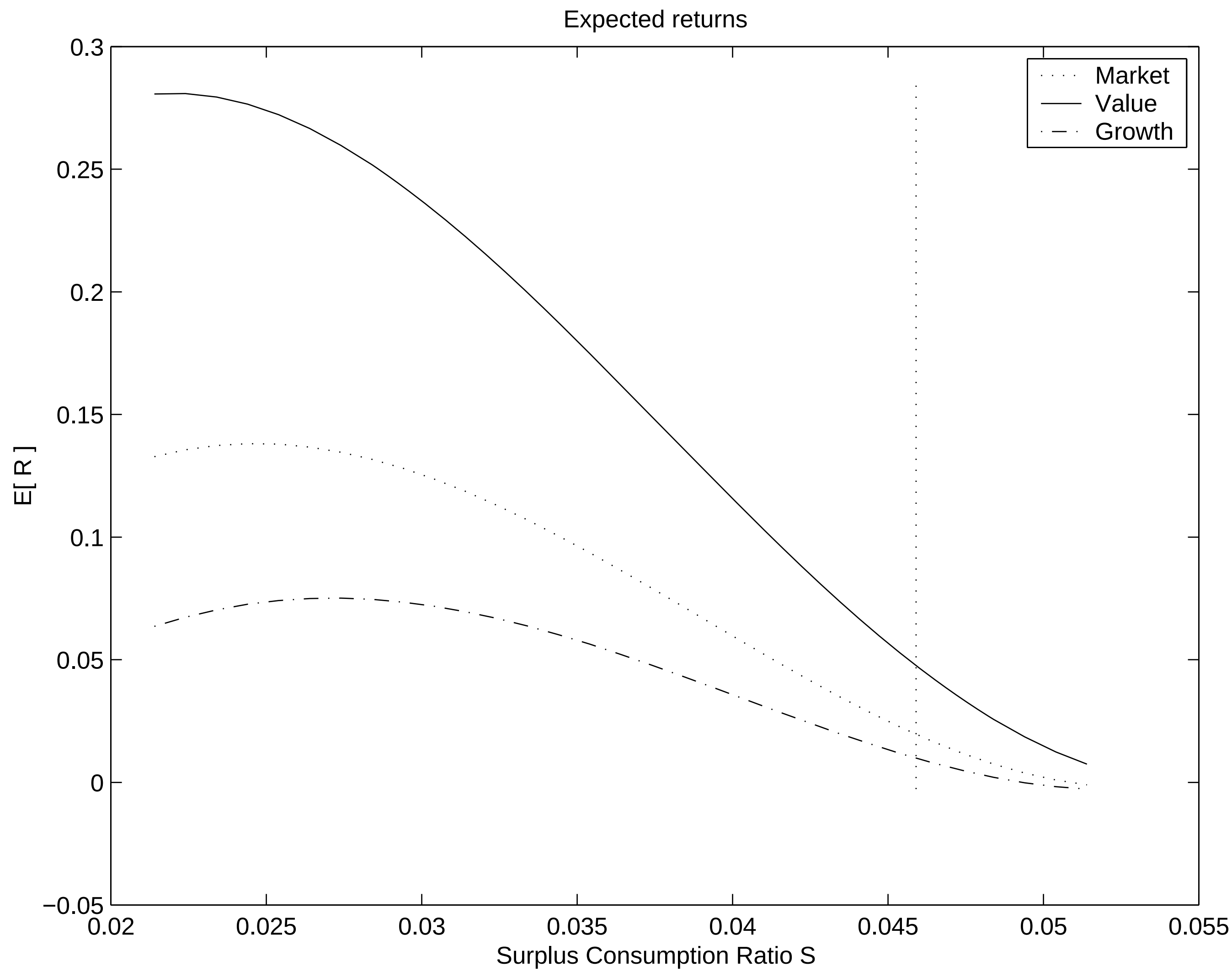


Figure: Simulations with cross-sectional dispersion in CF risk. Methodology: (a) Simulate prices, (b) sort portfolios by P/D, (c) take averages.

- The dynamics of the value premium
  - There are two "effects" in our setup:
    a. Cross-sectional differences in cash-flow risk, θ^i_{CF} and
    b. discount risk effects
  - These two effects interact to induce fluctuations in the value premium.
  - Intuition: Value stocks become relative $riskier$ in bad times.
  - This is exactly what the conditional asset pricing models of, say, Lettau and Ludvigson (2001) capture.


Expected returns
Market
Value
Growth
E[R]
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
0.02
0.025
0.03
0.035
0.04
0.045
0.05
0.055
Surplus Consumption Ratio S

Simulations

I. **Data**

- CRSP-COMPUSTAT
- Sample period: 1948-2001
- We are after two sets of moments:

(A) Time Series:

- Equity premium and volatility of returns.
- Predictability.

(B) Cross section

- The value premium.

- What is that we want to match?

II. Details of the simulation

- We simulate 10,000 years of quarterly data for 200 firms.
- We sort the 200 firms into 10 portfolios, sorted on P/D.
- Parameter choices are:

Table II
Model parameters used in the simulation

Panel A: Consumption and preference parameters

μ_c	σ_c	γ	ρ	$\gamma/\overline{S}$	$\min\{\gamma/S_t\}$	α	k
.02	.015	1.5	.072	48	27.75	77	.13

Panel B: Share process parameter

n	$\overline{\theta}_{CF}$	$\overline{s}^i$	ϕ	$\overline{\nu}$
200	.00345	.005	.07	0.55

III. Cash-flow effects, discount effects, and the value premium

- The model implies a steady state value of the local curvature of the utility function

$$-\frac{u_{CC}}{u_C}C = \frac{\gamma}{\overline{S}} = 48$$

- The model generates
  - A slightly low equity premium: 4.40%
  - A reasonable volatility of market returns: 13.6%
  - Predictability that matches well the one in the 1948-2001 sample.

Table III
Basic moments in simulated data

Panel A: Summary statistics for the aggregate portfolio

$E(R^M)$	vol(R^M)	r^f	vol(r^f)
4.35%	13.03%	.69%	4.36%

Panel B: Predictability regressions

Horizon	4	8	12	16
$\ln\left(\frac{D}{P}\right)$	.25	.38	.43	.47
t−stat.	(29.11)	(34.68)	(37.58)	(39.46)
R^2 (%)	5.74	7.82	7.57	7.06

Table III (cont.)
Basic moments in simulated data

Panel C: The value premium

	Growth									Value
Portf.	1	2	3	4	5	6	7	8	9	10
$\overline{R}$ (%)	3.07	3.58	4.37	4.77	5.27	5.45	5.84	6.00	6.43	8.23
$\overline{ln\,(P/D)}$	6.38	5.07	4.613	4.35	4.12	3.90	3.68	3.44	3.15	2.68
Avge(θ^i_{CF})$\times$ 100	$-.2858$	$-.1589$	$-.0665$	$-.0083$	.0295	.0568	.0787	.0958	.1128	.1431
Sharpe Ratio	.260	.271	.307	.313	.331	.328	.336	.330	.334	.366
CAPM β	.84	.91	.98	1.05	1.10	1.13	1.16	1.20	1.22	1.26
CAPM ret. (%)	3.67	3.94	4.28	4.55	4.78	4.91	5.05	5.21	5.29	5.50

(A) The value premium

(B) The value premium puzzle

(C) The Sharpe ratio is decreasing in P/D.

(D) Cash flows of value stocks is riskier

- What does our choice of θ^i_{CF} mean? Strong cash-flow effects, but more on this below.

IV. The dynamics of the value premium

- What are the value premium dynamics in the data?
  - Split sample in periods of low aggregate M/B ($< \overline{c}$), and the complementary
  - Compute average excess returns for M/B sorted portfolios.

