The Excess Comovement of International Stock Markets in Bad Times:

A Rational Expectations Equilibrium Model *

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Abstract

We present an intertemporal, rational expectations equilibrium model where the cross-sectional covariances and correlations of international market returns increase during bad times, as a consequence of an endogenous increase in the "uncertainty" about the global economy. We assume that the drift rates of the fundamental processes of international economies are jointly affected by an unobservable global business cycle indicator. We show that as investors strive to learn the state of the global economy, their uncertainty fluctuate, thereby affecting the cross-covariances and correlations of asset returns. Excess comovement during bad times is so obtained as a reflection of higher uncertainty. When estimated with data on seven major countries, the model is able to replicate well the historical pattern of international average covariances and correlations.

1 Introduction

There is abundant evidence that the covariance and the correlation across asset returns change over time (see e.g. Bollerslev, Engle and Wooldridge (1988), Bollerslev, Chou and Kroner (1992), Moskowitz (2002)). For example, Figure 1 reports the time series of the average covariance across the returns of seven international stock markets (see Section 5). As it is apparent, the average covariance changes dramatically over time and these changes are correlated with the business cycle (NBER recessions), denoted in the figure by the bold lines. The time-variation in covariance is not only due to changes in local markets volatilities, however, as Figure 2 shows that also average correlations vary substantially through time. Even though the relationship of the average correlation with the (U.S.) business cycle is less evident, evidence reported in Erb, Harvey and Viskanta (1994), as well as in Table 1 below, shows that the average correlation indeed tends to be higher during the US recessions. This evidence does not only exist for international markets, but also for the US market (see e.g. Moskowitz (2002)). Figure 3 shows that these variations are also economically important, since they induce a considerable variation in the weights of the global minimum-variance portfolio.[1]

[Figures 1 - 3 About Here]

To further illustrate the time-variation in these economic quantities, Table 1 provides a quantitative comparison of average covariances and correlations across the business cycle at monthly or quarterly frequency. The average conditional correlation among seven international stock markets varies from 0.7263 in recessions to 0.8546 during booms, while the average conditional covariances are between 0.0163 and 0.0560 in these two phases of the cycle, respectively.[2] Under both metrics, the difference is substantial. In particular, this findings are even more puzzling because at the fundamental level,

[1] For better visualization, we only present the portfolio weights for three of the seven countries.

[2] We assume that the currency risk is hedged and hence these numbers, as well as Figures 1 and 2, are in local currencies. The same pattern holds if the variables are converted into US Dollars with the existing exchange rate.

cross-covariances of output growths display relatively very little time-variation. Indeed, Table 1 shows that different measures of fundamental cash flows present much less significant relation to the business cycle fluctuations. Only at quarterly frequency they tend to show a stronger relation to the business cycle, but the variation is still significantly lower than for returns.[3] In addition, Table 1 also shows that the level of cross-correlation of fundamentals is much lower than the ones of stock returns.

[Table 1 About Here]

Although there is much evidence pointing at time-varying covariances and correlations of asset returns, much less is known on the economic reasons why this happens. In this article we present an intertemporal, rational expectations equilibrium model of asset pricing where a time-varying cross-sectional covariance of asset returns obtains due to changes in "uncertainty" on the global economic prospects of international economies. More specifically, we assume that for each country, the output process is given by a linear Brownian motion with constant variances and cross-covariances. Hence, at the fundamental level the cross-covariances and correlations are constant. However, we also assume that the drift rates of the fundamentals processes follow a *joint* two-state, regime shift model, which is to be interpreted as a global business cycle indicator. When the "regime" switches to the expansionary state, the drifts of what we call "*cyclical*" countries switch to a higher value, while when the regime switches back to the recessionary state they all switch to a lower value.[4] Thus, in our framework, global recessions are characterized by a sequence of reductions in output for most countries, while global booms are due to generalized and prolonged increases in GDP. We remark that during the cycle, however, the conditional variance-covariance matrix of fundamentals remains constant, in line with the finding of Table 1. It is only the drift rates that change over the global cycle. Finally, although the

[3] This fact is generated because of time aggregation. Our model is able to explain why this effect should be present in the data, since we claim that the instantenous shocks are poorly correlated and most of the correlation among markets is due to common and unobservable business cycle component.

