

Habit Formation, the Cross Section of Stock Returns and the Cash-Flow Risk Puzzle*

Tano Santos

Columbia University, CEPR and NBER

Pietro Veronesi

University of Chicago, CEPR and NBER

September 22, 2009

Abstract

Non-linear external habit persistence models, which feature prominently in the recent “equity premium” asset pricing and macroeconomics literature, generate counterfactual predictions in the cross section of stock returns. In particular, we show that in the absence of cross sectional heterogeneity in firms’ cash-flow risk these models produce a “growth premium,” that is, stocks with high price-to-fundamental ratios command a higher premium than stocks with low price-to-fundamental ratios. This implication is at odds with the well established empirical observation of a “value premium” in the cross section of stock returns. Substantial heterogeneity in firms’ cash-flow risk yields both a value premium as well as most of the stylized facts about the cross-section of stock returns, but it generates a “cash-flow risk puzzle:” Quantitatively, value stocks have to have “too much” cash-flow risk compared to the data to generate empirically plausible value premia.

*We thank seminar participants at Carnegie Mellon, UCLA, Princeton University, The Federal Reserve Bank of New York, Columbia Business School, London Business School, London School of Economics, and the Graduate School of Business of the University of Chicago for their comments and Gene Fama, Lars Hansen, John Heaton, and Wei Jiang for valuable suggestions. Errors, of course, remain our own. This paper has circulated previously under the title “Cash-flow Risk, Discount Risk and the Value Premium”

I. INTRODUCTION

The equity premium and the value premium puzzles constitute two of the focal points of the asset pricing literature. As is well known, the starting point of the first is the inability of standard consumption models to rationalize the observed level of the equity premium, the volatility and predictability of returns, and the low and stable risk free rate (see Panel A and B of Table I.) The value premium puzzle is instead concerned with the failure of the CAPM to explain the cross section of average returns of portfolios sorted according to book-to-market (see Panel C of Table I and Figure 1.) Surprisingly, these two puzzles are, for the most part, studied separately. This is unfortunate because, as we argue here, the two puzzles cannot be tackled independently: Any economic mechanism proposed to address one of them immediately has general equilibrium implications for the other.

In this paper, we focus on one important mechanism, habit persistence, which has featured prominently both in the asset pricing literature¹ and in the real business cycle literature.² In particular, we investigate a non-linear external habit formation model a la' Campbell and Cochrane (1999), a framework particularly successful in addressing the equity premium puzzles described above, and investigate the implications of this model for the established facts in the cross section of stock returns. We show that these implications are problematic and that for this reason the success of the non-linear habit formation mechanism has to be put on hold.

In particular, we show that when, importantly as we shall see, firms differ only in their expected dividend growth, habit persistence models counterfactually generate a “growth premium” rather than a “value premium,” a point also recently made by Lettau and Wachter (2007). The reason is at the heart of the habit formation model: The variation over time of the market price of consumption risk, which is responsible for the success of the model to explain the properties of the market portfolio, interacts with the timing of cash-flows to generate a term premium. Indeed, assets that pay far in the future are more sensitive to shocks in the stochastic discount factor than assets with front loaded cash-flows. For this reason the

¹Sundaresan (1989), Constantinides (1990), Abel (1990), Ferson and Constantinides (1991), Detemple and Zapatero (1991), Daniel and Marshall (1997), Campbell and Cochrane (1999), Heaton (1993 and 1995), Li (2001), and Wachter (2000). *All* these papers focus exclusively on the time series properties of the market portfolio and do not investigate implications for the risk and return properties of individual securities.

²For example Boldrin, Christiano, and Fisher (2001) use the habit persistence model of Constantinides (1991) to investigate whether this mechanism can be consistent with both key asset pricing and real business cycle facts. See also, among others, Boivin and Giannoni (2005), Christiano, Eichenbaum and Evans (2005), Giannoni and Woodford (2004), Ravn, Schmitt-Grohé, and Uribe (2006), and Smets and Wouters (2003 and 2004).

former are riskier and command a premium over the latter. We show that “growth stocks” are precisely those that pay far in the future and thus they command a counterfactual premium over value stocks.

In order to solve this “growth premium puzzle” induced by habit formation preferences, we introduce ex-ante heterogeneity in firms’ cash-flow risk, that is, firms differ from each other not only in their expected dividend growth, but also in the covariance of their cash flows with consumption growth itself. In this case, we find that the standard sorting procedure used in the literature to allocate stocks to portfolios according to their price-to-fundamental ratio *endogenously* selects as value stocks those with higher cash-flow risk, an implication empirically confirmed by a series of recent papers.³ This higher cash-flow risk of value stocks naturally translates into higher expected returns, as investors require a premium to hold stocks whose cash flows fall at the same time as the aggregate consumption. Using simulations, we document that if the heterogeneity in firms’ cash-flow risk is sufficiently large, then indeed stocks with low price dividend ratios, value stocks, do command a substantial premium compared to high price dividend ratio stocks.⁴ That is, a value premium appears. We show that, under these conditions, the model then not only matches the properties of the market portfolio, as Campbell and Cochrane (1999) find, but it is actually able to replicate most, if not all, of the stylized facts about the cross-section of stock returns, including (a) the failure the CAPM and thus the value premium *puzzle*, (b) the better performance of the conditional CAPM and (c) the better pricing ability of an HML factor as in Fama and French (1993). In addition, the model also yields a large variation of the value premium over the business cycle, an additional stylized fact that is well documented in the data.⁵

Obviously the remaining question is then a *quantitative* one: is the cash-flow risk required to match the cross-section of stock returns consistent with the data? Unfortunately not. The external habit persistence model investigated here succeeds in reproducing the times series and cross sectional properties of asset data at the expense of an implausible high (low) levels of

³See Cohen, Polk and Vuolteenaho(2008), Campbell, Polk and Vuolteenaho (2005), Bansal, Dittmar and Lundblad (2005), Parker and Julliard (2005), and Hansen, Heaton and Li (2008). Also Liew and Vassalou (2000) and Vassalou (2003) show that news about forecasts of GDP growth correlate with value stock returns.

⁴ In our model, “book value” is not well defined and so we use price-dividend ratios in lieu of market-to-book ratios throughout (see Santos and Veronesi (2006) and Lettau and Wachter (2007)). Fama and French (1996, Table II), Fama and French (1998, Table III), and Lettau and Wachter (2007, Table I) show that sorting by earnings-to-price or cash-flow to price generates as sizable a “value” premium as sorting by book-to-market.

⁵See for example Lettau and Ludvigson (2001) and Cohen, Polk, and Vuolteenaho (2003).

cash-flow risk for value (growth) stocks when compared to the data. That is, habit preferences induce a “cash-flow risk puzzle.” The intuition is relatively simple: Since habit preferences tend to generate a growth premium when stocks do not differ in cash-flow risk, a large cross-sectional difference in cash-flow risk is needed in order to “undo” the growth premium. An extensive simulation exercise documents the severity of the problem.

Outline of the paper. The article develops as follows. Section II introduces a general equilibrium model with multiple securities which is solved for prices and returns in Section III. This model generalizes the setting put forward in Menzly, Santos and Veronesi (2004), MSV henceforth, in order to have more flexible preferences and a more manageable process for firms’ cash flows while retaining the closed form solutions that are such an important analytical advantage when dealing with multiple securities. Section IV investigates qualitatively the implications of the model for the cross section of stock returns whereas Section V does the same quantitatively. It is in Section V.C where we introduce the cash-flow risk puzzle. In Section VI we use our model to shed new light on standard asset pricing models tests. In particular, we use our model to construct an HML factor as in Fama and French (1993) and provide an economic foundation for its success as a cross sectional predictor. In this section, we also show that the model is able to match to a surprising degree the time series variation in the value premium, which is at the heart of the recent interest in the conditional CAPM. Section VII concludes.

Related literature. Our work touches on several recent papers in the literature on the cross section of stock returns, but differs from them in several respects. First, Santos and Veronesi (2006) and Lettau and Wachter (2007) investigate the effect that cross sectional differences in cash-flow duration (as defined by the expected dividend growth) have on the cross section of expected returns. They both find that assets with low duration have high expected returns and low price dividend ratios whereas the opposite is true for high duration assets, that is the value premium. Our work departs from these two papers in two crucial aspects.

First, both papers make assumptions to avoid the natural growth premium that comes with the variation in the discount rate, which is necessary to match the properties of the market portfolio: Santos and Veronesi (2006) fail to match the volatility of the aggregate stock returns and Lettau and Wachter (2007) assume away general equilibrium restrictions, and instead assume that the variation in the discount factor is unpriced by market participants.

These assumptions ensure that differences in durations generate a value premium. Duration effects are also present in this paper, but the presence of the strong discount effects implied by the Campbell and Cochrane (1999) model make, as explained above, cross sectional differences in duration generate a *growth* premium rather than a value premium. Santos and Veronesi (2006) don't have discount effects and Lettau and Wachter (2007) simply assume that they go unpriced.⁶ Here we don't take this stand, but rather, and reasonably in our view, assume that discount effects are both present and priced and some other ingredient is needed to generate the value premium. Cross sectional differences in cash-flow risk are such an ingredient. Second, the combination of cross sectional differences in cash-flow risk and discount effects generates the empirically documented time series variation in the value premium, a regularity for which, to the best of our knowledge, there is no extant theoretical explanation. Neither Santos and Veronesi (2006) nor Lettau and Wachter (2007) address this issue.

Our paper also touches on the recent literature emphasizing cross-sectional differences in long-run risk across asset classes. For instance, Parker and Julliard (2005) and Bansal, Dittmar and Lundblad (2005) use cross sectional differences in the long-run covariance between returns, consumption growth and dividend growth, to offer a characterization of cross-sectional differences in one period returns. Clearly, the long run components of cash-flow risk are but one contribution to one period returns; transient components may also be very important. Recognizing this Hansen, Heaton and Li (2008) offer a characterization of the *long run* trade-off between risk and return. This long-run trade-off is key because transient components, which may be first order for one period returns, are negligible in the long run.⁷ In contrast, our definition of cash-flow risk is entirely unrelated to low frequency components in consumption growth, which are (mostly) absent from our paper, and rather it emphasizes contemporaneous covariances of consumption and dividend growth. More importantly, the discount effects that play so prominent a role in our calibration are entirely absent in this literature so that, once again, the time series variation of the value premium cannot be generated in the context of

⁶This is the fundamental reason why duration effects are enough to generate a value premium in these two papers. All assets have identical, and positive, cash-flow risk, but some have their dividends more front loaded than others. This has two consequences. First, and by assumption, the more front loaded the dividends the lower expected dividend growth and thus the lower the price-dividend ratio. Second, the more front loaded the dividend the riskier the asset as it would constitute a larger fraction of current consumption and thus the higher the premium. Thus these assets are “value” and a value premium arises in these two papers.

⁷More precisely for these authors, value stocks are riskier because their cash-flow growth process load relatively more than growth stocks on low frequency components of consumption growth.

these models. Finally, none of these papers offers an integrated view of the time series and the cross section of stock returns and try to relate the properties of the market portfolio with the cross sectional properties of portfolios that add up to the market itself. We argue here, in contrast with the cash-flow literature, that these two sides of the asset pricing puzzles have to be jointly considered otherwise the inferences on cash-flow parameters are misleading.⁸

Campbell and Vuolteenaho (2004) decompose shocks to market returns into shocks to expected discount rates and shocks to expected dividend growth rates. They show that value and growth load on these shocks differently and this, combined with the market price of risk associated with these shocks, generates a value premium and its corresponding puzzle. Differently from us, however, they neither connect the time series properties of the market portfolio with the magnitudes of the cash-flow risk needed to generate a value premium, nor do they address the time series variation of the value premium.

Our paper also relates to the literature on conditional asset pricing. For instance, Lettau and Ludvigson (2001) show that empirically a conditional version of the CAPM outperforms its unconditional counterpart. Their results provide empirical evidence supporting our model's implication that conditioning variables capture the time series variation in the value premium. In our setup, as in the data, the conditional CAPM performs better than the unconditional CAPM. Importantly, though, in our model the conditional CAPM is a misspecified asset pricing model, and so, with enough data, it can also be rejected.

The present paper is obviously related to MSV, but there are also many differences with that paper. First, our model is more general than the one in MSV and the additional flexibility is instrumental in the empirical performance of the model. In particular, while MSV only considers the log-utility case and have approximate formulas for the cross section of prices, in this paper we solve for the power utility case and obtain exact solutions. Second, whereas MSV are concerned with the time series predictability of industry portfolios, the present paper focuses on the cross-sectional predictability of value sorted portfolio. This focus allows us also to shed light on the vast literature on cross sectional predictability, something MSV did not touch upon and, in particular, the present paper is after a quantitative assessment of the cash-flow risk effects needed to generate a plausible value premium.

⁸See also Brennan, Wang, and Xia (2004) and Brennan and Xia (2005) for a partial equilibrium model that ties the time series to the cross-section of stock returns. An investment-based general equilibrium model of the cross-section is also put forward by Gomes, Kogan, and Zhang (2003) who build on the partial equilibrium model of Berk, Green and Naik (1999). See also Zhang (2005).

II. THE MODEL

We consider an endowment economy with n financial assets. Each asset has an instantaneous dividend stream denoted by D_t^i , for $i = 1, \dots, n$. The consumption good is immediately perishable and non-storable, which yields the equilibrium restriction

$$C_t = \sum_{i=1}^n D_t^i \tag{1}$$

This add-up, general equilibrium restriction is important in our setting, as it breaks the theoretical validity of the CAPM, as discussed below.⁹ Unfortunately, even relatively simple processes for D_t^i imply aggregate consumption processes that are difficult to work with and restrictive assumptions need to be made for tractability (see discussion in MSV, Santos and Veronesi (2006), Cochrane, Longstaff and Santa Clara (2008), and Martin (2008)). We follow MSV and Santos and Veronesi (2006), and make assumptions about aggregate consumption C_t , and the joint dynamics of the *shares* of aggregate consumption produced by each asset, denoted by

$$s_t^i = \frac{D_t^i}{C_t} \tag{2}$$

Assumption 1: Aggregate consumption is given by

$$\frac{dC_t}{C_t} = \mu_c(\mathbf{s}_t) dt + \boldsymbol{\sigma}'_c d\mathbf{B}_t$$

where \mathbf{B}_t is a $n \times 1$ vector of Brownian motions, and

$$\mu_c(\mathbf{s}_t) = \bar{\mu}_c + \mu_{c,1}(\mathbf{s}_t) \quad \text{and} \quad \mu_{c,1}(\mathbf{s}_t) = \mathbf{s}'_t \boldsymbol{\theta}_{CF}. \tag{3}$$

Above, $\mathbf{s}_t = (s_t^1, \dots, s_t^n)'$, $\boldsymbol{\theta}_{CF} = (\theta_{CF}^1, \dots, \theta_{CF}^n)'$, and $\boldsymbol{\sigma}_c = (\sigma_c, 0, \dots, 0)'$. The specification of θ_{CF}^i is explained below.

As in Campbell and Cochrane (1999), we assume consumption growth has constant volatility. Unlike them, however, we assume expected consumption growth has a predictable component that depends on the distribution of shares. We make this assumption for four reasons: First, it follows naturally from the general equilibrium restriction (1) in any model

⁹Several recent articles have emphasized the importance of market clearing conditions in finance, such as Santos and Veronesi (2006), Johnson (2006), Cochrane, Longstaff and Santa Clara (2008), and Martin (2008). None of these papers combines habit formation with multiple trees, nor investigates the properties of the cross-section of stock returns, except for Santos and Veronesi (2006), already discussed in the introduction.

that has dividend processes as primitives (see equation (29) the Appendix).¹⁰ Second, this assumption is consistent with the recent long-run risk literature, which documents a small persistent predictable component in consumption growth (see e.g. Bansal and Yaron (2005), Hansen, Heaton and Li (2008)). In our model, such a predictable component is also small.¹¹ Third, the specific assumption (3) allows us to obtain analytical formulas for asset prices, an important property given our focus on the cross section of stock returns with many assets. Finally, in our model the time variation of expected consumption growth breaks the theoretical validity of the CAPM, both conditionally and unconditionally, a property that we exploit to provide insights on the economic meaning of the tests of the CAPM provided in the literature.

Assumption 2: For each i , the share s_t^i follows the mean reverting process

$$ds_t^i = \phi (\bar{s}^i - s_t^i) dt + s_t^i \boldsymbol{\sigma}^i (\mathbf{s}_t)' d\mathbf{B}_t \quad (4)$$

where

$$\boldsymbol{\sigma}^i (\mathbf{s}_t) = \boldsymbol{\nu}_i - \sum_{j=1}^n s_t^j \boldsymbol{\nu}_j \quad (5)$$

The cash-flow model (4) imposes a structure on the relative size of firms, where “size” is measured as the fraction of total output produced by a given firm. In particular, it imposes the economically plausible assumption that no firm will take over the economy, as $s_t^i > 0$ for all i . In addition, the volatility $\boldsymbol{\sigma}^i (\mathbf{s}_t)$ in (5) ensures that $\sum_{i=1}^n s_t^i = 1$ for all t , which in turn implies that (1) is always satisfied. Although the form of the volatility $\boldsymbol{\sigma}^i (\mathbf{s}_t)$ in (5) seems ad hoc, it can actually be recovered from first principles in a model with multiple dividend processes each with constant volatility (see Appendix). Instead, the key simplifying assumption is the mean reversion component in the drift rate of (4).

II.A Expected cash-flow growth and cash-flow risk

Given Assumptions 1 and 2, we can apply Ito’s Lemma to $D_t^i = s_t^i C_t$ and obtain:

$$\frac{dD_t^i}{D_t^i} = \mu_{D,t}^i dt + \boldsymbol{\sigma}_D^i (\mathbf{s}_t)' d\mathbf{B}_t \quad (6)$$

¹⁰Such models also imply that the volatility of consumption is a weighted average of dividend volatilities, which we instead approximate to a constant.

¹¹ We show in simulations below that expected consumption growth fluctuates between a maximum of 2.22% and a minimum of 1.87%, a very mild variation compared to the 1.5% standard deviation of consumption growth that we assume. Indeed, for our baseline case, which has the maximum variation in expected consumption growth, when we regress future consumption growth on $\ln(P_t/C_t)$ in artificial data we find R^2 s that are puny, between 0.1% and 0.2% at the 3- and 4-year horizon, respectively.

where the dividend drift and volatility are given by

$$\mu_{D,t}^i = \bar{\mu}_c + \theta_{CF}^i + \phi \left(\frac{\bar{s}^i}{s_t^i} - 1 \right) \quad (7)$$

$$\sigma_D^i(\mathbf{s}_t) = \boldsymbol{\sigma}_c + \boldsymbol{\sigma}^i(\mathbf{s}_t) \quad (8)$$

In these formulas,

$$\theta_{CF}^i = \boldsymbol{\nu}'_i \cdot \boldsymbol{\sigma}_c$$

Two comments are in order: First, equation (7) shows that when the asset's *relative share*, \bar{s}^i/s_t^i , is high and thus the asset's relative contribution to total consumption is below its long term average, the asset has an expected dividend growth higher than the unconditional expected consumption growth $\bar{\mu}_c$ (adjusted for a small Ito term θ_{CF}^i).¹² In addition, the drift rate of dividends μ_D^i depends on a parameter θ_{CF}^i , which is asset specific and it depends on the correlation of the stock's share s_t^i with consumption growth, as shown below. While technically θ_{CF}^i is simply an Ito term obtained from the definition $D_t^i = s_t^i C_t$, we note that quantitatively it has a minimal impact on the average dividend growth itself: As we show in the calibration section, θ_{CF}^i is an order of magnitude smaller than the other two drift components.¹³

Second, in our model the stochastic discount factor is only driven by shocks to consumption growth. Thus, *cash-flow risk* is measured by the following covariance

$$\sigma_{CF,t}^i \equiv Cov_t \left(\frac{dD_t^i}{D_t^i}, \frac{dC_t}{C_t} \right) = \boldsymbol{\sigma}'_c \boldsymbol{\sigma}_c + \theta_{CF}^i - \mathbf{s}'_t \boldsymbol{\theta}_{CF} \quad (9)$$

The conditional cash-flow risk of asset i , $\sigma_{CF,t}^i$, will play a prominent role in this paper. The term $\theta_{CF}^i - \mathbf{s}'_t \boldsymbol{\theta}_{CF}$ is parametrically indeterminate, that is, adding a constant to all θ_{CF}^i leaves this term unaffected, as $\sum_{i=1}^n s_t^i = 1$. Thus we can impose the identifiability restriction

$$\sum_{j=1}^n \bar{s}^j \theta_{CF}^j = 0, \quad (10)$$

The expected covariance between asset i 's cash-flow growth and consumption growth is

$$\bar{\sigma}_{CF}^i = E [\sigma_{CF,t}^i] = E \left[Cov_t \left(\frac{dD_t^i}{D_t^i}, \frac{dC_t}{C_t} \right) \right] = \boldsymbol{\sigma}'_c \boldsymbol{\sigma}_c + \theta_{CF}^i. \quad (11)$$

¹²MSV find strong empirical support for the inverse relation between relative share and dividend growth in industry portfolios.