Table IV
The dynamics of the value premium

Panel A: Annualized average excess returns (%) in empirical data

Market-to-book of market portfolio $< \overline{c}$					Market-to-book of market portfolio $> \overline{c}$				
$\overline{c}$	1	10	10-1	$\overline{R}^M$	$\overline{c}$	1	10	10-1	$\overline{R}^M$
15%	13.18	23.57	10.38	15.40	15%	5.73	10.35	4.62	6.34
20%	10.57	21.70	11.14	13.41	20%	5.95	10.06	4.11	6.31
25%	5.51	19.16	13.64	9.89	25%	7.31	10.11	2.80	6.99
30%	6.97	19.49	12.51	10.50	30%	6.82	9.32	2.50	6.62
35%	8.19	18.65	10.45	11.14	35%	6.15	8.98	2.83	5.87

- What are the value premium dynamics implied by the model?

Table IV (cont.)
The dynamics of the value premium

Panel B: Annualized average excess returns (%) in simulated data

Price-dividend of market portfolio $< \overline{c}$					Price-dividend of market portfolio $> \overline{c}$				
$\overline{c}$	1	10	10-1	$\overline{R}^M$	$\overline{c}$	1	10	10-1	$\overline{R}^M$
15%	7.37	18.27	10.90	10.43	15%	2.30	6.46	4.15	3.27
20%	6.56	16.07	9.51	9.22	20%	2.19	6.26	4.07	3.13
25%	5.96	14.60	8.64	8.36	25%	2.10	6.10	4.00	3.01
30%	5.50	13.46	7.96	7.67	30%	2.02	5.98	3.96	2.92
35%	5.13	12.60	7.47	7.18	35%	1.95	5.87	3.92	2.82

V. The CAPM and other asset pricing models

(A) The CAPM

1. Time series evidence

Table V Panel A

Time series regression $R_t^i = \alpha + \beta^M R_t^M + \epsilon_t$

Panel A-2: Empirical data

	Growth									Value
Portf.	1	2	3	4	5	6	7	8	9	10
α	−.46	−.03	−.02	.07	.44	.78	.40	.99	1.07	1.20
$t(\alpha)$	(−2.00)	(−.18)	(−.14)	(.32)	(2.07)	(3.73)	(1.51)	(3.73)	(3.32)	(2.65)
β^M	1.13	1.02	1.01	.95	.88	.89	.88	.91	.92	.98
$t\left(\beta^M\right)$	(39.80)	(43.68)	(42.56)	(30.32)	(27.24)	(27.27)	(21.38)	(21.33)	(17.56)	(14.16)

Panel A-2: Simulated data

	Growth									Value
Portf.	1	2	3	4	5	6	7	8	9	10
α	−.15	−.09	.02	.06	.12	.13	.20	.20	.29	.68
$t(\alpha)$	(−14.25)	(−5.95)	(1.52)	(3.27)	(6.99)	(6.87)	(9.12)	(8.35)	(10.32)	(17.56)
β^M	.84	.91	.98	1.05	1.10	1.13	1.16	1.20	1.22	1.26

2. Fama-MacBeth regressions

Table VI
CAPM: Fama-MacBeth regressions (quarterly)

Panel A: Empirical data

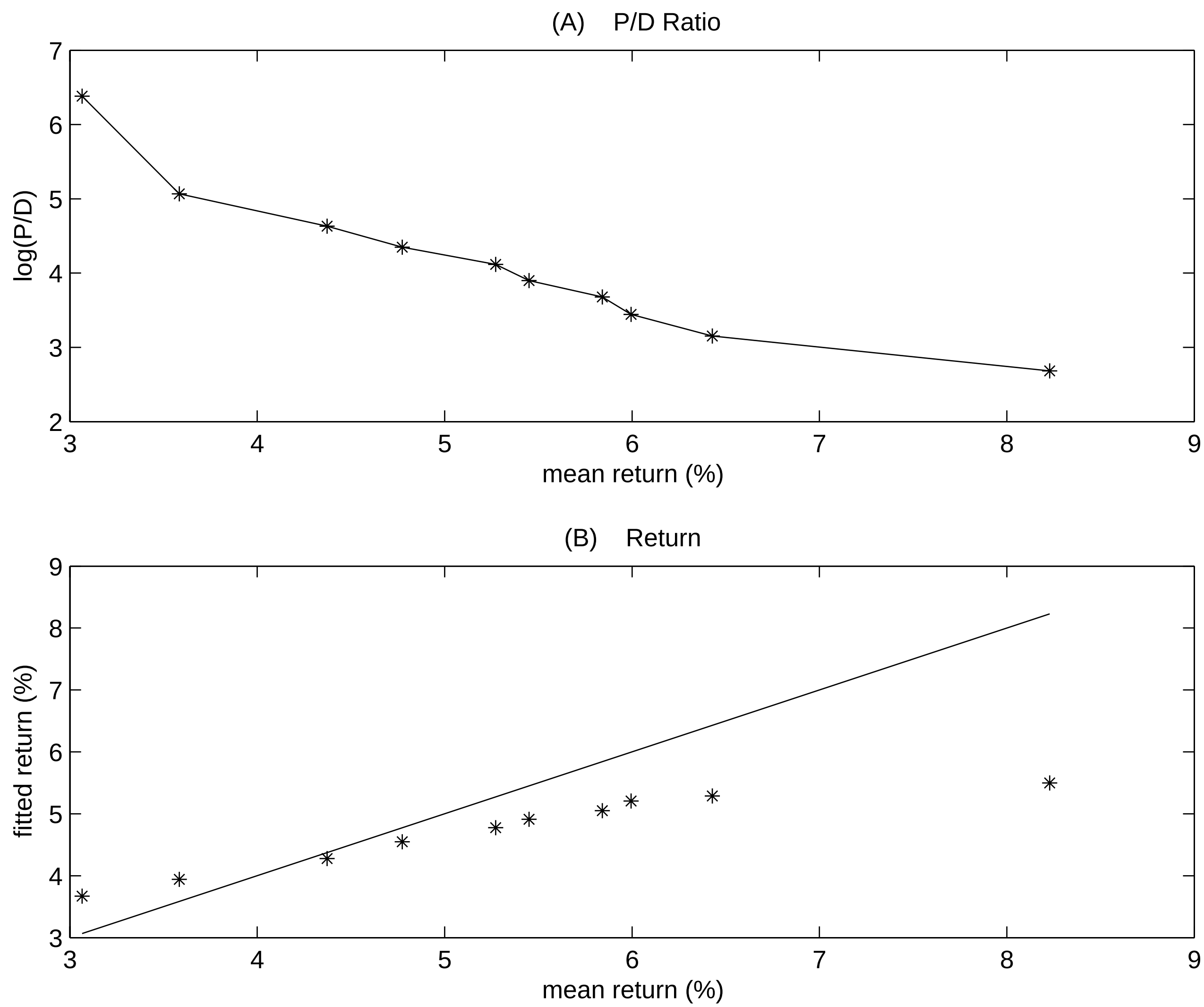
	Const.	Mkt.	Adj. R^2
1.	4.69	−2.52	11%
	(3.21)	(−1.65)	

Panel B: Simulated data

	Const.	Mkt.	Adj. R^2
5.	−1.45	2.56	91%
	(−19.93)	(32.45)	

