Information Acquisition in Financial Markets*

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Abstract

Previous work on information and financial markets has focused on a special set of assumptions: agents have exponential utility, and random variables are normally distributed. These assumptions are often necessary to obtain closed-form solutions. We present an example with alternative assumptions, and demonstrate that some of the conclusions from previous literature fail to hold. In particular, we show that in our example, as more agents acquire information, prices do not necessarily become more informative, and agents may have greater incentive to acquire information. Learning can therefore be a strategic complement, allowing for the possibility of multiple equilibria.

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Introduction

A common explanation for why individuals trade assets in stock markets is that they have access to different information. To motivate differences in information, it is typically assumed that information is costly to acquire, so that some agents will buy information and some will not. But this explanation raises a host of interesting questions: How much information will be acquired about stocks? How will this information be reflected in prices? How do informed and uninformed traders interact with one another? Unfortunately, answering these questions turn out to be remarkably challenging. The reason is that in an equilibrium where information is costly to acquire, agents who choose not to purchase information nevertheless extract some information from the prices they observe, and so their demand will depend on the distribution of equilibrium prices. But since prices must equate supply and demand in the market, the distribution of prices depends on the demand of agents who are not fully informed. In most cases, the system of equations that characterizes this fixed point condition is nonlinear, and has no simple closed-form solution.

One of the few cases in which there is a closed-form solution is if agents have exponential utility, and if all relevant random variables are normally distributed. Under these assumptions, it can be shown that the equilibrium conditions reduce to a system of linear equations. Not surprisingly, then, many of the existing models on the role of information in asset markets have relied heavily on these two assumptions. Examples include Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), Verrecchia (1982), Admati (1985), and Wang (1993, 1994). But while imposing these assumptions makes the problem tractable, it naturally raises questions about robustness. Are the predictions that come out of these models generalizable? or are they unduly restrictive? Some effort has gone into generalizing the rational expectations equilibria to allow for non-normal distributions and more general utility functions. For example, Ausubel (1990) generalizes both distributions and preferences, and Rochet and Vila (1994), following Kyle (1985), consider risk-neutral traders with more general distributional assumptions. But these papers are concerned primarily with existence and uniqueness of equilibria.[1] They do not address the acquisition or transmission of information.

This paper develops a simple example of endogenous information acquisition which illustrates that the predictions of the exponential utility and normal random variables framework may not be entirely robust. In particular, we show that the particular assumptions in our example yield sharply different results regarding the incentives for agents to acquire information, and the nature in which this information becomes revealed through prices. The conventional wisdom on information acquisition traces back to Grossman and Stiglitz (1980). They show that as more agents acquire information, prices become more informative, in the sense that it is easier for remaining agents to free-ride on the learning of others. The reason is that as more agents learn and act on their information, prices will rise in good states and fall in bad states. Uninformed agents can then just observe extreme prices and infer whether the state of the world is good or bad. This paper shows that this intuition breaks down if extreme prices are likely to arise for other reasons. For example, suppose traders know that there is a large (but not fully observed) exogenous supply of stocks that is unrelated to the value of the asset, say because the economy has hit hard times and liquidity constrained individuals need cash. This will cause low prices to prevail independently of the value of the asset. As a result, it is difficult to identify what an extremely low price means: is the price low because liquidity constrained individuals sold off a lot of stock, or because informed traders learned the fundamental value of the price was low? Hence, the fact that more agents learn and cause prices to be more extreme means that remaining agents will have a more difficult time identifying what prices reflect, and so their incentive to learn increases. The reason this effect is ruled out in the previous framework is that when random variables are normally distributed, extreme prices are by necessity low probability events, so this identification problem will not arise. While our example is quite stylized, it is still suggestive that different assumptions can have important consequences for the role of information in asset markets. For example, we show that our example allows for multiple

[1] Foster and Vaswinathan (1993) also consider risk-neutral traders with a more general class of distributions. However, their paper is concerned with price volatility.

equilibria, which are ruled out under conventional assumptions.

The paper is organized as follows. Section 1 lays out the key assumptions. Section 2 solves for the equilibrium. Section 3 demonstrates why learning can become a strategic complement and shows this can lead to multiple equilibria. Section 4 concludes.

1. Setup

We follow the original Grossman and Stiglitz (1980) framework. There are only two assets. One is a risky asset, whose price is P and which yields a payoff $\widetilde{\theta}$ to its owner (we adopt the convention that random variables are denoted with a tilde). The typical assumption imposes that $\widetilde{\theta}$ is normally distributed. We instead assume it to be binomial:

$$\widetilde{\theta} = \begin{cases} \overline{\theta} & \text{with probability } \rho \\ \\ \underline{\theta} & \text{with probability } 1-\rho \end{cases}$$

The net supply of this asset is exogenous and fixed for convenience at 1. The second asset is a safe asset (call it money) whose price is normalized to 1 and which yields a fixed payoff of $1+R$. To simplify notation, we set $R=0$.

