

Dynamic Asset Pricing Models: Recent Developments

Day 1: Asset Pricing Puzzles and Learning

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Introduction

A Canonical Model

- Aggregate dividends D_t are i.i.d.

$$\frac{dD_t}{D_t} = \mu_d dt + \sigma_d dB_t$$

- P_t = price of stock that is a claim on these dividends. r_t = risk free rate of return.
- A representative agent has infinite life, power utility over consumption, chooses C_t and asset allocation θ_t to

$$\max_{C_t, \theta_t} E_0 \left[\int_0^\infty e^{-\phi t} \frac{C_t^{1-\gamma}}{1-\gamma} dt \right]$$

- Equilibrium: $C_t = D_t$ and $\theta_t = 1 \implies \text{SDF} = \lambda_t = e^{-\phi t} C_t^{-\gamma}$

$$P_t = E_t \left[\int_t^\infty \frac{\lambda_\tau}{\lambda_t} D_\tau d\tau \right] = \frac{D_t}{R - \mu_d}$$

- where R = discount rate for risky stock

Puzzles in the Lucas Endowment Economy

- A large number of empirical regularities clash with this standard paradigm.

1. **Equity premium puzzle:** Stocks have averaged returns of about 7% over treasuries.

- This number is high compared to the volatility of consumption, of about 1-2%.
- The canonical model implies

$$\text{Expected Excess Return} = \gamma \text{Variance of Consumption Growth}$$

- Even assuming that γ is large, say $\gamma = 10$, we have

$$\text{Expected Excess Return} = 10 \times (.02)^2 = 0.4\%$$

- We are an order of magnitude off.

Puzzles in the Lucas Endowment Economy

2. **Volatility Puzzle 1:** Return volatility (about 16 %) is too high compared to the volatility of dividends (about 7%).

- The same classic canonical model has

$$\frac{P_t}{D_t} = \text{Constant}$$

- This implies

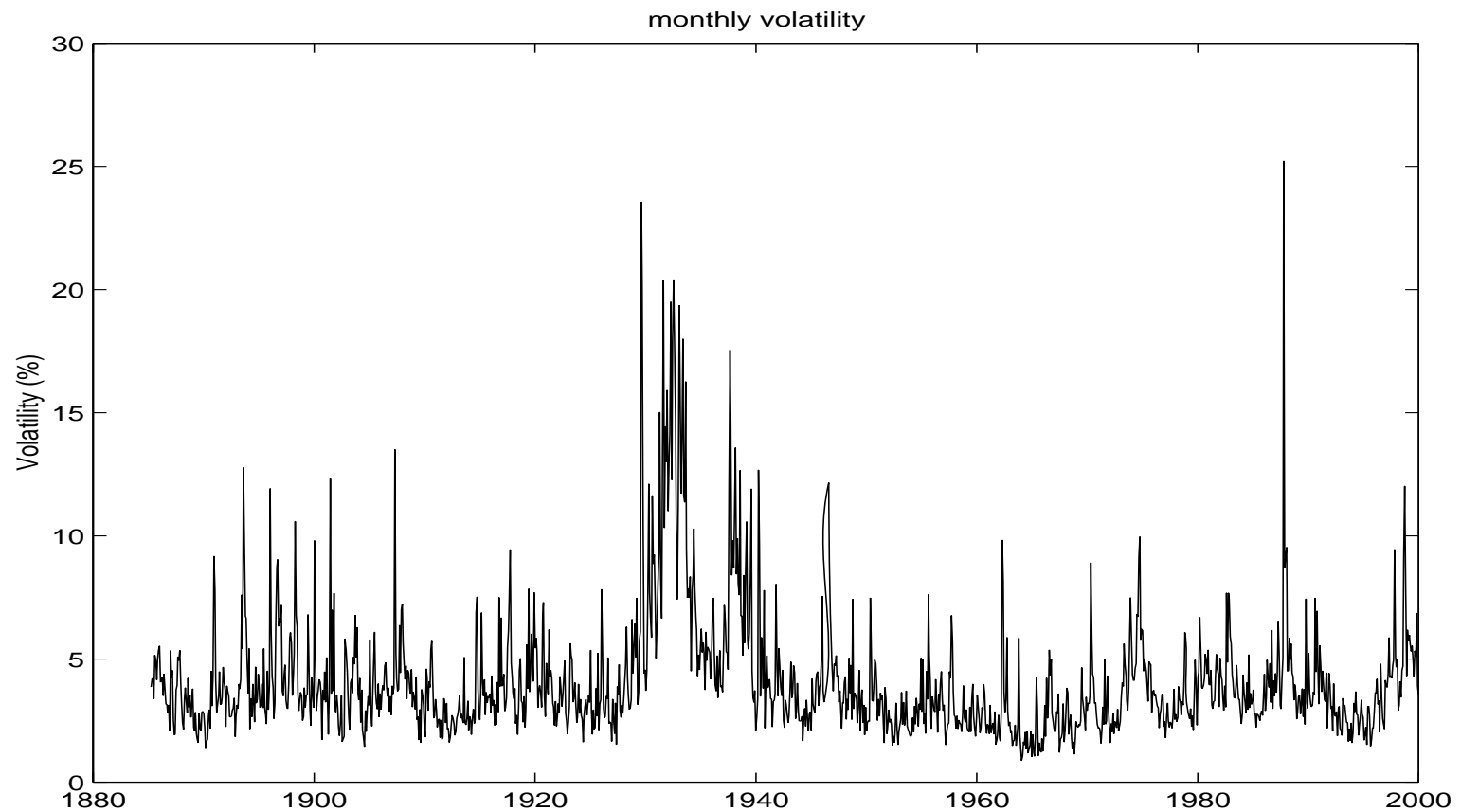
$$\text{Volatility of } \frac{dP_t}{P_t} = \text{Volatility of } \frac{dD_t}{D_t}$$

- Something else must be time varying to make the volatility higher.
- Indeed, the canonical model would imply a constant P/D ratio, which we know it is not.