A Pitfall

- Judging by t-stat and R^2, CAPM works well.
  - This is because the betas in the first pass regression indeed line up with average returns.
$$r_t^i = \alpha^i + \beta^i r_t^M + \epsilon_t^i$$
  - $\Longrightarrow$ In the second pass (cross-sectional) regression, R^2 and t-stat are high.
$$\lambda^i = \lambda_0 + \beta^i \lambda^M + \eta^i$$
  - *But magnitude of coefficient is off:*
$$\text{Implied premium } = 2.56 \times 4 = 10.4\% > 4.35\% (= E[dR^M])$$
- Pitfall: Finding a significant t-stat and high R^2 is missleading.
- Economic magnitudes of coefficients in Fama-Macbeth regressions are index of whether asset pricing model works or not.
- Tests of the magnitudes are harder, especially for conditional asset pricing models (below)
- Santos and Veronesi (2006) use simulations to gauge the magnitudes of coefficients,


(A) P/D Ratio
log(P/D)
7
6
5
4
3
2
3
4
5
6
7
8
9
mean return (%)
(B) Return
fitted return (%)
9
8
7
6
5
4
3
3
4
5
6
7
8
9
mean return (%)

(B) The Fama and French (1993) Model

1. Time series evidence

Table V Panel B

Time series regression $R_t^i = \alpha + \beta^M R_t^M + \beta^{HML} R_t^{HML} + \epsilon_t$

Panel B-1: Empirical data

	Growth									Value
Portf.	1	2	3	4	5	6	7	8	9	10
α	.20	.17	.02	−.12	.19	.28	−.40	.01	−.08	−.36
$t(\alpha)$	(1.13)	(1.05)	(.14)	(−.61)	(.87)	(1.58)	(−2.15)	(.09)	(−.43)	(−1.23)
β^M	1.04	.99	1.00	.98	.91	.96	.99	1.05	1.09	1.20
$t\left(\beta^M\right)$	(43.68)	(51.25)	(46.13)	(35.28)	(30.25)	(38.66)	(39.90)	(48.04)	(39.61)	(29.85)
β^{HML}	−.42	−.12	−.03	.12	.16	.31	.50	.61	.72	.97
$t\left(\beta^{HML}\right)$	(−12.13)	(−2.37)	(−.68)	(1.88)	(3.62)	(8.85)	(10.35)	(15.52)	(21.04)	(14.14)

Panel B-2: Simulated data

	Growth									Value
Portf.	1	2	3	4	5	6	7	8	9	10
α	−.01	.02	.07	.06	.09	.10	.11	.03	.07	.13
$t(\alpha)$	(−1.15)	(1.24)	(4.50)	(3.44)	(5.26)	(4.85)	(5.38)	(1.57)	(2.97)	(5.38)
β^M	.93	.97	1.01	1.05	1.08	1.11	1.11	1.10	1.09	.93
β^{HML}	−.28	−.21	−.09	−.01	.06	.08	.16	.31	.41	1.07

2. Fama-MacBeth regressions

Table VI
Fama and French (1993): Fama-MacBeth regressions (quarterly)

Panel A: Empirical data

	Const.	Mkt.	SMB	HML	Adj. R^2
2.	.36	1.63	−.31	1.05	80%
	(.23)	(.99)	(−.31)	(2.16)	

Panel B: Simulated data

	Const.	Mkt.	HML	Adj. R^2
6.	−.17	1.31	.94	99%
	(−1.64)	(11.85)	(28.69)	

(C) Conditional CAPM

1. Fama-MacBeth regressions

Table V
Conditional CAPM: Fama-MacBeth regressions (quarterly)

Panel A: Empirical data

	Const.	Mkt	Mkt×log(D/P)	Mkt×cay	Adj. R^2
3.	2.72	−.87	1.71		83%
	(2.24)	(−.65)	(2.46)		
4.	3.06	−1.37		.06	81%
	(2.48)	(−1.01)		(2.34)	

Panel B: Simulated data

	Const.	Mkt.	Mkt×log(D/P)	Adj. R^2
7.	.63	.38	1.16	98%
	(3.56)	(2.00)	10.11	

VI. Discussion: The size of the cash-flow risk effect

(A) Do value stocks have larger cash-flow risk?

- A key *implication* of our model is that value stocks are those with higher cash-flow risk: Is there evidence to support this implication?

Yes. For instance:

  - Cohen, Polk, and Vuolteenaho (2003), Parker and Julliard (2005), and Hansen, Heaton, and Li (2005) Campbell and Vuolteenaho (2005).
  - Example CPV (2003):

Table VII: Cash-flow betas

Cash-flow def.	Growth 1	2	3	4	5	6	7	8	9	Value 10
$\frac{X^p_{t+4,j+4} - X^p_{t-1,0}}{ME^p_{t-1,0}}$	.21	.66	1.46	1.61	.24	1.83	2.74	5.50	2.38	2.64
std. err.	(.19)	(.08)	(.52)	(.28)	(.61)	(.60)	(1.24)	(2.69)	(.60)	(1.65)
$\Sigma_{j=0}^4 \rho^j \Delta d^p_{t+j,j+1}$	.79	.90	.96	1.03	1.34	1.44	1.14	1.44	1.39	1.28
std. err.	(.19)	(.13)	(.10)	(.13)	(.28)	(.46)	(.31)	(.88)	(.77)	(.91)

(B) Sensitive analysis: Asset Pricing

- How sensitive are the results to the particular choice of θ^i_{CF} and $\overline{\nu}$?

1. Let's compute the basic return moments for several values of $\overline{\theta}_{CF}$:

$$\theta^i_{CF} \in [-\overline{\theta}_{CF}, \overline{\theta}_{CF}] \qquad \overline{\theta}_{CF}\,(\times 100) \in \{0, .1, .2, .3, .345\} \qquad \text{with} \qquad \overline{\nu} = .55$$

Table VII: Sensitivity with respect to $\overline{\theta}_{CF}$

Cash-flow risk	Market portfolio				Predictability				Value premium	
$\overline{\theta}_{CF} \times$ 100	$\overline{R}^M$	vol(R^M)	$\overline{r}^f$	vol(r^f)	b_{12}	R^2_{12}	b_{16}	R^2_{16}	10−1	CAPM 10−1
.0	9.90	24.16	1.16	5.44	.76	23.1	.78	22.4	−3.67	−3.86
.1	9.70	23.66	1.13	5.34	.74	21.9	.76	21.2	−3.27	−3.48
.2	8.99	21.95	1.01	5.05	.70	18.2	.72	17.3	−1.49	−1.70
.3	7.15	17.85	.81	4.60	.58	10.1	.59	9.0	2.83	2.19
.345	4.35	13.03	.69	4.36	.43	7.6	.47	7.1	5.16	1.83

– Why the equity premium, the volatility of returns and the predictability go down as we increase $\overline{\theta}_{CF}$?

* Intertemporal consumption smoothing effect.

* In our setup dc_t and $E_t\left[dc_t\right]$ are positively correlated.

* In habit persistence models ...

$$\downarrow dc_t \quad \Rightarrow \quad \downarrow S_t \quad \Rightarrow \quad \uparrow \frac{\gamma}{S_t} \quad \Rightarrow \quad \downarrow \frac{P_t}{C_t}$$

* ... but now

$$\downarrow dc_t \quad \Rightarrow \quad \downarrow E_t\left[dc_t\right] \quad \Rightarrow \quad \uparrow \frac{P_t}{C_t},$$

because the agent wants to smooth consumption intertemporally and desires to "transfer" consumption to the future, increasing prices in the process.

* This reduces the drop in prices $\Longrightarrow$ the volatility decreases, etc.

* This effect is stronger the larger the cash-flow risk effects:

$$\mu_{c,1}\left(\mathbf{s}_t\right) = \mathbf{s}_t'\theta_{CF}$$

2. Let's compute the basic moments for several values of $\overline{\nu}$. Let

$$\nu \in \{.25, .40, .55\} \qquad \text{with} \qquad \overline{\theta}_{CF} = .00345$$

– Recall that this parameter controls the volatility of the shares.