Agents in this economy are identical ex-ante. Each is endowed with an amount of money, which we set to 1. In addition, we assume agents are risk-neutral. The advantage of risk-neutrality is that agents only demand the asset with the higher expected returns. This simple structure on demand allows us to get around the inherent nonlinearity of equilibrium conditions discussed in the Introduction and still obtain a closed-form solution.

In addition, we make the following assumptions to characterize agents:

- **No Borrowing** - Agents can spend no more than their original endowment.

- **No Market Power** - There is a continuum of agents, with mass 1, so that no agent can affect the price through his demand.

- **Costly Information** - Agents can learn the fundamentals $\tilde{\theta}$ but they incur a utility cost of c.

The last two assumptions are quite standard. But since we move away from risk-aversion in our framework, we have to add a no borrowing constraint to ensure that net demand for the asset is finite.[2]

Finally, as Grossman and Stiglitz point out, we need to prevent prices from being fully revealing; otherwise, an equilibrium will fail to exist. We must therefore introduce noise into the system so that uninformed traders cannot directly infer the payoff θ from the price.[3] To this end, we model noise trading as having two components. First, we assume this group spends a constant amount of wealth w on assets. Thus, if the price of the asset is equal to P, they purchase $\frac{w}{P}$ shares of the asset. We assume that w is sufficiently large to keep the market "liquid":

$$w > \overline{\theta} \tag{1.1}$$

This assumption implies that for any price $P \leq \overline{\theta}$, noise traders can afford to buy up the entire net supply of assets (which recall was set fixed at 1). This prevents the price of the stock from bottoming out in the bad state of the world, since noise traders can absorb assets and keep the price propped up. Without this assumption, high prices would become fully revealing, since they would never arise in the bad state of the world.

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[2] Moreover, as Kyle (1989) points out, without this assumption our definition of equilibrium would imply a schizophrenia on the part of the agents, since by borrowing infinite amounts they are able to affect prices, and yet in equilibrium they act as price takers.

[3] Technically, we need to introduce an additional variable to prevent price from being an invertible function of $\widetilde{\theta}$.

In addition to this liquidity component, we need to add a stochastic component so that price does not depend only on $\widetilde{\theta}$. Hence, we assume that out of this amount $\frac{w}{P}$, a random amount $\widetilde{x}$ of assets must be sold off for liquidity reasons. That is, demand from the noise sector is given by

$$x^0(P) = \frac{w}{P} - \tilde{x}$$

Again, previous work has assumed $\widetilde{x}$ is normally distributed. We consider instead the exponential distribution, which has the important feature that it is asymmetric:

$$\tilde{x} \sim \exp(\mu) \tag{1.2}$$

Finally, it is important that we not allow agents to observe the volume of trade $\widetilde{x}$; otherwise, agents could invert the price function and infer $\widetilde{\theta}$ from the price.[4]

To recap, total demand in the market for assets is the sum of the demand of the risk-neutral agents and of the liquidity traders. The total supply of assets is equal to 1. The timing of the model is as follows. In the beginning of the period, $\widetilde{\theta}$ is determined. Traders must then choose whether to learn the fundamentals or not. Once this decision is made, $\widetilde{x}$ is determined but not revealed to the agents. Agents then trade given their information, and the market clears so that supply equals demand. After trade commences, agents cannot learn the fundamentals if they failed to do so earlier. At the end of the period, the holders of the asset receive the payoff $\widetilde{\theta}$.

2. Equilibrium

We now define and solve the equilibrium in the economy described above. To solve the model, we need to work backwards. Suppose a fraction z of traders learned the value of $\widetilde{\theta}$

[4] For an example of how agents can observe the quantity of trade without causing prices to be fully revealing, see Blume, Easley, and O'Hara (1994).

in the first stage. This gives us a complete description of the market, allowing us to define an equilibrium. In particular, an equilibrium in the asset market involves a price for every pair (x, θ), which we denote $P_z(x, \theta)$, along with demand schedules for both informed and uninformed.

We begin by characterizing demand schedules. By virtue of their risk-neutrality, agents invest all of their wealth in the asset that yields the higher expected payoff. Since informed traders know the value of $\widetilde{\theta}$, they will invest in the asset if and only if $P < \widetilde{\theta}$. With z such informed traders, we can represent the aggregate demand of informed traders for the asset by the following schedule:

$$x^I(\tilde{\theta}, P) \in \begin{cases} \dfrac{z}{P} & \text{if } P < \tilde{\theta} \\ \left[0, \dfrac{z}{P}\right] & \text{if } P = \tilde{\theta} \\ 0 & \text{if } P > \tilde{\theta} \end{cases} \tag{2.1}$$

Uninformed traders engage in a similar calculation, except that they do not observe $\tilde{\theta}$ and only get to form a conditional expectation of $\tilde{\theta}$ based on the price. Since there are $1 - z$ such agents, we can represent the aggregate demand of the uninformed by the following schedule:

$$x^U(P; P_z(\cdot, \cdot)) \in \begin{cases} \dfrac{1-z}{P} & \text{if } P < E(\tilde{\theta} | P_z(\cdot, \cdot) = P) \\ \left[0, \dfrac{1-z}{P}\right] & \text{if } P = E(\tilde{\theta} | P_z(\cdot, \cdot) = P) \\ 0 & \text{if } P > E(\tilde{\theta} | P_z(\cdot, \cdot) = P) \end{cases} \tag{2.2}$$

Now that we have characterized the demand functions for the agents, we can proceed to define an equilibrium for the asset market.