[4] We also study the behavior of other acyclical and countercyclical countries, whose drift rates are either not affected by the "regime" or move in opposite direction to the expansion and boom states.

formulation misses the possibilities that individual countries may undergo their own local cycles, the latter are captured in our framework by the residual unit-root processes in fundamentals. In other words, a country may experience a decline in output even without the switch to a lower state of its drift, but just because its production process was hit by a sequence of negative innovations.

The key assumption in our model is that consumers/investors do not observe the current "regime" (and consequently, nor the drift rates of countries' fundamentals) but they can observe the past realizations of output growth. Hence, they form a posterior probability on the two regimes. Depending on the actual paths of past output levels across countries, investors' uncertainty on the current regime changes over time, being at its maximum when they assign probability .5 to both states. This variation in uncertainty about the current regime leads to time-varying covariances and correlations across assets' returns. In addition, it also implies higher covariances and correlations during bad times, that is, at times when the global economy slows down.

To understand the intuition, assume first that all agents are risk-neutral. In this case the value of country i's stock market just equals the expected discounted value of its future output, discounted at the risk-free rate. Hence, it depends (linearly) on the current local output level, and the probability assigned to currently be in a global expansion, which we denote as π_t. This implies that *returns* will depend on the *changes* in output and the changes in this (subjective) probability π_t of being in the high state. When agents are confident to be in a expansion, then $\pi_t \approx 1$, and, in addition, under rational Bayesian learning news on the global economy have little effect on the updating of π_t itself. This occurs because agents confidence in the high state leads them to rationally give a lower weight to innovations. As a consequence, since π_t is almost insensitive to news, stock returns are mainly affected by the variation in local output. Thus, the cross-covariance of asset returns should be driven almost only by the cross-covariance of fundamentals. Suppose now, in contrast, that agents are uncertain on whether the global economy is an expansion or recessions, that is π_t is close to 0.5. This implies that now the posterior probability π_t reacts strongly to news on the health of the global economy, as these news have a higher weight in the posterior probability distribution. In other words, the relative volatility of π_t in the pricing function is now higher than in the case of almost perfect certainty. But this probability π_t is *common* across all of the countries' stock market indices, which then tend to display a higher cross-covariance and correlation of returns during uncertain periods (while, at the fundamentals level, the cross-covariance and correlation is unchanged). As we shall see below, an estimation of the model shows that recessionary periods are indeed characterized by a higher level of uncertainty (π_t close to 0.5), thereby explaining the excess comovement of international stock market returns during global recessions.

The above intuition was developed for the case where agents are risk-neutral, but a similar argument goes through when we add risk aversion, with additional effects on asset prices. In fact, we show that in this case the price function of the cyclical countries is increasing and convex in the probability π_t of being in an expansion. The intuition is the following: consider again the case where investors believe that the state is good, that is $\pi_t \approx 1$. A bad piece of news has two effects: The first is to decrease the expected future output which then decreases the market index (as in the risk-neutral case). The second is that uncertainty is higher, because π_t is now closer to .5. As explained above, this raises the cross-covariance of returns and hence it reduces the diversification possibilities of a global investor. As a consequence, since investors want to be compensated for bearing a higher risk, they require a higher discount on the expected future payouts. Thus, in the presence of bad news in good times, the price drops by more than it would in a present-value model. The opposite argument holds for $\pi_t \approx 0$, in which case good news in bad times would tend to increase the asset price only mildly. This effect is similar to the one studied in Veronesi (1999) for the individual stock market returns, whose results can be considered a special case of the ones obtained here. An implication of the model presented here is that global-level bad news obtained after a long expansionary period of the global economy would lead to a contemporaneous "over-reaction" of all (cyclical) international stock markets, with a different degree across countries which depend on their sensitivity of their fundamentals to the cycle index.

We also characterize the process for the covariance and correlation function of stock returns, as functions of the probability of being in an expansion, π_t. As mentioned, we find that for cyclical firms the covariance with the global market return is low for π_t close to 1 or 0, and high for levels of π_t that are close to 0.5. Instead, the behavior of correlations is more complex, as when covariances rise also individual market volatilities increase. This implies that correlations across market can either increase or decrease. However, given the parameters estimated from fundamentals, we will find that the model implies an increase in average correlation during bad times.