Table VII: Sensitivity with respect to $\overline{\nu}$

	Market portfolio				Predictability				Value premium	
$\overline{\nu}$	$\overline{R}^M$	vol(R^M)	$\overline{r}^f$	vol$\left(r^f\right)$	b_{12}	R^2_{12}	b_{16}	R^2_{16}	10−1	CAPM 10−1
.25	3.97	10.23	.67	4.20	.38	4.4	.43	4.1	7.10	6.70
.40	4.09	11.14	.68	4.23	.46	7.0	.51	6.5	6.29	4.57
.55	4.35	13.03	.69	4.36	.43	7.6	.47	7.1	5.16	1.83

– Changes in $\overline{\nu}$ do not affect the properties of the market portfolio but

– affect the ability of the CAPM to price the set of test portfolios. Why?

∗ The total wealth portfolio is not perfectly correlated with m_t.

∗ Higher idiosyncratic volatility of shares, higher variation in expected consumption growth, which is not correlated with shocks to consumption growth.

∗ Thus the worse performance of the CAPM

(C) Sensitivity Analysis: Dividend growth

- We have seen what our choices of $\overline{\theta}_{CF}$ and $\overline{\nu}$ imply for average returns?
- A natural question is what do these choices imply for:
  - the volatility of dividend growth,
  - the correlation coefficient between dividend and consumption growth and
  - the cash-flow betas of Cohen, Polk and Vuolteenaho (2003).

Table IX
The properties of the cash-flow process

$\overline{\nu}$	$\overline{\theta}_{CF} \times 100$	$[\underline{\rho}, \overline{\rho}]$	Avge(σ_D^i)	$\beta_{CF,1}^1$	$\beta_{CF,1}^{10}$	Avge(σ_R^i)
.25	0	[.04,.07]	24.88	1.04	.96	27.67
	.1	[−.21,.32]	24.29	.04	1.89	27.33
	.2	[−.48,.57]	22.44	−3.30	4.15	26.11
	.3	[−.76,.81]	18.92	−8.14	6.70	22.88
	.345	[−.89,.91]	16.39	−9.62	7.94	16.68
.40	0	[.02,.05]	40.04	1.09	.96	31.33
	.1	[−.13,.20]	39.65	.43	1.49	31.02
	.2	[−.29,.36]	38.50	−1.80	3.10	29.88
	.3	[−.46,.52]	36.55	−6.37	5.22	26.66
	.345	[−.53,.59]	35.40	−8.63	5.73	19.83
.55	0	[.01,.04]	56.20	1.17	.99	34.86
	.1	[−.10,.15]	55.87	.69	1.28	34.55
	.2	[−.21,.26]	54.96	−1.01	2.40	33.41
	.3	[−.32,.37]	53.47	−4.79	4.28	30.10
	.345	[−.37,.42]	52.60	−7.40	4.73	22.96

Conclusions

- The time varying market price of risk is helpful in addressing many of the time series properties of the market portfolio and interest rates (Campbell and Cochrane (1999)).
- This effect generates a counterfactual "growth premium" ...
- ... unless there is a sufficiently strong cross-sectional dispersion in cash-flow risk.
- We have shown that a model with substantial cross-sectional dispersion in cash-flow risk explains a large number of properties of the data:

(A) Time series properties of the market portfolio.

(B) The value premium and the value premium puzzle.

(C) The performance of the Fama and French (1993) model and, in particular, the role of HML and the performance of the conditional CAPM model.

(D) The dynamics of the value premium.

Recursive Preferences and Long Run Risk

- A different strand of literature focuses on recursive preferences.
  - Disentangle risk aversion from intertemporal substitution.
  - Could be useful, because we have seen that EIS generates a lot of troubles.
- Consider first the iso-elastic utility function

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}$$

- If C is stochastic, then $\gamma = -CU_{cc}/U_c$ is the coefficient of relative risk aversion.
- In an intertemporal model, with deterministic consumption $C_1, C_2, ...$ $\psi = 1/\gamma$ instead measures also the elasticity of intertemporal substitution.
- That is, the derivative of planned log consumption growth with respect to log interest rate

$$\psi = \frac{d\left(C_{t+1}/C_t\right)/\left(C_{t+1}/C_t\right)}{dR/R}$$

- This measures the willingness to exchange consumption today for consumption tomorrow, given the interest rate R.

Recursive Preferences and Long Run Risk

- There is no need to have such a tight relationship between the relative risk aversion coefficient and the elasticity of intertemporal substitutions.
  - Very different concepts: one applies to stochastic variables, the other to deterministic consumption paths.
- This separation is accomplished by the use of recursive utility functions.
  - For example, consider a simple two period model. At time $t = 0$ you know that your consumption is C_0.
  - However, at $t = 1$, you may receive the stochastic consumption $\widetilde{C}_1$.
  - Given the distribution of $\widetilde{C}_1$, you can think what is the level of certain consumption at time $t = 1$ that indeed is equivalent to $\widetilde{C}_1$.
  - Say this is $\overline{C}_1 = m\left(\widetilde{C}_1\right)$. Clearly, the function $m(.)$ measures the "risk-aversion."
  - Now, we can compare the consumption today C_0 and the deterministic consumption tomorrow $\overline{C}_1$ by using some conventional utility function defined on two commodities $W\left(C_1, \overline{C}_2\right)$.
  - Clearly, the function $W\left(C_1, \overline{C}_2\right)$ measures only the substitution preferences across the two periods and not the "risk aversion" component.

Recursive Preferences and Long Run Risk

- Recursive utility functions generalize the above.
- They are in fact defined by the following ingredients:

  I. V_t is the "utility" at time t. $\widetilde{V}_{t+1}$ denotes the fact that it is stochastic in the future (as of time t or before).

  II. A certainty equivalent function $m\left(.|\mathcal{F}_t\right)$ defined on the future stochastic utility $\widetilde{V}_{t+1}$

  III. An aggregator function $W\left(.,.\right)$ defined on current consumption and the certainty equivalent function.

- Specifically, we have that the utility at time t is given by

$$V_t = W\left(C_t, m\left[\widetilde{V}_{t+1}|\mathcal{F}_t\right]\right)$$

  – The certainty equivalent $m\left[\widetilde{V}_{t+1}|\mathcal{F}_t\right]$ "records" the risk aversion component;

  – The function $W\left(x,y\right)$ records the relative preference for a good x today or the "certainty equivalent" of utility $\widetilde{V}_{t+1}$, y, tomorrow.

Long Run Risk

- Aggregate dividends:

$$\frac{dD_t}{D_t} = g_t dt + \boldsymbol{\sigma}_D d\mathbf{B}_t$$

- Drift rate of dividends:

$$dg_t = (\eta - \eta_1 g_t)\, dt + \boldsymbol{\sigma}_g d\mathbf{B}_t$$

- In a nutshell, long run risk is the risk that is embedded in stocks due to their sensitivity to g_t.

- Let returns be given by

$$dR = (r(g_t) + \mu(g_t))\, dt + \boldsymbol{\sigma}_R(g_t)\, d\mathbf{B}_t$$

- where r, μ and σ_R will be determined in equilibrium.

Recursive Preferences in Continuous Time

- Consider a (representative) agent with Epstein - Zin (EZ) preferences.
- The agent maximizes

$$J_t = E_t \left[\int_t^\infty f\left(C_\tau, J_\tau\right) d\tau \right]$$

- subject to the usual wealth equation.
- The function $f(C,J)$ is the (normalized) *aggregator* of current consumption and continuation value.
- Under EZ preferences, we have

$$f\left(C,J\right) = \frac{\phi}{\rho} \alpha J \left(\left(\frac{C}{\left(\alpha J\right)^{\frac{1}{\alpha}}} \right)^{\rho} - 1 \right)$$

- where

$$\rho \; = \; 1 - \frac{1}{\psi}; \alpha = 1 - \gamma$$

- and $\gamma = RRA$ and $\psi = EIS$.