Puzzles in the Lucas Endowment Economy

3. Volatility Puzzle 2: Return volatility is not only high, but it is time varying.

- Historically, monthly market return volatility fluctuated between 20 - 25 % in the 30s to less than 2% in the middle of the 1960s.



Puzzles in the Lucas Endowment Economy

4. **Risk Free Rate Puzzle:** The usual canonical model implies that the interest rate is given by

$$r = \phi + \gamma\mu_c - \frac{1}{2}\gamma(\gamma + 1)\sigma_c^2$$

- If $\gamma = 10$ for instance, using $\mu_c = 2\%$, $\sigma_c = 1\%$ and $\phi = 2\%$ we find $r = 21\%$
- The problem is γ that is too high: If we set $\gamma = 2$ we obtain $r = 6\%$.
- Note the tension between equity premium puzzle (need γ high) and risk free rate puzzle (need γ low).

Puzzles in the Lucas Endowment Economy

5. Predictability 1: Stock returns are predictable by, say, the dividend price ratio.

- Predictability regression

$$\text{Cumulated Returns } (t \rightarrow t + \tau) = \alpha + \beta \log \left(\frac{D_t}{P_t} \right) + \epsilon_{t,t+\tau}$$

Table: Forecasting Regression

Sample	Horizon (qtrs)			
1948 - 2001	4	8	12	16
$\log(D/P)$.13	.20	.26	.35
NW t-stat	(2.13)	(1.65)	(1.34)	(1.29)
Adj. R^2	.09	.10	.11	.14
Sample	Horizon (qtrs)			
1948 - 1994	4	8	12	16
$\log(D/P)$.28	.48	.63	.78
NW t-stat	(4.04)	(4.00)	(4.49)	(5.41)
Adj R^2	.19	.32	.43	.54

Puzzles in the Lucas Endowment Economy

- This result raises a number of issues, such as:
 - (a) Why are stock return predictable?
 - (b) Why the regression coefficients (and significance) depend on the time interval used?
 - (c) What are the implication for an investor who is allocating his wealth between stocks and bonds to maximize his life time utility?

Puzzles in the Lucas Endowment Economy

6. **Predictability 2:** From a basic canonical model, we have

$$\text{Expected Excess Return} = \gamma \text{Variance of Stock Return}$$

- Data show that expected excess returns are time varying (predictability) and variance of stock return is time varying.
- Are they related? Most of the empirical literature shows that there is very little relation between the two.
- For instance, a simple regression

$$\text{Cumulated Returns } (t \rightarrow t + \tau) = \alpha + \beta (\text{Monthly Vol}) + \epsilon_{t,t+\tau}$$

Puzzles in the Lucas Endowment Economy

Table: Forecasting Regression

Sample	Horizon (qtrs)			
1925 - 1999	4	8	12	16
Volatility	-.32	-.30	.82	1.59
NW t-stat	(-.32)	(-.20)	(.62)	(1.28)
Adj. R^2	.00	.00	.00	.01
Sample	Horizon (qtrs)			
1948 - 1994	4	8	12	16
Volatility	1.05	1.1	1.00	2.69
NW t-stat	(1.56)	(0.81)	(0.87)	(1.41)
Adj. R^2	.01	.01	.00	.02

- Using more sophisticated models for volatility, some studies find a significantly positive relation, but some others find a significant negative relation. There is still a considerable debate.

Puzzles in the Lucas Endowment Economy

7. **Cross-sectional Predictability Puzzle:** Some type of stocks yield an average return that is not consistent with the canonical model.

- The canonical model implies that expected excess returns of asset i is given by:

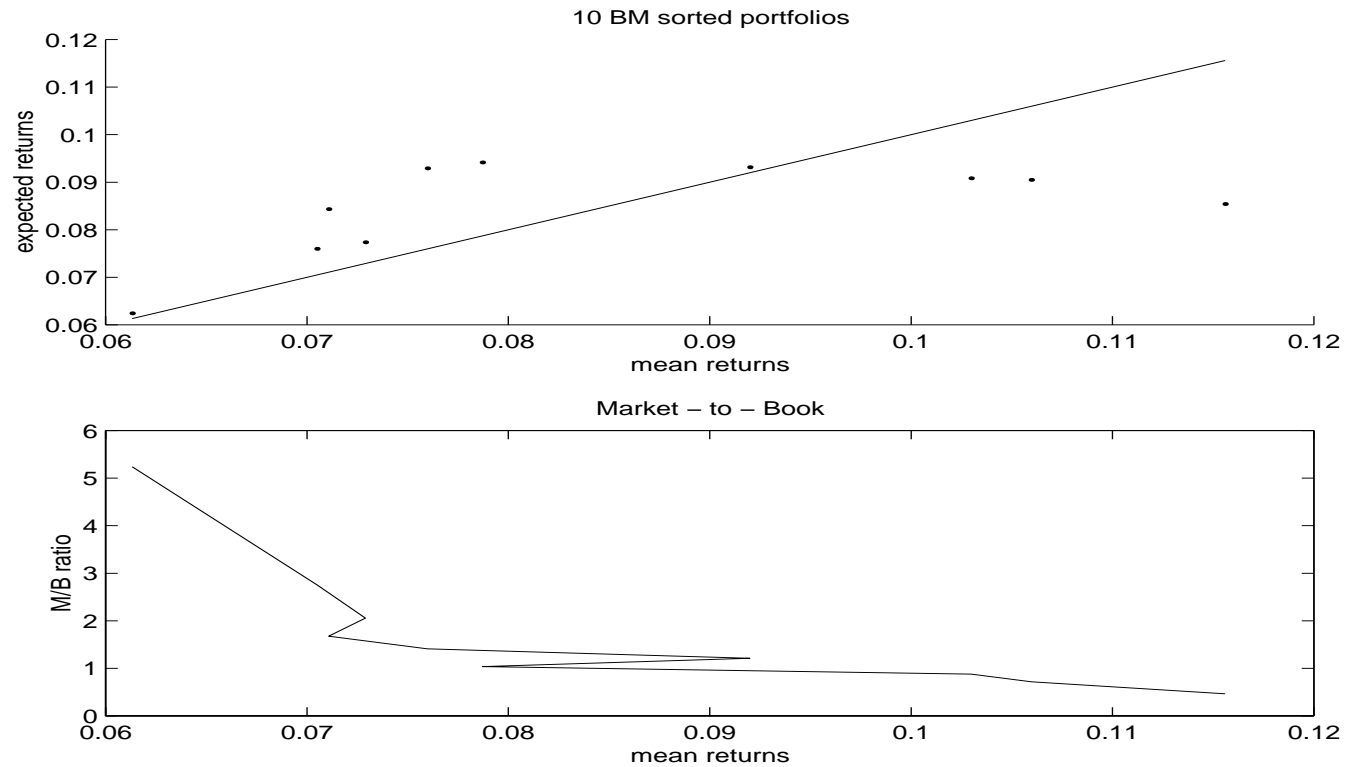
$$\begin{aligned} E \left[\text{Excess Return}_t^i \right] &= \gamma \text{Cov} \left(\text{Return}^i, \text{Consumption Growth} \right) \\ &= \beta^i E \left[\text{Excess Return of Mkt Portfolio} \right] \end{aligned}$$