We finally take the model to the data: We estimate a joint regime-switching model using fundamentals, either GDP growth or Industrial Production, for a set of international countries. The filter allow us to obtain a time-series of probabilities π_t over time. The nature of the estimate implies an *asymmetric* pattern for these probabilities, which tend to hover around 0.95 during expansions but above 0.3 during recessions. We find that this asymmetry, which is generated by the simple fact that recessions are of a shorter duration than booms, leads to "theoretical" levels of correlations and covariances that are very much in line with the findings in Table 1 above. In fact, the asymmetric behavior of the probability of being in expansion π_t, which is closer to 0.5 during recessions than during booms, nicely complements the previous analysis showing that uncertainty (again, $\pi_t \approx 0.5$) leads to a higher cross-covariance and cross-correlation of international stock market returns. We point out that our empirical results that recessions are characterized by a higher "uncertainty" are also in line with the evidence reported elsewhere, such as in David (1997) and Veronesi (1999). In addition, more recently David and Veronesi (2001) and Andersen, Bollerslev, Diebold and Vega (2002) exploit the information contained in survey data to show that (broadly defined) "economic uncertainty" increases during recessions or after bad news hit the market in good times.

1.1 Related Literature

As mentioned, a number of articles have investigated issues related to the time co-variation in asset returns, although they mainly concentrate on the empirical analysis. Moskowitz (2002) explores the relationship between the cross-section of stock expected returns and the time-variation in the conditional covariance across a large set of portfolios of US assets. Interestingly, he finds that the covariance across the portfolio returns changes quite dramatically and it is highly correlated with NBER recessions. Analogously to the evidence in Figure 2, also the average correlations change quite substantially over time, although they are correlated to NBER recession by a less extent. Similar results are obtained in David and Veronesi (2001), who investigate the relationship between the conditional covariance between stocks and treasury bonds and various measures of economic uncertainty.

At the international level, Ledoit, Santa-Clara and Wolf (2002) analyze the comovement among different stock markets and also conclude that the degree of correlation depends on the phase of the business cycle. Their results are similar to the ones contained in Table 1 above, as they show that covariances fluctuate significantly and correlations also present similar but less pronounced movements. There is further evidence that the covariance between stock markets tend to increase during periods of global crisis (see e.g. Erb, Harvey and Viskanta (1994)). The international markets tend to be more correlated when the countries are simultaneously in a recessionary state. Longin and Solnik (1995) also find that correlation increases in periods of high volatility, a finding that is also explained by our model. Their results also confirm that the covariance matrix is less stable than the correlation matrix.

Other papers tend to focus on the effect of the increasing market integration on the level of correlation across markets. In some cases, the temporary changes are not considered like in Longin and Solnik (1995), where market integration is treated like a trend. In contrast, Bekaert and Harvey (1995) apply a two-state Markov- switching process to describe the degree of market integration which affects the market price of risk. Fundamentals of each country are allowed to affect the probability of market integration, which may change drastically through time. Few are the papers that try to relate the correlation of stock returns to the relation among fundamentals like production, earnings or dividends. Dumas, Harvey and Ruiz (2001) ask what the correlation among equity returns should be for a measured commonality in country outputs. The intuition is that, if most of the variation in economic activity in two countries is associated with the world business cycle, then the two countries should have high equity correlations. Dumas et al. (2001) however do not try to explain time variation in correlation, but only to match the level of correlation based on output correlations under different hypothesis about market integration. In the context of the contagion literature, Kyle and Xiong (2001) shows that time-varying covariances may obtain because of changing risk aversion caused by wealth effects of financial intermediaries.

Additionally, some attention has been given to the effects of large stock return downturns on correlations. For instance, Ang and Chen (2001) analyze the asymmetries in stock returns. The correlation between US stocks and aggregate US market is much higher for downside moves, especially for extreme changes, than for upside moves. They conclude that a Markov switching process may describe well the behavior of stock expected returns.

The rest of the article proceeds as follows: Next section introduces the model while Section 3 obtains the equilibrium results. Section 4 includes the theoretical implications for asset covariances and correlations, while Section 5 reports evidence supporting the model by using a set of international countries. Section 6 concludes.

2 The economy

This section describes the model, which is an extension to the multi-asset case of Veronesi (1999). As we shall see, a number of novel results will emerge from the case with many assets, and we shall emphasize those. For brevity, other results that are more similar to the ones in Veronesi (1999), will only be briefly mentioned.

We consider an economy with a single physical consumption good, which can be allocated to investment or consumption, and a continuum of identical investors. We assume that there are n risky assets - identified here as the international stock markets - and a riskless asset. We make the following assumptions:

Assumption 1. Dividend Processes. The dividend (output) process of country i is given by the stochastic process

$$dD_t^i = \theta_t^i dt + \sigma_i d\xi_t^i \tag{1}$$

where θ_t^i is unobservable, σ_i is a constant and ξ_t^i is a Wiener process. In addition, $E\left[d\xi_t^i d\xi_t^j\right] = \rho_{ij} dt$.

