Rational IPO Waves

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ABSTRACT

We argue that the number of firms going public changes over time in response to time variation in market conditions. We develop a model of optimal IPO timing in which IPO waves are caused by declines in expected market return, increases in expected aggregate profitability, or increases in prior uncertainty about the average future profitability of IPOs. We test and find support for the model's empirical predictions. For example, we find that IPO waves tend to be preceded by high market returns and followed by low market returns.

The number of initial public offerings (IPOs) changes dramatically over time, as shown in Figure 1. For example, 845 firms went public in the United States in 1996, but there were only 87 IPOs in 2002. Although the fluctuation in IPO volume is well known (e.g., Ibbotson and Jaffe (1975)), its underlying causes are not well understood. Many researchers attribute time variation in IPO volume to market inefficiency, arguing that IPO volume is high when shares are "overvalued."[1] Such an argument assumes that the periodic market mispricing can somehow be detected by the owners of the firms going public, but not by the investors providing IPO funds. In contrast, we present a model in which fluctuation in IPO volume arises in the absence of any mispricing, and in which IPO volume is more closely related to recent changes in stock prices than to the level of stock prices.

******************** INSERT FIGURE 1 HERE ********************

We develop a model of optimal IPO timing in which IPO volume fluctuates due to time variation in market conditions. We define market conditions as having three dimensions: expected market return; expected aggregate profitability; and prior uncertainty about the post-IPO average profitability in excess of market profitability, henceforth referred to as "prior uncertainty." Market conditions indeed appear to vary in these dimensions. Time variation in expected market return is consistent with empirical evidence on return predictability.[2] Time variation in expected aggregate profitability is related to business cycles. Time variation in prior uncertainty seems plausible as well. For example, technological revolutions are likely to be accompanied by high prior uncertainty, because they make the prospects of new firms highly uncertain. We show, theoretically and empirically, that IPO volume responds to time variation in all three dimensions of market conditions. Moreover, we note that market conditions are related not only to IPO volume but also to stock prices, as represented by the firms' ratios of market to book value of equity (M/B). IPO volume is then naturally related to stock prices as well.

Our model considers a special class of agents, "inventors," who invent new ideas that can lead to abnormal profits. Inventors patent each idea and start a private firm that owns the patent. Inventors possess a real option to take their firms public, invest part of the IPO proceeds, and begin producing. They choose the best time to exercise this option. When market conditions are constant, it is optimal to go public as soon as the patent is secured. When market conditions vary over time, however, inventors may prefer to postpone their IPO in anticipation of more favorable market conditions.

We solve for the optimal time to go public and show that private firms are attracted to capital markets especially when market conditions are favorable in the sense that expected market return is low, expected aggregate profitability is high, and prior uncertainty is high.

At any point in time, private firms are waiting for an improvement in market conditions; that is, for a decline in expected market return or for an increase in expected aggregate profitability or prior uncertainty. When market conditions improve sufficiently, many inventors exercise their options to go public, thus creating a cluster of IPOs, or an "IPO wave."

To analyze the properties of IPO waves in our model, we calibrate the model to match some key features of the data on asset prices, profitability, and consumption, and simulate it over a long period of time. In the simulation, one idea is invented each period, so that IPO waves do not develop from the clustering of technological inventions in time. Instead, IPO waves are the result of clustering in the inventors' optimal IPO timing decisions.

Our model makes many empirical predictions. IPO waves caused by a decline in expected market return should be preceded by high market returns, because prices rise when expected return falls, and followed by low market returns, because expected return has fallen.[3] IPO waves caused by an increase in expected aggregate profitability should also be preceded by high market returns, because prices rise as cash flow expectations go up, and followed by high profitability, because expected profitability has risen. IPO waves caused by an increase in prior uncertainty should be preceded by increased disparity between newly listed firms and seasoned firms in terms of their valuations and return volatilities.

We test the model's implications by using data between 1960 and 2002. Our results support all three channels (discount rate, cash flow, and uncertainty) through which IPO waves are created in our model. We find that IPO volume is positively related to recent market returns, which suggests that many firms go public after expected market return declines or after expected aggregate profitability increases. This result is consistent with both the discount rate and cash flow channels. Additional support for the discount rate channel is provided by the findings that IPO volume is negatively related to future market returns and to recent changes in market return volatility. The cash flow channel is further supported by the facts that IPO volume is positively related to changes in aggregate profitability and to revisions in analysts' forecasts of long-term earnings growth. IPO volume is also positively related to recent changes in two empirical proxies for prior uncertainty.

Another testable implication of our model is that IPO volume is more closely related to recent changes in stock prices than to the level of stock prices. The relation between IPO volume and recent changes in prices is due to the endogeneity of IPO timing: Firms are induced to go public by improvements in market conditions, and these improvements lift stock prices at the same time. IPO volume is also positively related to the level of stock prices, as represented by the aggregate M/B ratio, but that relation is weaker. IPO volume is not necessarily high when the level of stock prices is high because the high price level is a result of cumulative improvements in market conditions, and many private firms that had been waiting for such improvements went public while prices were rising. Consistent with

these arguments, we find that IPO volume is significantly related to recent market returns, but unrelated to the level of the aggregate M/B ratio.

The evidence of no relation between the level of M/B and IPO volume does not support the behavioral story in which IPO waves arise when shares are overvalued. This story also does not predict our findings that IPO volume is negatively related to changes in market return volatility, and positively related to changes in aggregate profitability and to changes in the difference between the return volatilities of new and old firms. These findings do not disprove the behavioral story, but they suggest that our explanation for IPO waves, which predicts all of these facts, provides a plausible alternative to the mispricing story.

This paper is related to many earlier studies. Apart from the literature on market mispricing, cited earlier, this paper is related to the studies that link the volume of equity issuance to the asymmetry of information resulting from the adverse selection costs of issuing equity (e.g., Myers and Majluf (1984)).[4] Also related is the literature that focuses on the corporate control aspect of an IPO (e.g., Zingales (1995)).[5] This paper abstracts from both of these important corporate finance issues and shows that IPO volume can fluctuate also in the absence of asymmetric information and private benefits of control.

This paper is also related to the literature on irreversible investment under uncertainty.[6] In our model, the capital raised in the IPO is immediately invested, as it is in the model of Jovanovic and Rousseau (2001). In their model, the option to delay an IPO is valuable because waiting allows a private firm to learn about its own production function. In our model, this option is valuable due to time variation in market conditions. Finally, Boehmer and Ljungqvist (2004) find empirical support for our model in German data.

The paper is organized as follows. Section I describes the setting in which IPO decisions are made. Section II discusses the decision to go public and analyzes some properties of optimal IPO timing. Section III uses a simulated sample to investigate the properties of IPO waves in our model. Section IV tests the model's predictions empirically. Section V examines the relation between IPOs and investment. Section VI concludes.

I. Model

There are two classes of agents, inventors and investors, who have identical information and preferences but different endowments. Investors are endowed with a stream of consumption good. Inventors are endowed with the ability to invent patentable ideas that can deliver abnormal profits. When an inventor patents his idea, he starts a private firm that owns the patent but produces no revenue. Any time before the patent expires, the inventor can decide to make the investment that initiates production. To finance this investment, the private firm issues equity to investors in an IPO.

In this section, we describe the economic environment in which IPO decisions take place.

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This environment features time-varying market conditions, whose three dimensions are described in the next three subsections. We then solve for the market value of a firm, which is an essential input to the optimal IPO timing problem analyzed in Section II.

A. Time-Varying Profitability

After the IPO, firm i's profits are protected by a patent until time T_i. Let $\rho_t^i = Y_t^i / B_t^i$ denote the firm's instantaneous profitability at time t, where Y_t^i is the earnings rate and B_t^i is the book value of equity. Motivated by empirical evidence (e.g., Fama and French (2000)), we assume that firm profitability follows a mean-reverting process between the IPO and T_i:

$$d\rho_t^i = \phi^i \left(\overline{\rho}_t^i - \rho_t^i \right) dt + \sigma_{i,0} dW_{0,t} + \sigma_{i,i} dW_{i,t}, \tag{1}$$

where $W_{0,t}$ and $W_{i,t}$ are uncorrelated Wiener processes that capture systematic ($W_{0,t}$) and firm-specific ($W_{i,t}$) components of the random shocks that drive the firm's profitability. We also assume that the firm's average profitability $\overline{\rho}_t^i$ can be decomposed as

$$\overline{\rho}_t^i = \overline{\psi}^i + \overline{\rho}_t. \tag{2}$$

The firm-specific component $\overline{\psi}^i$, which we refer to as the firm's average excess profitability, reflects the firm's ability to capitalize on its patent, and is assumed to be constant over time. The common component $\overline{\rho}_t$, which we refer to as expected aggregate profitability, is assumed to exhibit mean-reverting variation:

$$d\overline{\rho}_t = k_L \left(\overline{\rho}_L - \overline{\rho}_t \right) dt + \sigma_{L,0} dW_{0,t} + \sigma_{L,L} dW_{L,t}, \tag{3}$$

where $W_{0,t}$ and $W_{L,t}$ are uncorrelated. Mean reversion in expected aggregate profitability reflects business cycles in the aggregate economy.

B. Time-Varying Prior Uncertainty

Average excess profitability $\overline{\psi}^i$ is unobservable. For any firm i that goes public at time t, all inventors and investors have the same prior belief about $\overline{\psi}^i$. Their prior uncertainty, $\widehat{\sigma}_t$, is assumed to be the same for all firms going public at time t, for simplicity. It seems plausible for prior uncertainty $\widehat{\sigma}_t$ to vary over time. For example, uncertainty about the $\overline{\psi}^i$'s of new firms is greater when the economy experiences technological advances whose long-term impact is uncertain. To model time variation in $\widehat{\sigma}_t$, we assume that $\widehat{\sigma}_t$ takes values in the discrete set $\mathcal{V} = \{v^1, ..., v^n\}$ and that it switches from one value to another in each infinitesimal interval Δ according to the transition probabilities $\lambda_{hk} \Delta = \Pr \left(\widehat{\sigma}_{t+\Delta} = v^k | \widehat{\sigma}_t = v^h \right)$.

Both inventors and investors begin learning about $\overline{\psi}^i$ as soon as firm i begins producing at its IPO. Both learn by observing realized profitability ρ_t^i, as well as $\overline{\rho}_t$, c_t (defined below),

and ρ_t^j for all firms j that are alive at time t. The prior distribution of $\overline{\psi}^i$ is assumed to be normal, so the posterior of $\overline{\psi}^i$ is also normal, with mean $\widehat{\psi}_t^i$ and variance $\widehat{\sigma}_{i,t}^2$. The dynamics of the posterior moments are given in Lemma 1 in the Appendix. Agents can observe $\overline{\rho}_t$.[7]

C. Time-Varying Expected Market Return

Let μ_t denote expected market return at time t. To generate time-varying μ_t, we work with a framework similar to that of Campbell and Cochrane (1999). In this framework, μ_t varies over time due to the time-varying risk aversion of the representative investor. All inventors and investors, indexed by k, have habit utility over consumption:

$$U\left(C_t^k, X_t, t\right) = e^{-\eta t} \frac{\left(C_t^k - X_t\right)^{1-\gamma}}{1-\gamma}, \tag{4}$$

where X_t is an external habit index, γ regulates the local curvature of the utility function, and η is a time discount parameter.

Let $C_t = \sum_k C_t^k$ denote aggregate consumption, $c_t = \log\left(C_t\right)$, and $S_t = \left(C_t - X_t\right)/C_t$ denote the surplus consumption ratio. Campbell and Cochrane assume that $s_t = \log\left(S_t\right)$ follows a mean-reverting process with time-varying volatility and perfect correlation with unexpected consumption growth. This specification allows Campbell and Cochrane to solve for market prices numerically. To obtain analytical solutions for prices, we assume that

$$s_t \equiv s\left(y_t\right) = a_0 + a_1 y_t + a_2 y_t^2, \tag{5}$$

where y_t is a state variable driven by the following mean-reverting process:

$$dy_t = k_y\left(\overline{y} - y_t\right) dt + \sigma_y dW_{0,t}. \tag{6}$$

Time variation in y_t generates time variation in both components of μ_t, the equity premium and the real risk-free rate. As shown in the Appendix, high y_t implies a low equity premium and a low risk-free rate in the plausible range. We show that time variation in either the equity premium or the risk-free rate leads to time variation in IPO volume.

We assume that markets are dynamically complete, in that shocks to the aggregate state variables y_t, $\overline{\rho}_t$, and $\widehat{\sigma}_t$ can be hedged using contingent claims. No contingent claims can hedge firm-specific shocks $dW_{i,t}$, but those shocks can be hedged using firm equity. Since markets are complete, inventors and investors can perfectly insure each other's consumption. Assuming that their initial endowments are equally valuable, inventors and investors choose identical consumption plans, thus justifying the existence of a representative agent with preferences given in equation (4). The stochastic discount factor (SDF) π_t is then unique:

$$\pi_t = U_C\left(C_t, X_t, t\right) = e^{-\eta t}\left(C_t S_t\right)^{-\gamma} = e^{-\eta t - \gamma\left(c_t + s_t\right)}. \tag{7}$$

In equilibrium, aggregate consumption is given by the sum of all endowments and net payouts in the economy. Computing this sum is complicated, because the payouts depend on the inventors' optimal IPO timing. Instead, for tractability, we assume that c_t follows

$$dc_t = (b_0 + b_1 \overline{\rho}_t)\, dt + \sigma_c dW_{0,t}. \tag{8}$$

As other recent studies, we assume that consumption is financed mostly by income that is outside our model, and the resulting process is given in equation (8). Consumption growth is allowed to depend on $\overline{\rho}_t$ because such a link is plausible ex ante, but none of our results rely on this link. The data-implied value of b_1 turns out to be small ($b_1 = 0.08$ in Table I), and $b_1 = 0$ leads to the same conclusions throughout.

D. The Market Value of a Firm

This subsection discusses a closed-form solution for the market value of a firm in the environment described above. Our pricing analysis extends the model of Pástor and Veronesi (2003a) to allow for time variation in market conditions.

After its IPO, firm i earns abnormal profits $(\overline{\psi}^i)$ until its patent expires at time T_i. We assume that any abnormal earnings after T_i are eliminated by competitive market forces, so that the firm's market value at T_i equals its book value, $M_{T_i}^i = B_{T_i}^i$. The firm is assumed to pay no dividends, to be financed only by equity, and to issue no new equity.[8] The firm's market value at any time t after the IPO but before T_i is $M_t^i = E_t\left[(\pi_{T_i}/\pi_t)B_{T_i}^i\right]$, with π_t given in equation (7). An analytical formula for M_t^i is provided in Proposition 1 in the Appendix, together with expressions for the firm's expected return and volatility.

The intuition behind the pricing formula is as follows. A firm's M/B is high if

(1) the firm's expected profitability is high

(2) the firm's discount rate is low

(3) uncertainty about the firm's average future profitability is high.

In (1), M/B increases with three cash-flow related quantities: expected aggregate profitability, $\overline{\rho}_t$; expected excess profitability, $\widehat{\psi}_t^i$; and current profitability, ρ_t^i. In (2), we find numerically that M/B increases with the state variable y_t in the calibrated model. Since high y_t implies a low risk aversion of the representative investor, it also implies a low expected market return and high M/B. In (3), M/B increases with $\widehat{\sigma}_{i,t}$, uncertainty about $\overline{\psi}^i$, as shown by Pástor and Veronesi (2003a). For more details on the pricing formula, see the Appendix.

Throughout, we say that market conditions improve (worsen) when expected market return falls (rises), expected aggregate profitability rises (falls), or prior uncertainty rises (falls). We note that improvements in market conditions raise M/B and vice versa.

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II. Optimal IPO Timing

This section analyzes the IPO decision. Figure 2 summarizes the sequence of events. At time t_i, a new idea is patented by an inventor.[9] Until the patent expires at time T_i, it enables the owner to earn average excess profitability $\overline{\psi}^i$. Production requires capital B^{t_i}, which is raised in an IPO. At some time τ_i, $t_i \leq \tau_i \leq T_i$, the inventor may decide to go public and file the IPO. The IPO itself takes place at time $\tau_i + \ell$, where the lag ℓ reflects the time required by the underwriter to conduct the "road show." In the IPO, the inventor sells the firm to investors for its fair market value, $M^i_{\tau_i+\ell}$, and pays a proportional underwriting fee, f. Part of the IPO proceeds, B^{t_i}, are immediately invested by the inventor and the production begins, generating the profits described in equation (1). Once the investment B^{t_i} is made, it is irreversible in that the project cannot be abandoned. The inventor's payoff from going public is $M^i_{\tau_i+\ell}(1-f) - B^{t_i}$, the market value of the patent net of fees.