The Bellman Equation

- The Bellman Equation is

$$0 = \max_{C,\theta} f(C,J) + J_g E[dg] + J_W E[dW] \quad (1)$$
$$+\frac{1}{2}\left(J_{gg}E\left[dg^2\right] + 2J_{gW}E[dgdW] + J_{WW}E\left[dW^2\right]\right) \quad (2)$$

- The solution strategy is as usual.

I. The FOC with respect to C and θ are

$$f_c = J_W$$

$$0 = J_W W \mu(g) + J_{gW} W \boldsymbol{\sigma}_R \boldsymbol{\sigma}'_g + J_{WW} W^2 \theta \boldsymbol{\sigma}_R \boldsymbol{\sigma}_R$$

II. Conjecture:

$$J(W,g) = F(g)\frac{W^\alpha}{\alpha}$$

III. Compute J_W, $J_W W$, etc.

The Solution

IV. Compute

$$C = \phi^{-\frac{1}{\rho-1}} F(g)^{\frac{\rho}{\alpha}\frac{1}{\rho-1}} W$$

$$\theta_t = \frac{1}{1-\alpha}\frac{\mu_t}{\boldsymbol{\sigma}_R\boldsymbol{\sigma}'_R} + \frac{1}{1-\alpha}\frac{\boldsymbol{\sigma}_g\boldsymbol{\sigma}'_R}{\boldsymbol{\sigma}_R\boldsymbol{\sigma}'_R}\frac{F_g}{F}$$

V. Resubstitute everything back into the Bellman Equation

$$\begin{aligned} 0 = {} & \alpha\left(\frac{1}{\rho}-1\right)\phi^{-\frac{1}{\rho-1}} F(g)^{\frac{1}{\alpha}\frac{\rho}{\rho-1}} - \frac{\phi}{\rho}\alpha + \frac{F_g}{F}(\eta-\eta_1 g_t) + \alpha\theta_t\mu(g) + \alpha r(g) \\ & + \frac{1}{2}\left(\frac{F_{gg}}{F}\boldsymbol{\sigma}_g\boldsymbol{\sigma}'_g + 2\frac{F_g}{F}\alpha\theta\boldsymbol{\sigma}_R\boldsymbol{\sigma}'_g + \alpha(\alpha-1)\theta^2\boldsymbol{\sigma}_R\boldsymbol{\sigma}'_R\right) \end{aligned}$$

VI. In a portfolio problem, we would substitute θ as well, and solve the resulting PDE. Here, instead, we use market clearing conditions.

– But the type of solution is similar.

Market Clearing

- Use the equilibrium condition $\theta_t = 1$ to obtain two equations

  I. Equity Premium

$$\mu_t = (1-\alpha)\,\boldsymbol{\sigma}_R\boldsymbol{\sigma}_R' - \boldsymbol{\sigma}_g\boldsymbol{\sigma}_R'\frac{F_g}{F}$$

  II. Bellman Equation

$$\begin{aligned} 0 \;=\; & \alpha\left(\frac{1-\rho}{\rho}\right)\phi^{-\frac{1}{\rho-1}}F\,(g)^{\frac{1}{\alpha}\frac{\rho}{\rho-1}} - \frac{\phi}{\rho}\alpha + \frac{F_g}{F}\,(\eta - \eta_1 g_t) \\ & + \frac{1}{2}\alpha\,(1-\alpha)\,\boldsymbol{\sigma}_R\boldsymbol{\sigma}_R' + \alpha r\,(g) + \frac{1}{2}\frac{F_{gg}}{F}\boldsymbol{\sigma}_g\boldsymbol{\sigma}_g' \end{aligned}$$

- We still need to determine $\boldsymbol{\sigma}_R\boldsymbol{\sigma}_R'$ and $r(g)$.
- Use market clearing conditions

$$C \;=\; D; \quad W = P$$

- Substitute in the consumption equation

$$C = \phi^{-\frac{1}{\rho-1}}F\,(g)^{\frac{\rho}{\alpha}\frac{1}{\rho-1}}\,W$$

Consumption Claim

- we obtain the price of a consumption claim

$$P_t = C_t \ \phi^{\frac{1}{\rho-1}} \ F(g_t)^K$$

where

$$K = \frac{\rho}{\alpha}\frac{1}{1-\rho}$$

- Use Ito's Lemma to find

$$\frac{dP}{P} = \mu_P dt + \boldsymbol{\sigma}_P d\mathbf{B}_t$$

- where

$$\begin{aligned} \mu_P &= \left(g_t + K\frac{F_g}{F}(\eta - \eta_1 g_t) + \frac{1}{2}\left(K(K-1)\left(\frac{F_g}{F}\right)^2 + K\frac{F_{gg}}{F}\right)\boldsymbol{\sigma}_g\boldsymbol{\sigma}'_g + K\frac{F_g}{F}\boldsymbol{\sigma}_g\boldsymbol{\sigma}'_D\right) \\ \boldsymbol{\sigma}_P &= \boldsymbol{\sigma}_R = \left(\boldsymbol{\sigma}_D + K\frac{F_g}{F}\boldsymbol{\sigma}_g\right) \end{aligned}$$

- We can substitute $\boldsymbol{\sigma}_R$ into the BE. But we still need the risk free rate $r(g)$.

Consumption Claim

- We know that

$$E\left[\frac{dP}{P}+\frac{C}{P}dt\right]-r(g)=\mu_t$$

- Thus from above

$$r(g) \;=\; \mu_P+\frac{C}{P}-\mu_t$$

  – Note: μ_P comes from Ito's Lemma above (dP/P), while μ_t comes from the equilibrium condition $\theta_t=1$.

- Finally, substitute *everything* back in the Bellman Equation to obtain

$$\begin{aligned} 0 \;=\; & \alpha\frac{1}{\rho}\phi^{-\frac{1}{\rho-1}}F\left(g\right)^{\frac{1}{\alpha}\frac{\rho}{\rho-1}}-\frac{\phi}{\rho}\alpha+\alpha g_t+\left(1+\alpha K\right)\frac{F_g}{F}\left(\eta-\eta_1 g_t\right)-\frac{1}{2}\alpha\left(1-\alpha\right)\boldsymbol{\sigma}_D\boldsymbol{\sigma}_D \\ & +\left(1+K\alpha\right)\alpha\frac{F_g}{F}\boldsymbol{\sigma}_D\boldsymbol{\sigma}_g'+\left(1+\alpha K\right)\frac{1}{2}\frac{F_{gg}}{F}\boldsymbol{\sigma}_g\boldsymbol{\sigma}_g'+\left(1+\alpha K\right)\frac{1}{2}\alpha K\left(\frac{F_g}{F}\right)^2\boldsymbol{\sigma}_g\boldsymbol{\sigma}_g' \end{aligned}$$

- It looks tough, but we can apply Campbell and Viceira log-linearization methodologies.

Log-Linear Solution

- Log linearization: The first term is

$$\frac{C}{W} = \phi^{-\frac{1}{\rho-1}} F\left(g\right)^{\frac{\rho}{\alpha}\frac{1}{\rho-1}}$$

- Approximate

$$\frac{C}{W} \approx h_0 + h_1 \left(c - w\right)$$

  – where $h_1 = e^{\overline{c-w}}$ and $h_0 = h_1(1 - log(h_1))$.