¹³In our model, θ_{CF}^i are uniformly distributed around the interval $[-\bar{\theta}_{CF}, \bar{\theta}_{CF}]$. The maximum level of $\bar{\theta}_{CF} = .0035$, which is much smaller than both the assumed average consumption growth $\bar{\mu}_c = 2\%$ and the fluctuations in expected dividend growth induced by the third term in (7), $\phi(\bar{s}^i/s_t^i - 1)$, which is over 10%.

The parameter θ_{CF}^i then regulates the relative cash-flow risk of individual assets. Notice that the benchmark level of risk of an asset is the riskiness of aggregate consumption: An asset is risky (safe) if its cash-flows are more (less) risky than aggregate consumption. This is a general equilibrium restriction as, by definition, the variance of consumption growth must be a weighted average of its covariances with individual dividend growth. Throughout we refer to either $\bar{\sigma}_{CF}^i$ or θ_{CF}^i as “cash-flow risk” as there is a one to one mapping between them.

We conclude this section by emphasizing that the present framework can be generalized to introduce more realistic features but, clearly, at the cost of additional complexity. For instance, we assume for simplicity that firms are infinitely lived and that agents know the long term average size \bar{s}^i . A plausible generalization is one in which \bar{s}^i is unknown, and agents learn about it over time as they observe different dividend and consumption realizations. In this case, the pricing function, equation (20) below, will depend on the expected long term share $E_t[\bar{s}^i]$ rather than on \bar{s}^i . This extension though would largely leave the results unaffected. Indeed, standard filtering results imply that the variation in $E_t[\bar{s}^i]$ would be *independent* of consumption shocks, as consumption does not yield any additional information on \bar{s}^i that is not already in the shares themselves (see also Pastor and Veronesi (2003)). It follows that this additional variation would be unpriced and thus would have no impact on firms’ expected returns. Second, since expectations move more slowly than signals, the ratio $E_t[\bar{s}^i]/s_t^i$ would still tend to move inversely with s_t^i , exactly as in the case in which \bar{s}^i is known. Since the ratio \bar{s}^i/s_t^i , as is formally shown in Proposition 2 and 3, is the key variable affecting the firm’s expected dividend growth, its price/dividend ratio and its expected return, it follows that the cross-sectional relation between price/dividend ratios and expected returns would not change if we were to assume that \bar{s}^i was unknown. Finally, the learning model also partially addresses our assumption of an infinitely lived firm: assuming \bar{s}^i are randomly selected at time 0, some firms would then converge to very low “sizes,” effectively disappearing from the economy.¹⁴ In summary then adding learning to the model to account for the fact that the agents are not likely to know the long run contributions of the different firms to the overall economy substantially complicates the analysis without largely affecting the relation between price/dividend ratios and expected returns which is the focus of this paper.

¹⁴We also solved the model assuming exponential distributed exit times (firm death), which lead to the usual increase in the time discount. The results remain the same, but the model becomes more challenging as to keep it stationary, we must have a flow of firms entering the economy.

II.B Preferences and Habit Dynamics

There is a representative investor who maximizes

$$E \left[\int_0^\infty u(C_t, X_t, t) dt \right], \quad (12)$$

where the instantaneous utility function is given by

$$u(C_t, X_t, t) = e^{-\rho t} \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} \quad (13)$$

In equation (13), the variable X_t denotes an *external* habit level and ρ denotes the subjective discount rate. In Campbell and Cochrane (1999) the fundamental state variable driving the attitudes towards risk is the *surplus consumption ratio*, $S_t = (C_t - X_t)C_t^{-1}$. To obtain closed form solutions for prices when there are multiple securities, MSV use a log habit model and specify instead the inverse surplus S_t^{-1} as a mean reverting process. MSV's modeling device though cannot be applied when $\gamma > 1$ and, moreover, they only obtain approximate formulas for the case $\theta_{CF}^i \neq 0$. The present paper offers a generalization of MSV that allows us to handle a large class of models. The key ingredient in this generalization is to focus on the process

$$G_t = \left(\frac{C_t}{C_t - X_t} \right)^\gamma = S_t^{-\gamma}. \quad (14)$$

To obtain a plausible, yet tractable, model for the dynamics of G_t , consider first the implications for G_t under the standard assumption that X_t is an exponentially weighted average of past consumption levels, as in Constantinides (1990) and Detemple and Zapatero (1991), $X_t = \lambda \int_{-\infty}^t e^{-\lambda(t-\tau)} C_\tau d\tau$. An application of Ito's Lemma to (14) yields the process

$$dG_t = [\mu_G(G_t) - \sigma_G(G_t) \mu_{c,1}(\mathbf{s}_t)] dt - \sigma_G(G_t) \sigma_c dB_t^1, \quad (15)$$

where $\mu_G(G_t)$ and $\sigma_G(G_t) > 0$ are complicated functions of G_t , provided in equations (31) and (32) in the Appendix. Equation (15) shows that a higher expected consumption growth $\mu_{c,1}(\mathbf{s}_t)$ implies a lower drift rate of G_t . Intuitively, an increase in the expected growth rate of consumption implies a high future level of consumption relative to the current habit X_t and thus a higher surplus consumption ratio S_t , and, given (14), a lower expected G_t . As in MSV and Campbell and Cochrane (1999), we make specific assumptions on $\mu_G(G_t)$ and $\sigma_G(G_t)$ in (15) to obtain a more manageable process. In particular, we assume

$$\mu_G(G_t) = k(\bar{G} - G_t) \quad \text{and} \quad \sigma_G(G_t) = \alpha(G_t - \lambda). \quad (16)$$

The first component of the drift of G_t is a mean reversion component and captures the basic idea of habit persistence models, namely that the habit X_t eventually “catches up” with C_t . The second component, as discussed above, links the drift rate of G_t to $\mu_{c,1}(\mathbf{s}_t)$. As for the diffusion component, and as in MSV, $\lambda \geq 1$ bounds G_t from below at λ and $\alpha > 0$ transmits the innovations in consumption growth, dB_t^1 , to the convexity of the utility function.¹⁵ Note that MSV’s model is a special case of (15) and (16) and obtains when $\gamma = 1$ and consumption growth is i.i.d., which is achieved by setting $\mu_{c,1}(\mathbf{s}_t) = 0$.

III. EQUILIBRIUM ASSET PRICES AND RETURNS

III.A The total wealth portfolio

The next proposition generalizes the results in MSV to the present model in what concerns the total wealth portfolio, that is, the claim to the aggregate consumption process.

Proposition 1: The price-consumption ratio, the expected excess return and diffusion terms of the total wealth portfolio are, respectively:

$$\frac{P_t^{TW}}{C_t} = \alpha_0^{TW}(\mathbf{s}_t) + \alpha_1^{TW}(\mathbf{s}_t) S_t^\gamma \quad (17)$$

$$E_t[dR_t^{TW}] = (\gamma + \alpha(1 - \lambda S_t^\gamma)) \left\{ \frac{S_t^\gamma \alpha (1 - \lambda S_t^\gamma)}{f_1^{TW}(\mathbf{s}_t) + S_t^\gamma} \sigma_c^2 + \sum_{j=1}^n w_{jt}^{TW} \sigma_{CF,t}^j \right\} \quad (18)$$

$$\sigma_{R,t}^{TW} = \frac{S_t^\gamma \alpha (1 - \lambda S_t^\gamma)}{f_1^{TW}(\mathbf{s}_t) + S_t^\gamma} \sigma_c + \sum_{j=1}^n w_{jt}^{TW} \sigma_D^j(\mathbf{s}_t), \quad (19)$$

where $\alpha_0^{TW}(\mathbf{s}_t)$, $\alpha_1^{TW}(\mathbf{s}_t)$, $f_1^{TW}(\mathbf{s}_t)$ and $\{w_{jt}^{TW}\}$ are given in the Appendix.

As in Campbell and Cochrane (1999) and MSV, the price-consumption ratio of the total wealth portfolio is increasing in the surplus consumption ratio S_t : A high S_t implies a low local curvature of the utility function, a “less risk averse” attitude of the representative agent, and thus a higher price-consumption ratio. Unlike Campbell and Cochrane (1999) and MSV, the price-consumption ratio now depends on the entire vector of shares \mathbf{s}_t . Intuitively, this effect stems from our assumption about consumption growth predictability (see the discussion

¹⁵Clearly, assumptions (16) then imply that habit X_t is no longer the weighted average of past consumption, as above, but a more complicated non-linear function of past consumption shocks. See Campbell and Cochrane (1999) for a discussion. See also Hansen (2008) for a discussion of the risk-return implications of our habit model proposed above.

after Assumption 1). The functions $\alpha_0^{TW}(\mathbf{s}_t)$ and $\alpha_1^{TW}(\mathbf{s}_t)$ are typically decreasing in expected consumption growth because in our setup the elasticity of intertemporal substitution is less than one. Thus, this component implies that an increase in $\mu_c(\mathbf{s}_t)$ results in lower prices.¹⁶ As for the expected excess returns, (18), and the volatility of returns, (19), we postpone the discussion of the intuition of these expressions until section V.B, when we assess quantitatively the implications of the model.

III.B Prices and returns for individual securities

The next proposition delivers closed-form solutions for individual stock prices:

Proposition 2: The price of asset i is given by

$$\frac{P_t^i}{D_t^i} = \alpha_0^i + \alpha_1^i S_t^\gamma + \alpha_2^i(\mathbf{s}_t) \left(\frac{\bar{s}^i}{s_t^i} \right) + \alpha_3^i(\mathbf{s}_t) S_t^\gamma \left(\frac{\bar{s}^i}{s_t^i} \right) \quad (20)$$

where α_0^i, α_1^i are positive constants and $\alpha_2^i(\mathbf{s}_t)$ and $\alpha_3^i(\mathbf{s}_t)$ are positive linear functions of the share vector \mathbf{s}_t given in the Appendix.

As before, a higher surplus consumption ratio S_t , which implies lower “risk aversion,” or a higher expected dividend growth, as measured by the relative share \bar{s}^i/s_t^i (see (7)), result naturally in higher price-dividend ratios. The last term in (20) shows that shocks to the surplus consumption ratio have a stronger effect on the price-dividend ratio the higher the asset’s expected dividend growth. This is linked to the duration effect that plays so prominent a role in what follows. Finally, as it was true for the total wealth portfolio and for the same reason, the price of each individual asset also depends on functions of the vectors of shares $\alpha_2^i(\mathbf{s}_t)$ and $\alpha_3^i(\mathbf{s}_t)$.

The next proposition presents a characterization of expected excess returns. The intuition and implications of Proposition 3 are given in depth in Section IV.

Proposition 3: The expected excess return of asset i is given by

$$E_t [dR_t^i] = \mu_{i,t}^{DISC} + \mu_{i,t}^{CF}$$

¹⁶To review the economic reasoning, a low elasticity of intertemporal substitution implies a taste for consumption smoothing. An increase in expected consumption growth yields a higher desire for current consumption, and thus lower savings. Because stocks and bonds are less desirable now for the representative consumer prices have to drop in order to encourage him to hold them, resulting, for example, in a decrease of the price-consumption ratio of the total wealth portfolio.

where

$$\mu_{i,t}^{DISC} = (\gamma + \alpha(1 - \lambda S_t^\gamma)) \left(\frac{S_t^\gamma}{f_1^i(\bar{s}_t^i, \mathbf{s}_t) + S_t^\gamma} \right) \alpha(1 - \lambda S_t^\gamma) \sigma_c^2 \quad (21)$$

$$\mu_{i,t}^{CF} = (\gamma + \alpha(1 - \lambda S_t^\gamma)) \left[\left(\frac{1}{1 + f_2^i(S_t, \mathbf{s}_t) \left(\frac{\bar{s}_t^i}{s_t^i} \right)} + \eta_{it}^i \right) \sigma_{CF,t}^i + \sum_{j \neq i} \eta_{jt}^i \sigma_{CF,t}^j \right] \quad (22)$$

with

$$f_1^i(\bar{s}_t^i/s_t^i, \mathbf{s}_t) = \frac{\alpha_0^i + \alpha_2^i(\mathbf{s}_t) (\bar{s}_t^i/s_t^i)}{\alpha_1^i + \alpha_3^i(\mathbf{s}_t) (\bar{s}_t^i/s_t^i)} > 0 \quad \text{and} \quad f_2^i(S_t, \mathbf{s}_t) = \frac{\alpha_2^i(\mathbf{s}_t) + \alpha_3^i(\mathbf{s}_t) S_t^\gamma}{\alpha_0^i + \alpha_1^i S_t^\gamma} > 0,$$

and η_{jt}^i is given in expression (39) in the Appendix.

IV. GROWTH VERSUS VALUE PREMIA

The key empirical observation in the cross sectional literature is that growth assets, which are those with high prices relative to fundamentals, say price dividend ratios, have on average lower returns than assets with low price dividend ratios, value stocks. In this section we investigate what is required of the model to generate qualitatively this fact. For this we make use of the results in both propositions 2 and 3 above.

IV.A Discount risk effects and the growth premium

We start by focusing on the component of the premia $\mu_{i,t}^{DISC}$ in (21), which is the part of the premium that is driven by variation of the aggregate discount – proxied by S_t^γ . To interpret this term further, notice first that

$$\frac{\partial P_t^i/P_t^i}{\partial S_t^\gamma/S_t^\gamma} = \frac{S_t^\gamma}{f_1^i(\bar{s}_t^i/s_t^i, \mathbf{s}_t) + S_t^\gamma}. \quad (23)$$

is the elasticity of prices to shocks in the variable driving the aggregate discount, which is S_t^γ . The volatility of these discount shocks is

$$\alpha(1 - \lambda S_t^\gamma) \sigma_c, \quad (24)$$

which is the diffusion component of dS_t^γ/S_t^γ , the inverse of our state variable G_t , as it follows from a basic application of Ito's Lemma to (15). Clearly, only the component of these shocks that covaries with the shocks to the stochastic discount factor is priced. From equation (33) in the Appendix, the diffusion term of the habit stochastic discount factor is

$$\sigma_m = -[\gamma + \alpha(1 - \lambda S_t^\gamma)] \sigma_c \quad (25)$$

The component of the asset’s premium that is linked to discount effects is then the product of (23), (24) and (25), which is expression (21) in Proposition 3.

Cross-sectional variation in the discount effects can only be driven by differences in the price elasticity (23), which is in turn driven by the behavior of the function $f_1(\bar{s}^i/s_t^i, \mathbf{s}_t)$. We have been unable to obtain a general characterization of this function, but for parameter values that are empirically relevant we find that

$$\frac{\partial f_1^i(\bar{s}^i/s_t^i, \mathbf{s}_t)}{\partial (\bar{s}^i/s_t^i)} < 0,$$

and thus assets with a higher expected dividend growth, as measured by the relative share \bar{s}^i/s_t^i , display stronger discount effects. The intuition is straightforward: stocks with a high expected dividend growth pay the bulk of their proceeds far in the future. Thus, minor variations in the aggregate discount rate – through the risk aversion of the representative investor – result in large percentage variations of the price of the asset. This variation is naturally priced and thus the higher required premium of assets with high relative shares.

IV.A.1 The growth premium

We can now relate these findings to the observation that when only discount effects are present a growth premium arises. For this it is useful to turn to Figure 2, where we plot $\mu_{i,t}^{DISC}$, as given by (21), as a function of our proxy for expected dividend growth, \bar{s}^i/s_t^i , for the case in which all firms have identical cash-flow risk, that is, $\theta_{CF}^i \approx 0$ for all i . To generate this plot, the level of surplus S_t is set to its steady state value \bar{S} and the parameters used are those of the calibration exercise discussed in detail in Section V.A. When all firms have identical cash-flow risk, expression (20) implies that sorting assets according to their price-dividend ratio is akin to sorting them on expected dividend growth, \bar{s}^i/s_t^i . Since low price dividend ratio stocks are those with low relative shares \bar{s}^i/s_t^i , value stocks are those located on the left-hand side of Figure 2 and thus have low expected excess returns. Similarly, high price dividend ratio stocks are those with high \bar{s}^i/s_t^i and thus growth stocks are on the right-hand side of Figure 2 and have high expected excess returns. Thus, if cross-sectional differences in cash-flow risk are “small,” so that $E_t[dR_t^i] \approx \mu_{i,t}^{DISC}$ for all stocks, growth stocks command a higher premium than value stocks, that is a “growth premium” is obtained.

To reinforce this point, we conduct an extensive simulation, that we describe in detail below, to reproduce the sorting procedure that is standard in the literature on the cross section of stock returns. Our purpose is to replicate Figure 1, where we plot average (log) market to

book of value sorted portfolios versus average excess returns. The equivalent in simulated data for the case in which firms have homogenous cash-flow risk is reported in the top panel of Figure 3. The figure clearly shows that stocks with high average price dividend ratio yield a higher average return, in contrast with the data in Figure 1. To summarize then, if discount effects were to be the only ones present, the cross section of excess returns would display a growth premium rather than the value premium that is observed empirically.

IV.B Cash-flow risk effects and the value premium

The source of premia related to cash-flow shocks, $\mu_{i,t}^{CF}$ given in (22), has two components to it. The first is related to shocks in the asset's dividends and the second is related to shocks in the dividends of the rest of the assets in the economy, which, as shown in (20), affect the price of asset i as well. The logic for the sources of the premia linked to cash-flow shocks is the same as in the discount effects case. First it can be easily shown that the elasticity of the price with respect to shocks to its own dividends is,

$$\frac{\partial P_t^i/P_t^i}{\partial D_t^i/D_t^i} = \frac{1}{1 + f_2^i(S_t, \mathbf{s}_t) \left(\frac{\bar{s}_t^i}{s_t^i} \right)} + \eta_{it}^i.$$

Recall also that we denote $\sigma_{CF,t}^i = cov_t(dD_t^i/D_t^i, dC_t/C_t)$ (see equation (9)). The first term of $\mu_{i,t}^{CF}$ is then the component of the dividend shocks that covaries with shocks to the stochastic discount factor multiplied by the effect that these shocks have on the price of asset i , as measured by the price elasticity. As for the second term in $\mu_{i,t}^{CF}$ it can be shown that

$$\frac{\partial P_t^i/P_t^i}{\partial D_t^j/D_t^j} = \eta_{jt}^i \quad \text{for } j \neq i.$$

As before this component of the premium results from the product of this (cross) elasticity and the priced component of the shock to asset j 's dividends, $\sigma_{CF,t}^j$.

How does the current level of expected dividend growth, as measured by \bar{s}_t^i/s_t^i , affect the cash-flow risk component of expected stock returns? Given the conditional covariance of the dividend of asset i with aggregate consumption, $\sigma_{CF,t}^i$, the first term of (22) is unambiguous: Since $f_2^i(S_t, \mathbf{s}_t) > 0$, if the asset is "risky," that is, if $\sigma_{CF,t}^i > 0$, then a high expected dividend growth translates in a lower premium stemming from current dividend volatility. The intuition is also clear: a stock that pays more in the future than today has a relatively low dividend compared to the future. Thus, the risk embedded in current dividends, $\sigma_{CF,t}^i$, has a relatively low impact on the total risk of the stock. In the limit, if the stocks does not pay *any* dividend

today, it cannot have any “cash-flow risk,” as there is zero current covariance of dividends with consumption. If instead the asset’s dividends covary negatively with consumption growth ($\sigma_{CF,t}^i < 0$), then a high expected dividend growth increases the risk premium. The argument, of course, is the converse of the previous one.

The effect that the current expected dividend growth of asset i has on the second term of the cash-flow risk component of stock return (22) is more difficult to tell. However, we found numerically that on average, the cash-flow component of expected return is increasing in $\bar{\sigma}_{CF}^i$, although variation in shares s_t^i generate small deviations from this increasing pattern.

Finally, we note that in our model the risk premium only depends on the relative share \bar{s}^i/s_t^i and not on the steady state dividend share \bar{s}^i per se. The reason is that there are two forces at play when considering the effect of \bar{s}^i on required premia. First, a stock with a high average dividend share \bar{s}^i is more exposed to consumption risk on average, but, second, it also has a higher average price. This higher price implies a lower *percentage* sensitivity of the stock to consumption shocks. Since risk premia depend on percentage returns, these two forces exactly offset each other in our model.¹⁷ The current level of dividends is instead key in determining the current cash flow risk, as it is the covariance between consumption growth and current dividends that has a direct bearing on the riskiness of the stock.

IV.B.1 The value premium

We showed in Section IV.A that the sole presence of discount effects generates a counterfactual growth premium. To see whether cash-flow effects can produce the desired value premium instead we turn to Figure 4. The first two panels report, respectively, the discount, $\mu_{i,t}^{DISC}$, and the cash-flow risk component, $\mu_{i,t}^{CF}$, of expected returns. Panel C adds up both components to obtain $E_t [dR_t^i]$. Let us start with Panel A, which reports the same quantity as in Figure 2, $\mu_{i,t}^{DISC}$, but for the case in which θ_{CF}^i differ across firms. Interestingly, we see that higher cash-flow risk increases the *level* of the discount component of the expected return. The reason is that a higher cash-flow risk decreases the price of the asset, on average. Thus, shocks to the aggregate discount (S_t^γ) have a larger percentage impact on the stock price, and

¹⁷That expected returns are independent of \bar{s}^i is also a feature of the standard asset pricing model. Indeed consider the case where the representative consumer has the standard power utility function, $s_t^i = \bar{s}^i$ for all t and, finally, let C_t follow a simple geometric Brownian motion. In this case $P_t^i = \bar{s}^i C_t K$ where K is a constant. Since the risk premium is $\mu^i = \gamma Cov(dP_t^i/P_t^i, dC_t/C_t)$ and dP_t^i/P_t^i is independent of \bar{s}^i the two assets have identical risk premia, independently of \bar{s}^i . Finally notice that the asset with a higher \bar{s}^i has a higher price.

thus imply a higher risk. Nonetheless, for given cash-flow risk level, θ_{CF}^i , the relation between $\mu_{i,t}^{DISC}$ and expected dividend growth \bar{s}^i/s_t^i is positive, as discussed in the previous section.