Definition: a rational expectations equilibrium for the economy given z is a set of functions $\left(P_z(\cdot, \cdot),\ x^I(\tilde{\theta}, P),\ x^U(P; P_z(\cdot, \cdot))\right)$ such that

1. **Utility Maximization:** x^I and x^U satisfy (2.1) and (2.2) respectively.

2. **Market Clearing:** For all pairs (x, θ), the supply of assets is equal to total demand, i.e.

$$x^0\left(P_z(x,\theta)\right) \;+\; x^I\left(\theta, P_z(x,\theta)\right) \;+\; x^U\left(P_z(x,\theta); P_z(\cdot,\cdot)\right) = 1 \qquad (2.3)$$

It turns out that low values of $\underline{\theta}$ lead to certain technical complications which we wish to avoid. We therefore limit our discussion to the case where $\underline{\theta} > \mu$.[5] Under this assumption, along with (1.1), we have the following proposition:

Proposition 1: Given that a fraction z of traders learns the value of $\tilde{\theta}$, there exists an equilibrium in the asset market, where

1. $P_z(x,\theta)$ is a piecewise hyperbolic function given by (A.2) and (A.3) in the Appendix, and which is illustrated in Figure 1.

2. Aggregate demand for informed traders x^I is given by (2.1).

3. Aggregate demand for uninformed traders x^U is given by

$$x^U\left(P; P_z(\cdot,\cdot)\right) \in \begin{cases} \dfrac{1-z}{P} & \text{if } P < P_z^* \\[2ex] \left[0, \dfrac{1-z}{P}\right] & \text{if } P = P_z^* \\[2ex] 0 & \text{if } P > P_z^* \end{cases}$$

where $P_z^* \in \left[\underline{\theta}, \overline{\theta}\right]$ is the unique solution to

$$P_z^* = E(\tilde{\theta} \mid P_z(\cdot,\cdot) = P_z^*) \qquad (2.4)$$

While the formal details of proving Proposition 1 are relegated to the Appendix, our construction is straightforward. The difficult part in proving Proposition 1 is deriving the

[5] For a discussion of where this case is violated, see Barlevy and Veronesi (1998).

demand function of the uninformed, precisely because it involves a fixed point condition. Our approach is to look for an equilibrium in which this function follows a cutoff rule, i.e. the uninformed invest in the asset if and only if its price is below some cutoff value P_z^*. Assuming that the uninformed use this cutoff rule, we can derive a price function which clears the market for this demand schedule. For example, if the price is high (i.e. $P > \overline{\theta}$), only noise traders are investing in the asset. For demand to equal supply, we must have $\frac{w}{P} - x = 1$, which gives us price as a function of x: $P = \frac{w}{x+1}$. For prices just below $\overline{\theta}$, informed traders will invest in the asset in the good state of the world but not in the bad state, and the price function shifts out in the good state. Similarly, at prices just below P_z^*, uninformed traders invest in the asset in both states of the world, so the price function shifts out by the same amount in both states of the world. The price function we construct allows us to calculate $E(\tilde{\theta} \mid P_z(\cdot,\cdot) = P)$. In the proof, we show that given this price function, a cutoff rule is in fact optimal, and we derive an equation characterizing the cutoff price P_z^*.

Having characterized an equilibrium price function $P_z(x,\theta)$, we can move back one step to determine the equilibrium fraction z of agents who choose to acquire information. Given z, any individual trader can figure out what the equilibrium outcome will be from Proposition 1. The trader knows that if he chooses not to learn, there is a possibility that he will make mistakes: if the state of the world is $\underline{\theta}$ but the price $P \in (\underline{\theta}, P_z^*)$, he will invest his wealth in the asset when money yields the higher return, while if the state of the world is $\overline{\theta}$ but the price $P \in \left(P_z^*, \overline{\theta}\right)$, he will invest his wealth in money when the asset offers the higher return. Since the distribution of prices given a fraction z of agents is learning is given by Proposition 1, the agent can explicitly calculate the expected gain from acquiring information and avoiding mistakes $g(z)$:

$$g(z) \quad = \quad \rho \int_{\frac{w+z}{\overline{\theta}}-1}^{\frac{w+z}{P_z^*}-1} \left[\frac{\overline{\theta}}{P_z(x,\overline{\theta})} - 1\right] \mu e^{-\mu x} dx +$$

$$(1-\rho)\int_{\frac{w+1-z}{P_z^*}-1}^{\frac{w+1-z}{\underline{\theta}}-1}\left[1-\frac{\underline{\theta}}{P_z(x,\underline{\theta})}\right]\mu e^{-\mu x}dx \qquad (2.5)$$