******************** INSERT FIGURE 2 HERE ********************

The inventor chooses the time to go public to maximize the value of his patent, because doing so allows him to maximize his lifetime expected utility from consumption given in equation (4). Given market completeness, standard results (Cox and Huang (1989)) imply that the maximization problem of inventor i can be written in its static form as

$$\max_{\{C^i_t, \tau_i\}} E_0 \left[\int_0^\infty e^{-\eta t} \frac{(C^i_t - X_t)^{1-\gamma}}{1-\gamma} \, dt \right] \tag{9}$$

subject to the budget constraint

$$E_0 \left[\int_0^\infty \frac{\pi_t}{\pi_0} C^i_t \, dt \right] \leq E_0 \left[\frac{\pi_{\tau_i+\ell}}{\pi_0} \left(M^i_{\tau_i+\ell}(1-f) - B^{t_i} \right) \right]. \tag{10}$$

The budget constraint states that the present value of the inventor's lifetime consumption cannot exceed the present value of his endowment, which is assumed to be positive. It is clearly optimal for the inventor to choose τ_i to maximize the value of his endowment; that is, to maximize the market value of the patent:

$$\max_{\tau_i} E_0 \left[\frac{\pi_{\tau_i+\ell}}{\pi_0} \left(M^i_{\tau_i+\ell}(1-f) - B^{t_i} \right) \right]. \tag{11}$$

This problem is analogous to computing the optimal exercise time of a call option. By securing a patent, the inventor acquires a real option to raise capital in an IPO and invest it in the patented technology. This option is American, as it can be exercised at any time before the patent expires. When deciding when to exercise the option, the inventor faces a tradeoff. On one hand, delaying the IPO is costly because delay forfeits abnormal profits that

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can be earned only until the patent's expiration. On the other hand, going public eliminates the time value of the option. This value is always positive because market conditions vary over time. In principle, market conditions can worsen so much after the IPO that the firm's cash flow does not provide a fair rate of return on the initial investment B^{t_i}. Retaining the option by delaying the IPO offers protection against such a scenario, which is why the option increases the market value of the patent.

Let τ_i^* denote the optimal time to exercise the option in equation (11). We solve for τ_i^* numerically. The market value of the patent at any time t, $t_i \leq t \leq \tau_i^* + \ell$, is

$$V\left(\overline{\rho}_t, y_t, \widehat{\sigma}_t, T_i - t\right) = E_t\left(\frac{\pi_{\tau_i^* + \ell}}{\pi_t}\left(M_{\tau_i^* + \ell}^i(1 - f) - B^{t_i}\right)\right). \tag{12}$$

The value of the patent, V, depends only on the aggregate quantities $\overline{\rho}_t$, y_t, and $\widehat{\sigma}_t$. Given market completeness, V can be replicated by trading in existing securities before the IPO. As a result, V must satisfy the standard Euler equation $E_t\left[d\left(\pi_t V_t\right)\right] = 0$. This condition translates into a system of partial differential equations, one for each possible uncertainty state $\widehat{\sigma}_t \in \mathcal{V} = \{v^1, ..., v^n\}$. Using the final condition that the patent is worthless at T_i, we work backwards to compute V_t for each combination of the state variables on a fine grid. The optimal stopping time τ_i^* is then chosen to maximize the patent value.

We note that the inventor faces no idiosyncratic risk before the IPO because the value of his patent, V, depends only on aggregate risks $(\overline{\rho}_t, y_t, \widehat{\sigma}_t)$ that can be fully hedged. The contingent-claims portfolio that replicates V is shorted by the inventor to finance his pre-IPO consumption. Since the inventor is hedged, he has no need to sell the patent. However, as soon as B^{t_i} is invested, new idiosyncratic risk is introduced in the economy, and the only way the inventor can hedge this risk is by selling the patent in an IPO.

According to this logic, the fact that the capital necessary for investment is raised in an IPO rather than by borrowing is a result, not an assumption. The inventor issues equity because he has a strong incentive to diversify. If he instead borrowed and began producing, his entire wealth would be driven by idiosyncratic shocks ($W_{i,t}$ in equation 1) that could not be hedged with existing securities, which is clearly suboptimal. The only security that can hedge this idiosyncratic risk is a share of the firm's equity, which is not traded before the IPO. Then, standard risk-sharing arguments imply that the inventor issues some equity in an IPO. It can be proved formally that it is optimal for the inventor to sell all of his ownership, as assumed above, but it is also easy to show that the model's implications are identical if the inventor retains any fraction of ownership after the IPO.

Finally, private firms in our model do not produce before their IPO, but many real-world IPOs are undertaken by mature firms that have produced for years before going public. Producing before the IPO is suboptimal in our model because it exposes the inventor to unhedgeable idiosyncratic risk, as explained above. Less strictly, this model envisions a private

firm whose pre-IPO production is small-scale relative to its post-IPO production.

A. When Do Firms Go Public?

The optimal timing of a private firm's IPO is driven by the firm's market value, as shown in equation (11), and this value depends crucially on market conditions, as shown in equation (12). It follows that market conditions are a key factor in the decision to go public. To analyze the dependence of the IPO decision on market conditions, we solve the IPO timing problem numerically, using the parameters from Section III.A.

Figure 3 plots the pairs of expected market return μ_t and expected aggregate profitability $\overline{\rho}_t$ for which the inventor optimally decides to go public. Each line denotes the locus of points that trigger the IPO decision, or the "entry boundary." Firms go public when μ_t and $\overline{\rho}_t$ lie inside the "entry region" northwest of the entry boundary. If the idea is invented when μ_t and $\overline{\rho}_t$ are inside the entry region, an IPO is filed immediately. Otherwise, the inventor waits until market conditions improve and files an IPO as soon as the entry boundary is reached. If the boundary is not reached before the patent expires, the firm never goes public.

******************** INSERT FIGURE 3 HERE ********************

Panel A considers a firm with $\widehat{\psi}_t^i = 0$ and a patent with $T = 15$ years to expiration.[10] The entry boundary is upward sloping, so if μ_t increases, $\overline{\rho}_t$ must also increase to trigger entry. The entry boundary moves southeast as prior uncertainty $\widehat{\sigma}_t$ increases. Both effects are intuitive. At any point in time, the inventor compares the option value of delaying the IPO with the value of the profits given up by waiting. He files an IPO when market conditions improve (i.e., μ_t decreases, $\overline{\rho}_t$ increases, or $\widehat{\sigma}_t$ increases) sufficiently so that the option to wait is no longer valuable enough to delay the IPO.

Panel B plots the entry boundaries for three different values of expected excess profitability $\widehat{\psi}_t^i$, with $\widehat{\sigma}_t = 0$. Higher values of $\widehat{\psi}_t^i$ expand the entry region by shifting the entry boundary southeast, which is intuitive because a more profitable patent has a higher opportunity cost of waiting for an improvement in market conditions.

Panels C and D focus on time to the patent's expiration, T. As time passes and T declines from 15 to 5 years, the entry boundary in Panel C moves southeast, lowering the hurdle for entry. Intuitively, the option to wait becomes less valuable as the patent's expiration approaches. However, this effect is reversed close to the patent's expiration, as shown in Panel D. The reason is the underwriting fee, f. As T declines toward zero, M/B at the IPO declines to one. When M/B is sufficiently close to one, the inventor does not exercise his option because his payoff net of fees would be negative. As a result, when T is sufficiently small, the hurdle for entry actually increases as time passes.

The endogeneity of IPO timing implies that the M/B ratios of IPOs tend to be high in our model, and also that these ratios typically decline after the IPO. IPOs take place when μ_t is low enough and $\overline{\rho}_t$ is high enough to be in the entry region (Figure 3). Low μ_t and high $\overline{\rho}_t$ help increase the M/B ratios of all firms, including IPOs. More often than not, μ_t is below and $\overline{\rho}_t$ above their long-term averages at the time of the IPO. As these mean-reverting variables move toward their central tendencies, the M/B ratios decline.

Prior uncertainty about $\overline{\psi}^i$ gives a second reason why M/B tends to decline after the IPO. As soon as the firm begins generating observable profits, the market begins learning about $\overline{\psi}^i$. This learning reduces posterior uncertainty, which leads to a gradual decline in M/B over the lifetime of a typical firm, as discussed by Pástor and Veronesi (2003a). Despite their projected decline, the high IPO valuations are perfectly rational, because IPOs are expected to earn a fair positive rate of return. The M/B ratios of IPOs do not fall because M is expected to go down, but because B is expected to go up faster than M, loosely speaking.

III. IPO Waves

This section extends the single-firm analysis of Section II to multiple firms. The main result here is that IPO waves develop naturally as a result of optimal IPO timing in time-varying market conditions. IPO waves can obviously also arise if technological inventions cluster in time. To preclude such an effect, we assume that the pace of technological innovation is constant, so that exactly one new idea is invented each month. We assume that inventors compete for ideas, so that each idea is patented as soon as it is invented. Inventors also immediately start a new private firm that owns the patent.

Private firms go public when market conditions improve sufficiently to reach the entry region in Figure 3. Recall the tradeoff: Delaying the IPO forfeits profits, but it preserves the option to wait. Improvements in market conditions weaken the incentive to delay an IPO for two reasons. First, they reduce the value of the option to wait for better market conditions because those conditions are mean-reverting. Second, they raise the opportunity cost of delaying the IPO by raising the value of the profits given up by waiting.

The premise of this paper is that IPO waves are caused by sufficiently large improvements in market conditions. Most of the time, there is a "backlog" of private firms waiting for market conditions to improve. After a sufficiently large improvement, many of these firms go public. The resulting IPO waves typically last several months, as all private firms rarely go public at exactly the same time because they differ in the time to expiration on their patents as well as in their firm-specific profitability.

The rest of this section analyzes the properties of IPO waves in a simulated environment, in which changes in market conditions are conveniently observable. We calibrate the model and simulate a long sample from it, allowing private firms to time their IPOs optimally. We

then analyze the relation between IPO waves and market conditions in simulated data.

A. Calibration

This subsection describes the parameters chosen to calibrate the model so that it matches some key features of the data on asset prices, profitability, and consumption. All parameters are summarized in Table I, together with some implied aggregate quantities. We use data on quarterly real aggregate consumption and aggregate profitability between 1966Q1 and 2002Q1 to estimate the parameters for c_t in equation (8) and for $\overline{\rho}_t$ in equation (3). Both series are described in the Appendix. We apply the Kalman filter to the discretized versions of the processes. The estimated parameters imply expected consumption growth of 2.37% and volatility of 0.94% per year. For profitability, we obtain $\overline{\rho}_L = 12.16\%$ per year, $k_L = 0.1412$, and $\sigma_{LL} = 0.64\%$ per year.[11] We set $\sigma_{L,0}$ equal to zero, very close to the unconstrained estimate, which implies zero correlation between $\overline{\rho}_t$ and y_t. As a result, all three state variables that drive IPO volume ($\overline{\rho}_t$, y_t, and $\widehat{\sigma}_t$) are independent of each other.

******************** INSERT TABLE I HERE ********************

The agents' preferences are characterized by the processes for s_t in equation (5), y_t in equation (6), and by the utility parameters η and γ. The parameters are chosen to match some basic empirical properties of the market portfolio. Since newly listed firms comprise a small fraction of the market (e.g., Lamont (2002)), we represent the market by a "long-lived firm" with instantaneous profitability of $\overline{\rho}_t$. The formulas for the long-lived firm's M/B ratio (M_t^m/B_t^m), expected return (μ_R^m), and volatility (σ_R^m) are given in the Appendix. The preference parameters are chosen to calibrate μ_R^m, σ_R^m, and M_t^m/B_t^m to their empirical values for the market, while producing reasonable properties for the real risk-free rate. Our values for $\overline{y}$ and σ_y imply the average equity premium of 6.8% and market volatility of 15% per year. The speed of mean reversion k_y implies a half-life of 9.5 years for y_t. The long-lived firm's ratio of dividends to book equity is set to 10% per year, which produces an average aggregate M/B of 1.7, equal to the time-series average in the data. The average risk-free rate is 3.3% per year. The volatility of the risk-free rate is 3.9%, which is slightly higher than in the data (as is common in models with habit utility), but still reasonable.

The parameters for individual firm profitability ρ_t^i in equation (1) are chosen to match the median firm in the data. We use $\phi^i = 0.3968$, estimated by Pástor and Veronesi (2003a), who also report an 8.34% per year median volatility of the AR(1) residuals for individual firm profitability. We decompose this volatility into $\sigma_{i,0} = 4.79\%$ and $\sigma_{i,i} = 6.82\%$ per year, which implies a M/B of 1.7 for a firm with 15 years to patent expiration and $\widehat{\psi}_t^i = 0$ when $\widehat{\sigma}_t = 0$, $y_t = \overline{y}$, and $\rho_t^i = \overline{\rho}_t = \overline{\rho}_L$. Finally, prior uncertainty $\widehat{\sigma}_t$ moves along the grid $\mathcal{V} = \{0, 1, ..., 12\}$

% per year. The transition probabilities are such that there is 10% probability in any given month of $\widehat{\sigma}_t$ moving up or down to an adjacent value in the grid. If $\widehat{\sigma}_t$ hits the boundary of the grid, there is a 20% probability of moving away from the boundary.

The parameters of the IPO timing model are specified as follows. The proportional underwriting fee is set equal to $f = 0.07$.[12] The lag between the IPO filing and the IPO itself is set equal to $\ell = 3$ months.[13] The capital required for production is assumed to be proportional to the book value of the long-lived firm, $B^{t_i} = qB^m_{t_i}$, with $q = 0.0235\%$.[14]

B. Simulation Evidence around IPO Waves

Using the parameters from the previous subsection, we simulate our model over a period of 10,000 years (120,000 months). One new idea is patented each month, with excess profitability $\widehat{\psi}^i_t$ drawn randomly from the set $\{-6, -4, \ldots, 4, 6\}$ % per year with equal probabilities. Each patent has $T = 15$ years to expiration.

We define IPO waves as follows. Following Helwege and Liang (2003), we calculate three-month centered moving averages in which the number of IPOs in each month is averaged with the numbers of IPOs in the months immediately preceding and following that month. We define "hot markets" as those months in which the moving average falls into the top quartile across the whole simulated sample. We then define IPO waves as all sequences of consecutive hot-market months.[15] In our simulated sample, there are 4,116 IPO waves whose length ranges from one to 17 months, with a median of three months. The maximum number of IPOs in any given month is 51, the median is one, and the average is 0.9.

Since we assume a three-month lag between an IPO filing and the IPO itself, we also define an IPO "pre-wave" as an IPO wave that is shifted back in time by three months. Each IPO wave in our model is driven by state variable changes that occur in the respective pre-wave. We let "b" denote the last month before the wave begins, and "e" denote the last month of the wave. An IPO wave begins at the end of month b and ends at the end of month e. A pre-wave begins at the end of month b-3 and ends at the end of month e-3.

Table II reports the averages of selected variables around IPO waves. Given the size of the simulated sample, we can treat all averages as population values, so no p-values are shown. Column 1 of Panel A reports the average change in the given variable during a pre-wave. First, IPO waves tend to be preceded by pre-wave declines in expected market return μ_t, in which the average pre-wave change is -0.99% per year. This decline is due to both components of μ_t, expected excess return (-0.46%) and the risk-free rate (-0.53%). Second, expected aggregate profitability $\overline{\rho}_t$ rises by 0.06% per year during a pre-wave, on average. Third, IPO waves are preceded by increases in prior uncertainty $\widehat{\sigma}_t$, in which the average pre-wave change is 0.33% per year. Table II thus illustrates the importance of all

three channels (discount rate, cash flow, and uncertainty) in generating IPO waves.

******************** INSERT TABLE II HERE ********************

The weakest of the three channels in Table II seems to be the cash flow channel, for two reasons. First, $\overline{\rho}_t$ exhibits relatively little variation, because aggregate profitability data that is used to calibrate the process for $\overline{\rho}_t$ is relatively stable over time. Second, $\overline{\rho}_t$ reverts to its mean relatively fast (e.g., faster than the variable y_t that drives μ_t), so changes in $\overline{\rho}_t$ are perceived as short-lived. The inventor's option to wait for an increase in $\overline{\rho}_t$ is thus less valuable, and $\overline{\rho}_t$ has a weaker effect on IPO volume than μ_t and $\widehat{\sigma}_t$ do.

IPO waves in our model are caused by changes in market conditions, not levels. Table II shows that market conditions are typically only slightly more favorable during the waves than outside the waves. The level of market conditions is reflected in the aggregate M/B, defined as the sum of earnings divided by the sum of book values across all firms. M/B rises during the pre-waves by 0.11 on average, which is consistent with IPO waves being produced by improvements in market conditions. However, the level of M/B during the waves is only slightly higher than it is outside the waves (1.78 vs 1.76, on average). The reason is that there is an interesting path dependence in IPO volume. Improvements in market conditions induce IPOs, thus depleting the backlog of private firms waiting to go public. After sufficiently large improvements, there is no backlog left, and IPO volume cannot exceed one per month when M/B is high. Similarly, the backlog of private firms builds up as market conditions get worse, and an improvement in unfavorable market conditions can induce much of the large backlog to go public when M/B is low.