Panel B plots the cash-flow component to expected return, $\mu_{i,t}^{CF}$, as a function of expected dividend growth, as proxied by \bar{s}^i/s_t^i , for various levels of θ_{CF}^i each corresponding to a line in the plot. As explained in the previous section, the cash-flow risk component of expected excess returns is decreasing in the expected dividend growth for stocks with high cash-flow risk.

Panel C reports the total expected return for each asset obtained by adding the cash-flow risk component $\mu_{i,t}^{CF}$ to the discount risk component, $\mu_{i,t}^{DISC}$. Value stocks (assets with low P/D ratio) have on average high risk ($\bar{\sigma}_{CF}^i$) and low expected dividend growth (\bar{s}^i/s_t^i). This combination corresponds to the area around the top-left corner of the plot in Panel C, that is, to high expected excess return. Conversely, growth stocks (assets with high P/D ratios) must have a combination of low $\bar{\sigma}_{CF}^i$ and high \bar{s}^i/s_t^i . This combination can be found on the bottom-right corner of the plot in Panel C, that is, low expected return. As can be seen then, value stocks will command a high premium and growth stocks a low (and even negative) premium. Thus, if cross-sectional differences in cash-flow risk are “large,” then value stocks have higher expected excess returns than growth stocks and a “value premium” is obtained.

To better illustrate this point, the top panel of Figure 5, as it was the case with Figure 3, again plots the average price-dividend ratios of price-dividend sorted portfolios against their average excess returns in simulated data, but now for the case in which firms have heterogeneous cash-flow risk. Our purpose is to assess to what extent the model can reproduce Figure 1, which is obtained from historical data. As in Figure 1 and in contrast with Figure 3, the presence now of heterogeneity in cash-flow risk generates a value premium: Low price-dividend ratio stocks, value stocks, are those that earn a high average excess return. The model is thus able to generate a value premium, although, clearly, the question is whether it can do so with a reasonable cross-sectional dispersion of cash-flow risk.

IV.C Conditional versus unconditional value premia

A novel theoretical implication of our framework is that the presence of discount risk effects, which are associated with the time series variation in risk preferences, affects the dynamics of the value premium, a feature for which there is already some empirical evidence (Cohen, Polk, and Vuolteenaho (2003), Table V). Essentially, discount risk effects interact with the cross-sectional dispersion in cash-flow risk to induce fluctuations in the value premium, as shown in Figure 6. This figure plots the expected excess returns of three assets against the

surplus consumption ratio, S_t . The dotted line shows the expected excess return of the market portfolio; the solid line corresponds to the expected excess return of a representative value stock with high cash-flow risk and low expected dividend growth; finally the dashed line corresponds to the premium of a representative growth stock with low cash-flow risk and high expected dividend growth. As can be seen, when the surplus consumption ratio is low (high), the value premium is high (low): Assets with a high value of θ_{CF}^i are particularly *riskier* when the representative agent's is highly risk averse which occurs whenever adverse consumption growth shocks depress the surplus consumption ratio, increasing in turn the market premium and its dividend yield. Thus in our model, the value premium has a strong predictable component, being high (low) when the market premium is high (low).

V. QUANTITATIVE IMPLICATIONS OF THE MODEL

In this section, we conduct a simulation study to evaluate the extent to which the model can match the standard return moments both in the time series and the cross-section, which can be found in Table I. The empirical data set is standard and is briefly described in the Notes to Table I. Panel A shows the mean and standard deviation for the returns on the market portfolio and the risk free rate. Panel B shows the predictability regressions of Fama and French (1988) and Campbell and Shiller (1988) for two different sample periods, which are meant to emphasize the sensitivity of these results to the particular period under consideration. Panel C shows the standard statistics for the cross section of book-to-market sorted (decile) portfolios. In particular, we report average excess returns for the ten value sorted portfolios for two sample periods, 1948-2001 (Panel C-1) and 1926-2001 (Panel C-2). The value premium is 5.50% in the 1948-2001 sample, which is very similar to the corresponding one in the longer sample. For the shorter sample we also report the average market-to-book, the Sharpe ratio, and the price-dividend ratio, the latter being the variable along which we are going to be sorting portfolios in simulated data as the model lacks a counterpart for the book value. Notice a strong feature of the data: Value stocks have higher Sharpe ratios than growth stocks and indeed from the highest market-to-book portfolio to the lowest, the Sharpe ratio almost doubles

An important regularity concerns the CAPM alphas and betas of these portfolios. For the postwar sample there is a *flat if not slightly negative* correlation between the CAPM betas and average returns, which is at the heart of the value premium puzzle. Indeed the alphas of value stocks are positive and statistically significant and the extreme growth portfolio is

negative and also statistically significant.

The evidence is somewhat different for the prewar sample. Panel C-2 reports the annualized monthly average excess returns for the ten value sorted portfolios for a sample period going back to 1926 as calculated by Ang and Chen (2007, Table 1 Panel A.) Relative to the earlier sample the CAPM betas correlate *positively* with average returns, rather than negatively, and this gives some hope for the CAPM to address the value premium.¹⁸

V.A Details of the simulation

We simulate 10,000 years of quarterly data for 200 firms that we then sort into ten portfolios according to their price-dividend ratio (see footnote 4) in an effort to mimic the standard procedure used in the cross-sectional literature. Table II contains the parameter values that are used throughout. We set the average consumption growth at 2% and its standard deviation at 1.5%, which should be measured against the value in the postwar sample of 1.22% and the one for the longer sample starting in 1889, which is 3.32% (see Campbell and Cochrane (1999), Table 2.) We choose $\gamma = 1.5$, which is between the values used by MSV, $\gamma = 1$, and Campbell and Cochrane (1999), $\gamma = 2$. This choice implies a steady state value of the local curvature of the utility function of $\gamma \bar{S}^{-1} = 48$, higher than the already high value of Campbell and Cochrane (1999) which is 35. The minimum value of this local curvature is 27.75. Finally the parameter k and α are similar to the values chosen by MSV.

As for the share process, all our results depend on the ratio \bar{s}^i/s_t^i and not the level \bar{s}^i , and so we set $\bar{s}^i = 1/200 = .005$, without loss of generality. The cross section of stock returns is sensitive though to parametric choices of other cash-flow parameters, $\bar{\theta}_{CF}$, ϕ , and ν_i . To avoid parameter proliferation, we restrict the share volatility $\nu_i = (\nu_{i,0}, 0, \dots, 0, \nu_{i,i}, 0, \dots)$. Given a value for the cash-flow risk parameter, θ_{CF}^i , the first entry by definition must be $\nu_{i,0} = \theta_{CF}^i/\sigma_c$. The second entry – the idiosyncratic part – is chosen constant across all assets according to the formula, $\nu_{i,i}^2 = \bar{\nu}^2 - \max(\nu_{i,0}^2)$, where $\bar{\nu}$ is a chosen parameter. In words, $\bar{\nu}$ is the maximum share volatility across assets.

We report first the results for our benchmark case where $\phi = .07$, which is the value that MSV (Table I) estimate for the market portfolio, and $\bar{\nu} = .55$ and $\bar{\theta}_{CF} = .345\%$, which as we will show shortly are values that allow us to match the moments reported in Table I. Section V.C contains a thorough discussion of the economic significance of these latter assumptions,

¹⁸On this point see also Campbell and Vuolteenaho (2004) and Fama and French (2006), who also show that notwithstanding the evidence above the CAPM is still rejected in the longer sample.

focusing especially on their impact on the main trade-off we highlight in this paper: the tension between discount effects and cash-flow risk effects in what concerns the cross-section of stock returns. In what follows, we refer to $\bar{\theta}_{CF}$ as the cash-flow risk parameter, but the reader should keep in mind that it is the *support* of the cash-flow risk parameters of individual assets.

V.B Simulation of the model

Table III is the counterpart to Table I but in simulated data.¹⁹ As can be seen the model does a reasonable job at capturing the main patterns of the empirical sample. Panels A shows that the model generates a sizable equity premium, though a bit low compared to other simulations of external habit persistence models, and high volatility of stock returns. As in MSV, the model yields a low risk-free rate though with volatility somewhat higher than its empirical counterpart. Panel B shows that the model produces the predictability at long horizons, though the R^2 s are lower than the ones observed in the empirical data.²⁰

Why is the equity premium lower in our model than in other external habit persistence models, such as Campbell and Cochrane (1999) and MSV? This is the result of a feature of our model that is absent from Campbell and Cochrane (1999) and MSV, namely, the small predictable variation in expected consumption growth due to general equilibrium restrictions (see the discussion after Assumption 1.) To gauge the intuition of this result it is useful to return to Proposition 1. Consider the case where there is a positive shock to consumption growth. This immediately translates into a higher price, which is now a claim to a larger dividend. In habit persistence models this positive consumption shock gives a second positive jolt to prices through the increase in the surplus consumption ratio, S , which lowers the representative agent's risk aversion. Because this makes stocks more volatile and riskier, they command a larger premium. This is the standard effect in habit persistence models and it corresponds to the first terms in (18) and (19) for the expected return and volatility, respectively.

In our framework though, shocks to consumption growth and shocks to expected consumption growth are positively correlated. Thus on average in the presence of a positive consumption growth shock, expected consumption growth is also high and this makes the total wealth portfolio less desirable to a representative consumer with a strong preference for

¹⁹We do not report t-statistics of simulation results, because the large sample (40,000 quarters) makes them meaningless. We take the simulated values as population moments to which compare empirical quantities.

²⁰Still, the riskless rate volatility is vastly lower than the one of traditional habit persistence models, such as Abel (1990) and Boldrin et al. (2001), who report a riskless rate volatility of 17.87%, and 24.6%, respectively.

intertemporal smoothing, as is standard in habit persistence models.²¹ This is a negative force on prices which partially undoes the positive effects discussed above. As a result the volatility is lower and so is the required premium when compared to the standard habit model with i.i.d. consumption growth. This is the second term in both (18) and (19). Notice that this effect is also bound to affect the strength of the predictability regression. It is important to note, however, that our model does not generate too much predictability of consumption growth, as already discussed in footnote 11.

Panel C of Table III reports the quantitative implications of the model for the cross section. The model generates a sizable value premium of a bit above 5%, though the average returns of individual portfolios are off by as much as the equity premium is. Interestingly, Sharpe ratios in simulated data show the same increasing pattern when moving from growth to value as in the empirical sample: Value stocks are good deals according to this metric both in the data and in the model. The model is able to generate not just the value premium but the value premium puzzle as well. Indeed notice that the CAPM alphas are negative for two growth portfolios (portfolios 1 and 2) and positive for the rest of the portfolios, which matches surprisingly well the empirical sample, in Table I. Importantly though the CAPM betas covary positively with average returns in the cross section, a pattern that is consistent with the 1926-2001 sample (see Panel C-2 of Table I), but not the postwar sample.

Why does the CAPM fail in our setting? As mentioned already, our model features a mild time variation in expected consumption growth through the term $\mu_{c,1}(\mathbf{s}_t) = \mathbf{s}_t' \boldsymbol{\theta}_{CF}$ in (3). The price/consumption ratio of the total wealth portfolio in (17) is a nonlinear function of both the surplus-consumption ratio S_t and expected consumption growth, $\frac{P_t^{TW}}{C_t} = \alpha_0^{TW}(\mathbf{s}_t) + \alpha_1^{TW}(\mathbf{s}_t) S_t^\gamma$. This result shows that general equilibrium restrictions imply that returns on the total wealth portfolio P^{TW} are determined by two types of shocks. First, consumption shocks dC , which affect the price P^{TW} through the level of consumption itself, the surplus consumption ratio S_t and their impact on the systematic component of the variation in shares \mathbf{s}_t . The second source of variation of the total wealth portfolio P^{TW} is the component of the variation in expected consumption growth that is orthogonal to consumption shocks. This is induced by the idiosyncratic components of shares \mathbf{s}_t . As these vary, expected consumption growth changes and so does the price P^{TW} , but this variation is not priced by the habit-based stochastic discount factor. In essence, the idiosyncratic component of the variation in

²¹Yang (2007) proposes a model that combines the Epstein-Zin utility framework with habit persistence precisely to allow for a more flexible specification of the intertemporal elasticity of substitution.

expected consumption growth breaks the perfect correlation between the total wealth portfolio return and the habit stochastic discount factor, which in turn invalidates the CAPM, both conditionally and unconditionally. We emphasize that, as discussed after Assumption 1, in multi-asset models the variation in expected consumption growth results from the general equilibrium restriction (1). It follows that in general we should not expect the CAPM to hold in a model with multiple trees.²²

In conclusion, our model is able to capture to a surprising degree the main characteristics of the return distribution both in the time series and the cross section and thus seems a useful lens through which to draw inferences about cash-flow parameters. Specifically, is the value of $\bar{\theta}_{CF}$ needed to match these moments “high” or “low”? We turn to this question next.

V.C The cash-flow risk puzzle

In this section, we assess what our choice of the cash-flow risk parameter $\bar{\theta}_{CF}$ means for the properties of the cash-flow process. Evaluating the assumed magnitude of the cash flow risk parameter is not easy, as it requires observing asset distributions that accrue to consumers, and then calculate the hard-to-estimate correlation with consumption growth. Here we get at this question by measuring instead the model implied “cash-flow betas,” as defined and estimated in empirical data by Cohen, Polk and Vuolteenaho (2009), and compare the model-implied values to their empirical counterparts.

Specifically, using data from 1928 to 1999, Cohen, Polk and Vuolteenaho (2009) regress different measures of firms’ cash-flows on the corresponding measures of market cash-flows as in, for example,

$$\sum_{j=0}^{R-1} \rho_{CPV}^j \Delta d_{t+j,j+1}^p = \beta_{CF,0}^p + \beta_{CF,1}^p \sum_{j=0}^{R-1} \rho_{CPV}^j \Delta d_{t+j}^{mkt} + \varepsilon_{t+R-1}^p \quad (26)$$

for each time t and each portfolio $p = 1, \dots, 10$. Here, $\Delta d_{t+j,j+1}^p$ is the dividend growth at time $t + j$ of the portfolio p which was formed $j + 1$ years earlier, that is, at $t - 1$. Similarly, Δd_{t+j}^{mkt} is the dividend growth of the market at time $t + j$. Finally, $\rho_{CPV}^j = .95$ is a discount, and R is the number of years over which the average growth rate is computed. They call the regression

²²We expressed the issue in terms of shares, but this finding is true more generally. Indeed, in the general set up in the Appendix we may assume $\mathbf{v}^i = [v_1^i, 0, \dots, 0, v_i^i, \dots]$. Shocks to consumption then depend only on dB_1 , the first Brownian Motion, as the others are diversified way. However, expected consumption growth depends on the shares, $\mu(\mathbf{s}_t) = \sum_i s_t^i \mu^i$, whose movement *does* depend on an aggregation of the idiosyncratic shocks.

coefficient $\beta_{CF,1}^p$ the cash-flow beta, as it measures essentially how portfolio cash-flows covary with aggregate cash-flows.

The empirical estimates of Cohen, Polk and Vuolteenaho (2009) are reported in Table IV Panel A. Notice first that empirically, irrespective of the cash-flow measure used, value stocks have higher cash-flow betas than growth stocks, though magnitudes differ across measures. If either (accumulated) return on equity, $\sum_{j=0}^4 \rho^j ROE_{t+j,j+1}^p$, or (accumulated) dividend growth is used as a measure of cash-flow growth, the regression coefficients roughly double when we go from growth to value stocks. If instead we use (accumulated) earnings growth relative to market value, $(X_{t+4,4}^p - X_{t-1,0}^p) / ME_{t-1,0}^p$, the coefficients increase by a factor of 10. Finally if (accumulated) earnings relative to market, $\sum_{j=0}^4 \rho^j (X_{t+j,j+1}^p / ME_{t+j-1,j}^p)$, is used they increase almost by a factor of 20.²³

Turning now to the model, the first line of Table IV-Panel B reports the cash-flow betas as estimated from the same regression (26) in simulated data. As one can see, the model generates the striking pattern uncovered by Cohen, Polk and Vuolteenaho (2009): Cash-flow betas increase with book-to-market (i.e., dividend yield in our model).²⁴ This result of our model stems from the fact that, on average, the procedure of sorting stocks based on their price/dividend ratio *endogenously* selects as value stocks those with higher cash flow risk, as illustrated in Section IV (see Figure 4). Indeed, this effect of the sorting procedure can be seen in the second line of Panel B, which shows that the average cash-flow risk parameter θ_{CF}^i is higher for the value portfolios than for the growth portfolios. We emphasize that the fact that value stocks have a higher cash flow risk than growth stocks in our model is a result and not an assumption, as it solely stems from the sorting procedure.

The *cash flow risk puzzle*, which we highlight in this paper, is the observation that even if the model qualitatively implies that cash-flow betas increase with the dividend yield, *quantitatively* the estimates of the cash-flow betas in simulated data are too high, in absolute

²³Other authors have also found that value stocks have a higher cash-flow risk than growth stocks. For instance, Bansal, Dittmar, and Lundblad (2005) regress market-to-book sorted portfolios' dividend growth on a moving average of consumption growth rates, and find that indeed cash-flow betas are larger for value sorted portfolios (see Table I, Panel A). Similarly, Hansen, Heaton and Li (2008) show that growth stocks have low long-run cash-flow covariation with consumption relative to value. We focus on Cohen et al. (2009) measure, as it is directly related to our setting, while the measure of cash-flow risk in Bansal et al. (2005) and Hansen et al. (2008) is related to the loading on the small predictable component in expected consumption growth.

²⁴Cohen, Polk and Vuolteenaho (2009) argue that this cross sectional dispersion of cash-flow betas can explain much of the long horizon returns of value sorted portfolios.

values, relative to their empirical counterparts. Indeed, notice two facts: First, our model produces a spread of cash-flow betas that is comparable to the empirical spread only when cash-flows are measured as the ratio of earnings to market. For any other measure, the empirically observed spread is much lower. Second, our model generates cash-flow betas for growth stocks that are negative and large in absolute values, which is also at odds with the data, as all empirical cash-flow betas, independently of the measure used, are positive.

Clearly the question is whether our choice of $\bar{\theta}_{CF}$ can be relaxed in order to reproduce the return moments of interest in the data while at the same time generating estimates of cash-flow betas more in line with the data. Tables A.1 and A.2 in the Appendix contain summary results of simulations under several alternative values of the cash flow parameters $(\bar{\theta}_{CF}, \phi, \bar{v})$ and these results are discussed in detail in Appendix C. Figure 7 summarizes these results. The figure plots the cash-flow beta spread in simulated data against the corresponding value premium for all these different parameterizations of the cash-flow processes. We also plot Cohen et al's estimates of these cash-flow betas (in diamonds) against the empirically observed value premium, which is denoted by the vertical dashed line. The key to our model is that there is a positive relation between the spread and the value premium. As we increase the spread in cash-flow risk, as measured by $\bar{\theta}_{CF}$, we increase the cash-flow beta spread, $\beta_{CF,1}^{10} - \beta_{CF,1}^1$, and the model does a better job at matching the value premium. The point of this paper is that the cash-flow beta spread needed to get the model to the vertical line is too high relative to the empirical measures estimated by Cohen, Polk and Vuolteenaho (2009). The only exception is when these authors use clean surplus earnings to market as their measure of cash-flows. Recall though that even this measure implies positive cash-flow betas for growth stocks whereas our model implies negative ones.

To summarize, a habit formation model calibrated to match the properties of the market portfolio needs too much dispersion in cash-flow risk in order to quantitatively deliver a reasonable value premium; this is what we refer to as the “cash-flow risk puzzle.” Although our results depend on the specifics of our model, we believe this is a general result pertaining to external habit formation models a la' Campbell and Cochrane (1999), as these models tend to generate a variation in the stochastic discount factor that induces a growth premium on stocks whenever there are no differences in cash-flow risk. Indeed, within a different model, Lettau and Wachter (2007) found a similar result, which they resolve by assuming an unpriced “sentiment” factor that drives the market price of risk. Here we take a different route. We maintain the habit formation specification, but assume instead that stocks differ in their cash-

flow risk, which generates several novel implications, such as that value stocks, endogenously, have a larger cash-flow risk than growth stocks, and that the CAPM fails because of the implied time variation in expected consumption growth. Next section highlights additional empirical predictions of the model for the cross section of stock returns.