An uninformed agent then compares the gain in (2.5) to c, and chooses to learn if and only if the gain is greater than the cost of learning. We define the fraction z as an equilibrium if

1. $z = 1$ and $g(1) \geq c$
2. $z = 0$ and $g(0) \leq c$

or

3. $z \in (0,1)$ and $g(z) = c$

i.e. either the gain exceeds the costs and all agents acquire information (case 1), the gain is less than the cost and no agent acquires information (case 2), or the gain is exactly equal to the cost so that some agents acquire information while other agents do not (case 3). The existence of an equilibrium fraction z is immediate consequence of the continuity of $g(z)$:

Proposition 2: There exists a z that satisfies one of the three conditions above.

3. Learning as a Strategic Complement

Having characterized an equilibrium, we are now ready to discuss our main result. In particular, we show that for certain parameter values, $g(z)$ is increasing in z. That is, as more agents learn, the gain to becoming informed increases. This goes against the prevailing view laid out in Grossman and Stiglitz. They argue that as more agents become informed, there is less incentive for remaining agents to acquire information. Their intuition is as follows. First, since informed traders gain from exploiting the uninformed, an increase in the ratio of informed traders to uninformed traders in the market erodes the gain to exploiting information: there are simultaneously fewer uninformed to exploit and more informed agents to compete away rents. Second, with more informed traders in the market, demand for the asset (and consequently the price of the asset) becomes more responsive to $\tilde{\theta}$; with more informed agents, demand is higher when $\tilde{\theta}$ is high and lower when $\tilde{\theta}$ is low. With more extreme prices, it becomes easier for the uninformed to free-ride and infer the value of $\tilde{\theta}$ from the actions of others. The presence of these two effects lead Grossman and Stiglitz to conjecture that in general, acquiring information in a market setting is a strategic substitute.

The reason that learning can become a strategic complement in our model is that under our distributional assumptions, the second effect described above ceases to hold. That is, while an increase in the fraction of informed traders still causes prices to be more extreme, it is no longer true that it is easier to infer $\widetilde{\theta}$ when prices are extreme. Quite the opposite: it may well be the case that as prices become more extreme, it is *more* rather than *less* difficult to identify $\widetilde{\theta}$.[6] Provided this effect is large enough, remaining agents will have greater incentive to acquire information when z increases.[7]

To motivate our finding, imagine what would happen to the market we describe during a recession. We would expect recessions to have two effects. First, during hard times, individuals who fall upon hard times will require more liquidity, so more noise traders are likely to sell the risky assets in favor of cash. In terms of the model, this would imply that high values of x are relatively more likely. Second, we would expect that risky assets are likely to experience low returns. Thus, lower values of θ to be more likely. Both of these effects tend to drive the price of the asset down, but for very different reasons. The first makes assets more attractive, since the sale of assets by liquidity constrained traders makes

[6] We purposely refer to "identification of $\widetilde{\theta}$" rather than to the "informativeness of prices" as previous work does. This is because our framework does not allow us to rank situations according to the usual criterion of Blackwell's sufficient experiments.

[7] It should be noted that the first effect still holds in our model, i.e. there is less opportunity to exploit information when z is large and there are few uninformed traders to profit from. While this condition will always hold when agents are acquiring information about a common asset, it might be violated in other contexts. For example, Vives (1985) shows that when firms engage in Cournot competition with goods that are complements, an increase in the precision of information for one firm helps the other firm. This is because firms would like to coordinate their actions and sell large quantities when demand is high. In this case, an increase in the proportion of informed agents allows remaining agents to exploit their own information better, and learning could potentially become a strategic complementarity.

the asset underpriced. The second makes assets less attractive, since the return on the asset is likely to be very low. In this environment, information on $\widetilde{\theta}$ would be relatively valuable, since it would allow the agent to distinguish which factor is causing the low price, and to assess whether the asset is worth investing in or not. Now, suppose we increase the fraction of informed traders z. As traders become more informed price become more extreme. In particular, in a recession, individuals are very likely to observe $\widetilde{\theta} = \underline{\theta}$, in which case they will not invest in the asset and drive its price further down. Thus, the effect of an increase in z is to make low prices more likely. But low prices are precisely the prices at which agents have the most difficulty inferring whether they should invest in the asset or not. Thus, as a consequence of having more informed traders in the market, remaining traders know that they are more likely to see low prices, in which case they would prefer to have more information on $\widetilde{\theta}$.

We now formalize this intuition. First, we want a low value of μ in the exponential distribution which governs $\widetilde{x}$. The lower this value, the more mass is shifted towards high values of $\widetilde{x}$. At the same time, we want low values of ρ. The lower this value, the more likely the bad state of the world $\underline{\theta}$ is to prevail. The next proposition confirms our intuitive claim that in these environments, learning becomes a strategic complement:

Proposition 3: There exists a neighborhood around $\rho = 0$ and $\mu = 0$ such that learning is always a strategic complement, i.e. $\dfrac{\partial g(z)}{\partial z} > 0$ for all $z \in [0, 1]$.