The relation between IPO volume and M/B is illustrated in Figure 4 on a randomly selected 100-year segment of the simulated data. The figure shows dramatic variation in IPO volume: There are periods as long as six years in which no IPOs take place, but also months of feverish IPO activity, with over 30 IPOs per month.[16] IPO waves invariably occur after increases in M/B, but not necessarily when M/B is high. Similarly, periods when no firms go public tend to be preceded by severe drops in M/B.

******************** INSERT FIGURE 4 HERE ********************

B.1. Proxies for Changes in Market Conditions

Changes in market conditions can be observed in our simulated environment, but not in the data. Therefore, we must construct observable proxies for our empirical analysis.

One key quantity that is unobservable in the data is the expected market return μ_t. Its risk-free rate component is observable, but the equity premium is not. One proxy for the

equity premium is market return volatility ($MVOL$). This volatility is highly correlated with the equity premium in our model because both variables decrease with y_t in the plausible range. Based on our long simulated time series, the correlation between $MVOL$ and the equity premium is 0.90, whereas $MVOL$'s correlations with $\overline{\rho}_t$ and $\widehat{\sigma}_t$ are 0.05 and zero, respectively. All correlations are computed for first differences because those are used in the empirical work. The second proxy for changes in μ_t is realized market return, motivated by the fact (e.g., Campbell (1991)) that market returns seem to respond more to news about discount rates than to news about cash flows. High realized market returns thus likely reflect declines in expected market return, and vice versa. In our simulation, realized market returns are indeed highly negatively correlated with changes in μ_t (-0.94).

Prior uncertainty $\widehat{\sigma}_t$ is also unobservable in the data. Both the M/B and the return volatility of IPOs are strongly positively related to $\widehat{\sigma}_t$, but neither the M/B nor the volatility of the long-lived firm depends on $\widehat{\sigma}_t$. This distinction suggests two proxies for $\widehat{\sigma}_t$. One proxy, $NEWVOL_t = \sigma_{R,t}^{ipo} - \sigma_{R,t}^{m}$, compares the return volatilities of IPOs and the long-lived firm. The second proxy compares their M/B ratios: $NEWMB_t = \log\left(M_t^{ipo}/B_t^{ipo}\right) - \log\left(M_t^m/B_t^m\right)$. The intuition that both $NEWVOL$ and $NEWMB$ should increase with $\widehat{\sigma}_t$ is confirmed in our long simulated sample. Both proxies have high positive correlations (0.80 and 0.59) with $\widehat{\sigma}_t$, but their correlations with the other two state variables are much lower: 0.09 with μ_t and zero with $\overline{\rho}_t$ for $NEWVOL$, -0.29 with μ_t and 0.09 with $\overline{\rho}_t$ for $NEWMB$. Thus, our proxies for changes in market conditions have solid theoretical motivation.

Table II examines the variation of these proxies around simulated IPO waves. $MVOL$ declines during the pre-waves by an average of 0.47% per year, which reflects a pre-wave decline in expected market return. $NEWVOL$ and $NEWMB$ both increase during the pre-waves by 2.34% per year and 0.07, respectively, which reflects a pre-wave increase in prior uncertainty.[17] Realized market returns should be unusually high before IPO waves, especially due to declines in expected market return. Indeed, Panel B shows that average return is significantly higher during the pre-waves than outside: 40.30% compared to 7.05% per year. Market returns during IPO waves and in the first three post-wave months are relatively low, about 9% for total returns, which is less than the 10.27% average outside a wave. There are two reasons behind the lower market returns. First, these returns are expected to be low if the wave is caused by a pre-wave decline in expected return. Second, market conditions typically begin deteriorating during the wave because of the endogeneity of IPO timing. If market conditions continued to get better, the wave would likely continue as well.

C. Regression Analysis

Table III analyzes the determinants of IPO volume in a regression framework. Each

column reports the coefficients from a regression of the number of IPOs on the variables listed in the first column. All variables are simulated from our calibrated model. Although the model is simulated at a monthly frequency, all variables are cumulated to the quarterly frequency so that Table III matches its empirical counterparts, Tables VI and VII. We do not report any p-values. All coefficients are highly statistically significant because the simulated sample is so large (40,000 quarters).

******************* INSERT TABLE III HERE *******************

We first examine the discount rate channel. As shown in column 1 of Table III, IPO volume increases after declines in expected market return over the previous two quarters. Column 6 shows that IPO volume also increases after declines in the risk-free rate. Column 5 shows that declines in *MVOL* tend to be followed by more IPOs. The results in column 4 also support the discount rate channel: IPO volume is positively related to past market returns, but negatively related to future and current returns. Realized returns are high while the expected market return drops, but they are low after the drop stops.

The cash flow and uncertainty channels are also supported by Table III. Column 2 shows that IPO volume is high after increases in $\bar{\rho}_t$. IPO volume is also high after increases in prior uncertainty $\widehat{\sigma}_t$, as shown in column 3, as well as after increases in *NEWMB* and *NEWVOL* (columns 8 and 9), both of which proxy for $\widehat{\sigma}_t$ in our empirical work. Moreover, IPO volume is positively related to the level of M/B in the previous quarter. This relation is significant statistically but not economically, as shown in Table II.

To be consistent with the subsequent empirical regressions, all regressions in Table III include a lag of IPO volume on the right-hand side. This lag is always significant, but its removal does not alter any of the relations noted above. When we compare the R^2's in the first three columns, the discount rate channel seems the strongest, and the cash flow channel the weakest. The R^2's are relatively low, between 0.04 and 0.12, because the true relations between IPO volume and the given variables are complex and nonlinear. We run linear regressions to be consistent with our empirical regressions, and also because they suffice to demonstrate the presence of all three channels that produce IPO waves in our model.

D. Robustness to Pre-IPO Idiosyncratic Risk

In our model, a private firm's IPO timing decision is driven by the firm's market value, which varies only with market conditions and with the passage of time (equations 11 and 12). The firm's value does not depend on firm-specific risk, because there is no production or learning before the IPO. In reality, though, private firms usually do face idiosyncratic risk, which creates firm-specific reasons for going public. This section explains why the main

predictions of our model obtain also in the presence of pre-IPO idiosyncratic risk.

In general, a private firm's decision to go public depends on the firm's own expected return, its own expected profitability, and its own prior uncertainty. We refer to these three elements as "firm conditions." Firm conditions clearly depend on market conditions. For example, if expected market return drops, expected individual stock returns must also drop, on average. In our model, firm conditions for private firms are in fact perfectly correlated with market conditions, because there is no pre-IPO idiosyncratic risk. The correlation is lower if such risk is present, which raises the question of whether firm conditions move together sufficiently to cause IPO waves. Measuring this comovement is difficult, because firm conditions are unobservable. Related evidence is provided by Vuolteenaho (2002), who finds that changes in expected returns are highly correlated across firms and concludes that these changes are "predominantly driven by systematic, marketwide components." More generally, the comovement in firm conditions must be significant because stock prices change if and only if firm conditions change, and stock prices do exhibit significant comovement. For example, of the 17,832 firms with more than three years of data on *CRSP* between January 1926 and December 2002, 96.2% have positive estimated market betas, and 74.2% of those betas are statistically significant. The average R^2 from the corresponding monthly market model regressions is 0.13. Note that for changes in market conditions to affect IPO volume, most of the variation in firm conditions does not need to be common; it is sufficient if a significant part of this variation is common.

To analyze theoretically how idiosyncratic risk affects IPO waves, we solve a modified version of our model in which private firms face idiosyncratic risk due to pre-IPO learning. We assume that agents observe signals about $\bar{\psi}^i$ before the IPO, so that the perception of $\bar{\psi}^i$ exhibits firm-specific pre-IPO variation. We simulate the modified model with signal precision chosen to make idiosyncratic risk more important than in the data.[18] As expected, the IPO volume in the simulation is less volatile than in our original model. This deviation from our model is realistic, because IPO volume is more volatile in our model than it is in the data (cf. Figures 1 and 4). More important, the IPO waves observed in the simulation have properties very similar to those obtained in our model. The discount rate and cash flow channels remain highly significant in the simulated regressions; only the uncertainty channel is weaker because higher uncertainty about pre-IPO signals increases the value of the option to wait (Cukierman (1980)). Therefore, our conclusions also hold in the presence of pre-IPO idiosyncratic risk. We focus on the simpler framework without pre-IPO learning because the modified framework is significantly more complicated and computationally challenging without adding any substantial new insights into the time variation in IPO volume.

IV. Empirical Analysis

This section empirically investigates the three channels (discount rate, cash flow, and uncertainty) through which time-varying IPO volume is created in our model.

A. Data

The data on the number of IPOs, obtained from Jay Ritter's website, cover the period January 1960 through December 2002. To avoid potential concerns about nonstationarity (see Lowry (2003)), we deflate the number of IPOs by the number of public firms at the end of the previous month.[19] In the rest of the paper, "the number of IPOs" and "IPO volume" both refer to the deflated series, whose values range from zero to 2.1% per month, with an average of 0.5%. The pattern of time variation in the deflated series looks very similar to the pattern in the raw series plotted in Figure 1.

The data on our proxies for changes in market conditions are also constructed monthly for January 1960 through December 2002, unless specified otherwise. We use all data available to us. Market returns (MKT) are total returns on the value-weighted portfolio of all NYSE, Amex, and Nasdaq stocks, extracted from *CRSP*. Market volatility ($MVOL$) is computed each month after July 1962 as standard deviation of daily market returns within the month. The aggregate M/B ratio (M/B), plotted in Figure 5, is the sum of market values of equity across all ordinary common shares divided by the sum of the most recent book values of equity. The real risk-free rate (RF) is the yield on a one-month T-bill in excess of expected inflation, where the latter is the fitted value from an AR(12) process applied to the monthly series of log changes in CPI from the Bureau of Labor Statistics. Aggregate profitability, measured as return on equity (ROE), is computed quarterly for 1966Q1 through 2002Q1 using *Compustat* data, as described in the Appendix. This measure of profitability follows the definition of ρ_t^i in Section I.A. Another measure of cash flow expectations is the $I/B/E/S$ summary data on equity analysts' forecasts of long-term earnings growth. These forecasts have horizons of five years or more, which makes them suitable, given the relatively long-term nature of $\bar{\rho}_t$. For each firm and each month, the average forecast of long-term earnings growth is computed across all analysts covering the firm. The forecast of average earnings growth ($IBES$) is then computed by averaging the average forecasts across all ordinary common shares. The resulting series is available for November 1981 through March 2002.

******************** INSERT FIGURE 5 HERE ********************

The proxies for prior uncertainty are constructed as follows. New firm excess volatility ($NEWVOL$) in a given month is computed by subtracting market return volatility from the median return volatility across all new firms, which are defined as those firms whose first

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appearance in the *CRSP* daily file occured in the previous month. A given firm's return volatility in each month is the standard deviation of daily stock returns within the month. The variable *NEWVOL* has 464 valid monthly observations in the 486-month period between July 1962 and December 2002. New firm excess M/B ratio (*NEWMB*) is computed for each month between January 1950 and March 2002 as follows. First, we compute the median M/B across all new firms, which are defined as firms that appeared in the *CRSP* monthly file in the previous year.[20] The variable *NEWMB* is computed as the natural logarithm of that median minus the log of the median M/B across all firms. The construction of M/B for individual firms is described in the Appendix. The variable *NEWMB* has eight missing values between January 1960 and March 2002. The monthly time series of *NEWVOL* and *NEWMB* are plotted in Figure 6.

******************** INSERT FIGURE 6 HERE ********************

Figure 6 shows that both *NEWMB* and *NEWVOL* rise sharply in the late 1990s and decline after 2000. The variable *NEWVOL* exhibits a remarkable pattern: In 1998, it triples from about 2% per day to about 6%, it remains around 6% through the end of 2000, and then it drops back to about 2% after 2000. Prior uncertainty was apparently unusually high in 1998 through 2000. This fact is not surprising, since long-term prospects of new firms are particularly uncertain when new paradigms are being embraced. The high prior uncertainty may have induced many firms to go public in the late 1990s, and it might also have contributed to the high valuations of many IPOs at that time.

B. Empirical Evidence around IPO Waves

Between January 1960 and December 2002, there are 16 IPO waves. Their lengths range from one to 21 months, with a median of five months. Some summary statistics for the 16 waves are shown in Tables IV and V. All variables except for the unitless *M/B* and *NEWMB* are in percent per year. In Table IV, all but three waves are preceded by above-average market returns during the pre-wave, as predicted by the model. Only one (one-month) wave is preceded by a negative return. For all but two waves, *MVOL* declines during the pre-wave, which is consistent with a pre-wave decline in expected market return. The wave that begins in 1993 appears to be due to the cash flow channel. The waves in 1991, 1992, and especially 1999 are preceded by increases in both *NEWVOL* and *NEWMB*, suggesting that these waves may have been caused at least in part by increases in prior uncertainty.

******************** INSERT TABLE IV HERE ********************

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Table V reports variable averages across the 16 waves. The *t*-statistics, given in parentheses, measure the significance of the difference between the averages within and outside the given period. For example, the *t*-statistic for average *MVOL* during a wave (-3.18) is computed by regressing *MVOL* on a dummy variable equal to one if the month is part of an IPO wave, and zero otherwise. A positive (negative) *t*-statistic indicates that the variable's average in the given period is bigger (smaller) than the average in the rest of the sample.

******************** INSERT TABLE V HERE ********************

The average pre-wave change in *MVOL* is significantly negative at -2.81% ($t = -2.27$), which is consistent with IPO waves being caused by declines in expected market return. The values of *M/B*, *ROE*, and *IBES* all increase before the waves, as the model predicts, but these increases are statistically insignificant. The value of *NEWVOL* increases significantly during pre-waves ($t = 2.27$), consistent with the uncertainty channel, but the value of *NEWMB* does not. The value of *RF* increases insignificantly during pre-waves, contrary to our model, which predicts a pre-wave decrease. Panel B shows that average market returns are high before IPO waves (e.g., 31.17% annualized with $t = 2.77$ two quarters before a wave), as predicted by the model. Market returns are low during and especially after IPO waves, but they are not significantly lower than in the rest of the sample. The return pattern is similar to the model-predicted pattern observed in Table II.

Since the averages in Table V are computed across only 16 IPO waves, only a few relations are statistically significant. More detailed empirical analysis is therefore performed in the following section, which focuses on IPO volume rather than on IPO waves alone.

C. Regression Analysis

Each column in Tables VI and VII corresponds to a separate regression, in which the number of IPOs in the current quarter is regressed on proxies for changes in market conditions. Lagged IPO volume is included on the right-hand side to capture persistence in IPO volume that is unexplained due to any potential misspecification in the regressions. Lowry (2003) also includes lagged IPO volume on the right-hand side of her regressions. She also always includes a first-quarter dummy that captures the apparent seasonality in IPO volume, and we follow her treatment. Both variables are significant in each regression.

******************** INSERT TABLE VI HERE ********************

******************** INSERT TABLE VII HERE ********************

First, we test the discount rate channel, in which IPOs are triggered by declines in

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expected market return. Column 1 of Table VI shows that IPO volume is positively related to total market returns over the previous two quarters ($t = 3.34$ and 3.25), which is consistent with both the discount rate and cash flow channels. Moreover, IPO volume is negatively related to market returns in the subsequent quarter ($t = -2.23$), which is consistent with the discount rate channel. This negative relation is also reported by Lamont (2002), Schultz (2003), and Lowry (2003). The relation with current returns is positive, not negative as in Table III, but this difference does not contradict the model. IPO waves in the data tend to last longer than our simulated IPO waves, so the actual IPO waves have more overlap than the simulated waves with the declines in expected market return that caused the waves and, therefore, also with high realized returns. Column 2 shows that IPO volume is negatively related to current ($t = -4.41$) as well as past ($t = -3.59$) changes in market volatility, which is again consistent with the discount rate channel. In column 3, changes in the risk-free rate are positively related to future IPO volume, not negatively as the model predicts. Combined with the results in columns 1 and 2, this positive relation suggests that IPO volume is strongly negatively related to recent changes in the equity premium.

Second, the cash flow channel is also supported by the data. Column 1 of Table VII shows that IPO volume is positively related to current ($t = 2.50$) as well as future changes in aggregate profitability, which suggests that firms go public when cash flow expectations improve. Column 2 reaches the same conclusion. IPO volume is higher ($t = 5.07$) when equity analysts upgrade their forecasts of long-term earnings growth.