- Taking logs in C/W

$$c - w = -\frac{1}{\rho - 1} \log\left(\phi\right) - K \log\left(F\left(g\right)\right)$$

- we then obtain the approximation

$$\begin{aligned} \frac{C}{W} &= \phi^{-\frac{1}{\rho-1}} F\left(g\right)^{\frac{\rho}{\alpha}\frac{1}{\rho-1}} \\ &\approx h_0 + h_1 \left(c - w\right) \\ &= h_0 - \frac{h_1}{\rho - 1} \log\left(\phi\right) - h_1 K \log\left(F\left(g\right)\right) \end{aligned}$$

An Approximate Solution to the PDE

- Substitute in the PDE

$$
\begin{aligned}
0 \;\approx\;& \alpha\frac{1}{\rho}h_0 - \frac{\alpha}{\rho}\frac{h_1}{\rho-1}\log\left(\phi\right) - \frac{\alpha}{\rho}h_1 K\log\left(F\left(g\right)\right) \\
& -\frac{\phi}{\rho}\alpha + \alpha g_t + \left(1+\alpha K\right)\frac{F_g}{F}\left(\eta - \eta_1 g_t\right) - \frac{1}{2}\alpha\left(1-\alpha\right)\boldsymbol{\sigma}_D\boldsymbol{\sigma}_D \\
& + \left(1+K\alpha\right)\alpha\frac{F_g}{F}\boldsymbol{\sigma}_D\boldsymbol{\sigma}_g' + \left(1+\alpha K\right)\frac{1}{2}\frac{F_{gg}}{F}\boldsymbol{\sigma}_g\boldsymbol{\sigma}_g' + \left(1+\alpha K\right)\frac{1}{2}\alpha K\left(\frac{F_g}{F}\right)^2\boldsymbol{\sigma}_g\boldsymbol{\sigma}_g'
\end{aligned}
$$

- The solution to this PDE has the form

$$F\left(g\right) = e^{A_0 + A_1 g}$$

- Use method of undetermined coefficients and find

$$A_1 \;=\; \frac{\alpha\left(1-\rho\right)}{h_1+\eta_1}$$

- and another equation for A_0.

The Results

I. Price consumption ratio

$$\frac{P_t}{C_t} = \phi^{-\psi} \exp\left(K A_0 + \left(\frac{1 - 1/\psi}{h_1 + \eta_1} \right) g_t \right)$$

- Notably: P/C is increasing in g_t iff $EIS = \psi > 1$
- Powerful additional variation in prices due to variation in g_t.
- E.g. With learning, D_t and g_t are positively correlated $\Longrightarrow$ higher premium than $EIS < 1$.

II. Diffusion term in dR

$$\boldsymbol{\sigma}_R = \boldsymbol{\sigma}_D + \frac{1 - 1/\psi}{h_1 + \eta_1} \boldsymbol{\sigma}_g$$

- The diffusion component of returns shows two sources of risk
  (A) Contemporaneous dividend shocks, from D_t
  (B) Long Run risk, from g_t
- Second component is higher for $EIS > 1$.

The Results

III. Equity premium

$$\mu_t = \gamma \boldsymbol{\sigma}_R \boldsymbol{\sigma}_R' - \frac{\gamma(1-1/\psi)}{h_1+\eta_1}\boldsymbol{\sigma}_R \boldsymbol{\sigma}_g'$$
$$= \gamma \boldsymbol{\sigma}_D \boldsymbol{\sigma}_D' + \left(\frac{2\gamma - \gamma/\psi - 1/\psi}{h_1+\eta_1}\right)\boldsymbol{\sigma}_D \boldsymbol{\sigma}_g' + \left(\frac{1-1/\psi}{h_1+\eta_1}\right)\left(\frac{\gamma-1/\psi}{h_1+\eta_1}\right)\boldsymbol{\sigma}_g \boldsymbol{\sigma}_g'$$

- The first equation shows that if $EIS > 1$, then the equity premium increase because $\boldsymbol{\sigma}_R \boldsymbol{\sigma}_R'$ increases, but it may decrease because of the Merton hedging demand component $\boldsymbol{\sigma}_R \boldsymbol{\sigma}_g'$

IV. Risk free rate

$$r = \phi + \frac{1}{\psi} g_t - \frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right)\boldsymbol{\sigma}_D \boldsymbol{\sigma}_D - \frac{1}{2}\gamma\left(\frac{1-\frac{1}{\psi}}{h_1+\eta_1}\right)^2 \boldsymbol{\sigma}_g \boldsymbol{\sigma}_g' + \left(\frac{\frac{1}{\psi}-\gamma}{h_1+\eta_1}\right)\boldsymbol{\sigma}_g \boldsymbol{\sigma}_D'$$

- The risk-free rate puzzle was due to γ multiplying g_t under CRRA utility.
- We can now increase γ without affecting the EIS, resolving in part the risk free puzzle.

Quantitative Results

- Can this model explain the various puzzles *quantitatively*?
  - Some, but not all.
  - The following table uses the parameters obtained by Bansal and Yaron (2005, JF).
  - In monthly units: $E[dC/C] = \eta/\eta_1 = .0015$, $\eta_1 = .0212$,$\sigma_c = .0078$, $\sigma_g = 0.3432 \times 10^{-3}$

		Consumption Claim		Risk Free Rate	
γ	ψ	μ_R	σ_R	r_f	$\sigma(r_f)$
7.5	0.5	-0.81	5.65	4.26	4.00
7.5	1.5	1.15	3.20	3.04	1.33
10	0.5	-1.28	5.70	3.65	4.00
10	1.5	1.55	3.20	2.85	1.33
45	0.5	-9.34	6.05	-5.53	3.99
45	1.5	6.71	3.14	0.29	1.33

- In addition, expected returns and volatility are constant.

Extension 1: Dividend Claim

- Consider an additional asset whose dividend follows the process

$$\frac{d\delta}{\delta} = (\mu_d + \lambda g_t)\, dt + \boldsymbol{\sigma}_\delta d\mathbf{B}_t$$

  - λ consumption leverage parameter (Abel (1990)).
    - Measure of long run cash flow risk.
  - $\sqrt{\boldsymbol{\sigma}_\delta \boldsymbol{\sigma}'_\delta}$ = dividend volaility.
    - Higher than consumption volatility.

- Same methodology as before.
- Price of dividend claim

$$\frac{S_t}{\delta_t} = \exp\left(A_0^\delta + \left(\frac{\lambda - 1/\psi}{h_1^\delta + \eta_1}\right) g_t\right)$$

Extension 1: Dividend Claim

- The diffusion of stock return

$$\boldsymbol{\sigma}_R^\delta = \boldsymbol{\sigma}_\delta + \left(\frac{\lambda - 1/\psi}{h_1^\delta + \eta_1}\right)\boldsymbol{\sigma}_g$$

  - A higher λ increases the volatility of stock returns

- The return premium of the dividend claim must be given by

$$\mu_R^\delta = \gamma \boldsymbol{\sigma}_R^\delta \boldsymbol{\sigma}'_R - \frac{\gamma\left(1 - 1/\psi\right)}{h_1 + \eta_1}\boldsymbol{\sigma}_R^\delta \boldsymbol{\sigma}'_g$$

  - A higher λ increases the equity risk premium.

Quantitative Results

- Can this model explain the returns and volatility *quantitatively*?
  - Yes.