Assumption 2. Regime Shifts. The drift rates $\theta_t^1, ..., \theta_t^n$ simultaneously shift between two states θ_P^i and θ_T^i (the subscripts are mnemonic of "Peak" and "Trough"). Formally, let x_t be a state variable evolving according to a two state, continuous time Markov chain with transition probability matrix between t and $t + \Delta$ given by

$$\mathsf{P}(\Delta) = \begin{pmatrix} 1 - \lambda\Delta & \lambda\Delta \\ \mu\Delta & 1 - \mu\Delta \end{pmatrix}$$

where the two states are denoted by x_T and x_P. Then, for all i, $\theta_T^i = \phi^i(x_T)$ and $\theta_P^i = \phi^i(x_P)$ for some map $\phi^i(.)$.

In other words, Assumptions 1 and 2 imply that each country output level is subject to small shocks (through $d\xi_t^i$'s) and big shocks, thought the movements of the drifts θ_t^i. As discussed in the introduction, the movement of x_t is to be interpreted as the global business cycle component that is common across various countries. Its movement affects the long term covariances between output levels, while the very short term covariance is captured by the correlations ρ_{ij}. Hence, notice that the shocks $d\xi_t^i$ cannot be interpreted as "idiosyncratic shocks," because they are not cross-sectionally independent.

Definition 1. Country i is termed as *cyclical* (*countercyclical*) if its drift rates are such that $\theta_P^i > \theta_T^i$ ($\theta_P^i < \theta_T^i$). It is termed *acyclical* if its drift rates are $\theta_P^i = \theta_T^i$.

Assumption 3. Preferences. Investors are endowed with a constant absolute risk aversion utility function

$$U(c,t) = -e^{-\rho t - \gamma c}$$

where ρ is the rate of intertemporal discount and γ is the constant absolute risk aversion coefficient.

Assumption 4. Asset Supply. The supply of the risky assets in each country is fixed and normalized to 1. The risk-free asset is elastically supplied and has an instantaneous rate of return equal to r.

2.1 The Dynamics of the Belief of a Global Expansion

Let $\mathcal{F}_t$ denote investors' information set at time t, which contains all the output realizations of all countries up to time t. We can compute the probability that the global economy is in an expansion as:

$$\pi_t = \Pr(x_t = x_P | \mathcal{F}_t)$$

Next lemma provides the law of motion of π_t under rational Bayesian learning:

Lemma 1 *Let $\mathbf{\Sigma}$ be the variance-covariance matrix of fundamentals, with ij-th element $[\mathbf{\Sigma}]_{ij} = \sigma_i \sigma_j \rho_{ij}$. The conditional posterior probability to be in an expansion evolves according to the stochastic process*

$$d\pi_t = (\lambda + \mu)(\pi^s - \pi_t)dt + \pi_t (1 - \pi_t) \mathbf{\Delta\theta}' \cdot \mathbf{\Sigma}^{-1} \cdot (d\mathbf{D}_t - E[d\mathbf{D}_t | \mathcal{F}_t]) \tag{2}$$

where $\pi^s = \frac{\mu}{\lambda + \mu}$ is the unconditional probability of an expansion, $\mathbf{D}_t = (D_t^1, ..., D_t^n)'$ is the n-dimensional vector of dividend payouts, and $\Delta\boldsymbol{\theta} = (\theta_P^1 - \theta_T^1, ..., \theta_P^n - \theta_T^n)'$ is the $n-$dimensional vector of drifts differentials.

Proof: See Liptser and Shyriaev (2001).