- where

$$\beta^i = \frac{\text{Cov} \left(\text{Return}^i, \text{Return Mkt Portfolio} \right)}{\text{Var} \left(\text{Return Mkt Portfolio} \right)}$$

- Portfolios of stocks that are sorted by Book-to-Market Ratio or by Size and Book to Market do not satisfy this relation.
- For instance, using Book-to-Market sorted portfolios, we obtain the following

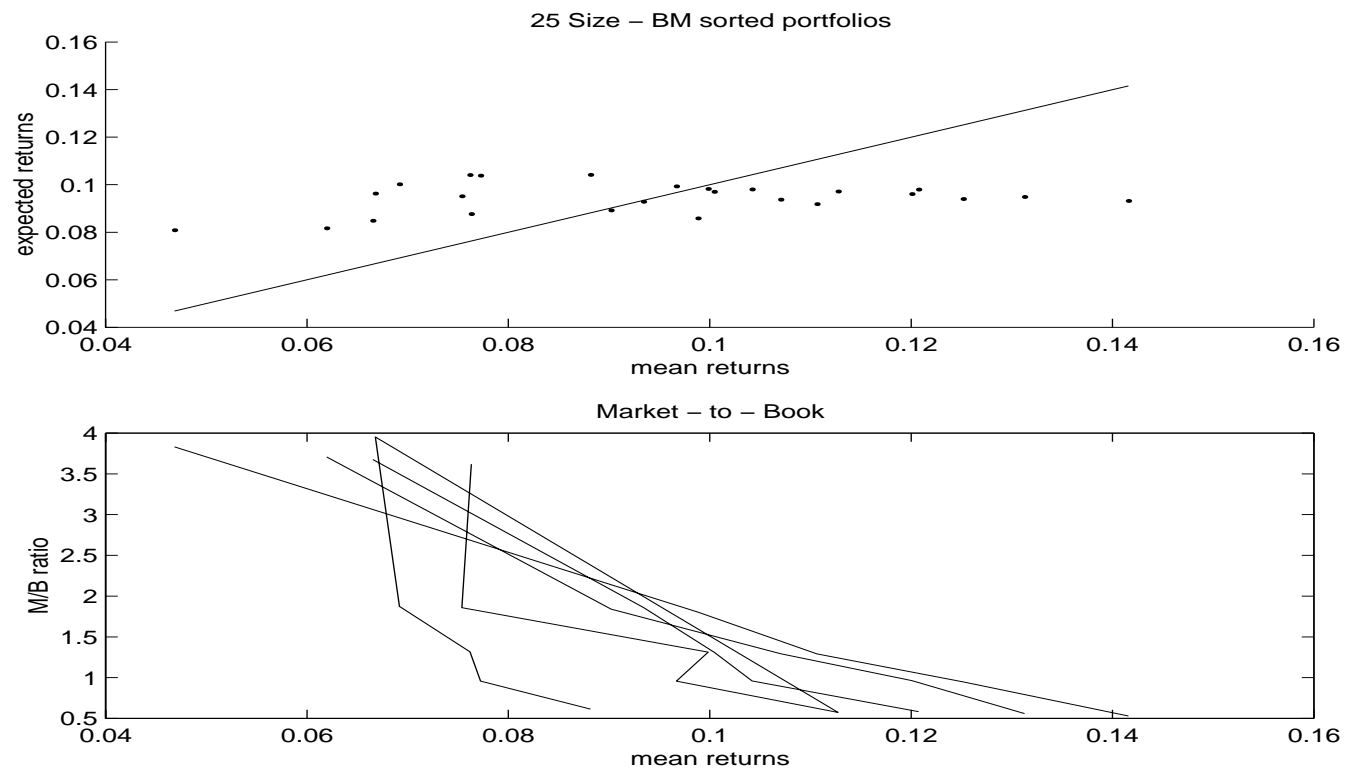
Puzzles in the Lucas Endowment Economy



- The top panel shows the the average return on B/M sorted portfolio on the x-axis, and the one implied by the CAPM (= $\beta \times$ Average Return of Market Portfolio) on the y-axis
- They should line up, but they don't