By contrast, consider the opposite case in which liquidity trading remains high (i.e. μ is close to zero) but ρ is high. In this case, as more agents learn, they are likely to observe $\widetilde{\theta} = \overline{\theta}$, in which case they will invest in the asset and drive its price up. But high prices are easy to identify as being due to fundamentals; since liquidity traders unload their stocks, high prices are very unlikely to arise, so observing a high price is a good indication that the state of the world is good. Consequently, there should be little value from learning the actual value of $\widetilde{\theta}$ when there are many informed traders. This would imply learning is likely to be a strategic substitute. In fact, this is what we find:

Proposition 4: There exists a neighborhood around $\rho = 1$ and $\mu = 0$ such that learning is always a strategic substitute, i.e. $\dfrac{\partial g(z)}{\partial z} < 0$ for all $z \in [0, 1]$.

Similarly, we would expect that for μ large (liquidity trading is unlikely) and ρ large (the high state of the world is likely), learning would once again become a complementarity since now it is difficult to identify what a high price means. We are unable to prove this formally, but we did confirm this with numerical simulations. Likewise, we confirm numerically that if μ is large but ρ is small, learning will again become a strategic substitute.

The above intuition helps explain why we fail to observe complementarities in the Grossman and Stiglitz model. In their framework, prices are a linear function of the underlying random variables, i.e.

$$P_z(x, \theta) = \alpha_z + \beta_z x + \gamma_z \theta \tag{3.1}$$

It follows that prices are also normally distributed. But since the normal distribution concentrates most of the probability mass in the center, extreme prices are unlikely events. As more agents become informed and price becomes more sensitive to $\widetilde{\theta}$, the identification problem always becomes easier and uninformed agents can indeed free-ride on the learning of others. One needs to move away from the normal distribution framework and towards distributions which place greater mass on the extreme in order to uncover our result.

Finally, we note that once learning becomes a strategic complement, there may be multiple equilibria in our model. Such a possibility is illustrated in Figure 2. It plots the gain function $g(z)$ for parameter values which generate complementarities.[8] If the cost of acquiring information c is at the level drawn in Figure 2, there will be three equilibrium levels of z: one at $z = 0$, and two interior equilibria. We find this feature interesting, since it could

[8] Specifically, the parameters in Figure 2 are $w = 1$, $\overline{\theta} = 1$, $\underline{\theta} = .98$, $\rho = .55$, and $\mu = 2$. Notice that $\underline{\theta} < \mu$, contrary to the assumption made just prior to Proposition 1. In fact, this assumption is unnecessarily restrictive, but simplifies our proofs considerably.

potentially explain large movements in stock prices as a movement from an equilibrium with low levels of information to one with high levels of information. That is, suppose that news arrives that the economy is in a recession (for example, unusually low employment figures are announced). This implies low prices are more likely both because of fundamentals and because of liquidity traders, generating a complementarity in learning which introduces multiple equilibria. In one equilibrium, few people acquire information, while at the other equilibrium, more individuals learn, information gets transmitted, and the market reacts strongly to the fundamentals. Hence, the effect of new information on stock prices might be amplified by movements to new equilibria where either more or fewer agents choose to acquire information at the same time.

4. Conclusion

This paper demonstrates that when we change the distributional and preference assumptions in the Grossman and Stiglitz (1980) model in a particular way, we can generate new predictions which are ruled out under the conventional framework of exponential utility and normally distributed random variables. In particular, we show that learning can become a strategic complement. This challenges previous intuition that as more agents learn, prices become more "informative" and outsiders can free-ride on the information available in the market. Even though our model still relies on very special assumptions, we believe it has certain intuitive appeal. For example, we would expect that recessions involve more liquidity trading and lower fundamental values, a condition which can generate learning complementarities. Large movements in stock prices, such as the 1987 crash, often trigger debates as to whether these movements are due to "smart" money or to panic selling. Investors typically track news much more closely during such downturns, precisely because these environments make the identification problem more difficult. While the assumptions we used to generate our findings are quite specific, we suspect that our results would continue to hold in more general cases. The crucial component for generating learning complementarities is that learning makes identification more complicated for uninformed agents. As long as learning leads agents to take actions whose outcomes are similar to outcomes that arise for completely different reasons, agents who are uninformed will be adversely affected when others around them become knowledgeable.

As a final note, our example serves to illustrate the possible fragility of the normal-linear model that has become so common in the literature on information and financial markets. While this paper describes one such example, we point out in Barlevy and Veronesi (1998) additional predictions of this same model which are ruled out in the traditional framework. We suspect that other results regarding the role of information in financial markets which hold under exponential utility and normally distributed random variables might also be fragile. We leave the further investigation of these issues to future work.