Third, prior uncertainty also seems to go up before firms go public. In columns 3 and 4 of Table VII, IPO volume is positively related to recent changes in the excess M/B ratio of new firms ($t = 3.18$ and 2.35), as well as to recent changes in the excess volatility of new firms ($t = 2.23$), both of which comove with prior uncertainty in our model.

Some of the proxies for changes in market conditions lose their statistical significance when realized market returns are included in the regression. The reason goes beyond the simple lost-degrees-of-freedom effect. In reasonably efficient markets, prices reflect much of the available information, and realized market returns are the best proxy for changes in market conditions; i.e., when market conditions improve, prices go up, and vice versa. Thus, it is not surprising that including market returns drives some of the weaker proxies below the threshold of significance. The role of these other proxies is only to provide additional evidence on the likely causes of the observed price changes.

The regressors in Tables VI and VII represent changes in market conditions, whereas the regressand is the level of IPO volume. Regressing levels on changes is appropriate because the level of IPO volume is driven by changes in market conditions in our model. Lowry (2003), who uses the same dependent variable as we do, also suggests using changes in the number of IPOs as a way of avoiding nonstationarity. Using this redefined dependent vari-

able leads to results that are almost identical to those reported here.

D. Rational vs. Irrational IPO Waves

Many recent studies blame time-varying IPO volume on market inefficiency, arguing that IPO volume is high when shares are overvalued. In this section, we examine the extent to which our empirical evidence is consistent with the simple behavioral story in which firms go public to take advantage of irrational overpricing.

In the mispricing story, IPO volume is high when the market is overvalued. Under the common behavioral assumption that misvaluation is reflected in M/B, this story predicts a positive relation between IPO volume and the level of aggregate M/B. Our rational model also predicts a positive relation (see Table III), but a weak one (see Table II), because IPO volume in our model is driven mainly by changes in market conditions, not levels. Column 4 of Table VI shows that IPO volume is not significantly related to the level of aggregate M/B at the end of the previous quarter.[21] Column 5 presents a horserace between the levels and changes, in that IPO volume is regressed on M/B as well as on market returns. In this regression, market returns remain highly significant and M/B remains insignificant. That is, IPO volume is high after a run-up in stock prices, but not necessarily when the level of prices is high. This evidence, which fits the intuition described in Section III, provides additional support for our model, but not for the overvaluation story.

Neither can our evidence related to the cash flow channel be easily explained by the mispricing story. One of our proxies for expected cash flow, *IBES*, might be subject to behavioral biases if analyst forecasts are biased. However, consider our second proxy, aggregate profitability (*ROE*). Column 1 of Table VII shows that IPO volume is positively related to current and future changes in *ROE*. This relation is not predicted by the mispricing story, in which IPO decisions do not reflect rational expectations of future cash flows.

The mispricing story also cannot fully explain our results related to the uncertainty channel. One proxy for prior uncertainty, *NEWMB*, can be subject to behavioral biases if we accept the idea that new firms can be more overvalued than seasoned firms. However, it is not obvious how the mispricing story could justify our result that IPO volume is positively related to changes in our second proxy, *NEWVOL*. Mispricing might affect the price level, but it is not clear why it should affect the return volatility of new firms.

Nor can the mispricing story account for all of our evidence related to the discount rate channel. One of our proxies for changes in expected market return, *MKT*, might be biased due to mispricing, but it is not clear why our second proxy, *MVOL*, should be biased. In the mispricing story, expected market return is driven by investor sentiment, and there is no obvious reason for market volatility to be related to investor sentiment. Therefore, the

mispricing story does not explain why IPO volume is significantly related to changes in *MVOL* in column 2 of Table VI. In summary, four of our empirical findings are consistent with our rational model, but they are not predicted by the mispricing story.

V. IPOs and Investment

This section examines the important role of investment in our model. The model features a link between a firm's decisions to go public and to invest. We discuss the plausibility of such a link, pointing to firm-level evidence on the extent to which IPO proceeds are invested, as well as to some evidence on the relation between IPO volume and investment in aggregate. We also discuss the relation between aggregate investment and market conditions.

The main purpose of an IPO in our model is to raise capital for investment. This description applies only to a subset of the observed IPOs, because many real-world IPOs happen for reasons other than investment, such as refinancing. However, as long as some firms go public to raise funds for investment, IPO volume should be affected by market conditions. Many firms indeed appear to invest their IPO proceeds. Mikkelson et al. (1997) report that 64% of the firms going public state in their offering prospectus that the reason for their IPO is to finance capital expenditures. Moreover, Jain and Kini (1994) report that the capital expenditures of IPOs grow by 142% in the two years around the IPO, on average, which significantly exceeds the contemporaneous investment growth for industry-matched seasoned firms. In fact, the industry-adjusted growth rate in the capital expenditures of firms going public is as large as 109% over the two-year period. Therefore, a link between the decisions to go public and to invest seems reasonable.[22]

The link between IPOs and investment seems present also in the aggregate data. Using data on real private nonresidential fixed investment between 1947 and 2002, obtained from the Bureau of Economic Analysis (BEA), we find that aggregate investment growth is significantly positively correlated with IPO volume. Lowry (2003) finds private firms' demands for capital to be a key empirical determinant of IPO volume, further supporting the link between IPOs and investment. Lowry also reports that the total amount raised in the IPOs is more volatile than the total amount invested, which is precisely what our model predicts. In our model, the firm invests only part of the IPO proceeds; the rest goes to the inventor as compensation for the patent, to pay for the inventor's pre-IPO consumption. The variation in IPO proceeds therefore exceeds the variation in the amount invested. The IPO decision is often de-linked from the investment decision in the leading explanations for IPO volume, such as market mispricing and asymmetric information, but the link is essential to obtaining the relation between IPO volume and market conditions documented in this paper.

Our focus is on IPO waves, but our model can also address a broader issue of cyclicality of investment. A public firm solving for the optimal time to make an irreversible invest-

ment is considering tradeoffs similar to those of our inventor, and "investment waves" might develop after market conditions improve. Consistent with this idea, several studies (e.g., Barro (1990), and Baker et al. (2003)) report a positive relation between investment and stock prices. Using the BEA data, we find that investment growth is positively related to recent market returns and negatively related to future market returns. Investment growth is also positively related to current and future changes in aggregate *ROE*. We conclude that aggregate investment is related to changes in market conditions, similar to IPO volume.

These results suggest that our model makes useful predictions not only for IPO volume, but also for aggregate investment. At the same time, we believe that our model is better suited for studying investment by new firms than for examining investment by public firms, for several reasons. First, public firms often invest simply to maintain a competitive stock of physical capital, rather than to embark on new projects with uncertain and perishable abnormal profits. This fact makes some features of our framework, such as prior uncertainty, less relevant for public firms. Indeed, aggregate investment growth seems unrelated to our proxies for prior uncertainty in the data. Second, the investment decisions of public firms may be affected by the firms' existing projects, a complication that is absent from our model in which inventors have only one project at a time. Third, we assume that learning about $\overline{\psi}^i$ starts when the production begins, which seems to better describe IPOs of start-up companies than investment by public firms. Learning about a public firm's new project can take place before the production begins, because investors can observe the firm's other projects, whose payoffs are presumably correlated with the new project's payoffs.

In addition, focusing on IPOs rather than on aggregate investment preserves market completeness. New projects introduce new idiosyncratic risk ($W_{i,t}$) in the economy. This risk is not spanned by the existing securities, so markets become incomplete unless a new security is issued that can perfectly hedge the new risk. Markets are dynamically complete in our model because each new project is accompanied by an issue of a claim on the project's cash flow. This issue has a natural interpretation as an IPO of a start-up company. The equity issued in the IPO provides a perfect hedge for the new project because it is a claim on that project only. In contrast, investments by public firms are not accompanied by issues of equity that would provide a perfect hedge. For example, the equity issued in an SEO is a claim to all projects of this public firm, not just the new project. Due to market incompleteness in that case, the SDF may not be unique, which could complicate the analysis.

VI. Conclusion

In their survey of the IPO literature, Ritter and Welch (2002) conclude that "market conditions are the most important factor in the decision to go public." We agree, and we point out three dimensions of market conditions that appear especially relevant. Ritter

and Welch also state that "perhaps the most important unanswered question is why issuing volume drops so precipitously following stock market drops." Our paper provides a simple answer. When market conditions worsen, stock prices drop and IPO volume declines because private firms choose to wait for more favorable market conditions before going public.

This answer is only one of many testable implications of our model of optimal IPO timing. We show by simulation that the model also implies that IPO waves should be preceded by high market returns, followed by low market returns, and accompanied by increases in aggregate profitability. In addition, IPO waves should be preceded by an increased disparity between new firms and old firms in terms of their valuations and return volatilities. IPO volume should be related to changes in stock prices, but less so to their levels. All of these implications are confirmed in the data.

Some implications of our model, such as the low post-wave market returns, are also consistent with the behavioral story in which firms go public in response to market over-valuation. However, several of our empirical findings are not predicted by this behavioral story. For example, this story does not predict that IPO volume should be related to recent changes in market return volatility or positively related to changes in aggregate profitability.

Behavioral biases have also been blamed for the high IPO valuations observed in the late 1990s, but those valuations need not have been irrational. IPO valuations in our model tend to be relatively high, partly because IPO timing is endogenous and partly due to prior uncertainty about the average future profitability of IPOs. According to its proxies, prior uncertainty was unusually high in the late 1990s. This high prior uncertainty may have attracted many firms to go public, and it might also have contributed to the high valuations of many IPOs at that time.

Many IPOs in the 1990s happened in technology-related industries. Industry clustering of IPOs obtains in a minor extension of our model. Instead of assuming that prior uncertainty is the same for all firms, we can assume that this uncertainty is more similar for firms in the same industry. Average excess profitability is also likely to be more correlated across firms in the same industry. Increases in industry-specific prior uncertainty or industry-specific excess profitability can lead to IPO waves concentrated in the given industry, without triggering IPOs in other industries. These implications can be tested empirically in future work.