		Dividend Claim		Risk Free Rate		
γ	ψ	μ_R	σ_R	r_f	$\sigma(r_f)$	$\overline{\log(P/D)}$
Panel A: $\lambda = 3$, $\eta_1 = 0.0212$						
7.5	0.5	1.34	13.11	4.26	4.00	3.30
7.5	1.5	3.90	16.45	3.04	1.33	3.10
10	0.5	1.96	13.11	3.65	4.00	3.30
10	1.5	5.13	16.21	2.85	1.33	2.89
Panel B: $\lambda = 3.5$, $\eta_1 = 0.0212$						
7.5	0.5	1.96	14.12	4.26	4.00	3.18
7.5	1.5	4.66	17.98	3.04	1.33	3.00
10	0.5	2.86	14.10	3.65	4.00	3.11
10	1.5	6.07	17.58	2.85	1.33	2.76

Extension 2: Stochastic Volatility

- Assume

$$\frac{dD_t}{D_t} = g_t dt + \sqrt{v_t}\boldsymbol{\sigma}_D d\mathbf{B}_t$$

$$\frac{d\delta}{\delta} = (\mu_d + \lambda g_t)\, dt + \sqrt{v_t}\boldsymbol{\sigma}_\delta d\mathbf{B}_t$$

where

$$dg_t = (\eta - \eta_1 g_t)\, dt + \sqrt{v_t}\boldsymbol{\sigma}_g d\mathbf{B}_t$$

$$dv_t = (n - n_1 v_t)\, dt + \sqrt{v_t}\boldsymbol{\sigma}_v d\mathbf{B}_t$$

- Use the same methodology.

Results

I. Price consumption ratio

$$\frac{P_t}{C_t} = \phi^{-\psi} \exp\left(KA_0 + \left(\frac{1-1/\psi}{h_1+\eta_1}\right) g_t + A_2^c v_t\right)$$

- $A_2^c < 0$: An increase in consumption volatility decreases the P/C ratio.

II. The consumption claim equity premium

$$\mu_t = v_t\left(\gamma \widetilde{\boldsymbol{\sigma}}_R \widetilde{\boldsymbol{\sigma}}'_R - \frac{\gamma\left(1-1/\psi\right)}{h_1+\eta_1}\boldsymbol{\sigma}_g \widetilde{\boldsymbol{\sigma}}'_R - A_2 \boldsymbol{\sigma}_v \widetilde{\boldsymbol{\sigma}}'_R\right)$$

where

$$\widetilde{\boldsymbol{\sigma}}_R = \boldsymbol{\sigma}_D + \frac{1-1/\psi}{h_1+\eta_1}\boldsymbol{\sigma}_g + KA_2\boldsymbol{\sigma}_v$$

Results

III. The price dividend ratio of dividend claim

$$\frac{S_t}{\delta_t} = \exp\left(A_0^\delta + \left(\frac{\lambda - 1/\psi}{h_1^\delta + \eta_1}\right) g_t + A_2^\delta v_t\right)$$

- $A_2^\delta < 0$: An increase in consumption volatility decreases the P/D ratio.

IV. The dividend claim equity premium

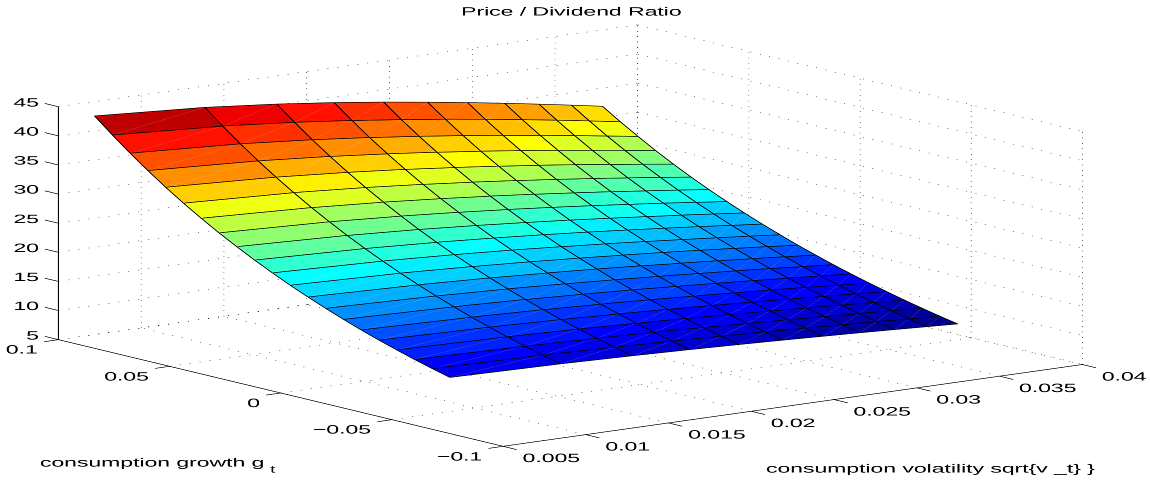
$$\mu_R^\delta = v_t \left(\gamma \widetilde{\boldsymbol{\sigma}}_R^\delta \widetilde{\boldsymbol{\sigma}}'_R - \frac{\gamma\,(1 - 1/\psi)}{h_1 + \eta_1} \widetilde{\boldsymbol{\sigma}}_R^\delta \boldsymbol{\sigma}'_g - A_2 \widetilde{\boldsymbol{\sigma}}_R^\delta \boldsymbol{\sigma}'_v\right)$$

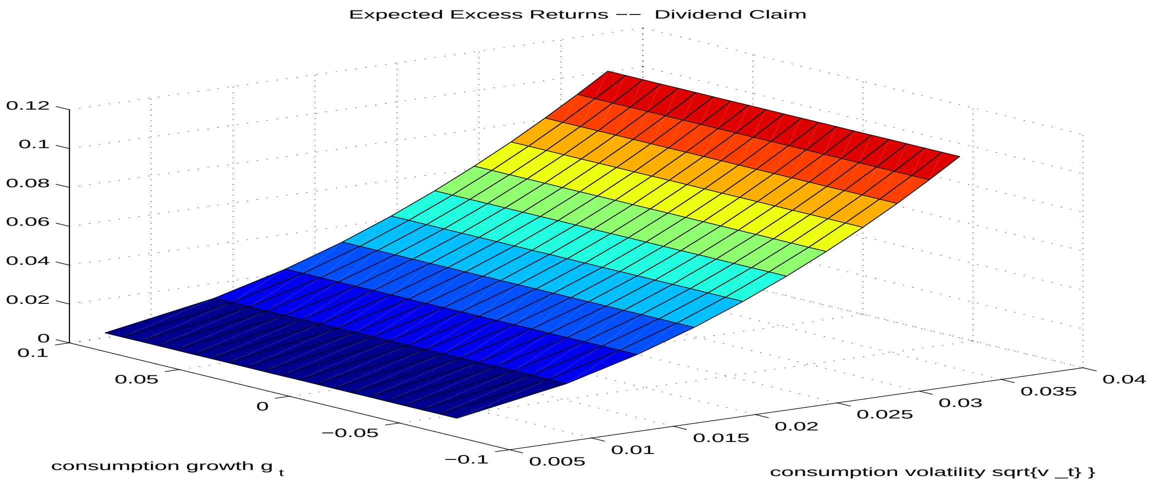
where

$$\widetilde{\boldsymbol{\sigma}}_R^\delta = \left(\boldsymbol{\sigma}_\delta + \left(\frac{\lambda - 1/\psi}{h_1^\delta + \eta_1}\right) \boldsymbol{\sigma}_g + A_2^\delta \boldsymbol{\sigma}_v\right)$$

Quantitative Results

- Using the parameters in Bansal and Yaron (2006)





Quantitative Results

- Especially along the volatility axis $\sqrt{v_t}$, there is a negative relation between P/D and $E_t[dR_t]$
  - $\Longrightarrow$ Predictability of stock returns

Recent Application: The Cross-Section of Stock Returns

- Bansal, Dittmar and Lundbland (2005, JF) show that value stocks have a higher cash flow risk λ
- They run a regression on quarterly data

$$g_{i,t} = \gamma_i \left(\frac{1}{K} \sum_{k=1}^{K} g_{c,t-k} \right) + u_{i,t} \qquad K = 8$$

where

- $g_{i,t} \rightarrow$ Demeaned log real dividend growth rate on portfolio i.
- $g_{c,t} \rightarrow$ Demeaned log real growth rate in aggregate consumption.