VI. UNDERSTANDING ASSET PRICING TESTS

As seen in the previous section, our model is flexible enough to replicate the standard moments of interest both in the time series and the cross section of stock returns, although at the expense of a large dispersion of cash-flow risk across firms. Notwithstanding this drawback, which is the analog for the cross-section of the high correlation between returns and consumption shocks in the original Campbell and Cochrane (1999) model (see e.g. their Table 7 and the discussion therein), we can build on this ability of the model to produce plausible magnitudes in the cross section to shed light on the economics behind many of the asset pricing tests that have been proposed in the literature, as well as obtain new testable predictions. First our model features a time varying value premium, a novel implication of our framework, and we start by showing evidence that this is indeed the case in the data and that our model matches to a surprising degree this time series variation. We then turn to some standard asset pricing tests and reinterpret them in light of our model. Thus for instance, we use our model to construct an HML like factor, as in Fama and French (1993), and show the reasons for its good performance as a successful predictor in the cross section. Similarly, we revisit some of the conditional CAPM tests that have been proposed recently in the literature. In our model, the CAPM does not hold either conditionally nor unconditionally, but we show that (misspecified) conditional CAPM tests can “look better” than their unconditional counterpart precisely because they capture the time series variation of the value premium that the model produces as well. In these additional exercises, we use the parameters $\phi = 0.07$, $\bar{\nu} = 0.55$ and $\bar{\theta}_{CF} = 0.345\%$, which is, as discussed our benchmark case.

VI.A The dynamics of the value premium

A novel implication of our framework is the fact that the value premium fluctuates as a result of the interaction of the two key ingredients of the model, the strong discount effects of habit persistence models with the cross sectional dispersion of cash-flow risk as measured by $\bar{\theta}_{CF} > 0$. To gauge the presence of this time series variation in the data, Table V Panel A

shows the average excess return of the first and tenth decile portfolio as a function of whether the market-to-book ratio of the market portfolio is above or below a certain percentile, denoted by \bar{c} for the 1948-2001 sample. Thus, the first line shows that the average excess rate of return of the first decile (growth) portfolio is 13.18% if the market-to-book of the market portfolio is *below* the 15th percentile of its empirical distribution and that of the tenth decile (value) portfolio is 23.57%. The value premium is then 10.38%. Instead when the market-to-book is *above* the 15th percentile the first decile portfolio has an average excess return of 5.73% and the tenth portfolio has one of 10.35% for a total value premium of 4.62%, which is considerably lower than the previous one. This pattern holds for any cut-off point: The value premium is higher whenever the market-to-book of the market portfolio is low which are also periods where the average excess return of the market is high, as shown in the columns headed by \bar{R}^M .

Panel B of Table V reports the same calculations as in Panel A, but in simulated data. The only difference is that, naturally, instead of using the market-to-book, we use the price dividend ratio of the market portfolio to identify the state. The pattern is indeed very similar with the only exception of the level of the premia which is, as already discussed, lower than in the data. The value premium is higher when the price-dividend ratio of the market portfolio is low than when it is high. For instance, when the price-dividend ratio of the market portfolio is below the 15th percentile the value premium is 10.90% whereas when it is above it is only 4.15%, very close to their empirical counterparts. In summary then, the discount risk effects needed to replicate the time series properties of the market portfolio interact with the cross-sectional dispersion in cash-flow risk to generate variation in the value premium: Value stocks are particularly risky during bad times, periods when the aggregate market premium and its dividend yield are high relative to their unconditional mean, both in the data and the model.²⁵

VI.B The CAPM and the Fama-MacBeth regressions

As discussed at the beginning of Section V, the CAPM is not able to price stocks sorted by market to book, the value premium puzzle. We already discussed the value premium puzzle in Section V.B and the performance of our model then; thus, for brevity, we do not repeat the

²⁵To further investigate the properties of the conditional variation in the value premium, we also ran a regression of the return on HML on the value spread, the difference between the dividend yield of the value and growth portfolio, as in Cohen, Polk, and Vuolteenaho (2003, Table V), and find that the value spread is statistically significant predictor of the return on HML for several horizons. In the interest of space, we do not report these results, which are available from the authors upon request.

comments here. However, there is one noteworthy point to make in relation to the standard tests of the CAPM via Fama MacBeth cross-sectional regressions. To illustrate the issue, the first line of Panel A of Table VII reports the classic results about the failure of the CAPM via Fama MacBeth regressions: the intercept is positive, the market premium is negative, and the cross-sectional R^2 is small. Line 5 of Panel B in the same table reports the performance of Fama-MacBeth cross-sectional regressions in artificial data. As can be seen, the CAPM produces a good fit with an R^2 of 91% and thus it appears that in our model the unconditional CAPM works well, in contrast with our earlier findings in Table III. The reason of this difference is that in our simulated data, the CAPM betas correlate positively with average excess returns (see Table III and Figure 5). Thus, cross-sectional regressions that impose no constraints on the level of estimated market premium tend to induce a good fit as measured by the R^2 . But this is misleading, as the rejection of the model comes from the comparison of the market premium implied by the cross-sectional regression, which is 10.24% ($= 2.56\% \times 4$), and the market premium computed from the time series of returns, which is 4.35% (see Table III Panel A.) That is, in general, the cross-sectional R^2 is a poor indicator of the performance of an asset pricing model.²⁶

VI.C Understanding HML and the Fama and French (1993) model

In their seminal paper, Fama and French (1993) advance a new factor, HML, that is able to correctly price value sorted portfolios. Since then, the Fama and French (1993) model has become a standard benchmark in asset pricing tests. How well does a HML factor work in our setup? To answer this question, we construct an HML factor in artificial data that is long the three top decile portfolios and short the bottom three. Table VI presents the results of time series regressions,

$$R_t^p = \alpha + \beta_M R_t^M + \beta_{HML} R_t^{HML} + \epsilon_t^p \quad \text{for } p = 1, 2, \dots, 10.$$

Panel A shows the results in the case of the empirical data. The results are well known. The intercepts go down considerably and only one of them is statistically significant; value (growth) stocks have a large (small) loading on HML and the inclusion of HML in the time

²⁶This message has recently been emphasized by Lewellen and Nagel (2006) and Daniel and Titman (2005): A small but slightly positive cross-sectional covariation between betas and average returns can result in the unwarranted support of asset pricing models that fail to impose economically based restrictions on the size of the premia of the proposed factors.

series regression collapses the betas on the market portfolio around 1 (see Fama and French (1993, page 21-26)).

Panel B shows the time series regression in simulated data. Turning first to the loadings on the market portfolio, notice that, as it was the case in the empirical sample, adding HML to the time series regressions has the effect of *reducing* the spread in the estimates of β_M^i and collapse them around 1. As Fama and French (1993) note this pattern is related to the negative correlation between the market and the returns on HML.

As for the loading on the HML portfolio notice that it has a strong cross-sectional variation which reflects the cross-sectional variation in the underlying cash-flow risk of the different portfolios. Indeed, the loading on HML of the growth portfolio is $-.28$, whereas that of the value portfolio is 1.07 . Also the size of the intercepts of the time series regressions drop considerably relative to the size of the intercepts when only the market portfolio is present.²⁷ Moreover, there is no longer any pattern in the variation of the intercept across decile portfolios, which shows that HML is capturing the systematic pattern of misspricing documented in Panel A.

The evidence in the Fama-MacBeth regression confirms the time series evidence. Line 2 of Table VII Panel A shows that HML enters significantly and the estimated size of the premium on HML is very close to the average excess return of the HML portfolio. This is also the case in our simulated regression, which is shown in line 6 of Panel B in Table VII. The coefficient on the loading on HML is very similar to its empirical counterpart and, once annualized, close to our estimated average excess return on the HML portfolio, which is 3.21%. Thus the inclusion of HML in the cross-sectional regression aligns the portfolios correctly, as the intercept is now close to zero and the (quarterly) market premium equals 1.31%, which annualized is 5.24%, still higher than the average market return in simulation (4.35%), but much smaller than the one obtained for the CAPM case.

VI.D Conditional asset pricing models

Conditional asset pricing models have been proposed recently to address the inability of the CAPM to explain the value premium. The idea, as advanced by Hansen and Richard (1987), is that the CAPM may fail unconditionally, but may hold conditionally and thus tests

²⁷Notice that the value-weighted sum of the alphas should be equal to zero. Given that the only negative alpha is that of the growth portfolio, it must be the case that some of the assets in the growth portfolio must have extreme prices. We thank Gene Fama for pointing out this to us.

of the CAPM that ignore conditioning information are misspecified. Researchers have reacted to this observations by using as proxies for investors' information set variables that are known to forecast returns in the time series.²⁸ Typically, this has led to tests of multifactor model where the additional factor, other than the market, is the market itself interacted with the proposed conditioning variables.

Lines 3 and 4 in Panel A of Table VII show that conditioning by the dividend yield of the market portfolio and the *cay* variable of Lettau and Ludvigson (2001) results also in a coefficient for the instrumented market that is strongly significant. In addition, the R^2 is an impressive 83% and 81% respectively. Line 7 of Panel B shows that our model does also well in this dimension. When we interact the returns of the market portfolio with the simulated dividend yield of the market portfolio, we obtain a coefficient of similar magnitude to its empirical counterpart.²⁹ These conditional asset pricing models capture the fact that value stocks become relatively riskier in bad times, a feature for which our model provides an explanation, as shown in Table V and Figure 6. In our setup the conditional CAPM does not hold, but it does better than its unconditional counterpart because it captures the conditional effects that arise out of the interaction of discount effects with the cross-sectional dispersion in θ_{CF}^i .

VII. CONCLUSIONS

Two sources of risk combine to determine the time series properties of the market portfolio and the cross-sectional properties of stock returns: discount risk and cash-flow risk. Campbell and Cochrane (1999) argue that time variation of the market price of risk - i.e. discount risk - is important to reconcile many empirical facts about the aggregate market portfolio. We show that this channel though imposes tight restrictions on the cash-flow properties of value versus growth stocks. Specifically, the natural growth premium that habit formation preferences generates on firms that only differ in their expected dividend growth requires a large cross-sectional dispersion in cash-flow risk across firms to generate a value premium.

Under this restriction, the model performs well, yielding most of the stylized facts about the time series and the cross section of stock returns. In particular, besides matching the conditional properties of the market portfolio, as in Campbell and Cochrane (1999), our model

²⁸See, among others, the conditional asset pricing models of Jagannathan and Wang (1996), Ferson and Harvey (1999), Lettau and Ludvigson (2001), and Santos and Veronesi (2006).

²⁹We do not report the results for *cay* as in our setting, *cay* is perfectly correlated with $\log(D/P)$.

also generates a sizable value premium, a countercyclical value premium, the failure of the CAPM and the better performance of factor models and conditional CAPM models.

Yet, in order to match the properties of both the aggregate market portfolio and the cross section of stock returns, the large dispersion of cash-flow risk that we must assume generates a “cash-flow risk puzzle:” In our simulations, as in the data, value stocks have (endogenously) higher cash-flow risk than growth stocks, but too much dispersion in cash-flow risk is required to generate the value premium observed in the data. The cash-flow risk puzzle may arise from various sources, such as the much larger noise in the cash-flow data compared to our simulated data, which would tend to decrease the size of the “cash-flow beta” due to attenuation bias. We view our model as a first step into understanding the sources of risk that explain both the time-series and the cross section of stock returns. Indeed, an important message of this paper is that we cannot study one set of empirical facts independently of the other: any story that attempts to quantitatively explain the cross-section of stock returns must also be consistent with the time series properties of the market portfolios. Otherwise, the parameterization that is used to obtain quantitative predictions at the cross-sectional level may be quite misleading.

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APPENDIX A. Some Additional Results

The Aggregation Problem: It is useful to see the nature of the difficulty of imposing a general equilibrium restriction $C_t = \sum_{j=1}^n D_t^j$ when working with multiple assets. To understand these restrictions and the nature of our Assumptions 1 and 2, define $\mathbf{D}_t = (D_t^1, \dots, D_t^n)'$ and assume that

$$\frac{dD_t^i}{D_t^i} = \mu_D^i(\mathbf{D}_t) dt + \boldsymbol{\nu}_i' d\mathbf{B}_t \quad (27)$$

for some generic drifts $\mu_D^i(\mathbf{D}_t)$. Assume that $\boldsymbol{\nu}_i$ is a $n \times 1$ constant vector, and $d\mathbf{B}_t$ is a $n \times 1$ vector of Brownian motions. From the general equilibrium restriction $C_t = \sum_{j=1}^n D_t^j$ and Ito's lemma, the process for aggregate consumption is

$$\frac{dC_t}{C_t} = \mu_c(\mathbf{s}_t) dt + \boldsymbol{\sigma}_c(\mathbf{s}_t)' d\mathbf{B}_t \quad (28)$$

where $\mathbf{s}_t = (s_t^1, \dots, s_t^n)' = (D_t^1/C_t, \dots, D_t^n/C_t)$ are shares of consumption produced by dividends, and

$$\mu_c(\mathbf{s}_t) = \sum_{i=1}^n s_t^i \mu_D^i \quad \text{and} \quad \boldsymbol{\sigma}_c(\mathbf{s}_t) = \sum_{i=1}^n s_t^i \boldsymbol{\nu}_i \quad (29)$$

The main difficulty in obtaining tractable expressions for asset prices lies in the dependence of $\mu_c(\mathbf{s}_t)$ and $\boldsymbol{\sigma}_c(\mathbf{s}_t)$ on the shares \mathbf{s}_t . Assumptions 1 and 2 in the body of the paper restrict the volatility of consumption to a constant, but retain the time variation in the drift rate of consumption, although in a specific form. Our specific assumptions not only allow us to obtain closed form solutions for assets, but the mild predictability in expected consumption growth also invalidates both the conditional and unconditional CAPM.

We note that in this setting, the cash flow risk is given by

$$Cov\left(\frac{dD_t^i}{D_t^i}, \frac{dC_t}{C_t}\right) = \boldsymbol{\nu}_i' \boldsymbol{\sigma}_c(\mathbf{s}_t) = \sum_{j=1}^n s_t^j \boldsymbol{\nu}_i' \boldsymbol{\nu}_j \quad (30)$$

Finally, an application of Ito's Lemma to $s_t^i = D_t^i/C_t$ when the processes of D_t^i and C_t are given by (27) and (28) shows that the volatility of the share process ds_t^i is as in expression (5).

The Habit Dynamics: If $X_t = \lambda \int_{-\infty}^t e^{\lambda(\tau-t)} C_\tau d\tau$ we have $dX_t = \lambda(C_t - X_t) dt$. Define then $G_t = f(C_t, X_t) = (C_t/(C_t - X_t))^\gamma$. We then have

$$\begin{aligned} f_C &= -\gamma G_t \left(G_t^{\frac{1}{\gamma}} - 1 \right) C_t^{-1} \\ f_{CC} &= \left\{ \gamma(\gamma-1) G \left(G_t^{\frac{1}{\gamma}} - 1 \right)^2 + 2\gamma \left(G_t^{\frac{1}{\gamma}} - 1 \right) G_t^{\frac{1}{\gamma}+1} \right\} C_t^{-2} \\ f_X &= \gamma G_t \frac{1}{(C_t - X_t)} \end{aligned}$$

where we used $G_t^{\frac{1}{\gamma}} = C_t/(C_t - X_t)$ and $G_t^{\frac{1}{\gamma}} - 1 = X_t/(C_t - X_t)$. Ito's Lemma then yields

$$dG_t = \left\{ \mu_G(G_t) - \sigma_G(G_t) \mu_{c,1}(\mathbf{s}_t) \right\} dt - \sigma_G(G_t) \boldsymbol{\sigma}_c d\mathbf{B}_t^1$$

where

$$\mu_G(G_t) = \gamma \lambda G_t + \frac{1}{2} \gamma(\gamma-1) G \left(G_t^{\frac{1}{\gamma}} - 1 \right)^2 \sigma_c^2 + \gamma \left(G_t^{\frac{1}{\gamma}} - 1 \right) G_t^{\frac{1}{\gamma}+1} \sigma_c^2 - \sigma_G(G_t) \bar{\mu}_c \quad (31)$$

$$\sigma_G(G_t) = \gamma G_t \left(G_t^{\frac{1}{\gamma}} - 1 \right) \quad (32)$$

APPENDIX B. Proof of Propositions

Our strategy to obtain prices and returns in our economy is standard. Given (13), the stochastic discount factor is given by

$$m_t = e^{-\rho t} (C_t - X_t)^{-\gamma} = e^{-\rho t} C_t^{-\gamma} G_t.$$

We use Ito's Lemma and our assumptions on the dynamics of C_t and $G_t = S_t^{-\gamma}$ to obtain

$$\frac{dm_t}{m_t} = -r_t^f dt + \boldsymbol{\sigma}'_m d\mathbf{B}_t,$$

where the first, and only non-zero, entry in the diffusion component vector, $\boldsymbol{\sigma}_m$, is given by

$$\sigma_m^1 = -[\gamma + \alpha(1 - \lambda S_t^\gamma)] \sigma_c. \quad (33)$$

Then we exploit our assumptions on the dynamics of C_t , $G_t = S_t^{-\gamma}$ and s_t^i to solve for

$$P_t^i = E_t \left[\int_t^\infty \left(\frac{m_\tau}{m_t} \right) D_\tau^i d\tau \right] = E_t \left[\int_t^\infty \left(\frac{m_\tau}{m_t} \right) s_\tau^i C_\tau d\tau \right] \quad (34)$$

in closed form. We then use (34) to compute returns and calculate the expected excess returns

$$E_t \left[dR_t^i \right] = -cov \left(\frac{dm_t}{m_t}, dR_t^i \right) = -\boldsymbol{\sigma}'_m \boldsymbol{\sigma}_R^i, \quad (35)$$

where $\boldsymbol{\sigma}_R^i$ is the diffusion component associated with the returns of asset i .

Proof of Proposition 1. This is a corollary to Proposition 2, and it is proved below.

Proof of Proposition 2. The pricing formula is

$$\begin{aligned} P_t^i &= E_t \left[\int_t^\infty e^{-\rho(\tau-t)} \frac{u_c(C_\tau, X_\tau)}{u_c(C_t, X_t)} D_\tau^i d\tau \right] \\ &= C_t^\gamma G_t^{-1} E_t \left[\int_t^\infty e^{-\rho(\tau-t)} C_\tau^{1-\gamma} G_\tau s_\tau^i d\tau \right] \end{aligned}$$

We divide the proof in two parts: First, we obtain a general pricing formula which depends on the state variables.

Second, we obtain analytical solutions for the coefficients of these state variables.

Part a.1: A pricing formula. For this proof, it is convenient to rewrite the share processes in its general form as

$$ds_t^i = \sum_{j=1}^n s_t^j \lambda_{ji} dt + s_t^i (\boldsymbol{\nu}^i - \mathbf{s}_t' \boldsymbol{\nu}) d\mathbf{B}_t$$

where $\lambda_{ji} = \phi \bar{s}^i$, for $i \neq j$, and $\lambda_{ii} = -\sum_{j \neq i} \lambda_{ij} = -\phi \sum_{j \neq i} \bar{s}^j = -\phi(1 - \bar{s}^i) = \phi \bar{s}^i - \phi$. Define the two quantities

$$q_t^i = C_t^{1-\gamma} G_t s_t^i \quad \text{and} \quad p_t^i = C_t^{1-\gamma} s_t^i$$

and the $2n \times 1$ vector $\mathbf{y}_t = [\mathbf{q}_t, \mathbf{p}_t]$. An application of Ito's Lemma and tedious algebra shows

$$d\mathbf{y}_t = \widehat{\boldsymbol{\Lambda}}_y \mathbf{y}_t dt + \boldsymbol{\Sigma}_{y,t} d\mathbf{B}_t$$

where

$$\widehat{\boldsymbol{\Lambda}}_y = \begin{bmatrix} \boldsymbol{\Lambda}' + \widehat{\boldsymbol{\Theta}}_q & \widehat{\boldsymbol{\Theta}}_{qp} \\ 0 & \boldsymbol{\Lambda}' + \widehat{\boldsymbol{\Theta}}_p \end{bmatrix},$$

$\boldsymbol{\Lambda} = \phi(\bar{\mathbf{s}} \times \mathbf{1}'_n)$, $\widehat{\boldsymbol{\Theta}}_i$ for $i = q, p, qp$ are diagonal matrices with ii element given by

$$\begin{aligned} \widehat{\theta}_q^i &= (1 - \gamma) \bar{\mu}_c - \frac{1}{2} \gamma (1 - \gamma) \sigma_c^2 - k - (1 - \gamma) \sigma_c^2 \alpha + (1 - \gamma) \theta^i - \alpha \theta^i \\ \widehat{\theta}_{qp}^i &= k \bar{G} + (1 - \gamma) \sigma_c^2 \alpha \lambda + \alpha \lambda \theta^i \\ \widehat{\theta}_p^i &= (1 - \gamma) \bar{\mu}_c - \frac{1}{2} \gamma (1 - \gamma) \sigma_c^2 + (1 - \gamma) \theta^i \end{aligned}$$

and $\Sigma_{y,t}$ is an appropriate matrix. Assuming existence of the expectation in the pricing function, we can apply Fubini's theorem

$$P_t^i = C_t^\gamma G_t^{-1} E_t \left[\int_t^\infty e^{-\rho(\tau-t)} y_\tau^i d\tau \right] = C_t^\gamma G_t^{-1} \int_t^\infty E_t \left[e^{-\rho(\tau-t)} y_\tau^i \right] d\tau$$