A. Appendix

Proof of Proposition 1: We conjecture that uninformed traders invest in the asset if the price of the asset is below a cutoff level, and then prove that for the equilibrium price function, a cutoff rule is in fact optimal. Denote this cutoff by P_z^*, where $P_z^* \in (\underline{\theta}, \overline{\theta})$. Aggregate demand for uninformed traders is then

$$x^U(P) \in \begin{cases} 0 & \text{if} \quad P > P_z^* \\ \left[0, \frac{1-z}{P}\right] & \text{if} \quad P = P_z^* \\ \frac{1-z}{P} & \text{if} \quad P < P_z^* \end{cases} \tag{A.1}$$

Using the market clearing condition, we can then compute the associated price function:

$$P_z(x, \underline{\theta}) = \begin{cases} \frac{w}{x+1} & \text{if} \quad x+1 < \frac{w}{P_z^*} \\ P_z^* & \text{if} \quad x+1 \in \left[\frac{w}{P_z^*}, \frac{w+1-z}{P_z^*}\right] \\ \frac{w+1-z}{x+1} & \text{if} \quad x+1 \in \left(\frac{w+1-z}{P_z^*}, \frac{w+1-z}{\underline{\theta}}\right) \\ \underline{\theta} & \text{if} \quad x+1 \in \left[\frac{w+1-z}{\underline{\theta}}, \frac{w+1}{\underline{\theta}}\right] \\ \frac{w+1}{x+1} & \text{if} \quad x+1 > \frac{w+1}{\underline{\theta}} \end{cases} \tag{A.2}$$

$$P_z(x, \overline{\theta}) = \begin{cases} \frac{w}{x+1} & \text{if} \quad x+1 < \frac{w}{\overline{\theta}} \\ \overline{\theta} & \text{if} \quad x+1 \in \left[\frac{w}{\overline{\theta}}, \frac{w+z}{\overline{\theta}}\right] \\ \frac{w+z}{x+1} & \text{if} \quad x+1 \in \left(\frac{w+z}{\overline{\theta}}, \frac{w+z}{P_z^*}\right) \\ P_z^* & \text{if} \quad x+1 \in \left[\frac{w+z}{P_z^*}, \frac{w+1}{P_z^*}\right] \\ \frac{w+1}{x+1} & \text{if} \quad x+1 > \frac{w+1}{P_z^*} \end{cases} \tag{A.3}$$

We now verify that given this price function, the demand schedule for uninformed traders is in fact a cutoff rule. Since uninformed traders are risk neutral, they will invest in the asset offering the highest expected return. Applying Bayes' rule and using the function $P_z(x, \theta)$, we get

$$E(\widetilde{\theta} \mid P_z(\cdot,\cdot) = P) = \begin{cases} \rho\overline{\theta} + (1-\rho)\underline{\theta} & \text{if} \quad P > \overline{\theta} \\ \overline{\theta} & \text{if} \quad P = \overline{\theta} \\ \frac{\rho(\overline{\theta}-\underline{\theta})}{\rho+(1-\rho)\exp\left[\frac{\mu z}{P}\right]} + \underline{\theta} & \text{if} \quad P \in \left(\underline{\theta}, \overline{\theta}\right) \\ \underline{\theta} & \text{if} \quad P = \underline{\theta} \\ \rho\overline{\theta} + (1-\rho)\underline{\theta} & \text{if} \quad P < \underline{\theta} \end{cases} \tag{A.4}$$

Clearly, the uninformed invest all of their wealth in money if $P > \overline{\theta}$ and in the asset if $P < \underline{\theta}$. For $P \in \left(\underline{\theta}, \overline{\theta}\right)$, the uninformed invest in the asset if and only if $E(\widetilde{\theta} \mid P_z(\cdot,\cdot) = P) > P$. Define

$$h(P) = \frac{1}{P}\left[\frac{\rho(\overline{\theta}-\underline{\theta})}{\rho+(1-\rho)\exp\left[\frac{\mu z}{P}\right]} + \underline{\theta}\right]$$

for $z, P > 0$. It follows that for $P \in \left(\underline{\theta}, \overline{\theta}\right)$, the uninformed invest in the asset if and only if $h(P) > 1$. Hence, we just need to show that (1) there exists a $P_z^* \in \left(\underline{\theta}, \overline{\theta}\right)$ such that $h(P_z^*) = 1$ and (2) that $h(P)$ is monotonically decreasing.