Cash-flow betas: Bansal et al. (2005)

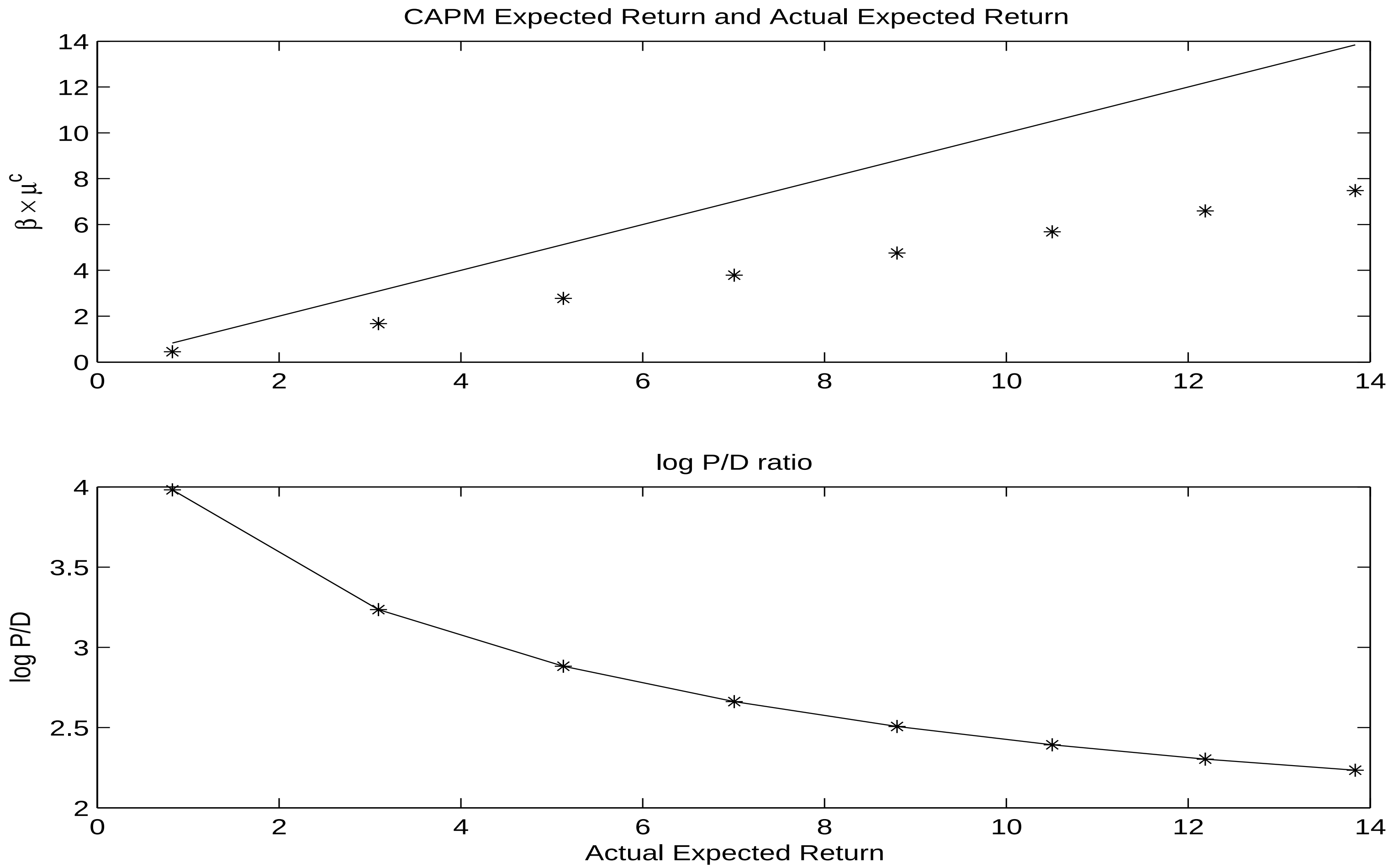
Cash-flow def.	Growth									Value
	1	2	3	4	5	6	7	8	9	10
γ_i	2.98	− 3.43	.02	−.28	.46	1.70	.78	4.45	4.74	8.44
std. err.	(2.90)	(2.27)	(2.39)	(2.81)	(1.81)	(1.51)	(1.14)	(1.66)	(3.08)	(4.08)

Cash Flow Risk and the Cross-Section of Stock Returns

- For $\lambda = 3$: $ER = 5.13\%$ and $\log(P/D) = 2.89$;
- For $\lambda = 8$: $ER = 13.83\%$ and $\log(P/D) = 2.23$;
  - Theoretically: high P/D correlates with low ER
  - $\Longrightarrow$ Value premium
  - References: Hansen, Heaton and Li (2005), Kiku (2005)
- This is good: But this per se' does *not* resolve the Value Premium *Puzzle*
  - One needs to show that market beta does not explain the return differential
  - Need of a full fledged calibration / simulation.
- For instance, the theoretical betas with respect to consumption claim are
  - $\lambda = 3$: $\beta = \left(\sigma_R^\delta \sigma'_R\right) / \left(\sigma_R \sigma'_R\right) = 1.79$
  - $\lambda = 8$: $\beta = \left(\sigma_R^\delta \sigma'_R\right) / \left(\sigma_R \sigma'_R\right) = 4.83$
- $\Longrightarrow$ value has a higher beta than growth.
- The question is then whether it is sufficiently high to justify the spread differential (in the model).

Long Run Risk and Value Premium Puzzle

- The following figure plots $E[dR^{\delta}]$ versus $\beta \times \mu^c$ for $\lambda = 1, ..., 8$



Long Run Risk and Value Premium Puzzle

- Delicate interpretation of these results:
  - Bansal et al (2005) estimates of λ are at the *portfolio* level.
  - I.e. these are the characteristics of mutual funds that pay dividends according to a specific trading strategy
    * Stocks are sorted by M/B and placed in bins.
    * Dividends are calculated as the total dividend payouts from these portfolios
    * Importantly, the amount reinvested in the portfolio at year end is equal to the total capital gain.
  - Characteristics of *portfolio* cash flows may differ from those of value and growth *firms*
    * E.g. Average growth rate of cash flows is 4% / year for value , while it is .76%/year for growth
    * Curious result: At the individual firm level, Fama and French show that value firms grow *less* than growth firms.
    * But portfolio cash flows are contaminated by re-investment policy.
  - Deeper investigation needed.

Conclusions

- Two leading models to explain asset returns in macro finance
  I. Habit preferences $\Longrightarrow$ variation in market price of risk.
  II. Long run risk $\Longrightarrow$ variation in the amount of risk.
- Habit preferences explain a wide variety of facts
  - But need to assume unrealistic amount of cash flow risk to overcome growth premium induced by "discount effects"
- Long run risk also explain a wide variety of facts
  - But research so far has only looked at portfolio cash flows, and not individual cash flows.
  - Moreover, it is not a general equilibrium model. Market clearing restrictions are not imposed.
- Long run risk is the hot topic of the moment. Habit has lost its allure.
- Additional applications
  - Lettau, Ludvigson and Wachter (Forthcoming, RFS): Lower consumption volatility pushed up prices in the 1990s.
  - Croce, Lettau and Ludvigson (2006): Learning, long run risk and the value premium