Future research can also endogenize the innovation process. We assume that new ideas arrive at a constant pace, but if capital must be raised to produce an idea, then low cost of capital might accelerate innovation, leading to more ideas and more IPOs. High expected aggregate profitability might also speed up innovation and produce more IPOs. These effects, if present, would link IPO volume more closely to the level of market conditions, and they would also amplify the variation in IPO volume obtained in our model.
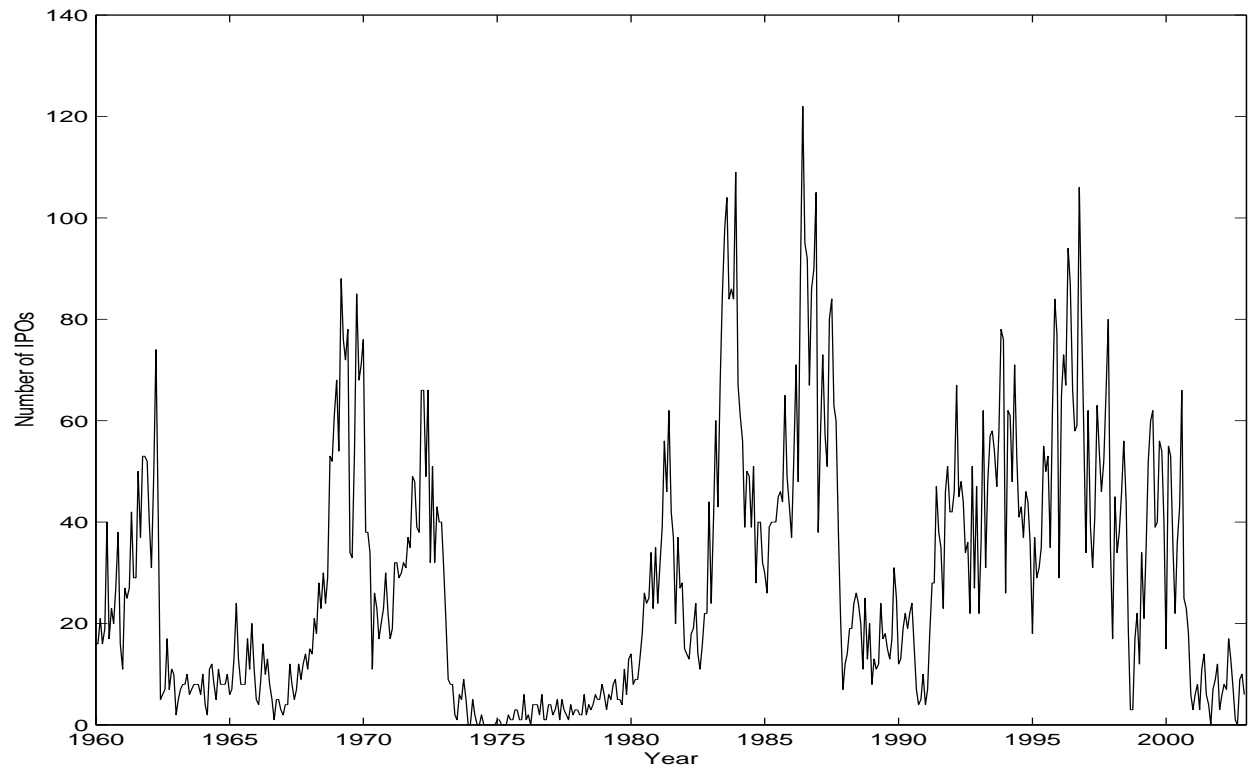
Figure 1. IPO volume. The figure plots the number of IPOs in each month between January 1960 and December 2002. The data is obtained from Jay Ritter's website.
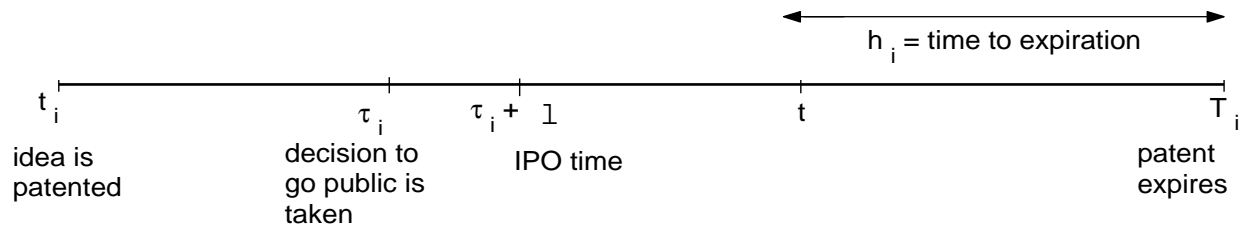
Figure 2. The timing of events in our model. At time t_i, an idea is patented by an inventor. The patent expires at time T_i. The inventor chooses whether to go public, and if so, when. If the inventor decides to go public at time τ_i, the IPO takes place at time $\tau_i + \ell$.
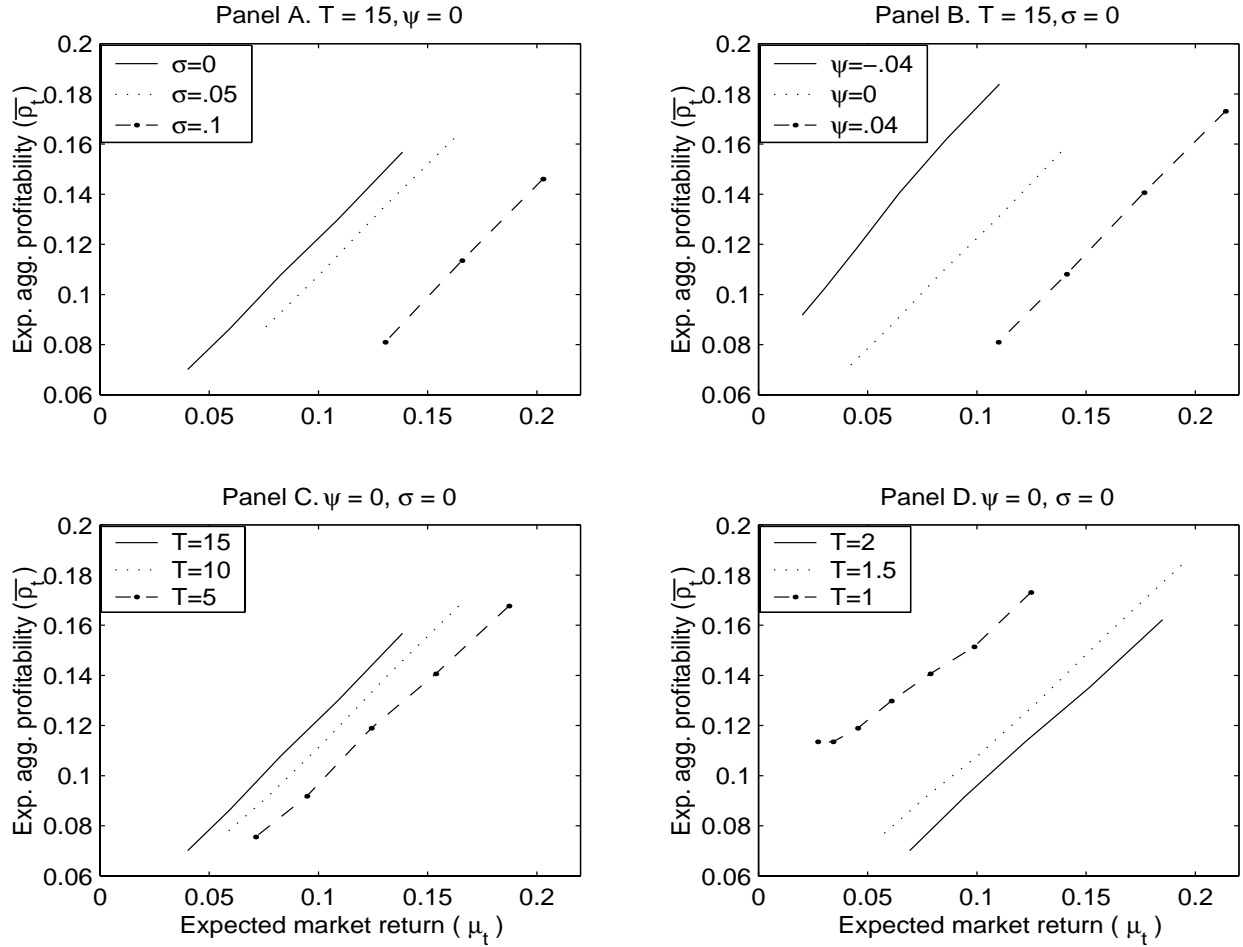
Figure 3. Optimal IPO timing. Each panel plots the entry boundary; i.e., the set of pairs of expected market return (horizontal axis) and expected aggregate profitability (vertical axis) that trigger the decision to go public. An IPO takes place in the parameter region northwest of each boundary. The entry boundaries are reported for three levels of prior uncertainty $\widehat{\sigma}_t$ per year (Panel A), firm-specific excess profitability $\hat{\psi}$ per year (Panel B), and time to the patent's expiration in years (Panels C and D).
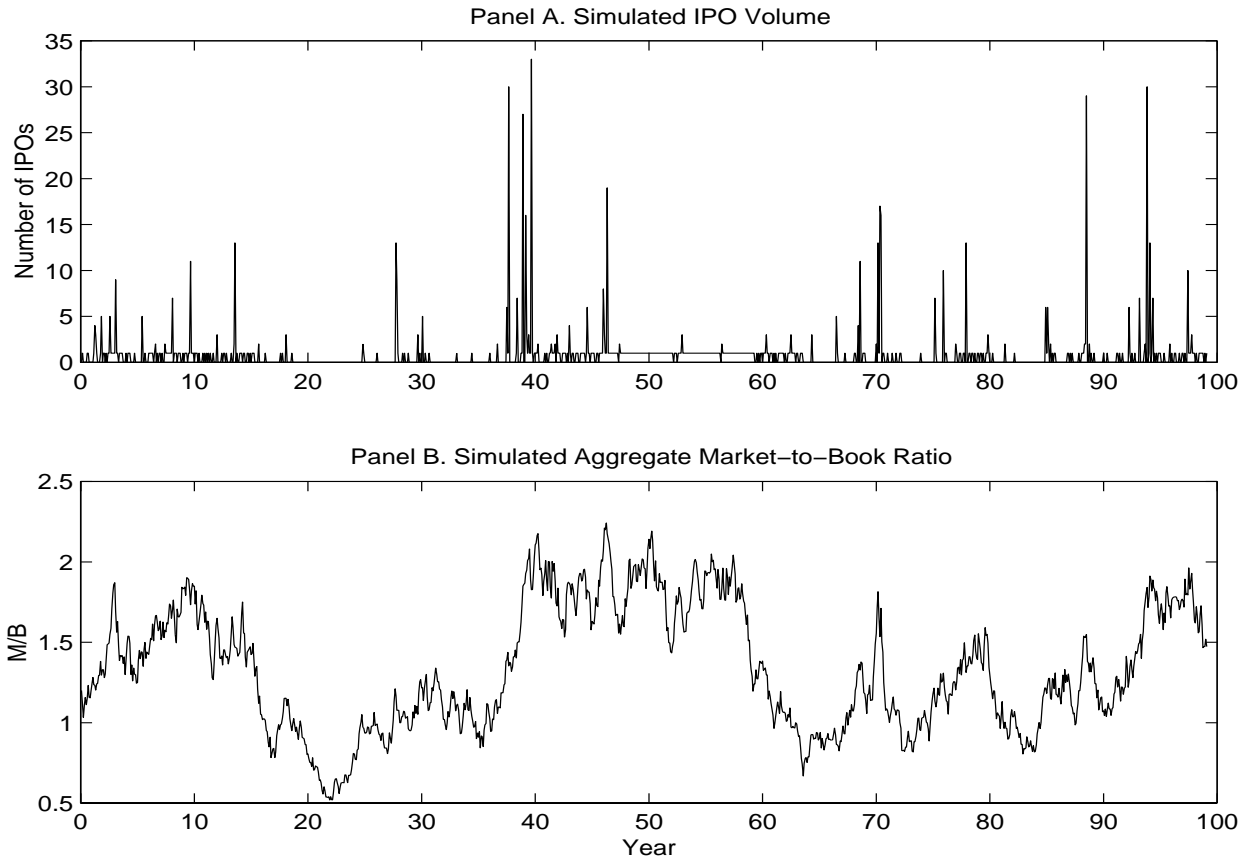
Figure 4. Simulated IPO waves. The figure plots the monthly number of IPOs and the aggregate M/B ratio in a randomly selected 100-year segment of our simulated data. All IPOs are optimally timed in the sense of Section II. The aggregate M/B ratio is defined as the sum of earnings divided by the sum of book values across all firms in the simulation.
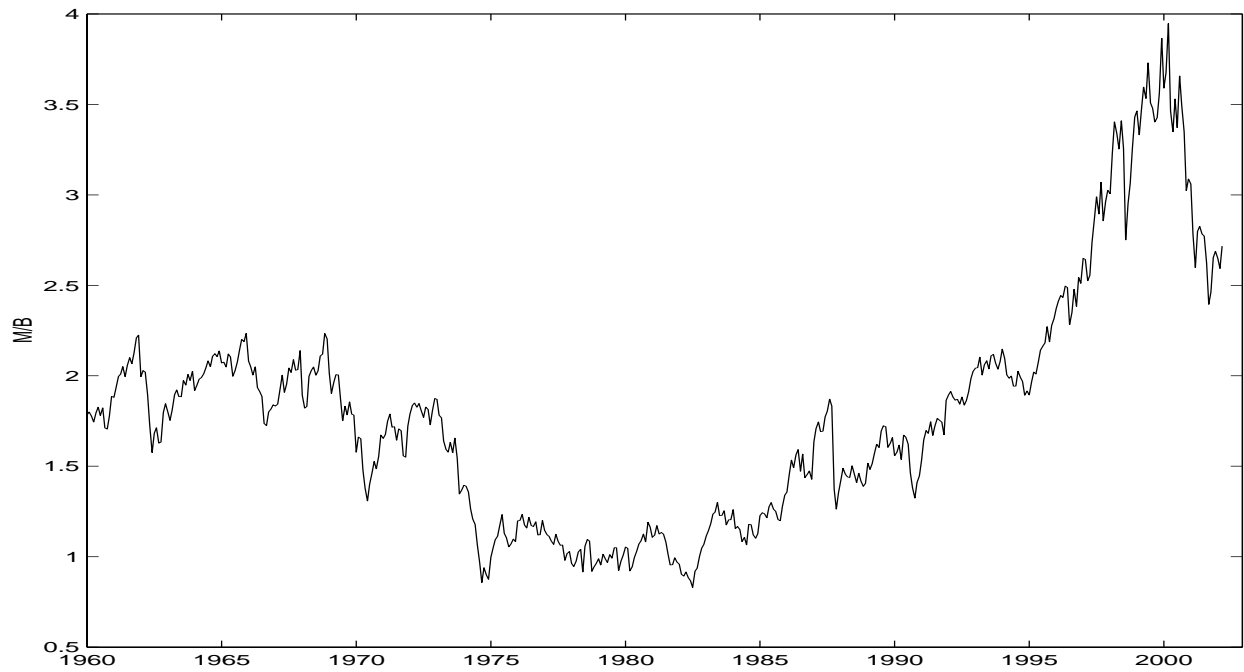
Figure 5. Monthly time series of the aggregate market-to-book ratio. Each month, the aggregate market-to-book ratio (M/B) is computed as the sum of market values of equity across all firms divided by the sum of the most recent book values of equity.

Panel A. New Firm Excess Volatility (NEWVOL)

Panel B. New Firm Excess M/B (NEWMB)
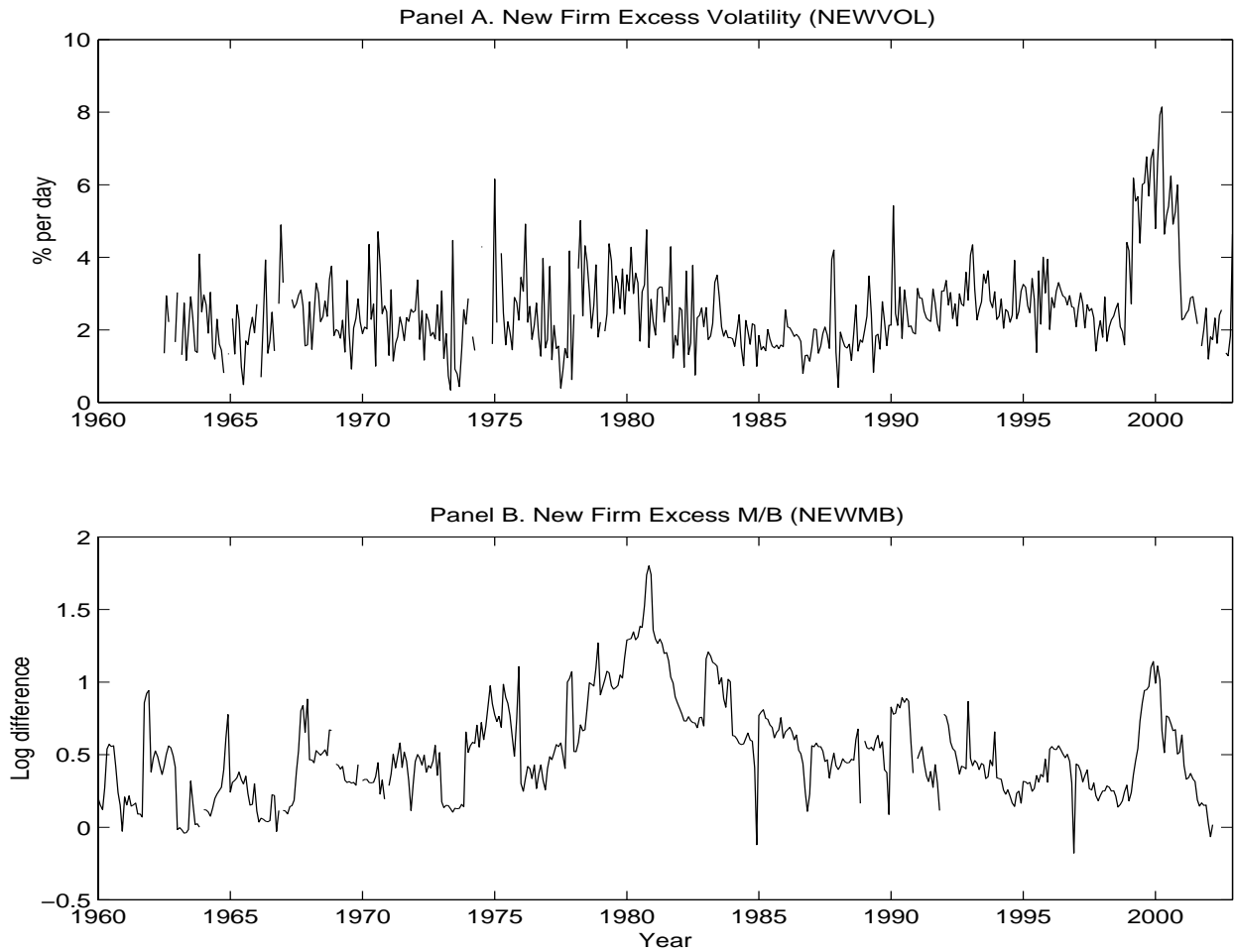
Figure 6. Monthly time series of proxies for prior uncertainty. Panel A plots *NEWVOL*, the median return volatility (standard deviation of daily returns) across all newly listed firms in excess of market return volatility. The values of *NEWVOL* are available between July 1962 and December 2002. Panel B plots *NEWMB*, the log median M/B across all newly listed firms in excess of the log median M/B across all firms. The values of *NEWMB* are available between January 1960 and March 2002.

Table I
Parameter Values in the Calibrated Model

The table reports the parameter values used to calibrate our model. The parameters of the processes for expected aggregate profitability and consumption growth are estimated from the consumption and aggregate profitability data using the Kalman filter. The value of $\sigma_{L,0}$ is restricted to zero to eliminate correlation across the three state variables ($\overline{\rho}_t$, y_t and $\widehat{\sigma}_t$). The parameters of the individual profitability process are calibrated to the median firm in our sample. The utility parameters (η and γ), the parameters defining the log surplus consumption ratio $s(y) = a_0 + a_1 y_t + a_2 y_t^2$, and those characterizing the state variable y_t are calibrated to match the observed levels of the equity premium, market volatility, aggregate M/B, and the interest rate. The transition probabilities $\lambda_{i,i\pm1}$ that characterize the uncertainty process $\widehat{\sigma}_t$ on the grid $\mathcal{V} = \{0, .1, ..., .12\}$ are chosen to obtain plausible dynamics for $\widehat{\sigma}_t$. The transition probability at the boundaries of the grid is denoted by λ_b. All entries are annualized.

Panel A. Parameters of Aggregate Profitability, Consumption Growth, and Individual Profitability

k_L	$\overline{\rho}_L$	σ_{LL}	$\sigma_{L,0}$	b_0	b_1	σ_c	ϕ^i	$\sigma_{i,0}$	$\sigma_{i,i}$
0.1412	12.16 %	0.64 %	0	1.40 %	0.0812	0.94 %	0.3968	4.79 %	6.82 %

Panel B. Parameters of the Utility Function, Surplus Consumption Ratio, and Prior Uncertainty

η	γ	k_y	$\overline{y}$	σ_y	a_0	a_1	a_2	$\lambda_{i,i\pm1}$	λ_b
0.0475	3.70	0.073	-0.0017	0.5156	-2.8779	0.2132	-0.0198	10%	20%

Panel C. Unconditional Moments from the Calibration

$E\left[R_t^{mkt}\right]$	$\sigma\left(R_t^{mkt}\right)$	$E\left[r_{f,t}\right]$	$\sigma\left(r_{f,t}\right)$	$E\left[M/B\right]$	$\sigma\left(M/B\right)$	$E\left[\widehat{\sigma}_t\right]$	$\sigma\left(\widehat{\sigma}_t\right)$	$E\left[\overline{\rho}_t\right]$	$\sigma\left(\overline{\rho}_t\right)$
6.8%	15%	3.3%	3.9%	1.7	.614	6.11%	3.5%	12.1%	1.2%

Table II
Simulation Evidence Around IPO Waves

The table reports averages of selected variables and market returns around simulated IPO waves. "b" stands for the beginning of an IPO wave, i.e., the end of the last month before the wave begins. "e" stands for the end of the wave's last month. "b(e)$\pm n$" denotes n months before or after the beginning (end) of a wave. A pre-wave is defined as the period that begins at the end of month b–3 and ends at the end of month e–3. Expected excess and total returns are computed for the market portfolio, the value-weighted portfolio of all existing simulated firms. Expected profitability stands for $\overline{\rho}$, and prior uncertainty stands for $\widehat{\sigma}$. The variable $MVOL$ is market return volatility, RF is the risk-free rate, M/B is the aggregate M/B ratio, $NEWVOL$ is the difference between the return volatility of a new firm and market volatility, and $NEWMB$ is the log difference between the M/B of a new firm and the M/B of the market. All variables except for M/B and $NEWMB$ are expressed in percent per year.