Puzzles in the Lucas Endowment Economy

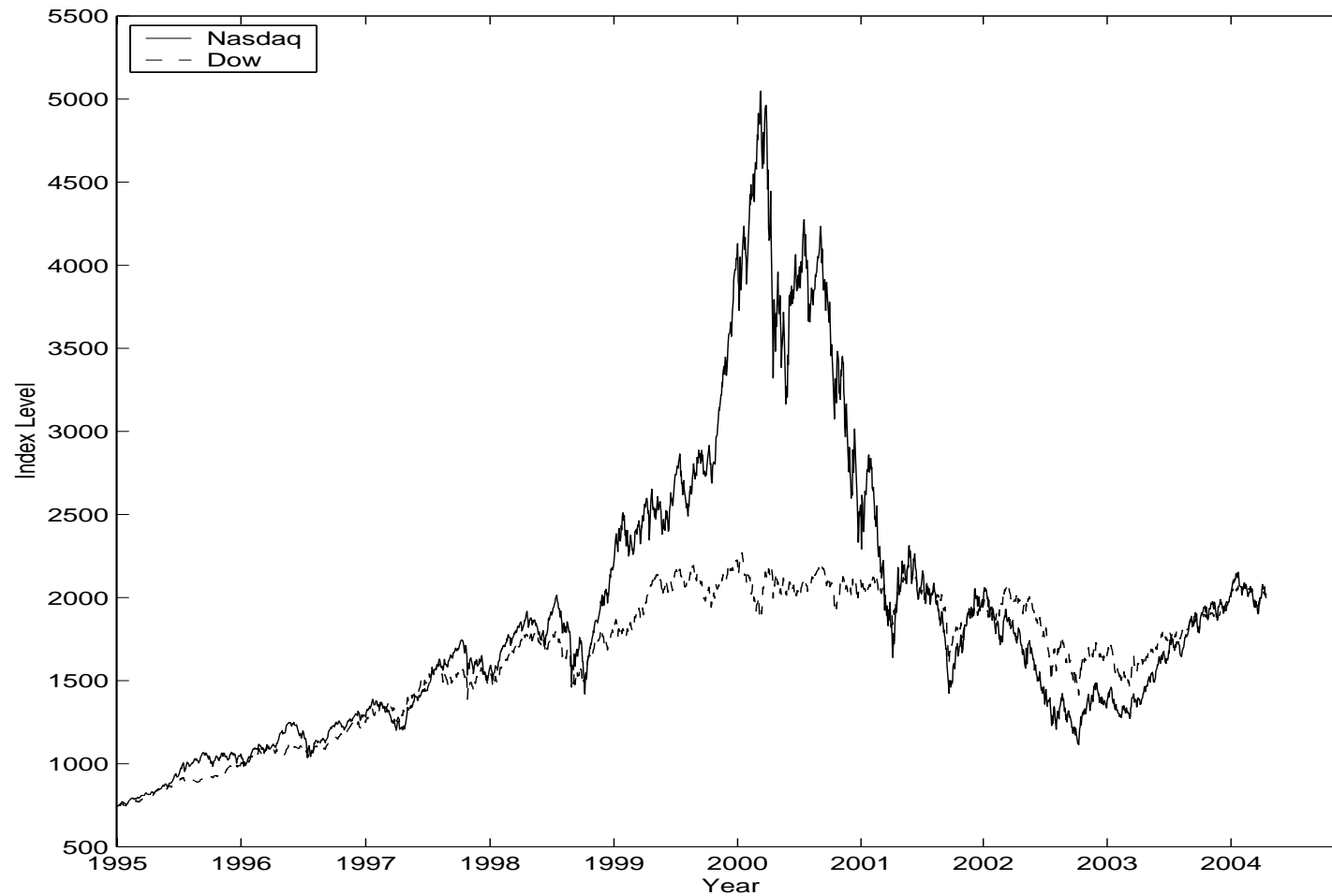
- It is even worse if one uses Size and Book-to-Market portfolios (the so-called FF 25 portfolios)



- Adding to this, momentum portfolios (sorted by past winners and losers) show similar and perhaps more striking pattern.

Puzzles in the Lucas Endowment Economy

8. Tech “Bubble”: Typical to talk about technology bubbles (e.g. late 1990s)



Puzzles in the Lucas Endowment Economy

- Was it a bubble?
- Why do stock prices tend to go up and then down around technological revolutions?
- Examples:
 - the early 1980s (biotechnology, PC)
 - the early 1960s (electronics)
 - the 1920s (electricity, automobiles)
 - the early 1900s (radio)

Overview of Lectures

- In these three days, we cover modern models of asset pricing to “explain” these facts.

1. Uncertainty and learning about future economic growth

- In the late 1990s, there was hope that Bayesian uncertainty may help address these facts.
- It doesn't.

2. Habit Formation Preferences

- Discuss how habit formation can successfully address most of the puzzles.
- This model turns out to be able to explain many other facts, in particular when it is “paired” with Bayesian uncertainty and learning about technological growth ...
- ... but it misses some other facts.

3. Risk for the long run

- Hot topic of the moment.
- Small predictable component in consumption growth may have large impact on prices
- Is this the solution?
 - Still a lot to be explored.

Economic Uncertainty, Information Quality, and Learning

- Questions: Can one or more of these puzzles be addressed by lack of information about some key parameter of the model?
- More precisely
 - What is the relationship between the *quality* of information and asset returns?
 - If information is noisy, is there a risk premium?
 - Does information quality affect the relationship between the risk premium and investors' risk aversion?
 - How does the precision of signal affect stock market volatility?
 - Can we infer how good investors' information is from the behavior of stock market returns?
- We now consider a simple modification of the above setting to answer this question (Reference: Veronesi (2000, JF))

A Model of Economic Bayesian Uncertainty

- Dividends grow according to the following process:

$$\frac{dD_t}{D_t} = \theta_t dt + \sigma_D dB_t^D$$

- Investors do not observe the drift θ_t , but they know its law of motion.
- **Assumption 1:** θ_t follows a Markovian process defined on a finite set $\Theta = \{\theta_i\}_{i=1}^n$.
- That is, for every $\theta_i, \theta_j \in \Theta$ there exists λ_{ij} such that in the infinitesimal interval Δ we have:

$$\Pr(\theta_{t+\Delta} = \theta_j | \theta_t = \theta_i) = \lambda_{ij} \Delta$$

- **Remark:** Although I assume that θ_t can take only a finite number of values, the assumption leaves unspecified the *number* of states.
- We can approximate any continuous-time, continuous-state stationary Markov process by choosing a sufficiently fine grid $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ on the real line and by carefully choosing the transition probabilities λ_{ij} .