Note that for all P, $\overline{\theta} \leq P \cdot h(P) \leq \overline{\theta}$. This implies that for $P > \overline{\theta}$, $h(P) < 1$ and for $P < \underline{\theta}$, $h(P) > 1$. By continuity, there exists a $P_z^* \in (\underline{\theta}, \overline{\theta})$ such that $h(P_z^*) = 1$. Next, differentiating $h(P)$ with respect to P yields

$$\frac{\partial h}{\partial P} = -\frac{\rho(\overline{\theta} - \underline{\theta})\left(\rho P + (1-\rho)(P - \mu z)\exp\left[\frac{\mu z}{P}\right]\right)}{P^3\left(\rho + (1-\rho)\exp\left[\frac{\mu z}{P}\right]\right)^2} - \frac{\theta}{P^2} \tag{A.5}$$

Since $z \in [0,1]$ and $\underline{\theta} > \mu$, $P - \mu z > 0$ for $P > \underline{\theta}$, and so $\frac{\partial h}{\partial P} < 0$. This establishes that uninformed demand does in fact follow a cutoff rule, as assumed.

Proof of Propositions 3 and 4: To prove Proposition 3, we note that at $\rho = 0$, $g(z) = 0$ for all z, which implies $\frac{\partial g(z)}{\partial z} = 0$. We want to show that for $\mu \to 0$, $\left.\frac{\partial^2 g(z)}{\partial \rho \partial z}\right|_{\rho=0} > 0$. By continuity, this implies that for $\rho \approx 0$ (but $\rho \neq 0$), the derivative of the gain function with respect to z is positive, i.e. $\frac{\partial g(z)}{\partial z} > 0$.

We substitute the price function (A.2)-(A.3) into the gain function in (2.5) to yield

$$\begin{aligned} g(z) &= \rho \int_{\frac{w+z}{\overline{\theta}}-1}^{\frac{w+z}{P_z^*}-1} \left[\frac{\overline{\theta}(x+1)}{w+z} - 1\right] \mu e^{-\mu x} dx + \\ &\qquad (1-\rho) \int_{\frac{w+1-z}{P_z^*}-1}^{\frac{w+1-z}{\underline{\theta}}-1} \left[1 - \frac{\underline{\theta}(x+1)}{w+1-z}\right] \mu e^{-\mu x} dx \end{aligned} \tag{A.6}$$

Differentiating $g(z)$ with respect to ρ yields

$$\begin{aligned} \frac{\partial g(z)}{\partial \rho} &= \int_{\frac{w+z}{\overline{\theta}}-1}^{\frac{w+z}{P_z^*}-1} \left[\frac{\overline{\theta}(x+1)}{w+z} - 1\right] \mu e^{-\mu x} dx \\ &\quad -\rho \left[\frac{\overline{\theta}}{P_z^*} - 1\right] \mu e^{-\mu\left(\frac{w+z}{P_z^*}-1\right)} \cdot \frac{w+z}{(P_z^*)^2} \cdot \frac{\partial P_z^*}{\partial \rho} && \text{(A.7)} \\ &\quad - \int_{\frac{w+1-z}{P_z^*}-1}^{\frac{w+1-z}{\underline{\theta}}-1} \left[1 - \frac{\underline{\theta}(x+1)}{w+1-z}\right] \mu e^{-\mu x} dx && \text{(A.8)} \\ &\quad +(1-\rho)\left[1 - \frac{\underline{\theta}}{P_z^*}\right] \mu e^{-\mu\left(\frac{w+1-z}{P_z^*}-1\right)} \frac{w+1-z}{(P_z^*)^2} \cdot \frac{\partial P_z^*}{\partial \rho} && \text{(A.9)} \end{aligned}$$

We then evaluate this derivative at $\rho = 0$. Noting that at $\rho = 0$, $P_z^* = \underline{\theta}$, we have that all but the first term drop out, leaving us with

$$\left.\frac{\partial g(z)}{\partial \rho}\right|_{\rho=0} = \int_{\frac{w+z}{\overline{\theta}}-1}^{\frac{w+z}{\underline{\theta}}-1} \left[\frac{\overline{\theta}(x+1)}{w+z} - 1\right] \mu e^{-\mu x} dx \tag{A.10}$$

Differentiating this expression with respect to z, we get

$$\begin{aligned}
\left.\frac{\partial^2 g(z)}{\partial z \partial \rho}\right|_{\rho=0} &= \frac{1}{\underline{\theta}}\left[\frac{\overline{\theta}}{\underline{\theta}} - 1\right] \mu e^{-\mu\left(\frac{w+z}{\underline{\theta}}-1\right)} - \int_{\frac{w+z}{\overline{\theta}}-1}^{\frac{w+z}{\underline{\theta}}-1} \left[\frac{\overline{\theta}(x+1)}{(w+z)^2}\right] \mu e^{-\mu x} dx \\
&> \left[\frac{\overline{\theta} - \underline{\theta}}{\underline{\theta}^2}\right] \mu e^{-\mu\left(\frac{w+z}{\underline{\theta}}-1\right)} - \frac{\overline{\theta}}{(w+z)^2} \mu e^{-\mu\left(\frac{w+z}{\overline{\theta}}-1\right)} \int_{\frac{w+z}{\overline{\theta}}-1}^{\frac{w+z}{\underline{\theta}}-1} (x+1) dx \\
&= \left[\frac{\overline{\theta} - \underline{\theta}}{\underline{\theta}^2}\right] \mu e^{-\mu\left(\frac{w+z}{\underline{\theta}}-1\right)} - \frac{\overline{\theta}}{2} \mu e^{-\mu\left(\frac{w+z}{\overline{\theta}}-1\right)} \left[\frac{\overline{\theta}^2 - \underline{\theta}^2}{\overline{\theta}^2 \underline{\theta}^2}\right] \\
&= \left[\frac{\overline{\theta} - \underline{\theta}}{\underline{\theta}^2}\right] \mu e^{-\mu\left(\frac{w+z}{\underline{\theta}}-1\right)} \left\{1 - \left[\frac{\overline{\theta} + \underline{\theta}}{2\overline{\theta}}\right] e^{-\mu(w+z)\left(\frac{\underline{\theta} - \overline{\theta}}{\overline{\theta}\underline{\theta}}\right)}\right\}
\end{aligned}$$