Panel A. Averages of first-column variables

	Avg change in pre-wave	Before wave b-6	Before wave b-3	Before wave b	Wave b+1:e	Wave e	After wave e+3	After wave e+6	Outside wave
Expected total return	-0.99	10.50	10.03	9.23	8.75	9.27	9.31	9.33	9.93
Expected profitability	0.06	12.12	12.14	12.18	12.21	12.17	12.17	12.17	12.16
Prior uncertainty	0.33	5.10	5.12	5.41	5.52	5.41	5.44	5.43	5.73
Expected excess return	-0.46	7.45	7.26	6.88	6.66	6.90	6.90	6.90	6.79
RF	-0.53	3.05	2.77	2.34	2.08	2.37	2.40	2.43	3.15
M/B	0.11	1.59	1.64	1.73	1.78	1.73	1.72	1.72	1.76
MVOL	-0.47	16.22	16.06	15.67	15.45	15.69	15.68	15.66	15.22
NEWVOL	2.34	43.62	43.30	45.46	46.18	45.48	45.77	45.74	48.73
NEWMB	0.07	0.22	0.24	0.30	0.33	0.30	0.30	0.30	0.29

Panel B. Average realized market returns

	Pre-wave b–2:e–3	Outside pre-wave	Before wave b–5:b–3	Before wave b–2:b	Wave b+1:e	After wave e+1:e+3	After wave e+4:e+6	Outside wave
Total return	40.30	7.05	21.99	32.38	8.99	9.10	9.59	10.27
Excess return	37.97	3.92	18.99	29.84	6.91	6.72	7.18	7.12

Table III
Simulation Evidence: Regressions of IPO Volume on Selected Variables

Each column represents a quarterly regression of IPO volume on the variables listed in the first column. All variables are taken from a 10,000-year-long sample simulated from our calibrated model. No t-statistics are given because all reported numbers are highly significant. The notation "Δ" denotes changes (first differences), and "$-n$" ("$+n$") denotes quarterly lags (leads). The notation ER denotes expected total market return, MKT is realized market return, $MVOL$ is market return volatility, RF is the risk-free rate, M/B is the aggregate M/B ratio, $NEWMB$ is the log difference between the M/B of a new firm and the M/B of the market, $NEWVOL$ is the difference between the return volatility of a new firm and market volatility, and IPO is the number of firms that went public this quarter. The units are chosen to ensure some significant digits for all coefficients in the table: MKT is measured in decimals per month, and all other variables except for the unitless M/B and $NEWMB$ are in percent per year.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	2.25	2.21	2.22	2.03	2.22	2.23	0.69	2.22	2.22
$\Delta ER - 2$	-0.24								
$\Delta ER - 1$	-0.65								
$\Delta \bar{\rho}\text{-}2$		0.05							
$\Delta \bar{\rho}\text{-}1$		0.46							
$\Delta \hat{\sigma}\text{-}2$			0.08						
$\Delta \hat{\sigma}\text{-}1$			0.34						
$MKT - 2$				4.30					
$MKT - 1$				9.92					
MKT				-1.50					
$MKT + 1$				-1.78					
$MKT + 2$				-1.53					
$\Delta MVOL - 2$					-0.15				
$\Delta MVOL - 1$					-0.98				
$\Delta RF - 2$						-0.25			
$\Delta RF - 1$						-0.87			
$M/B - 1$							0.93		
$\Delta NEWMB - 2$								0.07	
$\Delta NEWMB - 1$								6.54	
$\Delta NEWVOL - 2$									0.00
$\Delta NEWVOL - 1$									0.03
$IPO(t-1)$	0.19	0.20	0.20	0.18	0.20	0.20	0.16	0.20	0.20
T	40000	40000	40000	40000	40000	40000	40000	40000	40000
R^2	0.11	0.04	0.05	0.12	0.08	0.10	0.07	0.08	0.04

Table IV
IPO Waves Observed in the Data

The table reports some summary statistics for the 16 IPO waves observed in our sample period of January 1960 through December 2002. "Avg pre-wave market return" for a given wave denotes the average monthly total market return during the respective pre-wave. The variable $MVOL$ is market return volatility, M/B is the aggregate M/B ratio, RF is the risk-free rate, ROE is aggregate profitability (return on equity), $IBES$ is the average analyst forecast of long-term earnings growth, $NEWVOL$ is the difference between the median return volatility of new firms and market volatility, and $NEWMB$ is the log difference between the median M/B of new firms and the median M/B across all firms. All variables except for M/B and $NEWMB$ are expressed in percent per year. The values that are not available due to missing data are denoted by "n/a."

	Beginning of wave	End of wave	Number of IPOs	Avg pre-wave market return	Pre-wave change in						
					MVOL	*M/B*	*RF*	*ROE*	*IBES*	*NEWVOL*	*NEWMB*
1	196108	196205	480	10.56	n/a	0.02	0.22	n/a	n/a	n/a	0.33
2	196810	197002	1061	0.07	-1.33	-0.26	-0.37	0.34	n/a	11.90	-0.07
3	197110	197207	488	13.47	-4.21	0.11	0.64	0.36	n/a	3.40	-0.15
4	197209	197209	32	-2.70	-4.09	-0.04	0.10	0.00	n/a	24.32	-0.12
5	197211	197211	40	19.02	-1.19	0.06	-0.08	0.00	n/a	-7.98	-0.02
6	198103	198107	245	16.16	-7.72	-0.06	2.81	-3.45	n/a	31.90	-0.51
7	198302	198407	1223	22.72	-17.14	0.15	1.45	0.48	1.71	0.38	-0.18
8	198507	198511	249	12.78	-1.39	0.01	-0.15	-0.96	-0.29	-0.82	-0.19
9	198601	198709	1517	27.69	-0.58	0.57	-2.22	-1.82	-0.75	6.11	-0.13
10	199111	199112	93	25.04	-2.65	0.03	-0.42	0.00	0.26	9.88	0.05
11	199202	199205	206	23.86	-0.30	0.17	0.74	3.83	0.19	22.81	0.46
12	199304	199406	828	6.85	3.88	-0.02	-1.02	6.33	0.79	0.64	-0.54
13	199507	199507	50	31.20	-1.99	-0.01	0.79	-0.12	-0.05	-3.66	0.01
14	199510	199612	1068	22.15	-0.80	0.34	-0.09	-0.01	2.21	-3.06	0.23
15	199710	199711	145	33.76	1.52	0.03	-0.31	-1.17	-0.11	-1.93	-0.14
16	199906	199907	122	19.60	-4.16	0.26	0.11	0.91	-0.08	54.14	0.21

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Table V
Empirical Evidence Around IPO Waves

The table reports averages of selected variables and market returns around IPO waves. "b" stands for the beginning of an IPO wave; i.e., the end of the last month before the wave begins. "e" stands for the end of the wave's last month. "b(e)±n" denotes n months before or after the beginning (end) of a wave. A pre-wave is defined as the period that begins at the end of month b–3 and ends at the end of month e–3. The variable $MVOL$ is market return volatility, M/B is the aggregate M/B ratio, RF is the risk-free rate, ROE is aggregate profitability (return on equity), $IBES$ is the average analyst forecast of long-term earnings growth, $NEWVOL$ is the difference between the median return volatility of new firms and market volatility, and $NEWMB$ is the log difference between the median M/B of new firms and the median M/B across all firms. All variables except for M/B and $NEWMB$ are expressed in percent per year. The t-statistics, reported in parentheses, assess the significance of the difference between the given variable's averages in the given period and outside that period.

Panel A. Averages of selected variables

	Avg change in pre-wave	Before wave b-6	b-3	b	Wave b+1:e	e	After wave e+3	e+6	Outside wave
MVOL	-2.81 (-2.27)	12.36 (-1.14)	13.66 (-0.48)	12.70 (-0.97)	12.67 (-3.18)	12.73 (-0.96)	15.63 (0.54)	15.00 (0.21)	15.24 (3.18)
M/B	0.08 (1.79)	1.78 (-0.06)	1.87 (0.50)	1.94 (0.96)	1.81 (0.50)	1.93 (0.89)	1.90 (0.72)	1.97 (1.15)	1.78 (-0.50)
RF	0.14 (0.50)	1.46 (0.05)	1.55 (0.22)	1.77 (0.65)	2.22 (5.13)	2.37 (1.84)	1.95 (1.02)	2.06 (1.23)	1.17 (-5.13)
ROE	0.32 (0.54)	11.69 (-1.02)	11.59 (-1.15)	11.71 (-1.00)	12.24 (-1.01)	12.11 (-0.48)	13.10 (0.81)	12.02 (-0.60)	12.57 (1.01)
IBES	0.39 (1.32)	16.93 (-0.66)	16.88 (-0.72)	16.86 (-0.74)	16.81 (-2.90)	17.45 (-0.03)	17.58 (0.12)	17.57 (0.12)	17.83 (2.90)
NEWVOL	9.87 (2.27)	46.92 (-0.24)	42.26 (-1.06)	47.65 (-0.11)	44.82 (-1.97)	47.19 (-0.19)	50.12 (0.34)	49.87 (0.28)	49.46 (1.97)
NEWMB	-0.05 (-0.69)	0.44 (-0.91)	0.56 (0.50)	0.55 (0.41)	0.55 (1.18)	0.46 (-0.67)	0.49 (-0.31)	0.46 (-0.74)	0.51 (-1.18)

Panel B. Average realized market returns

	Pre-wave b–2:e–3	Outside pre-wave	Before wave b–5:b–3	b–2:b	Wave b+1:e	After wave e+1:e+3	e+4:e+6	Outside wave
Total return	15.20 (1.11)	9.13 (-1.11)	31.17 (2.77)	21.45 (1.45)	9.51 (-0.28)	0.49 (-1.35)	18.86 (1.09)	11.03 (0.28)
Excess return	9.54 (1.12)	3.44 (-1.12)	25.83 (2.81)	15.74 (1.44)	3.83 (-0.28)	-5.35 (-1.36)	13.42 (1.12)	5.35 (0.28)

Table VI
Empirical Evidence: Regressions of IPO Volume on Selected Variables

Each column represents a quarterly regression of IPO volume on the variables listed in the first column. The notation "Δ" denotes changes (first differences), and "$-n$" ("$+n$") denotes quarterly lags (leads). The variable MKT is realized market return, $MVOL$ is market return volatility, RF is the real risk-free rate, M/B is the aggregate M/B ratio, and IPO is scaled IPO volume. The units are chosen to ensure some significant digits for all coefficients in the table: IPO is measured in percent per month, MKT in decimals per month, $MVOL$ in percent per day, and RF in percent per month. The t-statistics, given in parentheses, are computed using standard errors that are robust to heteroskedasticity and serial correlation of residuals (Newey-West with five lags).

	(1)	(2)	(3)	(4)	(5)
Intercept	0.23	0.31	0.33	0.33	0.28
	(3.03)	(4.48)	(4.76)	(2.51)	(2.02)
$MKT - 2$	1.67				1.68
	(3.25)				(3.23)
$MKT - 1$	2.09				2.11
	(3.34)				(3.27)
MKT	2.06				2.03
	(4.51)				(4.50)
$MKT + 1$	-0.95				-0.98
	(-2.23)				(-2.25)
$MKT + 2$	-0.49				-0.52
	(-0.74)				(-0.77)
$\Delta MVOL - 2$		-0.31			
		(-1.91)			
$\Delta MVOL - 1$		-0.63			
		(-3.59)			
$\Delta MVOL$		-0.60			
		(-4.41)			
$\Delta RF - 2$			1.10		
			(3.10)		
$\Delta RF - 1$			0.28		
			(1.21)		
ΔRF			0.91		
			(2.53)		
$M/B - 1$				0.01	-0.03
				(0.12)	(-0.44)
$IPO(t - 1)$	0.84	0.87	0.84	0.84	0.84
	(23.09)	(21.47)	(20.37)	(19.21)	(22.75)
$Q1Dummy$	-0.48	-0.42	-0.38	-0.42	-0.43
	(-4.92)	(-4.52)	(-4.04)	(-4.91)	(-4.28)
T	169	159	171	169	157
R^2	0.78	0.75	0.73	0.72	0.79

Table VII
Empirical Evidence: Regressions of IPO Volume on Selected Variables

Each column represents a quarterly regression of IPO volume on the variables listed in the first column. The notation "Δ" denotes changes (first differences), and "$-n$" ("$+n$") denotes quarterly lags (leads). The variable ROE is aggregate profitability (return on equity), $IBES$ is the average analyst forecast of long-term earnings growth, $NEWVOL$ is the difference between the median return volatility of new firms and market volatility, $NEWMB$ is the log difference between the median M/B of new firms and the median M/B across all firms, and IPO is scaled IPO volume. The units are chosen to ensure some significant digits for all coefficients in the table: IPO is measured in percent per month, ROE in percent per month, $IBES$ in percent per year, and $NEWVOL$ in percent per day. The t-statistics, given in parentheses, are computed using standard errors that are robust to heteroskedasticity and serial correlation of residuals (Newey-West with five lags).

	(1)	(2)	(3)	(4)	(5)
Intercept	0.34	0.57	0.31	0.37	0.37
	(3.92)	(5.04)	(4.33)	(4.67)	(3.77)
ΔROE	0.90				0.19
	(2.50)				(0.46)
$\Delta ROE + 1$	0.55				0.31
	(1.43)				(0.76)
$\Delta ROE + 2$	0.64				1.00
	(1.90)				(2.40)
$\Delta IBES - 2$		-0.16			
		(-0.87)			
$\Delta IBES - 1$		-0.43			
		(-1.37)			
$\Delta IBES$		0.78			
		(5.07)			
$\Delta NEWMB - 2$			0.46		0.48
			(2.35)		(2.28)
$\Delta NEWMB - 1$			0.52		0.62
			(3.18)		(2.78)
$\Delta NEWMB$			0.11		-0.11
			(0.59)		(-0.49)
$\Delta NEWVOL - 2$				0.12	0.12
				(2.23)	(2.87)
$\Delta NEWVOL - 1$				0.03	-0.00
				(0.57)	(-0.09)
$\Delta NEWVOL$				0.01	-0.04
				(0.19)	(-0.94)
$IPO(t-1)$	0.84	0.79	0.87	0.84	0.86
	(18.30)	(11.16)	(19.79)	(18.75)	(16.56)
$Q1Dummy$	-0.26	-0.67	-0.51	-0.43	-0.46
	(-2.22)	(-4.59)	(-5.40)	(-4.42)	(-3.26)
T	142	79	136	144	105
R^2	0.72	0.70	0.76	0.71	0.77

Appendix A: Data Construction

Aggregate quarterly consumption data is obtained from *NIPA*. Consumption is defined as real per capita consumption expenditures on nondurables plus services, seasonally adjusted. The series is deflated by the personal consumption expenditure deflator (PCE), also taken from *NIPA*.

The following data is obtained from *CRSP* and *Compustat*. Quarterly aggregate profitability (ROE) is computed as the sum across stocks of earnings in the current quarter divided by the sum of book values of equity at the end of the previous quarter. Quarterly earnings, which are generally available from 1966Q1, denote income before extraordinary items available for common (*Compustat* item 25) plus deferred taxes from the income account (item 35, if available). If either value is indicated as .A (annual) or .S (semi-annual) in the quarterly file, these values are divided by four (if .A) or two (if .S). When quarterly book equity is missing, it is replaced with the most recent annual book equity. Following Fama and French (1993), annual book equity is constructed as stockholders' equity plus balance sheet deferred taxes and investment tax credit (item 35) minus the book value of preferred stock. Depending on availability, stockholder's equity is computed as *Compustat* item 216, or 60+130, or 6-181, in that order, and preferred stock is computed as item 56, or 10, or 130, in that order. Quarterly book equity, which is generally available from 1972Q1, is constructed analogously. Stockholders' equity is item 60, or 59+55, or 44-54, preferred stock is item 55, and deferred taxes and tax credit is item 52. If the quarterly values are indicated as .A (annual) or .S (semi-annual) in the SAS datafile, the respective annual or semiannual values are used. Monthly ROE values are interpolated from quarterly values. Market equity is computed monthly by multiplying the common stock price by common shares outstanding, both obtained from *CRSP*. M/B ratio is computed as market equity divided by book equity from the most recent quarter. We eliminate the values of market equity and book equity smaller than $1 million, as well as M/B ratios smaller than 0.01 and larger than 100. All variables that require *Compustat* data (e.g. ROE, M/B) are constructed through the end of 2002Q1.