A Model of Economic Bayesian Uncertainty

- Examples:

1. **Business Cycle:** Use only two states $\theta_1 < \theta_2$. See Veronesi (1999), David and Veronesi (2000) and Locarno and Massa (2000), Cagetti, Hansen, Sargent and Williams (2000).

2. **Pure Jump Process:**

$$d\theta_t = (J_t - \theta_t) dQ_t^{p(\theta_t)}$$

– where $dQ_t^{p(\theta_t)}$ denotes a Poisson process with intensity $p(\theta_t)$ and J_t is a random variable with any density $f(\theta)$.

3. **Pure Drift Uncertainty:** $p(\theta) = 0$ in the previous case.

4. **Mean Reverting Processes:**

$$d\theta_t = k(\bar{\theta} - \theta_t) dt + \sigma_\theta dB_t^\theta$$

– where $dB_t^\theta dB_t^D = 0$.

5. **Mean Reverting Process with Jumps:**

$$d\theta_t = k(\bar{\theta} - \theta_t) dt + \sigma_\theta dB_t^\theta + (J_t - \theta_t) dQ_t^{p(\theta_t)}$$

– This is a very complex filtering problem, used in Veronesi (2003).

Economic Uncertainty and Information Quality

- Investors observe a noisy signal:

$$de_t = \theta_t dt + \sigma_e dB_t^e$$

- where B_t^e is a standard Brownian motion independent of B_t^D .
- This form of the signal is the continuous time analog of the standard “signal equals fundamentals plus noise”, i.e. $e_t = \theta_t + \varepsilon_t$ with ε_t normally distributed, in a discrete time model (see e.g. Detemple (1986)).

$$h_e = 1/\sigma_e = \textit{precision of the external signal}$$

- Similarly, $h_D = 1/\sigma_D = \textit{precision of dividend signal}$

Economic Uncertainty, Information Quality, and Learning

- Denote investors' information set at time t by \mathcal{F}_t , and let

$$\pi_t^i = \text{Prob}(\theta_t = \theta_i | \mathcal{F}_t) \quad (1)$$

- Define $\boldsymbol{\pi}_t = (\pi_t^1, \dots, \pi_t^n)$. This distribution summarizes investors' overall information at time t .
- The expected drift rate at time t $m_t^\theta \equiv E(\theta | \mathcal{F}_t) = \sum_{i=1}^n \pi_t^i \theta_i$
- The evolution of π_t^i is given by
- **Lemma 1:** (a) For all $i = 1, \dots, n$:

$$d\pi_t^i = [\boldsymbol{\pi}_t \boldsymbol{\Lambda}]_i dt + \pi_t^i (\theta_i - m_t^\theta) (h_D d\tilde{B}_t^D + h_e d\tilde{B}_t^e) \quad (2)$$

- where $\boldsymbol{\Lambda}$ is such that $[\boldsymbol{\Lambda}]_{ij} = \lambda_{ij}$ for $i \neq j$ and $[\boldsymbol{\Lambda}]_{ii} = -\sum_{j \neq i} \lambda_{ij}$.
- In this equation

$$d\tilde{B}_t^D = h_D \left(\frac{dD_t}{D_t} - m_t^\theta dt \right); \quad d\tilde{B}_t^e = h_e (de_t - m_t^\theta dt)$$

Economic Uncertainty and Information Quality

- To understand the effect of information quality on asset prices, it is convenient to look at a specific example.
- Assume θ follows a pure jump process.

$$d\theta_t = (J_t - \theta_t)dQ_t^p \quad \text{with} \quad J_t \sim F = (f_1, \dots, f_n) \quad (\text{stationary distribution})$$

- Then if $\theta(t) = \theta_\ell$:

$$d\pi_i = [p(f_i - \pi_i) + k\pi_i(\theta_i - m_\theta)(\theta_\ell - m_\theta)] dt + \sqrt{k}\pi_i(\theta_i - m_\theta)dB_t$$

- where $m_\theta \equiv E(\theta|\mathcal{F}(t))$, $k = h_D^2 + h_e^2$, and $dB_t = \frac{1}{\sqrt{k}}(h_D dB_t^D + h_e dB_t^e)$
 1. More precise signals \implies posterior π_t more concentrated around the true state θ_ℓ ;
 2. Less precise signals \implies posterior π closer to stationary distribution $F = (f_1, \dots, f_n)$;

Economic Uncertainty and Stock Prices

- The price function is:

$$\frac{P_t}{D_t} = \sum_{i=1}^n \pi_t^i C(\theta_i)$$

- where $C(\theta)$ is defined as follows. Define the constant

$$H = \sum_{i=1}^n \frac{f_i}{\phi + p + (\gamma - 1)\theta_i + \frac{1}{2}\gamma(1 - \gamma)\sigma_D^2} \quad (3)$$

then

$$C(\theta) = \frac{1}{\left(\phi + p + (\gamma - 1)\theta + \frac{1}{2}\gamma(1 - \gamma)\sigma_D^2\right) (1 - pH)} \quad (4)$$

- $C(\theta)$ is monotone and convex
- $C(\theta)$ is *decreasing* if and only if $\gamma > 1$.