But as $\mu \to 0$, the expression $\left\{1 - \left[\frac{\overline{\theta}+\underline{\theta}}{2\overline{\theta}}\right] e^{-\mu(w+z)\left(\frac{\underline{\theta}-\overline{\theta}}{\overline{\theta}\underline{\theta}}\right)}\right\}$ becomes positive, so $\left.\frac{\partial^2 g(z)}{\partial z \partial \rho}\right|_{\rho=0} > 0$, which is what we wanted to show.

To prove Proposition 4, we can apply a symmetric argument to show that

$$\left.\frac{\partial^2 g(z)}{\partial z \partial \rho}\right|_{\rho=1} < \left[\frac{\overline{\theta} - \underline{\theta}}{\overline{\theta}^2}\right] \mu e^{-\mu\left(\frac{w+1-z}{\overline{\theta}}-1\right)} \left\{1 - \left[\frac{\overline{\theta} + \underline{\theta}}{2\underline{\theta}}\right] e^{-\mu(w+1-z)\left(\frac{\overline{\theta} - \underline{\theta}}{\overline{\theta}\underline{\theta}}\right)}\right\}$$

As $\mu \to 0$, $\left\{1 - \left[\frac{\overline{\theta}+\underline{\theta}}{2\underline{\theta}}\right] e^{-\mu(w+z)\left(\frac{\underline{\theta}-\overline{\theta}}{\overline{\theta}\underline{\theta}}\right)}\right\}$ becomes negative, so $\left.\frac{\partial^2 g(z)}{\partial z \partial \rho}\right|_{\rho=1} < 0$, completing the proof.

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Figure 1: Equilibrium Price Function

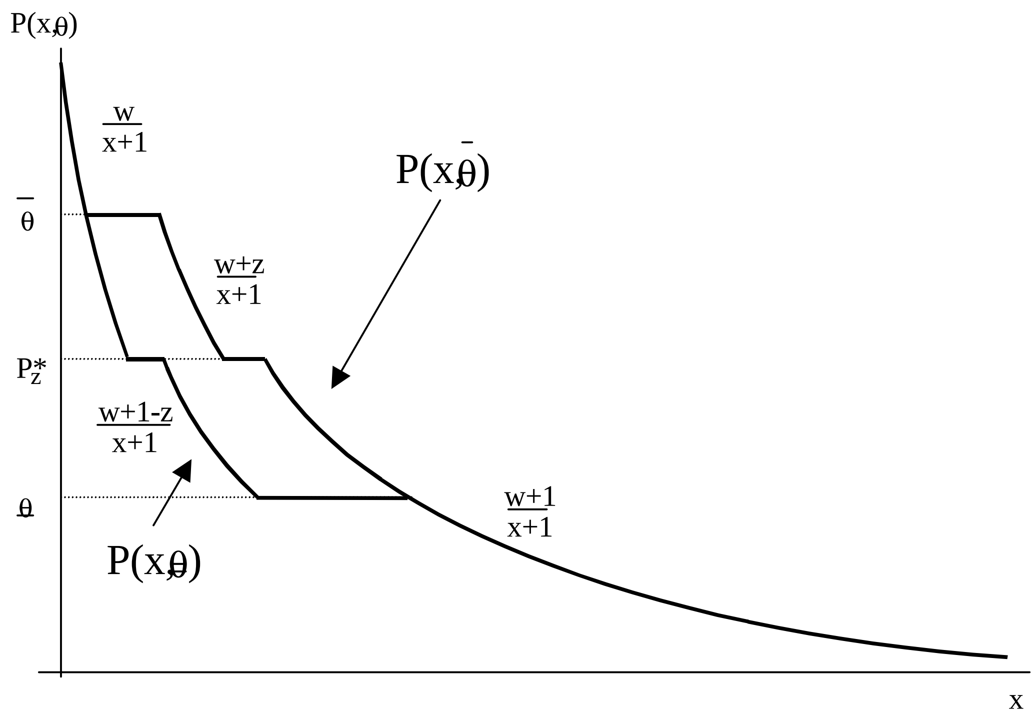

Figure 2: Multiple Equilibria