Appendix B: Preferences and the Stochastic Discount Factor

This appendix describes the properties of the process of log surplus consumption

$$\log\left(S_t\right) \equiv s_t \equiv s\left(y_t\right) = a_0 + a_1 y_t + a_2 y_t^2. \tag{A1}$$

The process for y_t implies a normal unconditional distribution for y_t with mean $\overline{y}$ and variance $\sigma_y^2/2k_y$. Let $y_D = \overline{y} - 4\sigma_y/\sqrt{2k_y}$ and $y_U = \overline{y} + 4\sigma_y/\sqrt{2k_y}$ be the boundaries between which y_t lies 99.9% of the time. To ensure that log surplus s_t conforms to the economic intuition of a habit formation model, we impose the following parametric restrictions: $a_2 < 0$, $a_1 > -2a_2 y_U$ and $a_0 < 1/4\left(a_1^2/a_2\right)$. These restrictions ensure that for all t, $s_t < 0$, and thus $S_t \in (0,1)$, and that $s\left(y\right)$ is increasing in y for all $y \in [y_D, y_U]$. Log surplus follows the process

$$ds_t = \mu_s\left(y\right)dt + \sigma_s\left(y\right)dW_{0,t}, \tag{A2}$$

whose parameters are given by

$$\begin{aligned}
\mu_s\left(y\right) &= k_y(\overline{y} - y_t)\left(a_1 + 2a_2 y\right) + a_2\sigma_y^2 \\
\sigma_s\left(y\right) &= \left(a_1 + 2a_2 y\right)\sigma_y.
\end{aligned}$$

The restrictions above imply that $\sigma_s(y)$ is positive and decreasing in y, for all $y \in [y_D, y_U]$. Since s increases with y in the relevant range, surplus is perfectly correlated with innovations to aggregate consumption, and its volatility is higher for low surplus levels.

Given the dynamics of consumption in equation (8) and surplus in equation (A2), the process for the stochastic discount factor $\pi_t = U_C(C_t, X_t, t) = e^{-\eta t}(C_t S_t)^{-\gamma} = e^{-\eta t - \gamma(c_t + s_t)}$ is given by

$$d\pi_t = -r_t \pi_t dt - \pi_t \sigma_{\pi,t} dW_{0,t}, \tag{A3}$$

where

$$r_t = R_0 + R_1 \bar{\rho}_t + R_2 y_t + R_3 y_t^2 \tag{A4}$$

with

$$
\begin{aligned}
R_0 &= \eta + \gamma b_0 + \gamma a_1 k_y \bar{y} - \frac{1}{2}\gamma^2 \sigma_c^2 + (\gamma a_2 - \frac{1}{2}\gamma^2 a_1^2)\sigma_y^2 - \gamma^2 a_1 \sigma_c \sigma_y \\
R_1 &= \gamma b_1 \\
R_2 &= \gamma\left(2a_2 k_y \bar{y} - a_1 k_y - \gamma a_2\left(2\sigma_c \sigma_y + 2a_1 \sigma_y^2\right)\right) \\
R_3 &= 2a_2 \gamma\left(-k_y - \gamma a_2 \sigma_y^2\right)
\end{aligned}
$$

and

$$\sigma_{\pi,t} = \gamma\left(\sigma_c + (a_1 + 2a_2 y_t)\sigma_y\right). \tag{A5}$$

The parameter restrictions imposed earlier imply that $\sigma_{\pi,t}$ decreases as y_t (and hence also the surplus S_t) increases. As a result, expected returns and return volatility are low when y_t is high.

Appendix C: Learning

LEMMA 1: Suppose the prior of $\bar{\psi}^i$ at time t_0 is normal, $\bar{\psi}^i \sim \mathcal{N}\left(\widehat{\psi}_{t_0}^i, \widehat{\sigma}_{t_0}^2\right)$, and the priors are uncorrelated across firms. Let $\mathcal{I}_t$ denote the set of firms that are alive at time t. Then the posterior of $\bar{\psi}^i$ at any time $t > t_0$ conditional on $\mathcal{F}_t = \{(\rho_s^j, c_s, \bar{\rho}_s) : t_0 \leq s \leq t, j \in \mathcal{I}_t\}$ is also normal, $\bar{\psi}^i|_{\mathcal{F}_t} \sim \mathcal{N}\left(\widehat{\psi}_t^i, \widehat{\sigma}_{i,t}^2\right)$, where

(a) The mean squared error $\widehat{\sigma}_{i,t}^2 = E\left[\left(\bar{\psi}^i - \widehat{\psi}_t^i\right)^2 | \mathcal{F}_t\right]$ is nonstochastic and given by

$$\widehat{\sigma}_{i,t}^2 = \frac{1}{\frac{1}{\widehat{\sigma}_{t_0}^2} + \frac{(\phi^i)^2}{\sigma_{i,i}^2}(t - t_0)}. \tag{A6}$$

We note that the uncertainty about $\bar{\psi}^i$ declines deterministically over time due to learning.

(b) The conditional mean $\widehat{\psi}_t^i = E[\bar{\psi}_t^i | \mathcal{F}_t]$ evolves according to the process

$$d\widehat{\psi}_t^i = \widehat{\sigma}_{i,t}^2 \frac{\phi^i}{\sigma_{i,i}} d\widetilde{W}_{i,t}, \tag{A7}$$

where $\widetilde{W}_{i,t}$ is the idiosyncratic component of the Wiener process capturing the agents' perceived expectation errors (see equation (A8) below).

Proof: Consider the vector $\mathbf{Z}_t = (c_t, \overline{\rho}_t, \rho_t^1, ..., \rho_t^n)$ of signals to identify the unobservable variables, stacked in another vector $\overline{\psi} = (\overline{\psi}^1, ..., \overline{\psi}^n)'$. The assumptions in the text imply

$$d\mathbf{Z}_t = \left(\mathbf{A} + \mathbf{B}\mathbf{Z}_t + \mathbf{C}\overline{\psi}\right) dt + \mathbf{b}d\mathbf{W}_t,$$

where $\mathbf{W}_t = (W_{0,t}, W_{L,t}, W_{1,t}, ..., W_{n,t})$ and

$$\mathbf{A} = \begin{pmatrix} b_0 \\ k_L\overline{\rho}_L \\ \vdots \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 & b_1 & 0 & 0 & 0 \\ 0 & -k_L & 0 & 0 & 0 \\ 0 & \phi^1 & -\phi^1 & 0 & 0 \\ 0 & \vdots & & \ddots & 0 \\ 0 & \phi^n & 0 & 0 & -\phi^n \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & \cdots \\ 0 & 0 & \cdots \\ \phi^1 & & \\ & \ddots & \\ & & \phi^n \end{pmatrix}, \mathbf{b} = \begin{pmatrix} \sigma_c & 0 & \cdots & \cdots & \cdots & 0 \\ \sigma_{L,0} & \sigma_{L,L} & 0 & \cdots & \cdots & 0 \\ \sigma_{1,0} & 0 & \sigma_{1,1} & & & \\ \sigma_{2,0} & \vdots & & \sigma_{2,2} & & \\ \vdots & \vdots & & & \ddots & \\ \sigma_{n,0} & 0 & & & & \sigma_{n,n} \end{pmatrix}.$$

From Liptser and Shiryayev (1977), the posterior of $\overline{\psi}$ is given by $\overline{\psi} \sim \mathcal{N}\left(\widehat{\psi}_t, \widehat{\mathbf{\Sigma}}_t\right)$, where $d\widehat{\psi}_t = \widetilde{\mathbf{\Sigma}}_t d\widetilde{\mathbf{W}}_t$ with $\widetilde{\mathbf{\Sigma}}_t = \widehat{\mathbf{\Sigma}}_t \mathbf{C}' (\mathbf{b}')^{-1}$, $\frac{d\widehat{\mathbf{\Sigma}}_t}{dt} = -\widetilde{\mathbf{\Sigma}}_t \widetilde{\mathbf{\Sigma}}_t'$, and

$$d\widetilde{\mathbf{W}}_t = \mathbf{b}^{-1} \{dZ_t - E[dZ_t|\mathcal{F}_t]\} = \mathbf{b}^{-1}\left\{dZ_t - \left[\mathbf{A} + \mathbf{B}\mathbf{Z}_t + \mathbf{C}\widehat{\psi}_t\right]dt\right\} \tag{A8}$$

is a Brownian motion with respect to $\mathcal{F}_t$. The claim is proved using the fact that $\widetilde{\mathbf{\Sigma}}_t$ is diagonal. See Pástor and Veronesi (2003b). $\blacksquare$

Appendix D: Pricing

LEMMA 2: Let $\widetilde{b}_t$ follow the process

$$d\widetilde{b}_t = \left(\zeta_0\overline{\rho}_t + \zeta_1\rho_t^i - \zeta_2\right) dt,$$

where ρ_t^i and $\overline{\rho}_t$ follow the processes in equations (1) and (3), and ζ_i are constants. We define $\mathbf{Y}_t = \left(v\widetilde{b}_t - \gamma c_t, y_t, \overline{\rho}_t, \rho_t^i, \widehat{\psi}_t^i\right)'$ and $g(\mathbf{Y}_T) = e^{Y_{1,T} - \gamma a_1 Y_{2,T} - \gamma a_2 Y_{2,T}^2}$, where v is a constant, $Y_{i,t}$ denotes the i-th element of $\mathbf{Y}_t$, and γ, a_1, and a_2 are taken from equations (4) and (5). Then

$$E_t\left[e^{-\eta(T-t)}g(\mathbf{Y}_T)\right] \equiv H(\mathbf{Y}_t, t) = e^{K_0(t;T) + \mathbf{K}(t;T)'\cdot\mathbf{Y}_t + K_6(t;T)Y_{2,t}^2}, \tag{A9}$$

where $K_0(t;T)$, $\mathbf{K}(t;T) = (K_1(t;T), .., K_5(t;T))'$, and $K_6(t;T)$ satisfy a system of ordinary differential equations (ODE)

$$\frac{dK_6(t;T)}{dt} = -2K_6^2(t;T)\sigma_y^2 + 2K_6(t;T)k_y \tag{A10}$$

$$\left(\frac{d\mathbf{K}(t;T)}{dt}\right)' = -\mathbf{K}(t;T)'\cdot\left[\mathbf{B}_Y + 2K_6(t;T)\left[\mathbf{\Sigma}_{Y,t}\mathbf{\Sigma}_{Y,t}'\right]_2 \mathbf{e}_2\right] - 2K_6(t;T)k_y\overline{y}\mathbf{e}_2 \tag{A11}$$

$$\frac{dK_0(t;T)}{dt} = \eta - \mathbf{K}(t;T)'\cdot\mathbf{A}_Y - \frac{1}{2}\mathbf{K}(t;T)'\mathbf{\Sigma}_{Y,t}\mathbf{\Sigma}_{Y,t}'\mathbf{K}(t;T) - K_6(t;T)\sigma_y^2 \tag{A12}$$

subject to the final condition $K_6 (T; T) = -\gamma a_2$, $\mathbf{K} (T; T) = (1, -\gamma a_1, 0, 0, 0)$, and $K_0 (T; T) = 0$. In the above, $\mathbf{e}_2 = (0, 1, 0, .., 0)$ and

$$
\mathbf{A}_Y = \begin{pmatrix} -\gamma b_0 - v\zeta_2 \\ k_y \overline{y} \\ k_L \overline{\rho}_L \\ 0 \\ 0 \end{pmatrix} ; \mathbf{B}_Y = \begin{pmatrix} 0 & 0 & -\gamma b_1 + v\zeta_0 & v\zeta_1 & 0 \\ 0 & -k_y & 0 & 0 & 0 \\ 0 & 0 & -k_L & 0 & 0 \\ 0 & 0 & \phi^i & -\phi^i & \phi^i \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} ; \mathbf{\Sigma}_{Y,t} = \begin{pmatrix} -\gamma \sigma_c & 0 & 0 \\ \sigma_y & 0 & 0 \\ \sigma_{L,0} & \sigma_{L,L} & 0 \\ \sigma_{i,0} & 0 & \sigma_{i,i} \\ 0 & 0 & \frac{\phi^i}{\sigma_{i,i}} \widehat{\sigma}_{i,t}^2 \end{pmatrix}.
$$

Proof: From the definition of the vector $\mathbf{Y}_t$, we have

$$
d\mathbf{Y}_t = (\mathbf{A}_Y + \mathbf{B}_Y \mathbf{Y}_t) dt + \mathbf{\Sigma}_{Y,t} d\widetilde{\mathbf{W}}_t.
$$

The Feynman-Kac theorem implies that $H(\mathbf{Y}_t, t)$ from (A9) solves the partial differential equation

$$
\frac{\partial H}{\partial t} + \sum_{i=1}^{5} \left(\frac{\partial H}{\partial Y_i}\right) [\mathbf{A}_Y + \mathbf{B}_Y \mathbf{Y}_t]_i + \frac{1}{2} \sum_{i=1}^{5} \sum_{j=1}^{5} \frac{\partial^2 H}{\partial Y_i \partial Y_j} \left[\mathbf{\Sigma}_{Y,t} \mathbf{\Sigma}'_{Y,t}\right]_{ij} = \eta H \qquad \text{(A13)}
$$

subject to the boundary condition

$$
H(\mathbf{Y}_T, T) = g(\mathbf{Y}_T). \qquad \text{(A14)}
$$

It is easy to verify that the exponential quadratic function (A9) indeed satisfies (A13) subject to (A14), as long as $K_0(t; T)$, $\mathbf{K}(t; T)$, and $K_6(t; T)$ are the solutions to the system of ODEs in (A10) through (A12) under the final conditions presented in the claim of the Lemma.∎

PROPOSITION 1: Let $h_i = T_i - t$ be the time to expiration of the patent of public firm i. Then

(a) The firm's ratio of market value of equity to book value of equity is given by

$$
\frac{M_t^i}{B_t^i} = Z^i \left(y_t, \overline{\rho}_t, \rho_t^i, \widehat{\psi}_t^i, h_i\right) = e^{Q_0\left(h_i; \widehat{\sigma}_{i,t}^2\right) + \mathbf{Q}(h_i)' \cdot \mathbf{N}_t + Q_5(h_i) y_t^2}, \qquad \text{(A15)}
$$

where $\mathbf{N}_t = \left(y_t, \overline{\rho}_t, \rho_t^i, \widehat{\psi}_t^i\right)$ is the vector of state variables characterizing firm i, $Q_0\left(h_i; \widehat{\sigma}_{i,t}^2\right) = K_0(T_i - h_i; T_i)$, $Q_1(h_i) = K_2(T_i - h_i; T_i) + \gamma a_1$, $Q_i(h_i) = K_{i+1}(T_i - h_i; T_i)$ for $i = 2, .., 4$, $Q_5(h_i) = K_6(T_i - h_i; T_i) + \gamma a_2$, and $K_i(.; T_i)'$ are given in Lemma 2 for the parameterization $\zeta_0 = \zeta_2 = 0$ and $\zeta_1 = v = 1$. Analytical, although rather complicated, formulas for these functions are available in Pástor and Veronesi (2003b).

(b) The firm's excess stock returns follow the process

$$
dR_t^i = \mu_R^i(y_t, h_i) dt + \sigma_{R,0}^i(y_t, h_i) d\widetilde{W}_{0,t} + \sigma_{R,L}^i(h_i) d\widetilde{W}_{L,t} + \sigma_{R,i}^i(\widehat{\sigma}_{i,t}, h_i) d\widetilde{W}_{i,t}, \qquad \text{(A16)}
$$

where $d\widetilde{W}_{j,t}$'s are the Wiener processes given in equation (A8), and

$$
\begin{align}
\mu_R^i(y_t, h_i) &= \sigma_{R,0}^i(y_t, h_i) \sigma_{\pi,t} \tag{A17} \\
\sigma_{R,0}^i(y_t, h_i) &= Q_3(h_i) \sigma_{i,0} + Q_2(h_i) \sigma_{L,0} + (Q_1(h_i) + 2Q_5(h_i) y_t) \sigma_y \tag{A18} \\
\sigma_{R,L}^i(h_i) &= Q_2(h_i) \sigma_{L,L} \tag{A19} \\
\sigma_{R,i}^i(\widehat{\sigma}_{i,t}, h_i) &= Q_3(h_i) \sigma_{i,i} + Q_5(h_i) \frac{\phi^i}{\sigma_{i,i}} \widehat{\sigma}_{i,t}^2. \tag{A20}
\end{align}
$$

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Note: It can be shown that the firm's M/B is increasing in $\widehat{\sigma}_{i,t}^2$. Inspection of the ODEs in equations (A10) through (A12) shows that $\widehat{\sigma}_{i,t}^2$ enters only the ODE defining K_0 and hence only Q_0 and no other Q_i's. Q_0 is increasing in $\widehat{\sigma}_{i,t}^2$, so that the firm's M/B is increasing in $\widehat{\sigma}_{i,t}^2$ as well. This fact is shown more explicitly in Pástor and Veronesi (2003b).