The expectation in the integral can be computed as follows: Let ω be the vector of eigenvalues of $\widehat{\Lambda}_y$, $\left[e^{\omega(\tau-t)} \right]$ the diagonal matrix with ii element given by $e^{\omega_i(\tau-t)}$ and \mathbf{U} the matrix of associated eigenvectors. Then, we can write

$$E_t \left[e^{-\rho(\tau-t)} y_\tau^i \right] = \boldsymbol{\nu}_i \cdot \mathbf{U} \cdot \left[e^{\omega(\tau-t)} \right] \cdot \mathbf{U}^{-1} \cdot \mathbf{y}_t e^{-\rho(\tau-t)} = \sum_{k=1}^{2n} \sum_{j=1}^{2n} u_{ik} e^{(\omega_k - \rho)(\tau-t)} [u_{jk}^{-1}] y_{jt}$$

where $[u_{jk}^{-1}]$ is the jk element of U^{-1} . Substituting into the expectation, and taking the integral, we find

$$\int_t^\infty E_t \left[e^{-\rho(\tau-t)} y_\tau^i \right] d\tau = \sum_{k=1}^{2n} \sum_{j=1}^{2n} \frac{u_{ik} [u_{jk}^{-1}]}{\rho - \omega_k} y_{jt} = \sum_{j=1}^{2n} b_j^i y_{jt}$$

where

$$b_j^i = \sum_{k=1}^{2n} \frac{u_{ik} [u_{kj}^{-1}]}{\rho - \omega_k}$$

Below, we obtain these coefficients in closed form. Note, however, that by substituting $y_{jt} = q_{jt}$ for $j = 1, \dots, n$ and $y_{jt} = p_{j-n,t}$ for $j = n+1, \dots, 2n$ we obtain

$$\begin{aligned} P_t^i &= C_t^\gamma G_t^{-1} E_t \left[\int_t^\infty e^{-\rho(\tau-t)} y_\tau^i d\tau \right] = C_t^\gamma G_t^{-1} \left(\sum_{j=1}^n b_{1j}^i q_{j,t} + \sum_{j=1}^n b_{2j}^i p_{j,t} \right) \\ &= C_t^\gamma G_t^{-1} \left(C_t^{1-\gamma} G_t \sum_{j=1}^n b_{1j}^i s_t^j + C_t^{1-\gamma} \sum_{j=1}^n b_{2j}^i s_t^j \right) \\ &= C_t \sum_{j=1}^n \left(b_{1j}^i + b_{2j}^i S_t^\gamma \right) s_t^j \end{aligned}$$

Part a.2: Analytical formulas for $b_{1,j}^i$ and $b_{2,j}^i$. We finally obtain a closed form formula for b_j^i 's, and thus, of b_{1j}^i and b_{2j}^i . First, note that we can write

$$b_j^i = \boldsymbol{\nu}_i \cdot \mathbf{U} \cdot (\boldsymbol{\Omega}^{-1}) \cdot \mathbf{U}^{-1} \boldsymbol{\nu}_j$$

where $\boldsymbol{\Omega}$ is the matrix with the eigenvalues of $\mathbf{I}\rho - \widehat{\Lambda}_y$ on the principal diagonal. But then, since $\mathbf{U} \cdot (\boldsymbol{\Omega}^{-1}) \cdot \mathbf{U}^{-1} = (\mathbf{I}\rho - \widehat{\Lambda}_y)^{-1}$ we have that for $i = 1, \dots, n$ and $j = 1, \dots, 2n$

$$b_j^i = \boldsymbol{\nu}_i \cdot (\mathbf{I}\rho - \widehat{\Lambda}_y)^{-1} \cdot \boldsymbol{\nu}_j$$

We now explicitly compute these quantities. Define $\mathbf{B} = (\mathbf{I}\rho - \widehat{\Lambda}_y)^{-1}$, so that

$$\mathbf{B} (\mathbf{I}\rho - \widehat{\Lambda}_y) = \mathbf{I}$$

Making this explicit, for every $i = 1, \dots, n$ (row) we have

$$\sum_{j=1}^{2n} b_j^i (\mathbf{I}\rho - \widehat{\Lambda}_y)_j = \boldsymbol{\nu}_i$$

where $(\mathbf{I}\rho - \widehat{\Lambda}_y)$ is the j th row of $(\mathbf{I}\rho - \widehat{\Lambda}_y)$ and $\boldsymbol{\nu}_i$ is a $(1 \times 2n)$ row vector with 1 in i th position, and zero elsewhere. For every i , we have a system of equations that pins down b_j^i for all $j = 1, \dots, 2n$. We now solve this system of equation. To limit the number of indices involved, we do this exercise for $i = 1$. Of course, the methodology works for every i . For $i = 1$ we have then the following two systems of equations. The first holds for $j = 1, \dots, n$ and the second for the remaining n rows:

$$b_1^1 \left(\rho - \phi \bar{s}^1 + \phi - \widehat{\theta}_q^1 \right) - \sum_{j=2}^n b_j^1 \phi \bar{s}_j = 1 \quad (\text{row 1})$$

$$-b_1^1 \phi \bar{s}^1 + b_2^1 \left(\rho - \phi \bar{s}^2 + \phi - \widehat{\theta}_q^2 \right) - \sum_{j=3}^n b_j^1 \phi \bar{s}_j = 0 \quad (\text{row 2})$$

⋮

$$- \sum_{j=1}^{n-1} b_j^1 \phi \bar{s}_j + b_n^1 \left(\rho - \phi \bar{s}^n + \phi - \widehat{\theta}_q^n \right) = 0 \quad (\text{row } n)$$

$$-b_1^1 \theta_{qp}^1 + b_{n+1}^1 \left(\rho - \phi \bar{s}^1 + \phi - \widehat{\theta}_p^1 \right) - \sum_{j=2}^n b_{n+j}^1 \phi \bar{s}_j = 0 \quad (\text{row } n+1)$$

$$-b_2^1 \theta_{qp}^2 - b_{n+1}^1 \phi \bar{s}_1 + b_{n+2}^1 \left(\rho - \phi \bar{s}^2 + \phi - \widehat{\theta}_p^2 \right) - \sum_{j=3}^n b_{n+j}^1 \phi \bar{s}_j = 0 \quad (\text{row } n+2)$$

⋮

$$-b_n^1 \theta_{qp}^n - \sum_{j=1}^{n-1} b_{n+j}^1 \phi \bar{s}_j + b_{2n}^1 \left(\rho - \phi \bar{s}^n + \phi - \widehat{\theta}_p^n \right) = 0 \quad (\text{row } 2n)$$

The first set of equation is readily solved. In fact, we can write

$$b_1^1 = \alpha_q^1 + \alpha_q^1 \times \phi \sum_{j=1}^n b_j^1 \bar{s}^j$$

$$b_k^1 = \alpha_q^k \times \phi \sum_{j=1}^n b_j^1 \bar{s}^j \quad \text{for } k = 2, \dots, n$$

where

$$\alpha_q^i = \frac{1}{\left(\rho + \phi - \widehat{\theta}_q^i \right)}$$

Multiply both sides of each row $k = 1, \dots, n$ by \bar{s}^k and sum across rows to obtain

$$\sum_{j=1}^n b_j^1 \bar{s}^j = \bar{s}^1 \alpha_q^1 + \sum_{j=1}^n \bar{s}_j^1 \alpha_q^j \left(\phi \sum_{j=1}^n b_j^1 \bar{s}^j \right)$$

Define the constants

$$H_q = \sum_{j=1}^n \bar{s}_j^j \alpha_q^j \quad \text{and} \quad K_q = \frac{1}{1 - \phi H_q}$$

Solving for $\sum_{j=1}^n b_j^1 \bar{s}^j$ we obtain the quantity

$$\sum_{j=1}^n b_j^1 \bar{s}^j = \bar{s}^1 \alpha_q^1 K_q$$

Thus

$$b_1^1 = \alpha_q^1 + \alpha_q^1 \times \phi \bar{s}^1 \alpha_q^1 K_q \quad (36)$$

$$b_k^1 = \alpha_q^k \times \phi \bar{s}^1 \alpha_q^1 K_q \quad \text{for } k = 2, \dots, n \quad (37)$$

Hence, the first term in the P/C ratio obtained earlier, i.e.

$$\frac{P_t^1}{C_t} = \sum_{j=1}^n b_{1j}^1 s_t^j + \sum_{j=1}^n b_{2j}^1 s_t^j S_t^\gamma$$

is given by

$$\sum_{j=1}^n b_{1j}^1 s_t^j = \alpha_q^1 s_t^1 + \phi \bar{s}^1 \alpha_q^1 K_q \sum_{j=1}^n \alpha_q^k s_t^j$$

where recall that for $j = 1, \dots, n$ we defined earlier $b_{1j}^1 = b_j^1$.

We now turn to the second system of equations, which for $k = 1, \dots, n$ can be rewritten as

$$b_{n+k}^1 = \alpha_p^k \phi \sum_{j=1}^n b_{n+j}^1 \bar{s}^j + b_k^1 \alpha_p^k \hat{\theta}_{qp}^k$$

with

$$\alpha_p^k = \frac{1}{(\rho + \phi - \hat{\theta}_p^k)}$$

and b_k^1 given in (36) - (37). Substitute b_k^1 first, to obtain

$$\begin{aligned} b_{n+1}^1 &= \alpha_p^1 \phi \sum_{j=1}^n b_{n+j}^1 \bar{s}^j + \alpha_{pq}^1 + \alpha_{pq}^1 \times \phi \bar{s}^1 \alpha_q^1 K_q \\ b_{n+k}^1 &= \alpha_p^k \phi \sum_{j=1}^n b_{n+j}^1 \bar{s}^j + \alpha_{pq}^k \phi \bar{s}^1 \alpha_q^1 K_q \end{aligned}$$

where

$$\alpha_{pq}^k = \alpha_p^k \hat{\theta}_{qp}^k \alpha_q^k$$

As before, for $k = 1, \dots, n$ multiply both sides by \bar{s}^k and sum across k 's to obtain

$$\sum_{k=1}^n \bar{s}^k b_{n+k}^1 = \alpha_{pq}^1 \bar{s}^1 + \left(\sum_{k=1}^n \bar{s}^k \alpha_p^k \right) \phi \sum_{j=1}^n b_{n+j}^1 \bar{s}^j + \left(\sum_{k=1}^n \bar{s}^k \alpha_{pq}^k \right) \phi \bar{s}^1 \alpha_q^1 K_q$$

Let

$$H_p = \left(\sum_{k=1}^n \bar{s}^k \alpha_p^k \right)$$

and solve for $\sum_{k=1}^n \bar{s}^k b_{n+k}^1$ to find

$$\sum_{k=1}^n \bar{s}^k b_{n+k}^1 = \alpha_{pq}^1 \bar{s}^1 K_p + \left(\sum_{k=1}^n \bar{s}^k \alpha_{pq}^k \right) \phi \bar{s}^1 \alpha_q^1 K_q K_p$$

where

$$K_p = \frac{1}{(1 - \phi H_p)}$$

Substitute back into b_{n+1}^1 and b_{n+k}^1 and find

$$\begin{aligned} b_{n+1}^1 &= \alpha_{pq}^1 + \bar{s}^1 g_1^1 \\ b_{n+k}^1 &= \bar{s}^1 g_k^1 \end{aligned}$$

where for $k = 1, \dots, n$

$$g_k^1 = \alpha_q^1 \phi \left\{ \alpha_p^k \left(\alpha_p^1 \hat{\theta}_{pq}^1 K_p + \left(\sum_{j=1}^n \bar{s}^j \alpha_{pq}^j \right) \phi K_q K_p \right) + \alpha_{pq}^k K_q \right\}$$

Thus, the second part in the price-consumption ratio is given by

$$\sum_{j=1}^n b_{2j}^1 s_t^j = \alpha_{pq}^1 s_t^1 + \bar{s}^1 \sum_{k=1}^n g_k^1 s_t^k$$

Generalizing the above derivations for every $i = 1, \dots, n$, we can finally write

$$\frac{P_t^i}{D_t^i} = \alpha_0^i + \alpha_1^i S_t^\gamma + \alpha_2^i(\mathbf{s}_t) \left(\frac{\bar{s}^i}{s_t^i} \right) + \alpha_3^i(\mathbf{s}_t) \left(\frac{\bar{s}^i}{s_t^i} \right) S_t^\gamma$$

where

$$\begin{aligned} \alpha_0^i &= \alpha_q^i = \frac{1}{(\rho + \phi - \widehat{\theta}_q^i)} \\ \alpha_1^i &= \alpha_{pq}^i = \frac{\widehat{\theta}_{pq}^i}{(\rho + \phi - \widehat{\theta}_q^i)(\rho + \phi - \widehat{\theta}_p^i)} \\ \alpha_2^i(\mathbf{s}_t) &= \phi \alpha_q^i K_q (\mathbf{s}_t' \boldsymbol{\alpha}_q) \\ \alpha_3^i(\mathbf{s}_t) &= \mathbf{s}_t' \mathbf{g}^i \end{aligned}$$

where

$$g_k^i = \alpha_q^i \phi \left\{ \alpha_p^k \left(\alpha_p^i \widehat{\theta}_{pq}^i K_p + (\bar{s}' \boldsymbol{\alpha}_{pq}) \phi K_q K_p \right) + \alpha_{pq}^k K_q \right\}$$

and

$$\begin{aligned} \widehat{\theta}_q^i &= (1 - \gamma) \bar{\mu}_c - (1 - \gamma) \left(\frac{1}{2} \gamma + \alpha \right) \sigma_c^2 - k + (1 - \gamma - \alpha) \theta^i \\ \widehat{\theta}_{qp}^i &= k \bar{G} + (1 - \gamma) \sigma_c^2 \alpha \lambda + \alpha \lambda \theta^i \\ \widehat{\theta}_p^i &= (1 - \gamma) \bar{\mu}_c - \frac{1}{2} \gamma (1 - \gamma) \sigma_c^2 + (1 - \gamma) \theta^i \end{aligned}$$

Proof of Proposition 3: The diffusion component of stock returns is given by

$$\boldsymbol{\sigma}_{R,t}^i = \frac{S_t^\gamma \alpha (1 - \lambda S_t^\gamma)}{f_1^i \left(\frac{\bar{s}^i}{s_t^i}, \mathbf{s}_t \right) + S_t^\gamma} \boldsymbol{\sigma}_c + \left(\frac{1}{1 + f_2^i(S_t, \mathbf{s}_t) \left(\frac{\bar{s}^i}{s_t^i} \right)} + \eta_{it}^i \right) \boldsymbol{\sigma}_D^i(\mathbf{s}_t) + \sum_{j \neq i} \eta_{jt}^i \boldsymbol{\sigma}_D^j(\mathbf{s}_t) \quad (38)$$

In fact, we can write

$$P_t^i = C_t \left(\alpha_0^i s_t^i + \alpha_2^i(\mathbf{s}_t) \bar{s}^i + \left(\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t) \bar{s}^i \right) S_t^\gamma \right)$$

Define by $\widetilde{S}_t = S_t^\gamma = G_t^{-1}$. Using Ito's Lemma, it is immediate to see that the diffusion of $d\widetilde{S}$ is given by

$$\boldsymbol{\sigma}_S(S^\gamma) = S_t^\gamma \alpha (1 - \lambda S_t^\gamma) \boldsymbol{\sigma}_c$$

Thus, an application of Ito's Lemma shows that the diffusion term of P_t^i is given by

$$\begin{aligned} \boldsymbol{\sigma}_{R,t}^i &= \boldsymbol{\sigma}_c + \frac{(\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t) \bar{s}^i) S_t^\gamma \alpha (1 - \lambda S_t^\gamma)}{(\alpha_0^i s_t^i + \alpha_2^i(\mathbf{s}_t) \bar{s}^i + (\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t) \bar{s}^i) S_t^\gamma) S_t^\gamma} \boldsymbol{\sigma}_c \\ &+ \sum_{k=1}^n \left\{ \frac{(\alpha_0^i + \alpha_1^i S_t^\gamma) 1_{\{i\}} + \phi \alpha_q^i K_q \alpha_q^k + g_k^i}{(\alpha_0^i s_t^i + \alpha_2^i(\mathbf{s}_t) \bar{s}^i + (\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t) \bar{s}^i) S_t^\gamma)} \right\} s_t^k \boldsymbol{\sigma}^k(\mathbf{s}_t) \end{aligned}$$

where $1_{\{i\}}$ is the indicator function for $k = i$. Since $\sigma_D^i(\mathbf{s}_t) = \sigma_c + \sigma^i(\mathbf{s}_t)$, and since by construction

$$\sum_{k=1}^n \left\{ \frac{(\alpha_0^i + \alpha_1^i S_t^\gamma) 1_{\{k=i\}} + \phi \alpha_q^i K_q \alpha_q^k + g_k^i}{(\alpha_0^i s_t^i + \alpha_2^i(\mathbf{s}_t) \bar{s}^i + (\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t) \bar{s}^i) S_t^\gamma)} \right\} s_t^k = 1$$

we can rewrite

$$\begin{aligned} \sigma_{R,t}^i &= \frac{S_t^\gamma \alpha (1 - \lambda S_t^\gamma)}{f_1^i(\bar{s}^i/s_t^i; \mathbf{s}_t) + S_t^\gamma} \sigma_c + \sum_{k=1}^n \left\{ \frac{(\alpha_0^i + \alpha_1^i S_t^\gamma) 1_{\{k=1\}} + \phi \alpha_q^i K_q \alpha_q^k + g_k^i}{(\alpha_0^i s_t^i + \alpha_2^i(\mathbf{s}_t) \bar{s}^i + (\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t) \bar{s}^i) S_t^\gamma)} \right\} s_t^k \sigma_D^k(\mathbf{s}_t) \\ &= \frac{S_t^\gamma \alpha (1 - \lambda S_t^\gamma)}{f_1^i(\bar{s}^i/s_t^i; \mathbf{s}_t) + S_t^\gamma} \sigma_c + \left\{ \frac{1}{1 + f_2(S; \mathbf{s}_t) \left(\frac{\bar{s}^i}{s_t^i}\right)} + \eta_{i,t}^i \right\} \sigma_D^i(\mathbf{s}_t) + \sum_{k \neq i} \eta_{k,t}^i \sigma_D^k(\mathbf{s}_t) \end{aligned}$$

where

$$\begin{aligned} f_1^i(\bar{s}^i/s_t^i; \mathbf{s}_t) &= \frac{\alpha_0^i + \alpha_2^i(\mathbf{s}_t) (\bar{s}^i/s_t^i)}{\alpha_1^i + \alpha_3^i(\mathbf{s}_t) (\bar{s}^i/s_t^i)} \\ f_2(S; \mathbf{s}_t) &= \frac{\alpha_2^i(\mathbf{s}_t) + \alpha_3^i(\mathbf{s}_t) S_t^\gamma}{\alpha_0^i + \alpha_1^i S_t^\gamma} \end{aligned}$$

and

$$\eta_{k,t}^i = \frac{(\phi \alpha_q^i K_q \alpha_q^k + g_k^i) s_t^k}{(\alpha_0^i s_t^i + \alpha_2^i(\mathbf{s}_t) \bar{s}^i + (\alpha_1^i s_t^i + \alpha_3^i(\mathbf{s}_t) \bar{s}^i) S_t^\gamma)} \quad (39)$$

Note that also that

$$f_1' < 0 \text{ if and only if } \frac{\alpha_2^i(\mathbf{s})}{\alpha_3^i(\mathbf{s})} < \frac{\alpha_0^i}{\alpha_1^i} = \frac{1}{\alpha_p^i \hat{\theta}_{pq}}$$

Finally, the expected return is obtained from $\sigma_{R,t}$ by using the formula

$$E_t \left[dR_t^i \right] = -Cov_t \left(dR_t^i, \frac{dm}{m_t} \right)$$

Q.E.D.