Equation (2) is rather intuitive: The drift rate of $d\pi_t$, $(\lambda + \mu)(\pi^s - \pi_t)$, shows that in the long-run, probabilities move around the unconditional average frequency of expansion states, π^s. The speed at which convergence to this central tendency occurs depends on the frequency of regime switches. For example, low values of $\lambda + \mu$ imply that probabilities take longer time to pass through π^s, therefore being more concentrated around 0 or 1. The diffusion term is also intuitive: The last parenthesis in (2) contains the vector of expectation errors. Hence, these are the "news" that hit the global economy and that affect beliefs. These expectation errors are weighted by the inverse of their variance covariance matrix, which therefore represents the "precision" of signals. Consider for example the case where $\mathbf{\Sigma}$ is diagonal and one variance term σ_i is very small. Then $[\mathbf{\Sigma}^{-1}]_{ii}$ is correspondingly very large implying that news on asset i have a big impact on the posterior probability. Hence, π_t would quickly converge to 0 or 1 as soon as a shift in regime occurs. However, this happens provided that the corresponding drift differential $\theta_P^i - \theta_T^i$ is non-zero. If for example $\theta_P^i = \theta_T^i$, then the small value of σ_i provides no information whatsoever on the underlying cycle index x_t. Hence, intuitively we can consider the term $\boldsymbol{\Delta\theta}' \cdot \boldsymbol{\Sigma}^{-1}$ as representing an $n-$dimensional vector of "signal to noise" ratios, each of which provides a weight to the corresponding innovation $dD_t^i - E\left[dD^i|\mathcal{F}_t\right]$, $i = 1, ..., n$.

Finally, the term $\pi_t(1-\pi_t)$ represents the impact that this weighted average of news on fundamentals has on the belief π_{t+dt}: If uncertainty is high, so that $\pi_t \approx 0.5$, then $\pi_t(1-\pi_t)$ is maximized and news have a high impact on the beliefs. Thus, beliefs are highly volatile when uncertainty is high. In contrast, if agents are relatively confident in a high or low state, $\pi_t \approx 1$ or 0, then $\pi_t(1-\pi_t) \approx 0$ and news have a low impact on beliefs. The changes in the volatility of beliefs, and hence of expectations of future output, will be the main driving force of the time-variation in the cross-covariances and cross-correlations.

3 A Rational Expectations Equilibrium

Let W_t be investors wealth at time t. Let P_t^i be the price of asset i and X_t^i be the demand for asset i. Define the return on a zero investment portfolio long one share of the risky asset i financed by borrowing at the risk-free rate by $dQ_t^i = \left(D_t^i - rP_t^i\right)dt + dP_t^i$. The dynamic budget constraint of the global investor can then be written as

$$dW_t = (rW_t - c_t)\,dt + \sum_{i=1}^{n} X_t^i dQ_t^i \tag{3}$$

Definition 2: A *Rational Expectations Equilibrium* is given by a set of price functions $P^1(W, \mathbf{D}, \pi)$,... , $P^n(W, \mathbf{D}, \pi)$, demand functions $X^1(W, \mathbf{D}, \pi)$, ..., $X^1(W, \mathbf{D}, \pi)$, and a consumption function $c(W, \mathbf{D}, \pi)$ such that investors solve the intertemporal maximization problem

$$J(W_0, \pi_0, t) = \max_{c, X^1, .., X^n} E_0\left[\int_0^\infty U(c, s)ds\right]$$

subject to the budget constraint (3) and the transversality condition $\lim_{\tau\to\infty} E_t\left[J\left(W_{t+\tau}, \pi_{t+\tau}, t+\tau\right)\right] =$

0, and such that the market clearing conditions hold, $\mathbf{X}_t (W_t, \mathbf{D}_t, \pi_t) = \mathbf{1}$.

3.1 Equilibrium Prices

3.1.1 The case of risk-neutrality

To build intuition, it is worth starting from the computation of asset prices in the case where investors are risk-neutral. This will help us understand what additional implications the assumption of risk aversion would yield for the cross-section of asset returns:

Proposition 2 *Suppose that investors are risk-neutral, so that for all* $i = 1, ..., n$, *we have* $\overline{P}_t^i = E_t \left[\int_t^\infty e^{-r(\tau - t)} D_\tau^i d\tau \right]$. *Then*

$$\overline{P}_t^i = p_D D_t^i + p_{1i} + p_{\pi i} \pi_t \tag{4}$$

where $p_D = 1/r$ *and*

$$p_{\pi i} = \frac{\left(\theta_P^i - \theta_T^i\right)}{r\left(r + \lambda + \mu\right)}, \; p_{i1} = \frac{\theta_T^i}{r^2} + \frac{\left(\theta_P^i - \theta_T^i\right)}{r^2 \left(r + \lambda + \mu\right)} \mu \tag{5}$$