Proof: From the pricing formula and Lemma 2:

$$
\begin{aligned}
M_t^i = \pi_t^{-1} E_t\left[\pi_{T_i} B_{T_i}^i\right] &= e^{\gamma c_t + \gamma a_1 y_t + \gamma a_2 y_t^2} E_t\left[e^{-\eta(T_i - t)} e^{b_{T_i}^i - \gamma c_{T_i} - \gamma a_1 y_{T_i} - \gamma a_2 y_{T_i}^2}\right] \\
&= e^{\gamma c_t + \gamma a_1 y_t + \gamma a_2 y_t^2} H\left(\mathbf{Y}_t, t\right).
\end{aligned}
$$

Since $\mathbf{B}_Y$ has only zeros in its first column, we have $\left[\mathbf{K}\left(t; T_i\right)' \cdot \mathbf{B}_Y\right]_1 = 0$ in equation (A11). This implies $\frac{dK_1(t; T_i)}{dt} = 0$ and thus $K_1\left(t; T_i\right) = 1$ for $t \leq T_i$. By substituting in $H\left(\mathbf{Y}_t, t\right)$, we obtain

$$
M_t^i = e^{\gamma c_t + \gamma a_1 y_t + \gamma a_2 y_t^2} \times H\left(\mathbf{Y}_t, t\right) = B_t^i \times e^{\gamma a_1 y_t + \gamma a_2 y_t^2} \times e^{K_0(t; T_i) + \sum_{i=2}^5 K_i(t; T_i) Y_{i,t} + K_6(t; T_i) Y_{2,t}^2}.
$$

This expression leads immediately to claim (a) on redefinition of variables. The proof of claim (b) follows from an application of Ito's lemma to M_t^i, and the equilibrium condition $\mu_R = -cov\left(\frac{dM_t^i}{M_t^i}, \frac{d\pi_t}{\pi_t}\right)$. See Pástor and Veronesi (2003b) for details. ∎

The Long-Lived Firm:

Let B_t^m denote the long-lived firm's book value and D_t^m its dividends at time t. The firm's dividend yield, $c^m = D_t^m / B_t^m$, is constant, and its instantaneous profitability is $\overline{\rho}_t$. The firm's market value is $M_t^m = E_t\left[\int_t^\infty \pi_s / \pi_t D_s^m ds\right]$. Since $E_t\left[\pi_s D_s^m\right] = c^m E_t\left[\pi_s B_s^m\right]$, Fubini's theorem, Lemma 2, and the same argument as in the proof of Proposition 1 yield the pricing formula:

$$
\frac{M_t^m}{B_t^m} \equiv c^m \int_0^\infty Z^m\left(s, \overline{\rho}_t, y_t\right) ds, \tag{A21}
$$

where $Z^m\left(s, \overline{\rho}_t, y_t\right) = e^{Q_0^m(s) + Q_1^m(s) y_t + Q_2^m(s) \overline{\rho}_t + Q_3^m(s) y_t^2}$, and $Q_0^m(s) = K_0(0; s)$, $Q_1^m(s) = K_2(0; s) + \gamma a_1$, $Q_2^m(s) = K_3(0; s)$, and $Q_3^m = K_6(0; s) + \gamma a_2$. Here, $K_i(0; s)$'s are as in Lemma 2 for the parametrization $\zeta_0 = \zeta_2 = v = 1$, and $\zeta_1 = 0$. Pástor and Veronesi (2003b) provide analytical, although complicated, formulas for these coefficients. Excess returns of the long-lived firm follow

$$
dR_t^m = \mu_R^m\left(y_t, \overline{\rho}_t\right) dt + \sigma_{R,0}^m\left(y_t, \overline{\rho}_t\right) d\widetilde{W}_{0,t} + \sigma_{R,L}^m\left(y_t, \overline{\rho}_t\right) d\widetilde{W}_{L,t}, \tag{A22}
$$

where

$$
\begin{aligned}
\mu_R^m\left(y_t, \overline{\rho}_t\right) &= \sigma_{R,0}^m\left(y_t, \overline{\rho}_t\right) \sigma_{\pi,t} \\
\sigma_{R,0}^m\left(y_t, \overline{\rho}_t\right) &= F_{\overline{\rho}}^m(t) \sigma_{L,0} + \left(F_{y,1}^m(t) + F_{y,2}^m(t) y_t\right) \sigma_y \\
\sigma_{R,L}^m\left(y_t, \overline{\rho}_t\right) &= F_{\overline{\rho}}^m(t) \sigma_{L,L}.
\end{aligned}
$$

In the above,

$$
\begin{aligned}
F_{\overline{\rho}}^m(t) &= \frac{\int_0^\infty Q_2^m(s) Z^m\left(s, \overline{\rho}_t, y_t\right) ds}{\int_0^\infty Z^m\left(s, \overline{\rho}_t, y_t\right) ds} \\
F_{y,1}^m(t) &= \frac{\int_0^\infty Q_1^m(s) Z^m\left(s, \overline{\rho}_t, y_t\right) ds}{\int_0^\infty Z^m\left(s, \overline{\rho}_t, y_t\right) ds} \\
F_{y,2}^m(t) &= 2\frac{\int_0^\infty Q_3^m(s) Z^m\left(s, \overline{\rho}_t, y_t\right) ds}{\int_0^\infty Z^m\left(s, \overline{\rho}_t, y_t\right) ds}.
\end{aligned}
$$

Appendix E: Payoff Computation

When the IPO decision is made at time τ, the expected payoff at time $\tau + \ell$ is

$$EPay^i_{\tau,\tau+\ell} = E_\tau \left(\frac{\pi_{\tau+\ell}}{\pi_\tau} \left(M^i_{\tau+\ell}\left(1-f\right) - B^{t_i} \right) \right) = B^{t_i} \left\{ \left(1-f\right) E_\tau \left(\frac{\pi_{\tau+\ell}}{\pi_\tau} \frac{M^i_{\tau+\ell}}{B^{t_i}} \right) - E_\tau \left(\frac{\pi_{\tau+\ell}}{\pi_\tau} \right) \right\}.$$
(A23)

Using equation (A15) with $\widetilde{h} = T - \left(\tau + \ell\right)$, we have

$$M^i_{\tau+\ell} = B^{t_i} e^{Q_0\left(\widetilde{h}_i; \widehat{\sigma}^2_{\tau+\ell}\right) + \mathbf{Q}\left(\widetilde{h}_i\right)' \cdot \mathbf{N}_{\tau+\ell} + Q_5\left(\widetilde{h}_i\right) y^2_{\tau+\ell}}.$$

The initial profitability at the time of the IPO is unknown at τ, so we assume it equal to its unconditional expectation $\rho^i_{\tau+\ell} = \overline{\rho}_{t+\ell} + \widehat{\psi}^i_{\tau+\ell}$. Then

$$E_\tau \left(\pi_{\tau+\ell} \frac{M^i_{\tau+\ell}}{B^{t_i}} \right) = e^{\left(Q_3\left(\widetilde{h}\right)+Q_4\left(\widetilde{h}\right)\right)\widehat{\psi}^i_{\tau+\ell}} \times e^{-\eta\tau - \gamma a_0} \times E_\tau \left[e^{Q_0\left(\widetilde{h}_i; \widehat{\sigma}^2_{t+\ell}\right)} \right]$$
(A24)

$$\times E_\tau \left(e^{-\eta\ell} e^{-\gamma c_{\tau+\ell} + \left(Q_2\left(\widetilde{h}\right)+Q_3\left(\widetilde{h}\right)\right)\overline{\rho}_{\tau+\ell} + \left(Q_1\left(\widetilde{h}\right)-\gamma a_1\right)y_{\tau+\ell} + \left(Q_5\left(\widetilde{h}\right)-\gamma a_2\right)y^2_{\tau+\ell}} \right).$$

The term $e^{\left(Q_3\left(\widetilde{h}\right)+Q_4\left(\widetilde{h}\right)\right)\widehat{\psi}^i_{\tau+\ell}}$ can be taken out of the expectation, because agents are assumed to know their prior mean $\widehat{\psi}^i_{\tau+\ell}$ at time τ. Prior uncertainty is stochastic between τ and $\tau + \ell$, but it is independent of everything else, so $E_\tau \left[e^{Q_0\left(\widetilde{h}_i; \widehat{\sigma}^2_{\tau+\ell}\right)} \right]$ can be computed separately. Since $\widehat{\sigma}^2_t$ follows a continuous time Markov Chain process, we have $E_\tau \left[e^{Q_0\left(\widetilde{h}_i; \widehat{\sigma}^2_{\tau+\ell}\right)} | \widehat{\sigma}^2_\tau = v^2_j \right] = \left[\mathbf{\Lambda}\left(\ell\right) \right]_j \mathbf{E}\left(v\right)$, where $\mathbf{\Lambda}\left(\ell\right) = \mathbf{W}^{-1} diag\left(e^{\omega_j \ell} \right) \mathbf{W}$, $\left[\mathbf{E}\left(v\right)\right]_i = e^{Q_0\left(\widetilde{h}_i; v^2_i\right)}$, ω_j are the eigenvalues of the infinitesimal transition matrix $\mathbf{\Lambda}$, and $\mathbf{W}$ is the matrix of corresponding eigenvectors.

The last term in equation (A24) can be written as $E_\tau \left(e^{-\eta\ell} \widetilde{g}\left(\mathbf{Y}_{\tau+\ell}\right) \right)$ with

$$\widetilde{g}\left(\mathbf{Y}_{\tau+\ell}\right) = e^{Y_{1,\tau+\ell} + \left(Q_1\left(\widetilde{h}\right)-\gamma a_1\right)Y_{2,\tau+\ell} + \left(Q_2\left(\widetilde{h}\right)+Q_3\left(\widetilde{h}\right)\right)Y_{3,\tau+\ell} + \left(Q_5\left(\widetilde{h}\right)-\gamma a_2\right)Y^2_{2,\tau+\ell}}$$
(A25)

and $v = 0$ in Lemma 2. Thus, Lemma 2 provides us with a solution of this expectation. The only difference is that the final conditions of the functions K_i are given by $K_6\left(\tau + \ell; \tau + \ell\right) = \left(Q_5\left(\widetilde{h}\right) - \gamma a_2 \right)$, $\mathbf{K}\left(\tau + \ell; \tau + \ell\right) = \left(1, Q_1\left(\widetilde{h}\right) - \gamma a_1, Q_2\left(\widetilde{h}\right) + Q_3\left(\widetilde{h}\right), 0, 0 \right)$, $K_0\left(\tau + \ell; \tau + \ell\right) = 0$. Pástor and Veronesi (2003b) report analytical, although complex, formulas for these functions.

Finally, we can compute $E_\tau \left(\frac{\pi_{\tau+\ell}}{\pi_\tau} \right)$ immediately from Lemma 2, under the assumption $\nu = 0$. ∎

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Notes

^{1}See Ritter (1991), Loughran, Ritter, and Rydqvist (1994), Loughran and Ritter (1995), Rajan and Servaes (1997, 2003), Pagano, Panetta, and Zingales (1998), Baker and Wurgler (2000), and Lowry (2003).

^{2}See, for example, Keim and Stambaugh (1986), Campbell (1987), and Fama and French (1988).

^{3}Schultz (2003) argues that equity issuers time the market ex post but not ex ante, so that IPO volume is correlated with future returns ex post but not ex ante. In contrast, in our model, firms go public after declines in expected market return, so that high IPO volume predicts low market returns also ex ante.

^{4}See, for example, Lucas and McDonald (1990), Choe, Masulis, and Nanda (1993), Bayless and Chaplinsky (1996), Hoffmann-Burchardi (2001), and Lowry (2003). For other information-based models, see Persons and Warther (1997), Chemmanur and Fulghieri (1999), Subrahmanyam and Titman (1999), Stoughton, Wong, and Zechner (2001), Lowry and Schwert (2002), Benveniste, Busaba, and Wilhelm (2002), and Alti (2003).

^{5}For example, Benninga, Helmantel, and Sarig (2003) model the tradeoff between private benefits of control and the diversification benefit of going public, and derive implications for optimal IPO timing.

^{6}See, for example, Cukierman (1980), Bernanke (1983), Brennan and Schwartz (1985), McDonald and Siegel (1986), Ingersoll and Ross (1992), Dixit (1989), Abel et al (1996), and Berk (1999). See also Shleifer (1986), Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2001), and Novy-Marx (2003).

^{7}Unobservable $\overline{\rho}_t$ can be incorporated at the cost of a significant increase in complexity but with little benefit given the objectives of this paper. It can be shown that higher uncertainty about $\overline{\rho}_t$ increases expected cash flow but also increases the discount rate, resulting in a relatively small net effect on prices. Veronesi (2000) discusses these effects in a different framework.

^{8}These assumptions are made mostly for analytical convenience; relaxing them would add complexity with no obvious new insights. Given these assumptions, the clean surplus relation implies that book equity grows at the rate equal to the firm's profitability: $dB_t^i = Y_t^i dt = \rho_t^i B_t^i dt$.

[9]The patent need not be interpreted literally; it can be thought of as a competitive advantage.

[10]This choice of T seems reasonable. According to the U.S. law, patents issued before June 8, 1995 typically last for 17 years from the date of issuance, while patents granted after June 8, 1995 last for 20 years from the date of filing. The effective life of a patent is often shorter than 20 years because some products such as drugs require various regulatory approvals before coming to the market, but patent extensions can frequently be obtained to compensate for the time lost in regulatory review (see Schwartz (2001)).

[11]The speed of mean reversion k_L implies a half-life of about 4.9 years. That is, given any starting value $\overline{\rho}_0$, it takes on average 4.9 years for $\overline{\rho}_t$ to cover half the distance between $\overline{\rho}_0$ and its central tendency $\overline{\rho}_L$.

[12]Chen and Ritter (2000) find that in 91% of the U.S. IPOs raising between $20 and $80 million (and in 77% of all IPOs) between 1995 and 1998, the gross spreads received by underwriters were exactly 7%. IPO underpricing can also be incorporated by using a bigger f without affecting our qualitative results.

[13]Lowry and Schwert (2002) report that the average time between the IPO filing and offer dates between 1985 and 1997 is 72 days. The median is 63 days, the minimum 11 days, and the maximum 624 days.

[14]Every month between January 1960 and December 2002, the book value of new lists (ordinary common shares that first appear on *CRSP* in that month) is divided by the total book value of equity. The time-series average of the monthly ratios is 0.0235%, excluding the spikes in July 1962 and December 1972 when Amex and Nasdaq were added to *CRSP*. The exact value of q is not important for any of our conclusions. As long as q is reasonably small, the long-lived firm accounts for the bulk of the market portfolio.

[15]Rarely, a month with zero IPOs can be designated as the first or last month of a wave if the large IPO volume in the neighboring month inflates the moving average. Such months are excluded from the wave.

[16]Since new ideas arrive at the rate of one per month, the average number of IPOs in our simulations is just under one per month (because some patents never go public). In the data, the number of IPOs between January 1960 and December 2002 averages 28.78 per month. Thus, to convert IPO volume in the simulation into a comparable number in the data, one must multiply it roughly by a factor of 30.

[17]Computing *NEWVOL* and *NEWMB* requires at least one IPO in the given month. Since only one idea is invented each month, our simulated sample includes many months with zero IPOs, especially before IPO waves. To avoid missing observations in the months with the biggest improvements in market conditions, we assume that one firm with $T = 15$ and $\widehat{\psi}_t^i = 0$ is born in any month t into the current market conditions summarized by y_t, $\overline{\rho}_t$, and $\widehat{\sigma}_t$. This assumption is made for the purpose of constructing *NEWVOL* and *NEWMB* only, and it provides a cleaner assessment of these proxies for $\widehat{\sigma}_t$ than any obvious alternatives.

[18]We simulate the modified model in the same way as our basic model, and run market model regressions using private firm returns and market returns computed in the simulation. The average R^2 is just under 0.10, which is below the 0.13 value obtained in the data.

[19]All individual stock price data is obtained from the Center for Research in Security Prices (*CRSP*) at the University of Chicago. We define public firms as ordinary common shares (*CRSP* sharecodes 10 or 11) with positive market values. The number of *CRSP*-listed firms jumps in July 1962 and December 1972 due to the addition of Amex and Nasdaq firms. Following Lowry (2003), we use the actual number of public firms after December 1972, but estimate the number of public firms prior to that by assuming that this number grew at the compounded growth rate of 0.45% per year before December 1972.

[20]This definition of new firms ensures availability of their valid M/B ratios. Few firms have valid M/B ratios in the first few months after listing because M/B is computed using lagged book equity, which is often available only on an annual basis and generally available only after market equity becomes available. For both *NEWMB* and *NEWVOL*, we require at least three new firms to compute a valid median.

[21]Lowry (2003) finds a relation on the border of significance using a different measure of M/B, the equal-weighted average of M/Bs of individual firms.

[22]Pagano, Panetta, and Zingales (1998) find that Italian firms tend to invest especially before their IPOs. Pre-IPO investment can be easily obtained in our model if we allow for "time to build." Instead of investing at time $\tau^i + \ell$ (at the IPO), suppose the inventor invests at time τ^i (when he decides to go public) using borrowed money. The loan is repaid from the IPO proceeds at time $\tau^i + \ell$. Also suppose that it takes ℓ months to build the production technology, so that production does not begin until time $\tau^i + \ell$. This modified model produces results identical to ours, except that investment precedes the IPO by ℓ months.