Proof of Proposition 1: The price consumption ratio of the total wealth portfolio can be obtained by simply adding the prices of individual securities. In particular, we find

$$\begin{aligned} \alpha_0^{TW}(\mathbf{s}_t) &= \sum_{i=1}^n \alpha_q^i s_t^i + \sum_{i=1}^n \phi \bar{s}^i \alpha_q^i K_q \sum_{j=1}^n \alpha_q^k s_t^j = (1 + \phi K_q \bar{s}' \alpha_q) \alpha_q' \mathbf{s}_t \\ \alpha_1^{TW}(\mathbf{s}_t) &= \sum_{i=1}^n \alpha_{pq}^i s_t^i + \sum_{i=1}^n \phi \bar{s}^i \sum_{k=1}^n \left\{ \alpha_p^k (\alpha_{pq}^i K_p + \bar{s} \alpha_{pq} \phi \alpha_q^i K_q K_p) + \alpha_{pq}^k \alpha_q^i K_q \right\} s_t^k \end{aligned}$$

Algebra shows

$$\begin{aligned} \alpha_0^{TW}(\mathbf{s}_t) &= \frac{1}{1 - \phi H_q} \alpha_q' \mathbf{s}_t \\ \alpha_1^{TW}(\mathbf{s}_t) &= \frac{1}{1 - \phi H_q} ((\alpha_p \mathbf{s}_t) K_p \phi \bar{s} \alpha_{pq} + \alpha_{pq} \mathbf{s}_t) \end{aligned}$$

We now turn to the computation of the volatility and expected returns of the total wealth portfolio. An application of Ito's Lemma to $P_t^{TW} = C_t (\alpha_0^{TW}(\mathbf{s}_t) + \alpha_1^{TW}(\mathbf{s}_t) S_t^\gamma)$ implies that the diffusion part of the TW

portfolios is given by

$$\begin{aligned}
\sigma_{P,t}^{TW} &= \sigma_c + \frac{\alpha_1^{TW}(\mathbf{s}_t)}{\alpha_0^{TW}(\mathbf{s}_t) + S_t^\gamma \times \alpha_1^{TW}(\mathbf{s}_t)} S_t^\gamma \alpha (1 - \lambda S_t^\gamma) \sigma_c \\
&\quad + \frac{\alpha_q + S_t^\gamma (K_p \phi \bar{\mathbf{s}} \alpha_{pq} \alpha_p + \alpha_{pq})}{\alpha'_q \mathbf{s}_t + S_t^\gamma \times ((\alpha_p \mathbf{s}_t) K_p \phi \bar{\mathbf{s}} \alpha_{pq} + \alpha_{pq} \mathbf{s}_t)} \mathbf{I}(\mathbf{s}_t) \sigma(\mathbf{s}_t) \\
&= \sigma_c + \frac{S_t^\gamma \alpha (1 - \lambda S_t^\gamma)}{f_1^{TW}(\mathbf{s}_t) + S_t^\gamma} \sigma_c + \sum_{j=1}^n \frac{\{\alpha_q^j + S_t^\gamma (K_p \phi \bar{\mathbf{s}} \alpha_{pq} \alpha_p^j + \alpha_{pq}^j)\} s_t^j}{\sum_{k=1}^n \{\alpha_q^k + S_t^\gamma \times (K_p \phi \bar{\mathbf{s}} \alpha_{pq} \alpha_p^k + \alpha_{pq}^k)\} s_t^k} (\boldsymbol{\nu}^j - \mathbf{s}' \cdot \boldsymbol{\nu}) \\
&= \frac{S_t^\gamma \alpha (1 - \lambda S_t^\gamma)}{f_1^{TW}(\mathbf{s}_t) + S_t^\gamma} \sigma_c + \sum_{j=1}^n w_{jt}^{TW} \sigma_D(\mathbf{s}_t)
\end{aligned}$$

with

$$f_1^{TW}(\mathbf{s}_t) = \frac{\alpha_0^{TW}(\mathbf{s}_t)}{\alpha_1^{TW}(\mathbf{s}_t)}$$

and where

$$w_{jt}^{TW} = \frac{\{\alpha_q^j + S_t^\gamma (K_p \phi \bar{\mathbf{s}} \alpha_{pq} \alpha_p^j + \alpha_{pq}^j)\} s_t^j}{\sum_{k=1}^n \{\alpha_q^k + S_t^\gamma \times (K_p \phi \bar{\mathbf{s}} \alpha_{pq} \alpha_p^k + \alpha_{pq}^k)\} s_t^k}$$

are weights such that $\sum_j w_{jt}^{TW} = 1$. Given the form of the stochastic discount factor, we obtain

$$\begin{aligned}
E_t \left[dR_t^{TW} \right] &= -Cov_t \left(dR_t^{TW}, \frac{dm_t}{m_t} \right) \\
&= (\gamma + \alpha (1 - \lambda S_t^\gamma)) \left\{ \frac{S_t^\gamma \alpha (1 - \lambda S_t^\gamma)}{f_1^{TW}(\mathbf{s}_t) + S_t^\gamma} \sigma_c + \sum_{j=1}^n w_{jt}^{TW} \sigma_{CF,t}^{jc} \right\}
\end{aligned}$$

Q.E.D.

APPENDIX C. Simulation Results under Different Parameterizations

In this appendix we discuss the simulations results under different parameterizations, contained in Tables A.1 and A.2.

(a) Return characteristics under different parameterizations

Columns 1 to 3 of Table A.1 report the combination of cash-flow parameters used in simulations. Columns 4 to 11 report the results for the total wealth portfolio and risk free rate, while columns 12 and 13 report the value premium. In bold face we report the benchmark case discussed in the previous sections.

The properties of the total wealth portfolio return (columns 4 and 5), interest rate (columns 6 and 7) and return predictability (columns 8 to 11) are empirically reasonable across parameter choices, although some differences exist across the various cash-flow parameter combinations. These differences are driven by general equilibrium restrictions, as the properties of the aggregate portfolio depend upon the properties of individual stocks returns, and thus on the characteristics of the individual cash-flow dynamics. The main impact of the general equilibrium restrictions on the total wealth portfolio is that induces a small predictable component in expected consumption growth (see the discussion in Section V.B.) In particular, as we increase the cash-flow risk $\bar{\theta}_{CF}$, the term $\mu_{c,1}(\mathbf{s}_t) = \mathbf{s}'_t \boldsymbol{\theta}_{CF}$ in (3), which governs the forecastable component in expected consumption growth, varies more and as a result consumption growth becomes more predictable. This higher predictability of consumption growth decreases the average equity premium. To reiterate the intuition, our model features a low elasticity of intertemporal substitution and this translates into a declining price/consumption ratio in the presence of an increase in expected consumption growth; because the latter is positively correlated with consumption shocks, it follows that the equity premium declines as $\bar{\theta}_{CF}$ increases. This argument is also behind the lower long return predictability in our model relative to other external habit persistence models.

Turning now to the cross-section of stock returns, Column 12 of Table A.1 reports the difference in average return between the value portfolio (portfolio 10, with low P/D ratio) and the growth portfolio (portfolio 1, with high P/D ratio). We see that for each level of ϕ and for each level of share volatility $\bar{\nu}$, as we increase the cash-flow risk parameter from $\bar{\theta}_{CF} = 0$ to $\bar{\theta}_{CF} = 0.345\%$, the value premium moves from negative (i.e. a growth premium) to positive. The intuition behind this result was discussed in Sections IV.B and IV.C and summarized in Figures 2 and 4. Table A.1 shows that indeed the model is able to quantitatively match the value premium for several parametric specifications, in addition to the one used in our benchmark case (in boldface).

Column 13 of Table III shows the CAPM fitted value premium. For each simulated portfolio i we compute the CAPM implied expected return, $\bar{r}^i = \beta^i \bar{r}^m$, where \bar{r}^m is the simulated average market excess return, and $\beta^i = cov(r_t^i, r_t^m) / var(r_t^m)$ is the market beta. The last column reports the difference in CAPM expected return between the value portfolio (portfolio 10) and the growth portfolio (portfolio 1). If the CAPM gave a good description of the model we would obtain a spread identical to the difference in average return in Column 12. The last column of Table A.1 shows that this is not the case when $\bar{\theta}_{CF}$ is high. Why does the CAPM fail in our setting when $\bar{\theta}_{CF}$ is high? In our model a high $\bar{\theta}_{CF}$ yields a mild time variation in expected consumption growth through the term $\mu_{c,1}(s_t) = s_t' \theta_{CF}$, a time variation which, as discussed earlier in Section V.B, invalidates the CAPM.

At this point it is perhaps useful to return to the evidence concerning the CAPM in the longer sample. As we showed in Table I Panel C-2, and unlike the postwar sample, betas and average returns correlate positively in the cross section in the 1927-2001 sample and the CAPM can, at least partially, explain some of the value premium. If generating a big gap between the simulated value premium and the CAPM fitted value premium is not an objective, one can greatly improve the performance of our model, particularly in what refers to the market portfolio. Indeed, consider the parameterization $\phi = .07$, $\bar{\nu} = .40$ and $\bar{\theta}_{CF} = .3\%$. In this case notice that as shown in Table A.1 the value premium is a respectable 4.91%, close to the empirical counterpart in the long sample of 5.76%, but more importantly the simulated equity premium is a robust 7.06% and the volatility is 17.37%, which are much closer to the empirically observed values (see Table I). Thus the model is quite capable of matching the fundamental moments in the time series and the cross section at the expense of a relatively good CAPM fit of the value premium, which is consistent with the evidence in the long sample. But as we show next the cash-flow risk puzzle obtains in all these alternative parameterizations.

(b) Cash-flow characteristics under different parameterizations

Table A.2 shows the properties of the cash-flow processes under the different parameterizations. Our purpose is to evaluate whether different parameterizations of the model imply cash-flow betas which are closer to their empirical counterparts, as opposed to the benchmark case. Columns 6 and 7 of Table A.2 report the cash-flow beta for the growth portfolio (portfolio 1) and the value portfolio (portfolio 10), respectively, while Column 8 report their spread. For each level of ϕ and each volatility $\bar{\nu}$, as we increase the cash-flow risk dispersion parameter $\bar{\theta}_{CF}$, the cash-flow beta of the growth portfolio decreases, while the one of the value portfolio increases, in line with the empirical evidence. As shown in Table A.1, in order for the model to yield a value premium of 5% or more, we must have a dispersion of cash-flow risk $\bar{\theta}_{CF}$ of at least 0.2%, independently of the other cash-flow risk parameters, ϕ and $\bar{\nu}$. The last column of Table A.2 shows that for such values, the implied cash flow beta spread is too high.

An alternative way of seeing this “excessive” cash-flow risk that the model seems to require to undo the strong discount effects implied by external habit persistence models, is to look at the implied dispersion in the correlation coefficients between dividend and consumption growth across individual assets. The minimum and maximum correlation in simulation is contained in columns 4 and 5 in Table A.2. Again, for those parameter combinations in which a value premium arises the dispersion of correlations is large: For instance, when $\phi = 0.07$, $\bar{\nu} = 0.55$ and $\bar{\theta}_{CF} = 0.345\%$, we have that the dispersion of correlations between dividend growth and consumption across assets is $[\underline{\rho}, \bar{\rho}] = [-37.44\%, 42.44\%]$, which are rather large. This can be compared, for instance, with the correlation between aggregate dividend and consumption growth in quarterly data which is at most .2 and thus the presence of substantial idiosyncratic risk should imply lower correlation coefficient for individual assets.

Table I
Basic moments in empirical data: 1948-2001

Panel A: Summary statistics for the market portfolio										
	\overline{R}^M	$\text{vol}(R^M)$			\overline{r}^f	$\text{vol}(r^f)$				
	7.71%	16.25%	1.48%					2.15%		

Panel B: Predictability regressions										
	Panel B-1: Sample 1948-2001				Panel B-2: Sample 1948-1995					
Horizon	4	8	12	16	4	8	12	16		
$\ln(\frac{D}{P})$.13	.2	.26	.35	.28	.48	.63	.78		
t-stat.	(2.13)	(1.65)	(1.34)	(1.29)	(4.04)	(4.00)	(4.49)	(5.41)		
R^2	.09	.10	.11	.14	.19	.32	.43	.54		

Panel C-1: The value premium										
	Growth									Value
Portf.	1	2	3	4	5	6	7	8	9	10
\overline{R} (%)	6.86	7.77	7.67	7.63	8.53	9.96	8.39	11.00	11.39	12.36
$\overline{ME/BE}$	5.05	2.68	2.00	1.63	1.38	1.18	1.01	.86	.70	.45
$\overline{P/D}$	43.47	31.38	26.87	24.65	22.65	21.62	20.64	19.95	20.00	21.77
Sharpe Ratio	.352	.450	.452	.461	.555	.640	.522	.657	.644	.600
CAPM β	1.13	1.02	1.01	.95	.88	.89	.88	.91	.92	.98
CAPM α	-.46	-.03	-.02	.07	.44	.78	.40	.99	1.07	1.20
$t(\alpha)$	(-2.00)	(-.18)	(-.14)	(.32)	(2.07)	(3.73)	(1.51)	(3.73)	(3.32)	(2.65)

Panel C-2: The value premium 1926-2001										
	Growth									Value
Portf.	1	2	3	4	5	6	7	8	9	10
\overline{R} (%)	7.08	8.28	8.16	7.56	9.12	9.12	10.08	11.52	12.96	12.84
CAPM β	1.01	.98	.95	1.06	.97	1.07	1.13	1.14	1.31	1.42
CAPM α	-.08	.05	.06	-.06	.11	.06	.09	.21	.21	.14
$t(\alpha)$	(-1.16)	(.84)	(1.08)	(-.92)	(1.61)	(.72)	(.88)	(1.90)	(1.50)	(.76)

Notes to Table I. *Panel A:* Summary statistics for the market portfolio. \overline{R}^M is the annualized average excess returns of the market portfolio over the three month Treasury Bill. $\text{vol}(R^M)$ is the annualized standard deviation of the returns on the market portfolio. \overline{r}^f is the average real risk-free rate, as measured by three-month Treasury Bill rate at the end of each quarter minus the expected (CPI) inflation computed from an AR(4). $\text{vol}(r^f)$ is the real risk-free rate annualized standard deviation. *Panel B:* Predictability quarterly regressions of excess returns at the 1, 2, 3, and 4 year horizon on the log of the price dividend ratio of the market portfolio. t-stat denotes the Newey-West t- statistic where the number of lags is the double of the forecasting horizon. *Panel C-1:* Summary statistics for the cross section of stock return for the sample period 1948-2001. \overline{R} is the annualized average excess returns of each of the decile portfolios, $\overline{ME/BE}$ is the average market-to-book and $\overline{P/D}$ the average price dividend ratio. CAPM β is obtained by running time series regressions of excess return on each of the ten decile portfolios sorted on ME/BE on the market excess return, where ME is the market equity and BE is the book value for the sample period 1948-2001. CAPM α denotes the intercepts of the time series regression and the $t(\alpha)$ is the the heteroskedasticity corrected t- statistic. Quarterly dividends, returns, market equity and other financial series are obtained from the CRSP-COMPUSTAT database. The construction of the BE/ME sorted portfolios follows the standard procedure of Fama and French (1992): Each year t portfolios are sorted into 10 BE/ME sorted portfolios using book-to-market ratios for year $t - 1$. Returns on each of these portfolios are calculated from July of year t to June of year $t + 1$. *Panel C-2:* Annualized monthly returns on value sorted decile portfolios and their corresponding betas for the sample period 1926:07-2001:12 from Ang and Chen (2007), Table 1 Panel A.

Table II
Model parameters used in the simulation

Panel A: Consumption and preference parameters							
μ_c	σ_c	γ	ρ	γ/\bar{S}	$\min\{\gamma/S_t\}$	α	k
.02	.015	1.5	.072	48	27.75	77	.13
Panel B: Share process parameters							
n	$\bar{\theta}_{CF}$			\bar{s}^i	ϕ		$\bar{\nu}$
200	[0, 0.1%, 0.2%, 0.3%, 0.345%]			.005	[0.05, 0.07, 0.1]		[0.25, 0.4, 0.55]

Notes to Table II. *Panel A:* μ_c is the annual average growth rate of the consumption process, σ_c is the standard deviation of consumption growth, γ is the coefficient controlling the local curvature of the utility function, ρ is the subjective discount rate, \bar{G} , λ , α and k are the parameters controlling the dynamics of the process $G_t = S_t^{-\gamma}$, where $S_t = (C_t - X_t)C_t^{-1}$ is the surplus consumption ratio and the process for G_t is given by

$$dG_t = [k(\bar{G} - G_t) - \alpha(G_t - \lambda)\mu_{c,1}(\mathbf{s}_t)] dt - \alpha(G_t - \lambda)\sigma_c dB_t^1. \quad (40)$$

Panel B: The share process for $i = 1, 2, \dots, n$ is

$$ds_t^i = \phi(\bar{s}^i - s_t^i) + s_t^i \boldsymbol{\sigma}^i(\mathbf{s}_t) d\mathbf{B}'_t$$

$n = 200$ is the number of assets in our artificial economy. θ_{CF}^i is the parameter controlling the cash-flow risk. Each assets is assigned a value of θ_{CF}^i , which are distributed uniformly in the range above. \bar{s}^i is the fraction that each assets contributes to consumption in the steady state and ϕ is the speed of mean reversion of the share process. Finally, $\boldsymbol{\sigma}^i(\mathbf{s}_t) = \boldsymbol{\nu}^i - \mathbf{s}'_t \boldsymbol{\nu}$ where $\boldsymbol{\nu}^i$ are vectors with $\nu_{i,0} = \theta_{CF}^i/\sigma_c$, $\nu_{i,i} = \sqrt{\bar{\nu}^2 - \nu_{0,i}^2}$, and the remaining entries equal to zero. The simulation consists of 10,000 years of daily data.

Table III
Basic moments in simulated data

Panel A: Summary statistics for the aggregate portfolio										
	\bar{R}^M	$\text{vol}(R^M)$	\bar{r}^f	$\text{vol}(r^f)$						
	4.35%	13.03%	.69%	4.36%						
Panel B: Predictability regressions										
Horizon	4	8	12	16						
$\ln\left(\frac{D}{P}\right)$.10	.17	.22	.27						
R^2	0.03	0.04	0.05	0.06						
Panel C: The value premium										
	Growth									Value
Portf.	1	2	3	4	5	6	7	8	9	10
\bar{R} (%)	3.07	3.58	4.37	4.77	5.27	5.45	5.84	6.00	6.43	8.23
$\ln(P/D)$	6.38	5.07	4.613	4.35	4.12	3.90	3.68	3.44	3.15	2.68
Sharpe Ratio	.260	.271	.307	.313	.331	.328	.336	.330	.334	.366
CAPM β	.84	.91	.98	1.05	1.10	1.13	1.16	1.20	1.22	1.26
CAPM α	-.15	-.09	.02	.06	.12	.13	.20	.20	.29	.68
CAPM fitt. ret. (%)	3.67	3.94	4.28	4.55	4.78	4.91	5.05	5.21	5.29	5.50

Notes to Table III. Moments of interest in simulated data, which consists of 10,000 years of quarterly data for 200 firms. To construct the portfolios we sort on simulated price dividend ratios following the standard procedures in the literature. The parameters for the simulation are the ones reported in Table II with $\phi = .07$, $\bar{v} = .55$ and $\bar{\theta}_{CF} = .00345$. *Panel A:* Summary statistics for the market portfolio. \bar{R}^M is the annualized average excess returns of the market portfolio over the three month Treasury Bill. $\text{vol}(R^M)$ is the annualized standard deviation of the returns on the market portfolio. \bar{r}^f is the average risk free rate and $\text{vol}(r^f)$ is its annualized standard deviation. *Panel B:* Predictability quarterly regressions of excess returns at the 1, 2, 3, and 4 year horizon on the log of the price dividend ratio of the market portfolio. *Panel C:* Annualized average returns \bar{R} , average log price-dividend ratio, $\ln(P/D)$, CAPM β , and CAPM α . CAPM fitted returns are the returns resulting from multiplying the CAPM betas in by the average excess return of the market portfolio reported in Panel A. $\text{Avge}(\theta_{CF}^i) \times 100$ refers to the average θ_{CF}^i (multiplied by 100) for the assets in the corresponding decile portfolio.

Table IV
Cash-flow betas

Panel A: Empirical data from Cohen, Polk, and Vuolteenaho (2008 (forthcoming))												
Cash-flow definition	Growth										Value	
	1	2	3	4	5	6	7	8	9	10	10-1	
$\sum_{j=0}^4 \rho^j ROE_{t+j,j+1}^p$.72	.91	.94	.96	.95	.96	.97	1.11	1.28	1.51	.79	
std. err.	(.50)	(.31)	(.12)	(.25)	(.14)	(.12)	(.14)	(.24)	(.32)	(.30)	(.21)	
$\sum_{j=0}^4 \rho^j \frac{X_{t+j,j+1}^p}{ME_{t+j-1,j}^p}$.35	.66	.93	1.18	1.25	1.61	1.91	2.92	4.09	11.05	10.70	
std. err.	(.32)	(.31)	(.17)	(.17)	(.28)	(.68)	(1.00)	(2.19)	(3.23)	(10.58)	(4.17)	
$\frac{\sum_{j=0}^4 \rho^j X_{t+j,j+1}^p}{ME_{t-1,0}^p}$.48	.72	.96	1.13	1.22	1.40	1.50	2.00	3.13	7.63	7.14	
std. err.	(.18)	(.18)	(.17)	(.15)	(.19)	(.35)	(.41)	(1.18)	(2.21)	(8.85)	(3.45)	
$\frac{X_{t+4,j+4}^p - X_{t-1,0}^p}{ME_{t-1,0}^p}$.21	.66	1.46	1.61	.24	1.83	2.74	5.50	2.38	2.64	2.43	
std. err.	(.19)	(.08)	(.52)	(.28)	(.61)	(.60)	(1.24)	(2.69)	(.60)	(1.65)	(.57)	
$\sum_{j=0}^4 \rho^j \Delta d_{t+j,j+1}^p$.79	.90	.96	1.03	1.32	1.42	1.12	1.44	1.37	1.20	.41	
std. err.	(.19)	(.14)	(.10)	(.13)	(.27)	(.45)	(.30)	(.91)	(.74)	(.92)	(.41)	

Panel B: Simulated data												
	Growth										Value	
	1	2	3	4	5	6	7	8	9	10	1-10	
$\sum_{j=0}^4 \rho^j \Delta d_{t+j,j+1}^p$	-7.40	-4.84	-2.47	-1.04	-0.05	0.75	1.44	2.05	2.64	4.73	12.13	
$\text{Avge}(\theta_{CF}^i) \times 100$	-.2858	-.1589	-.0665	-.0083	.0295	.0568	.0787	.0958	.1128	.1431	.4289	

Notes to Table IV. *Panel A:* This panel reports the results of Cohen, Polk, and Vuolteenaho (2009, forthcoming *Journal of Finance*, Table II Panel B) for the following regressions with annual data for each of the ten decile portfolios sorted on market-to-book,

$$\begin{aligned}
 \sum_{j=0}^4 \rho^j ROE_{t+j,j+1}^p &= \beta_{CF,0}^p + \beta_{CF,1}^p \sum_{j=0}^4 \rho^j ROE_{t+j}^M + \epsilon_4^p \\
 \sum_{j=0}^4 \rho^j \frac{X_{t+j,j+1}^p}{ME_{t+j-1,j}^p} &= \beta_{CF,0}^p + \beta_{CF,1}^p \sum_{j=0}^4 \rho^j \frac{X_{t+j,j+1}^M}{ME_{t+j-1}^M} + \epsilon_4^p \\
 \frac{\sum_{j=0}^4 \rho^j X_{t+j,j+1}^p}{ME_{t-1,0}^p} &= \beta_{CF,0}^p + \beta_{CF,1}^p \frac{\sum_{j=0}^4 \rho^j X_{t+j,j+1}^M}{ME_{t-1}^M} + \epsilon_4^p \\
 \frac{X_{t+4,j+4}^p - X_{t-1,0}^p}{ME_{t-1,0}^p} &= \beta_{CF,0}^p + \beta_{CF,1}^p \left(\frac{X_{t+4}^M - X_{t-1}^M}{ME_{t-1}^M} \right) + \epsilon_4^p \\
 \sum_{j=0}^4 \rho^j \Delta d_{t+j,j+1}^p &= \beta_{CF,0}^p + \beta_{CF,1}^p \sum_{j=0}^4 \rho^j \Delta d_{t+j}^M + \epsilon_4^p.
 \end{aligned}$$

ROE denotes the ratio of clean surplus earning ($X_t = BE_t - BE_{t-1} + D_t$ where BE_{t-1} is the beginning of the period book equity and D_t are the dividends from CRSP) to BE_{t-1} . ME_{t-1} denotes the market value at the beginning of the period and $\Delta d_{t+j,j+1}^p$ is the log of dividend growth of decile portfolio p . The first subscript refers to the year of observation and the second to the number of years after the portfolio formation in the sorting procedure. Similar quantities are defined for the market portfolio. GMM standard errors computed using the Newey-West formula with four lags and leads are reported in parenthesis. ρ is a constant, linked to one minus the dividend yield, set at .95. *Panel B:* The first line reports the regression in simulated data which corresponds to (41) above. The second line, $\text{Avge}(\theta_{CF}^i) \times 100$, reports the average cash-flow parameter for each of the ten decile portfolios.