Economic Uncertainty and the Stock Price

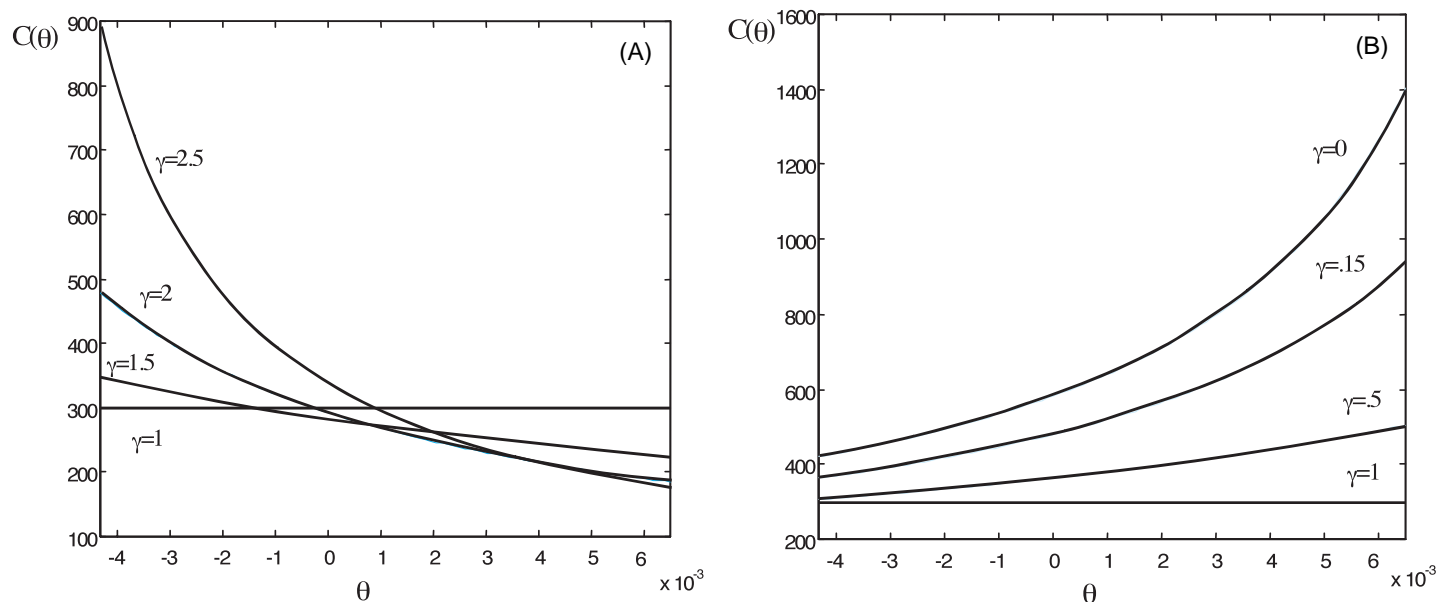


Figure 1. The function $C(\theta)$. (A) plots the function $C(\theta)$ for various values of investors' coefficient of risk aversion $\gamma \geq 1$. This function represents investors' marginal valuation of the stock as a multiple of the current dividend when they condition on the true drift of the dividends process being θ . (B) plots the same function for values of $\gamma \leq 1$.

- Why $C(\theta)$ is decreasing for $\gamma > 1$?
 - Because of low Elasticity of Intertemporal Substitution (EIS)
 - * Low EIS \implies desire of consumption smoothing
 - * \implies Higher $\theta \implies$ higher future consumption \implies lower savings today \implies lower prices (and higher r)

Economic Uncertainty and Stock Prices

- The previous result has an intriguing implication:
 - \implies A mean preserving spread on $\pi \implies$ an increase in P/D .
 - \implies Higher uncertainty increases the P/D ratio.
- Intuition:
 - Higher uncertainty increases the probability of high θ and low θ
 - Compounding effect: 1/2 - 1/2 chance of growing at 10% or 0% is more valuable than sure probability of growing at 5%.
- (Note that the discount adjusts appropriately according to GE model)
- Pastor and Veronesi (2003, JF) document this uncertainty effect on individual stocks, when uncertainty is proxied by firm age.
- Pastor and Veronesi (2006, JFE) uses this intuition to “explain” the tech bubble in the late 1990s.

Economic Uncertainty and Stock Returns

- Let's denote the total excess returns by

$$dR = \frac{dS_t + D_t dt}{S_t} - r_t dt \quad (5)$$

- **Proposition:** The equilibrium excess returns follow the process:

$$dR = \mu_R dt + (\sigma_D + h_D V_\theta(\boldsymbol{\pi}_t)) d\tilde{B}_t^D + V_\theta(\boldsymbol{\pi}_t) h_e d\tilde{B}_t^e \quad (6)$$

where

$$\mu_R = \gamma (\sigma_D^2 + V_\theta(\boldsymbol{\pi}_t)) \quad (7)$$

$$V_\theta(\boldsymbol{\pi}_t) = \frac{\sum_{i=1}^n \pi_t^i C_i (\theta_i - m_t^\theta)}{\sum_{i=1}^n \pi_t^i C_i} = \sum_{i=1}^n \bar{\pi}_t^i \theta_i - \sum_{i=1}^n \bar{\pi}_t^i m_t^\theta = \bar{m}_t^\theta - m_t^\theta \quad (8)$$

- where

$$\bar{\pi}_t^i = \frac{\pi_t^i C_i}{\sum_{j=1}^n \pi_t^j C_j} \quad (9)$$

Economic Uncertainty and Stock Returns

- The function $V_\theta(\boldsymbol{\pi}_t)$ characterizes both expected returns μ_R and volatility.
- $V_\theta(\boldsymbol{\pi}_t)$ is a measure of both level of uncertainty about the true growth rate θ as well as of the impact of this uncertainty on the investors' own valuations of the asset.
 - For example, when $\gamma = 1$ we have that $C_i = C_j$ for all i and j . Hence, $V_\theta(\boldsymbol{\pi}) = 0$: Investors may be uncertain, but they do not care (because they are myopic).
- It is possible to characterize $V_\theta(\boldsymbol{\pi}_t)$ and we obtain the following
 - **Proposition:**
 1. If $\gamma > 1$, then higher uncertainty decreases the risk premium. That is, a mean preserving spread on investors' beliefs $\boldsymbol{\pi}_t$ decreases μ_R .
 2. If either $m_t^\theta > \sigma_D^2 + \theta_1$ or $\pi_t^1 < \bar{\pi}_1^*$ where $\bar{\pi}_1^*$ is a given constant (quite high), the expected excess return μ_R decreases with γ for γ sufficiently high. Hence, μ_R is bounded above.
 3. If $m_t^\theta > \sigma_D^2 + \theta_1$, there is $\bar{\gamma}$ such that $\mu_R < 0$ for $\gamma > \bar{\gamma}$. Moreover, a mean-preserving spread on $\boldsymbol{\pi}$ decreases $\bar{\gamma}$.