Table V
The dynamics of the value premium

Panel A: Annualized average excess returns (%) in empirical data									
Market-to-book of market portfolio < \bar{c}					Market-to-book of market portfolio > \bar{c}				
\bar{c}	1	10	10-1	\bar{R}^M	\bar{c}	1	10	10-1	\bar{R}^M
15%	13.18	23.57	10.38	15.40	15%	5.73	10.35	4.62	6.34
20%	10.57	21.70	11.14	13.41	20%	5.95	10.06	4.11	6.31
25%	5.51	19.16	13.64	9.89	25%	7.31	10.11	2.80	6.99
30%	6.97	19.49	12.51	10.50	30%	6.82	9.32	2.50	6.62
35%	8.19	18.65	10.45	11.14	35%	6.15	8.98	2.83	5.87

Panel B: Annualized average excess returns (%) in simulated data									
Price-dividend of market portfolio < \bar{c}					Price-dividend of market portfolio > \bar{c}				
\bar{c}	1	10	10-1	\bar{R}^M	\bar{c}	1	10	10-1	\bar{R}^M
15%	7.37	18.27	10.90	10.43	15%	2.30	6.46	4.15	3.27
20%	6.56	16.07	9.51	9.22	20%	2.19	6.26	4.07	3.13
25%	5.96	14.60	8.64	8.36	25%	2.10	6.10	4.00	3.01
30%	5.50	13.46	7.96	7.67	30%	2.02	5.98	3.96	2.92
35%	5.13	12.60	7.47	7.18	35%	1.95	5.87	3.92	2.82

Notes to Table V. *Panel A:* Annualized average excess returns in empirical data of the growth (portfolio 1) and value (portfolio 10) portfolios depending on whether the market-to-book of the market portfolio is below or above the \bar{c} percentile of its empirical distribution. *Panel B:* Annualized average excess returns in simulated data of the growth (portfolio 1) and value (portfolio 10) portfolios depending on whether the simulated price-dividend ratio of the market portfolio is below or above the \bar{c} percentile of its distribution in simulated data. \bar{R}^M is the average excess return on the market portfolio in empirical data (Panel A) and simulated data (Panel B).

Table VI
The Fama-French (1993) model: Time series regressions (quarterly)

$$R_t^p = \alpha + \beta^M R_t^M + \beta^{HML} R_t^{HML} + \epsilon_t^p \text{ for } p = 1, 2, \dots, 10$$

Panel A: Empirical data										
Portf.	Growth									Value
	1	2	3	4	5	6	7	8	9	10
α	.20	.17	.02	-.12	.19	.28	-.40	.01	-.08	-.36
$t(\alpha)$	(1.13)	(1.05)	(.14)	(-.61)	(.87)	(1.58)	(-2.15)	(.09)	(-.43)	(-1.23)
β^M	1.04	.99	1.00	.98	.91	.96	.99	1.05	1.09	1.20
$t(\beta^M)$	(43.68)	(51.25)	(46.13)	(35.28)	(30.25)	(38.66)	(39.90)	(48.04)	(39.61)	(29.85)
β^{HML}	-.42	-.12	-.03	.12	.16	.31	.50	.61	.72	.97
$t(\beta^{HML})$	(-12.13)	(-2.37)	(-.68)	(1.88)	(3.62)	(8.85)	(10.35)	(15.52)	(21.04)	(14.14)
Panel B: Simulated data										
Portf.	Growth									Value
	1	2	3	4	5	6	7	8	9	10
α	-.01	.02	.07	.06	.09	.10	.11	.03	.07	.13
β^M	.93	.97	1.01	1.05	1.08	1.11	1.11	1.10	1.09	.93
β^{HML}	-.28	-.21	-.09	-.01	.06	.08	.16	.31	.41	1.07

Notes to Table VI. Time series regressions in empirical (Panel A) and simulated (Panel B) data of returns on each of the book-to-market sorted portfolios on the market excess return and the returns on HML, where β^{HML} is the regression coefficient on HML. The HML portfolio is constructed by taking long and short position in the top and bottom three decile portfolios, respectively.

Table VII
Asset pricing models: Fama-MacBeth regressions (quarterly)

Panel A: Empirical data							
	Const.	Mkt.	SMB	HML	Mkt \times log(D/P)	Mkt \times cay	Adj. R^2
1.	4.69 (3.21)	-2.52 (-1.65)					11%
2.	.36 (.23)	1.63 (.99)	-.31 (-.31)	1.05 (2.16)			80%
3.	2.72 (2.24)	-.87 (-.65)			1.71 (2.46)		83%
4.	3.06 (2.48)	-1.37 (-1.01)				.06 (2.34)	81%
Panel B: Simulated data							
	Const.	Mkt.	HML	Mkt \times log(D/P)	Adj. R^2		
5.	-1.45	2.56					91%
6.	-.17	1.31	.94				99%
7.	.63	.38		1.16			98%

Notes to Table VII. *Panel A:* Fama-MacBeth regressions in empirical data. Line 1, CAPM regressions where Mkt. represents the average excess return of the market portfolio. Line 2, Fama and French (1993) model, where SMB is the return on “small minus big” and HML is the return on “high minus low”. Line 3, conditional CAPM regression where the dividend yield, log(D/P), of the market portfolio is used as a conditioning variable. Line 4 conditional CAPM regression where the variable *cay* of Lettau and Ludvigson (2001) is used as a conditioning variable. *Panel B:* Fama-MacBeth regressions in simulated data. *t*-statistic in parenthesis and Adj. R^2 is the adjusted R^2 .

Table A.1
The market portfolio and the value premium in simulations: Robustness

Cash-flow risk			Market portfolio				Predictability				Value premium	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
ϕ	$\bar{\nu}$	$\bar{\theta}_{CF} \times 100$	\bar{R}^M	$\text{vol}(R^M)$	\bar{r}^f	$\text{vol}(r^f)$	b_{12}	R_{12}^2	b_{16}	R_{16}^2	10 - 1	CAPM 10 - 1
0.05	0.25	0.00	9.90	24.16	1.16	5.44	0.64	24.69	0.74	28.40	-2.54	-2.59
0.05	0.25	0.30	6.32	15.57	0.78	4.48	0.43	10.82	0.50	12.22	8.59	8.48
0.05	0.40	0.00	9.90	24.16	1.16	5.44	0.64	24.69	0.74	28.40	-3.28	-3.38
0.05	0.40	0.30	6.40	15.97	0.79	4.52	0.43	11.03	0.49	12.57	6.61	5.84
0.05	0.55	0.00	9.90	24.16	1.16	5.44	0.64	24.69	0.74	28.40	-3.71	-3.82
0.05	0.55	0.30	6.61	17.03	0.81	4.71	0.36	9.98	0.42	11.35	4.53	2.81
0.07	0.25	0.00	9.90	24.16	1.16	5.44	0.64	24.69	0.74	28.40	-2.44	-2.53
0.07	0.25	0.10	9.69	23.62	1.12	5.32	0.63	23.92	0.73	27.51	-1.27	-1.42
0.07	0.25	0.20	8.95	21.79	1.00	4.97	0.61	21.25	0.70	24.36	2.40	2.34
0.07	0.25	0.30	7.02	17.22	0.79	4.46	0.53	14.14	0.60	15.89	7.51	7.45
0.07	0.25	0.35	3.97	10.23	0.67	4.20	0.22	4.23	0.27	4.98	7.10	6.70
0.07	0.40	0.00	9.90	24.16	1.16	5.44	0.64	24.29	0.74	28.40	-3.22	-3.37
0.07	0.40	0.10	9.69	23.63	1.12	5.33	0.63	23.87	0.73	27.45	-2.56	-2.77
0.07	0.40	0.20	8.96	21.83	1.00	4.99	0.61	21.16	0.70	24.26	-0.07	-0.25
0.07	0.40	0.30	7.06	17.37	0.80	4.49	0.52	14.17	0.60	15.95	4.91	4.62
0.07	0.40	0.35	4.09	11.14	0.68	4.23	0.25	5.63	0.30	6.69	6.29	4.57
0.07	0.55	0.00	9.90	24.16	1.16	5.44	0.64	24.69	0.74	28.40	-3.67	-3.86
0.07	0.55	0.10	9.70	23.66	1.13	5.34	0.63	23.19	0.73	27.34	-3.27	-3.48
0.07	0.55	0.20	8.99	21.95	1.01	5.05	0.60	20.92	0.69	23.95	-1.49	-1.70
0.07	0.55	0.30	7.15	17.85	0.81	4.60	0.49	13.66	0.57	15.43	2.83	2.19
0.07	0.55	0.35	4.35	13.03	0.69	4.36	0.22	5.39	0.27	6.48	5.16	1.83
0.10	0.25	0.00	9.90	24.16	1.16	5.44	0.64	24.69	0.74	28.40	-2.21	-2.34
0.10	0.25	0.35	6.23	15.43	0.70	4.21	0.48	11.05	0.55	12.17	7.38	7.29
0.10	0.40	0.00	9.90	24.16	1.16	5.44	0.64	24.69	0.74	28.40	-3.04	-3.23
0.10	0.40	0.35	6.26	15.56	0.70	4.23	0.48	11.23	0.55	12.41	5.30	4.97
0.10	0.55	0.00	9.90	24.16	1.16	5.44	0.64	24.69	0.74	28.40	-3.51	-3.80
0.10	0.55	0.35	6.33	15.93	0.70	4.30	0.47	11.29	0.54	12.51	3.57	2.89

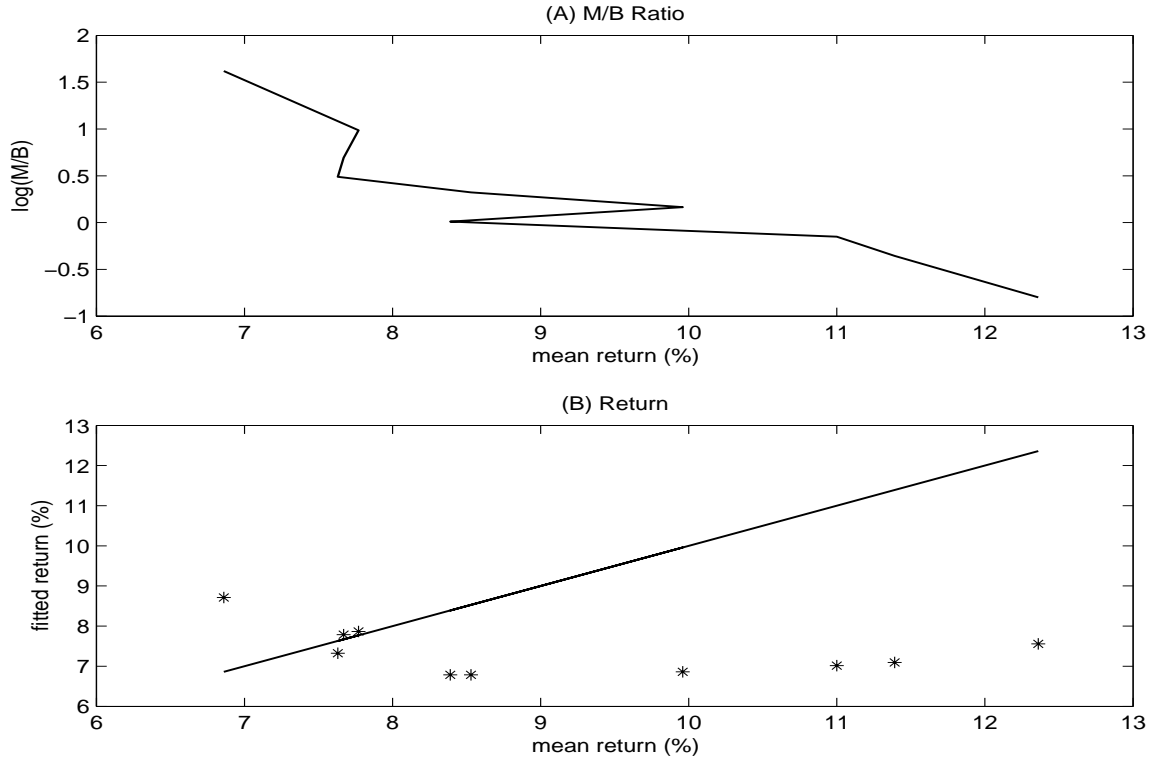
Notes to Table A.1. This table reports basic moments of the returns for three different values of $\bar{\nu}$, which determines the maximum volatility of share process across assets, and the measure of cash-flow risk, $\bar{\theta}_{CF} \geq 0$, which determines the support on which the cash-flow risk parameters of individual firms are uniformly distributed, $\theta_{CF}^i \in [-\bar{\theta}_{CF}, \bar{\theta}_{CF}]$. \bar{R}^M is the annualized average excess returns of the market portfolio over the three month Treasury Bill. $\text{vol}(R^M)$ is the annualized standard deviation of the returns on the market portfolio. \bar{r}^f is the average risk free rate and $\text{vol}(r^f)$ is its annualized standard deviation. All these numbers are in percentages. b_{12} and b_{16} are the regressions coefficients of the quarterly predictability regressions of excess returns on the log of the price dividend ratio of the market portfolio for the three and four year horizons. R_{12}^2 and R_{16}^2 are the corresponding R^2 s. 10-1 denotes the value premium, in percentages, defined as the difference between the average return on the value portfolio, portfolio 10, and the growth portfolio, portfolio 1. CAPM 10-1 is the fitted CAPM value premium, where the betas are calculated the standard way in simulated data and the market premium is the corresponding \bar{R}^M in each line. The numbers in bold correspond to the benchmark case presented in more detail in Table II.

Table A.2
The properties of the cash-flow process in simulations: Robustness

ϕ	$\bar{\nu}$	$\bar{\theta}_{CF} \times 100$	$[\underline{\rho}$	$\bar{\rho}]$	$\beta_{CF,1}^1$	$\beta_{CF,1}^{10}$	10-1
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0.05	0.25	0.00	4.61	7.17	1.06	0.97	-0.009
0.05	0.25	0.30	-76.14	80.65	-8.69	6.89	15.58
0.05	0.40	0.00	2.39	4.89	1.13	0.93	-0.20
0.05	0.40	0.30	-45.59	51.54	-7.33	5.38	12.71
0.05	0.55	0.00	1.41	3.83	1.27	0.98	-0.29
0.05	0.55	0.30	-31.57	36.52	-5.68	4.55	10.23
0.07	0.25	0.00	4.62	7.17	1.04	0.96	-0.08
0.07	0.25	0.10	-20.59	32.18	0.04	1.89	1.85
0.07	0.25	0.20	-47.93	57.09	-3.30	4.15	7.45
0.07	0.25	0.30	-76.25	80.83	-8.14	6.70	14.84
0.07	0.25	0.35	-88.89	91.05	-9.62	7.94	17.56
0.07	0.40	0.00	2.37	4.89	1.09	0.96	-0.13
0.07	0.40	0.10	-13.11	20.28	0.43	1.49	1.06
0.07	0.40	0.20	-29.05	36.23	-1.80	3.10	4.90
0.07	0.40	0.30	-45.78	51.91	-6.37	5.22	11.59
0.07	0.40	0.35	-53.42	58.85	-8.63	5.73	14.36
0.07	0.55	0.00	1.39	3.84	1.17	0.99	-0.18
0.07	0.55	0.10	-9.54	14.60	0.69	1.28	0.59
0.07	0.55	0.20	-20.60	25.98	-1.01	2.40	3.41
0.07	0.55	0.30	-32.10	37.34	-4.79	4.28	9.07
0.07	0.55	0.35	-37.44	42.44	-7.40	4.73	12.13
0.10	0.25	0.00	4.63	7.18	1.05	0.96	-0.09
0.10	0.25	0.35	-89.06	91.24	-8.94	7.75	16.69
0.10	0.40	0.00	2.37	4.91	1.08	0.98	-0.10
0.10	0.40	0.35	-53.47	59.03	-7.43	5.73	13.16
0.10	0.55	0.00	1.37	3.85	1.04	1.00	-0.04
0.10	0.55	0.35	-37.78	43.02	-5.92	4.76	10.68

Notes to Table A.2. For each value of $\bar{\nu}$ and $\bar{\theta}_{CF}$ the table reports several moments of the cash-flow process in simulated data. $[\underline{\rho}, \bar{\rho}]$ stands for the range of the correlation coefficients between individual dividend growth and consumption growth. $\beta_{CF,1}^1$ and $\beta_{CF,1}^{10}$ correspond to the regression coefficients of the time series regression, see equation (41) in the Notes to Table IV in simulated data for the Growth ($p = 1$) and Value ($p = 10$) portfolios. In bold the benchmark parameterization discussed in the text.

Figure 1: The Cross-Section of Stock Returns in the Data

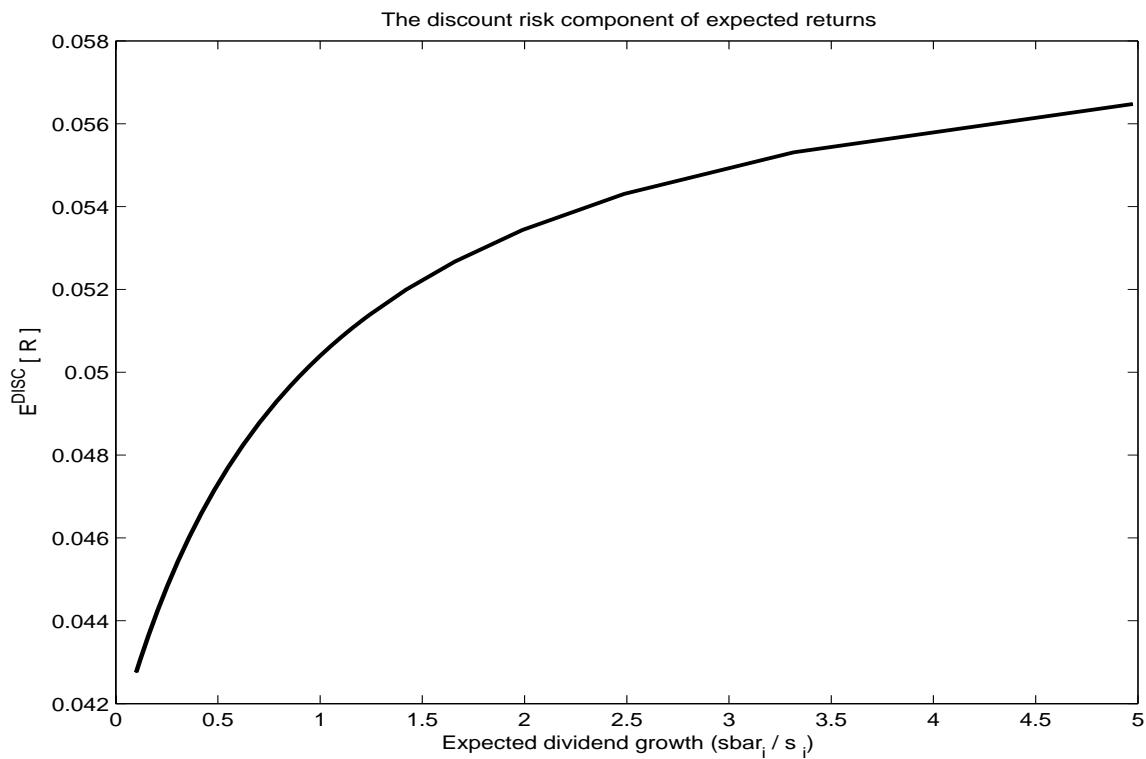


Panel A reports the average log M/B ratio for the 10 market-to-book sorted portfolios, plotted against the average excess return of the portfolios. The stars in Panel B are the portfolios predicted return according to the CAPM plotted against the realized average return of the portfolios. The predicted return are given by

$$\bar{R}_i = \beta^i \times \bar{R}_M$$

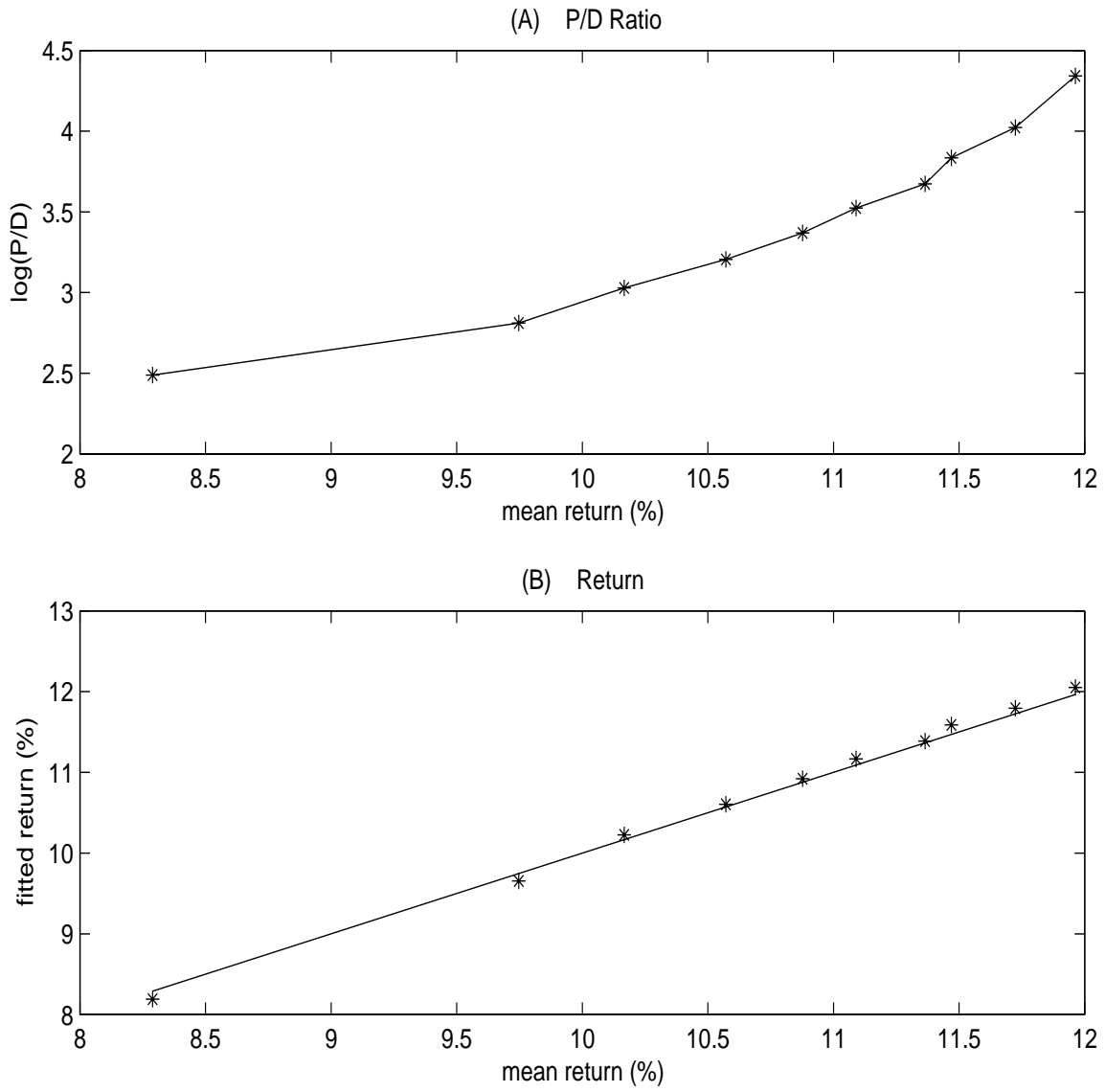
where \bar{R}_i is the predicted return of portfolio i , \bar{R}_M is the market average excess return, and β^i is the coefficient of the regression of each portfolio excess return on the market excess return.

Figure 2: Expected Returns and Expected Dividend Growth



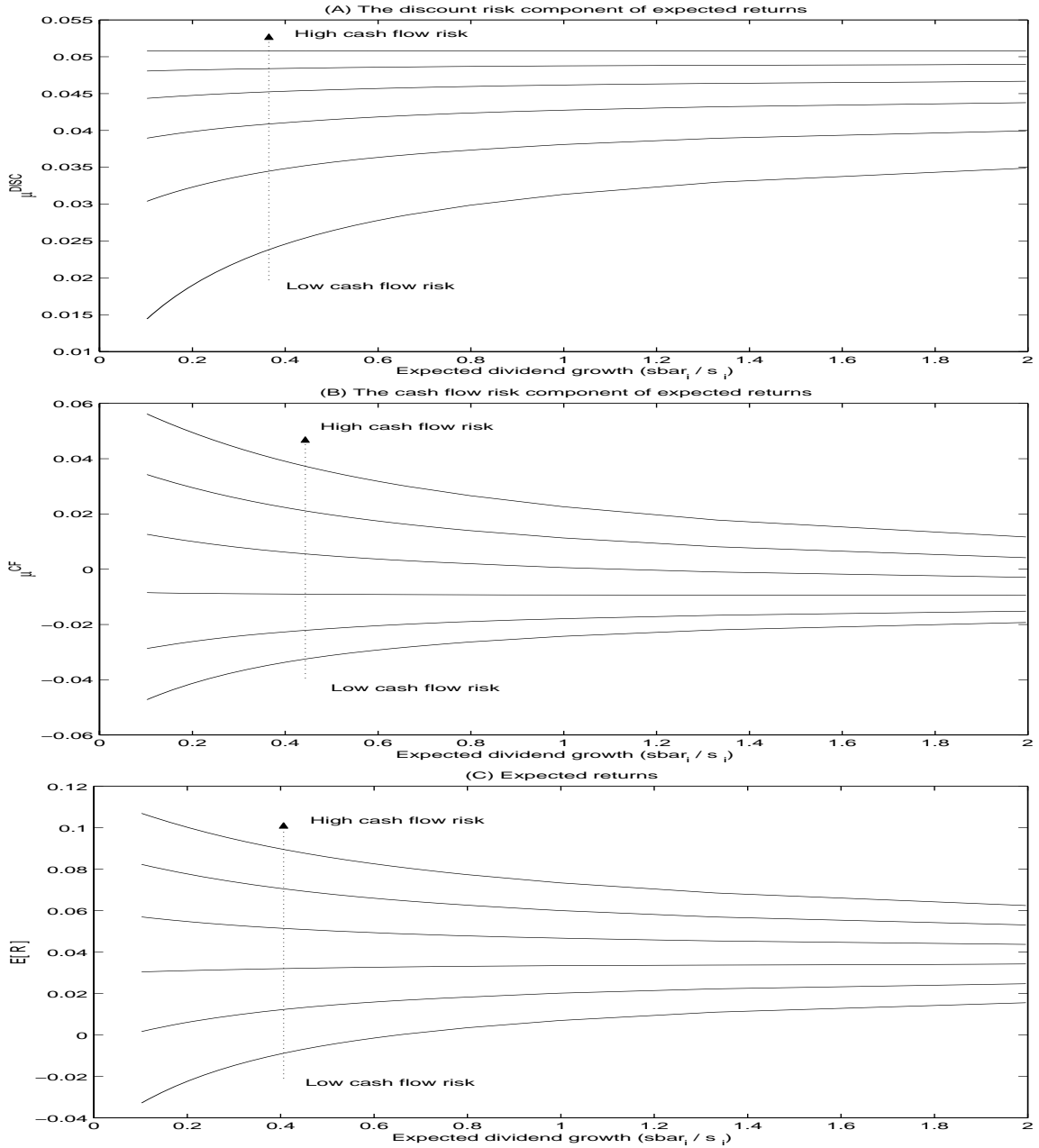
This figure plots theoretical discount component of individual stock returns plotted against the relative share \bar{s}^i/s_t^i , which proxies for expected dividend growth. This quantity is computed for the case in which all firms have the same cash flow risk $\sigma_{CF}^i = \sigma_c^2$.

Figure 3: The Cross-Section of Stock Returns with only Discount Risk Effects



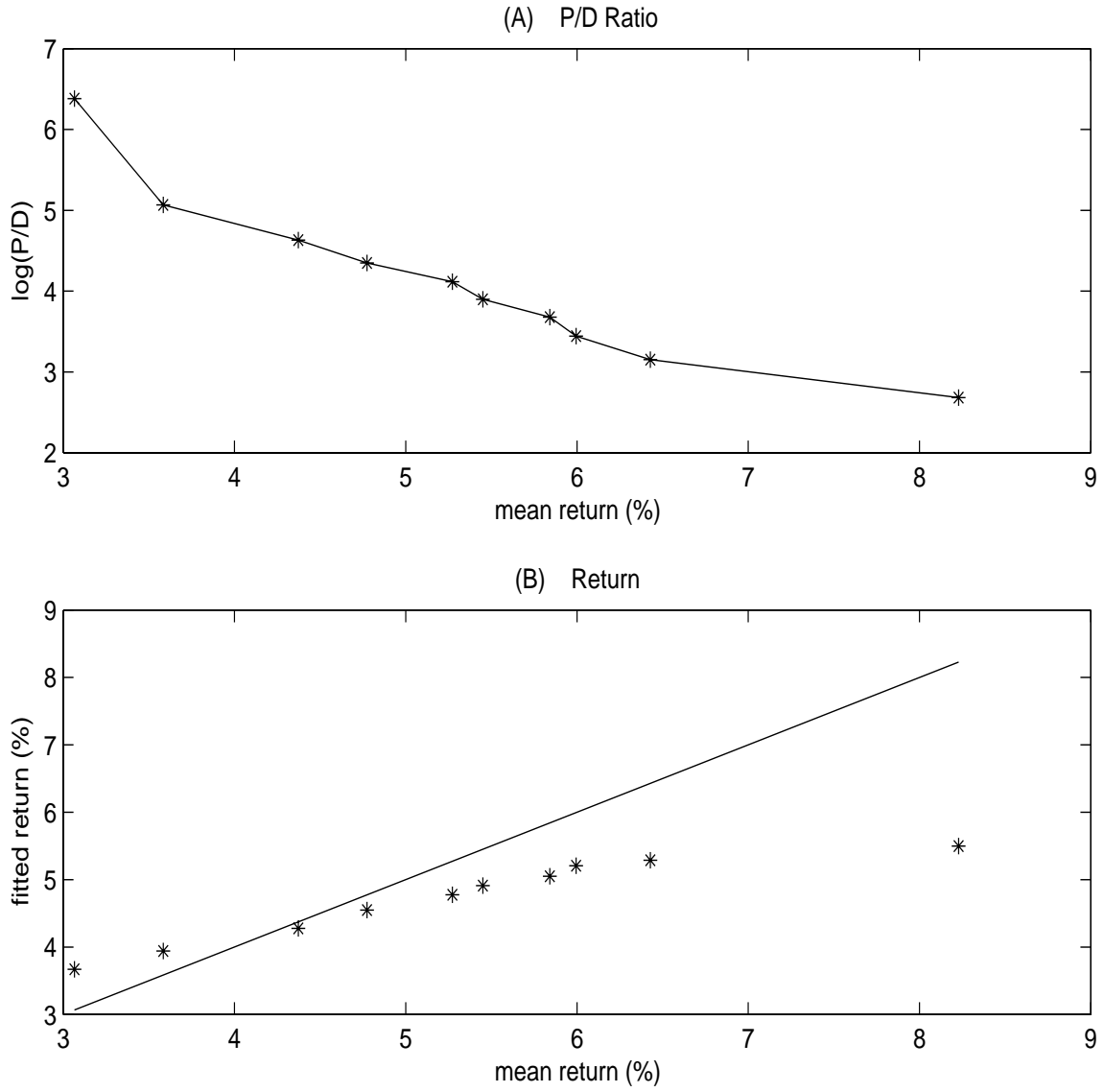
The top panel plots the average log price-dividend ratio (y-axis) of P/D sorted portfolios versus their unconditional average return (x-axis) in artificial data, under the assumption that assets have no cross-sectional differences in cash flow risk, $\sigma_{CF}^i = \sigma_{CF}^j = \sigma_c^2$. Under the same assumptions, the bottom panel plots the “fitted” average return according to the CAPM, i.e. $E[\text{Return}^i] = \beta_{CAPM}^i E[\text{Return}^{mkt}]$, on the y-axis against the average return on the x-axis.

Figure 4: Expected Returns and Expected Dividend Growth



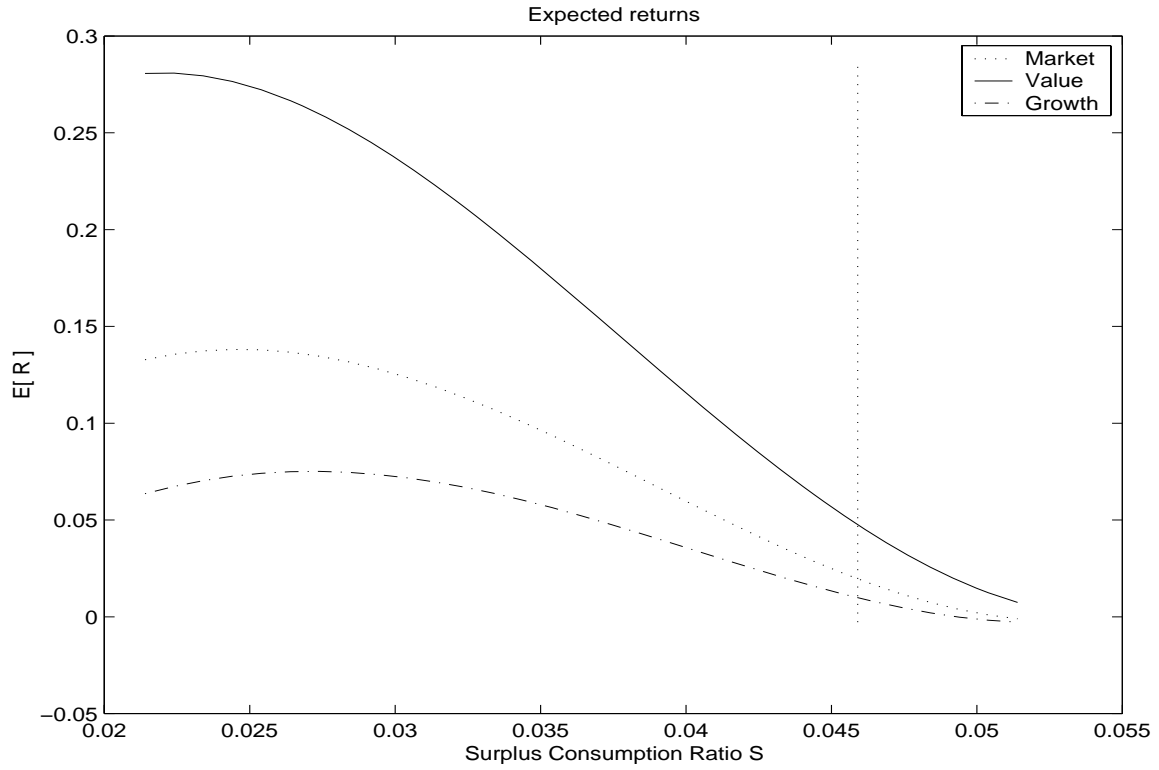
The top panel plots the theoretical discount component of individual stock returns plotted against the relative share \bar{s}^i/s_i^i , which proxies for expected dividend growth. This quantity is computed for various levels of the asset unconditional cash flow risk $\sigma_{CF}^i = E[cov(dD^i/D^i, dC/C)]$. The middle panel plots the cash flow risk component of stock returns, plotted against the relative share \bar{s}^i/s_i^i , again for various levels of unconditional cash flow risk. The bottom panel reports the total conditional expected return for individual assets.

Figure 5: The Cross-Section of Stock Returns in Simulated Data



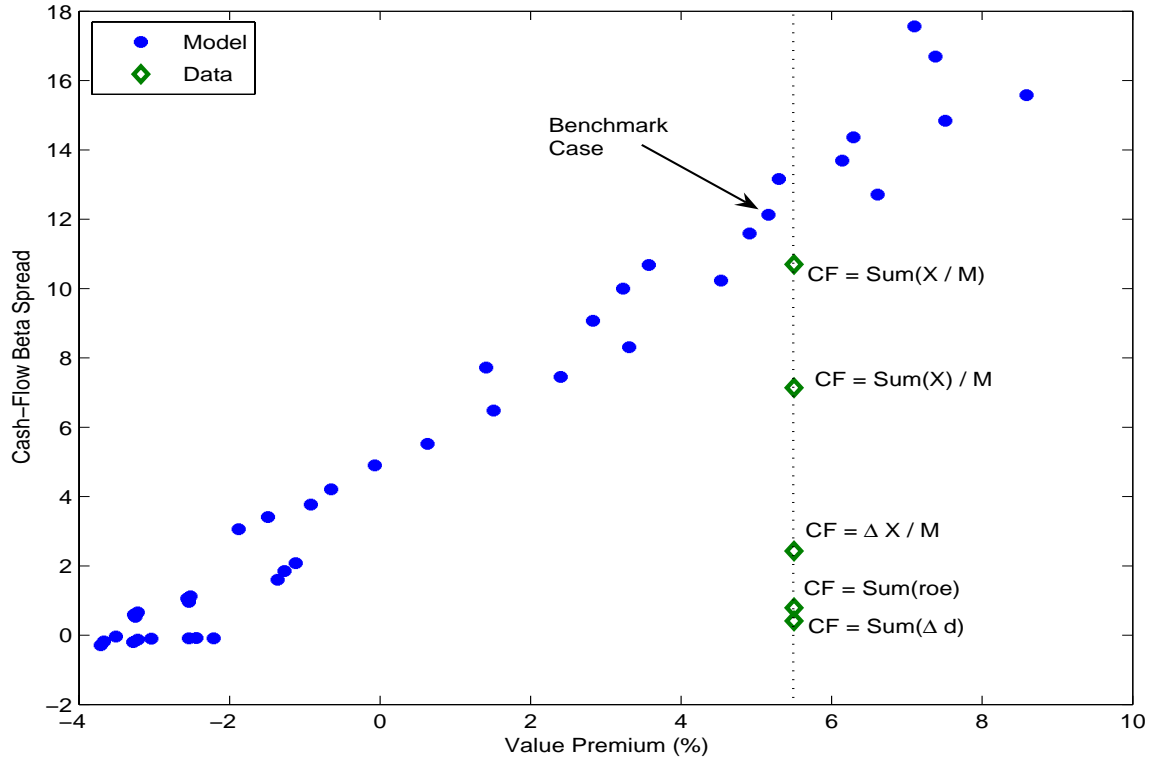
The top panel plots the average log price-dividend ratio (y-axis) of P/D sorted portfolios versus their unconditional average return (x-axis) in artificial data, under the assumption that assets differ cross-sectionally in their cash flow risk parameter θ_{CF}^i . Under the same assumptions, the bottom left panel plots the “fitted” average return according to the CAPM, i.e. $E[\text{Return}^i] = \beta_{CAPM}^i E[\text{Return}^{mkt}]$, on the y-axis against the average return on the x-axis.

Figure 6: Expected Returns and Surplus Consumption Ratio



This figure shows the theoretical expected return for the market portfolio (dotted line), a representative value stock (solid line), and a representative growth stock (dash dotted line), plotted against values of the surplus consumption ration S_t . The vertical dotted line is the median value of the surplus consumption ratio S_t . The representative value (growth) stock is chosen with low (high) expected dividend growth and high (low) cash flow risk θ_{CF} .

Figure 7: The Cash Flow Risk Puzzle



This figure shows the relation between the value premium (x -axis) and cash-flow beta spread (y -axis) implied by the model for many different parameterizations that lead to reasonable properties of the market portfolio and the interest rate. The value premium is the difference in average returns between the value portfolio and the growth portfolio. The cash-flow beta is defined as in Cohen, Polk and Vuolteenaho (2009), and reviewed in the notes to Table IV. The cash flow beta spread is the difference in cash flow betas between the value portfolio and the growth portfolio. The diamonds corresponds to the data points from Table IV for the five different measures of the cash flows. In particular, X is the clean-surplus earnings, M is the market value, ΔX is the accumulated earnings, roe is the return on equity, and Δd is the dividend growth. The (weighted) sum is taken over five consecutive periods, as in Table IV.