Economic Uncertainty and Stock Returns: Intuition

- Part 1 shows that there is no premium for uncertainty. Actually, quite the opposite holds.

- Intuition?

- Recall

$$P_t = D_t \times \sum_{i=1}^n \pi_t^i C(\theta_i)$$

* $D_t \downarrow \implies P_t \downarrow$ because of the first term.

* $D_t \downarrow \implies E_t[\theta] \downarrow \implies$ consumption smoothing $\implies P_t/D_t \uparrow$ because of the second term.

- Higher uncertainty, second effect stronger.

- Since the premium is given by

$$\mu_R = \gamma \text{Cov}_t \left(dR_t, \frac{dC_t}{C_t} \right)$$

- the stronger the second effect, the lower is the covariance between dR_t and $dC_t/C_t = dD_t/D_t$.
- \implies The equity premium is lower for higher economic uncertainty.

Economic Uncertainty, Risk Aversion and Stock Returns

- Similarly, higher risk aversion \implies lower EIS \implies second effect stronger
 - \implies Equity premium is bounded above.

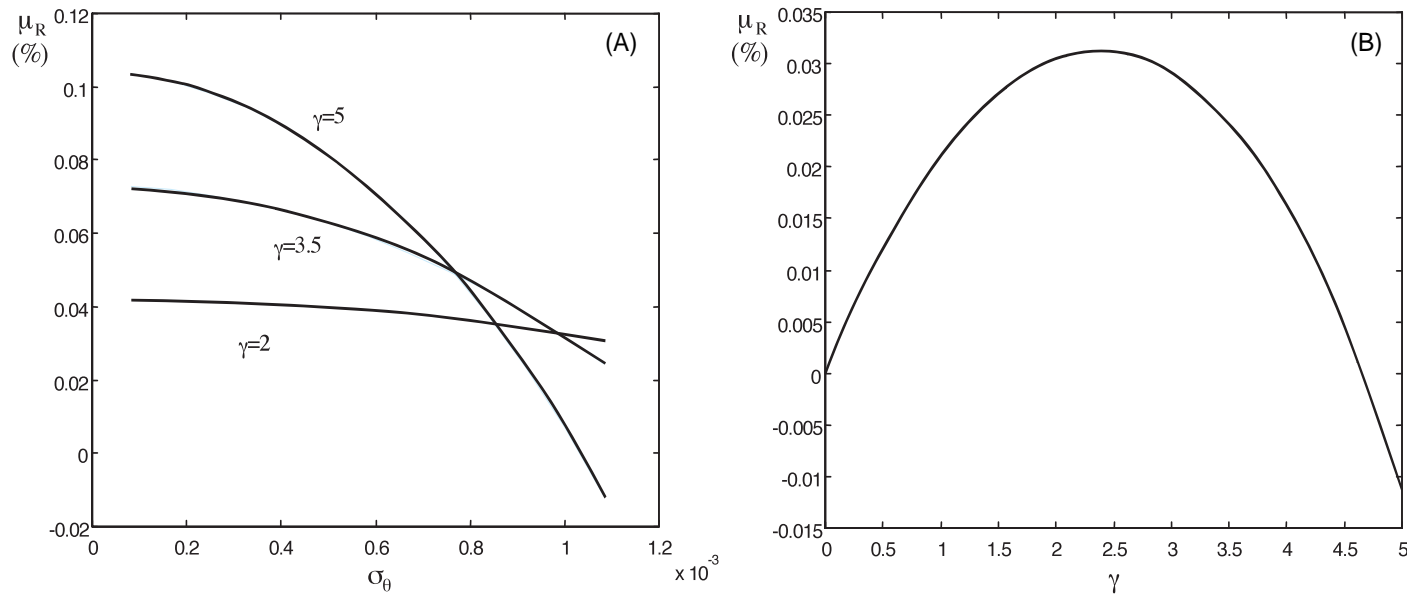


Figure 2. Expected returns and investors' uncertainty. (A) plots the conditional risk premium μ_R against the standard deviation of investors' beliefs $\sigma_\theta = \sqrt{\sum_{i=1}^n \pi_i (\theta_i - m_\theta)}$, which proxies for investors' uncertainty, for various coefficients of risk aversion. (B) plots the conditional risk premium μ_R against the coefficient of risk aversion γ for a level of $\sigma_\theta = 0.11\%$.

Economic Uncertainty and Return Volatility

- Return Volatility is given by

$$\sigma_R^2(\boldsymbol{\pi}_t) = \sigma_D^2 + V_\theta(\boldsymbol{\pi}_t) [2 + (h_e^2 + h_D^2)V_\theta(\boldsymbol{\pi}_t)] \quad (10)$$

- The following proposition then holds:

- **Proposition:**

1. σ_R is a U-shaped function of γ with $\sigma_R = \sigma_D$ for $\gamma = 1$.
2. a mean-preserving spread on $\boldsymbol{\pi}_t$ increases σ_R if $\sigma_R > \sigma_D$. The effect is ambiguous if $\sigma_R < \sigma_D$.
3. Under some conditions, if $h_e > h_D$ then $\sigma_R > \sigma_D$ for a coefficient of risk aversion sufficiently high.

Equity Premium and Return Volatility

- From the results about risk premium and return volatility, it is clear that the relationship between return volatility and expected returns is ambiguous and depends on the degree of investors' uncertainty.

- This statement can be made precise by noticing that we can write:

$$\mu_R(\boldsymbol{\pi}_t) = \gamma\sigma_R^2(\boldsymbol{\pi}_t) - \gamma V_\theta(\boldsymbol{\pi}_t) [1 + (h_e^2 + h_D^2)V_\theta(\boldsymbol{\pi}_t)] \quad (11)$$

- The second term in equation (11) can be positive or negative depending on the magnitude of $V_\theta(\boldsymbol{\pi}_t)$.
- Specifically, for log-utility or when signals are very precise, $V_\theta(\boldsymbol{\pi}_t)$ is approximately zero and hence a linear positive relationship results.
- In contrast, when $\gamma > 1$ and signals are not precise, the second term in equation (11) is positive for $-1/(h_e^2 + h_D^2) < V_\theta(\boldsymbol{\pi}_t) < 0$ and it is negative for $V_\theta(\boldsymbol{\pi}_t) < -1/(h_e^2 + h_D^2)$.
- Since the magnitude of V_θ changes over time due to investors' fluctuating level of uncertainty, equation (11) implies that there is no precise relationship between expected excess returns and conditional volatility.

Equity Premium and Return Volatility

- Higher uncertainty / volatility is related to lower expected return.

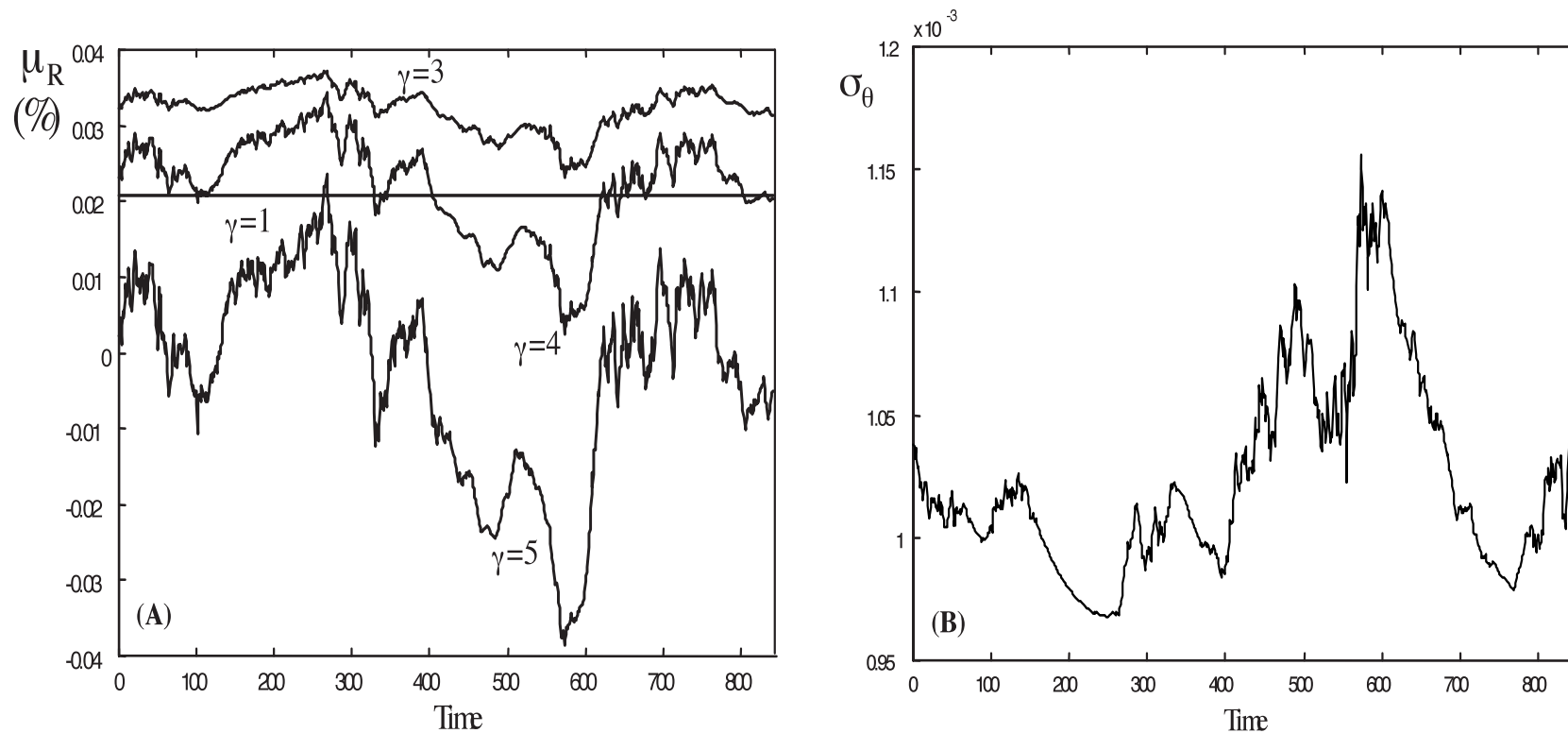


Figure 3. Expected returns and investors' uncertainty over time. (A) plots the conditional risk premium μ_R over time as the result of one Monte Carlo simulation of dividends and posterior distributions. μ_R is computed for the coefficients of risk aversion $\gamma = 1, 3, 4, 5$. (B) plots the standard deviation of investors' beliefs σ_θ across time.

Conclusion

- It is not obvious how economic uncertainty affects stock prices and stock returns.
- The result that P/D ratio increases with uncertainty is rather general.
- The result that expected return decreases with uncertainty crucially depends on $EIS < 1$.
 - There used to be an agreement among macroeconomists that $EIS < 1$.
 - Recent research and new estimates seem to suggest that $EIS > 1$ for stock holders.
 - In this case, expected return and uncertainty would be positively related, as intuition has it.

- What are the ways out?
 - Recent research:
 1. Habit persistence preferences
 2. Risk for the long run
 3. Preferences for robustness

- How far can we go?
- What about the cross-section?
 - See next classes.