Proof: Special case of Proposition 3. ¥

Equation (4) shows that the price of each country index depends on the level of its local cash payout, D_t^i, but also on the probability of being in a global expansion π_t. The presence of this common belief in the pricing function of all (cyclical) countries is the source of time-variation in the cross-sectional covariances and correlations. Indeed, to further our intuition we can use formula (4) to immediately compute the covariance between the return of stock market i and the global market portfolio. Let us denote by $\overline{P}_t^M$ the value of the global market portfolio under risk-neutrality, given by $\overline{P}_t^M = E_t \left[\int_t^\infty e^{-r(\tau - t)} D_\tau^M d\tau \right]$, where D_t^M is the total global payout distributed to the investors, $D_t^M = \sum_{j=1}^n D_t^j$. Clearly, from linearity we obtain immediately $\overline{P}_t^M = \sum_{j=1}^n \overline{P}_t^j$. Let us also denote by $d\overline{Q}^M$ the total dollar return of a unit investment in the global market portfolio fully financed at the risk free rate $d\overline{Q}_t^M = \left(D_t^M - r\overline{P}_t^M \right) dt + d\overline{P}_t^M$. From proposition 2, the value of the market portfolio under risk-neutrality is given by

$$\overline{P}_t^M = p_D D_t^M + p_1 + p_\pi \pi_t$$

where $p_1 = \sum_{j=1}^{n} p_{1j}$ and $p_\pi = \sum_{j=1}^{n} p_{\pi j}$. Since the covariance between the total global output D_t^M and the one of country i is given by $E\left[dD_t^i dD_t^M\right] \equiv \sigma_i \sigma_M \rho_{i,M}$, with $\sigma_M^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j \rho_{ij}$ and $\rho_{i,M} = \sum_{j=1}^{n} (\sigma_j / \sigma_M) \rho_{i,j}$, using Ito's lemma we find that the instantaneous covariance between the return on the market index i and the global market portfolio (in the case of risk neutrality) is:

$$\overline{\sigma}_{iM}(\pi_t) \equiv E_t\left[d\overline{Q}_t^i d\overline{Q}_t^M\right] = \frac{1}{r^2}\sigma_i \sigma_M \rho_M + a_i \pi_t^2 (1-\pi_t)^2 + b_i \pi_t (1-\pi_t) \tag{6}$$

where a_i and b_i are two constants given by

$$a_i = \frac{\Delta\theta^i \Delta\boldsymbol{\theta}' \cdot \mathbf{1}_n}{r^2 (r+\lambda+\mu)^2} \Delta\boldsymbol{\theta}' \cdot \boldsymbol{\Sigma}^{-1} \cdot \Delta\boldsymbol{\theta} \text{ and } b_i = \frac{2\Delta\theta^i \Delta\boldsymbol{\theta}' \cdot \mathbf{1}_n}{r^2 (\lambda+\mu+r)} \tag{7}$$

As it can be seen, the covariance between the risk neutral excess return of asset i and the risk-neutral excess return of the global market portfolio depends on π and hence it is time-varying. In particular, it is minimized at $\pi = 0$ and $\pi = 1$ and it has a unique maximum in the interior of the unit interval. The source of variation is the higher sensitivity of the revisions of beliefs, $d\pi$, to news during periods of high uncertainty, as discussed in Section 2.1.

3.1.2 The case of risk aversion

Of course, because of risk-neutrality, the time-variation in the conditional covariance of asset returns uncovered in equation (6) has no effect on the expected excess return on asset i, $E_t\left[dQ_t^i\right]$, which is zero. However, as one can imagine, in the presence of risk aversion, the predictable increases (decreases) in the expected covariance between asset i and the global market portfolio would decrease (increase) the diversification opportunities offered by that asset to the global investor. This effect feeds back into the price of the asset, as each agent would require a higher (lower) discount to hold the asset, inducing some non-linearities in the equilibrium pricing functions. Next proposition characterizes these non-linearities:

Proposition 3 *There exists a REE where for all $i = 1, ..., n$*

$$P_t^i = p_{0i} + S_i(\pi_t) + p_D D_t^i + p_{1i} + p_{\pi i} \pi_t \tag{8}$$

where $p_{0i} = -\frac{\gamma}{r^2}\sigma_i\sigma_M\rho_{i,M}$, $p_D = \frac{1}{r}$ *and* p_{1i} *and* $p_{\pi i}$ *are given above in equation (5). In addition,* $S_i(\pi)$ *is the solution of the ordinary differential equation*

$$S_i''(\pi)\,P_3(\pi) = S_i'(\pi)P_2(\pi) + S_i(\pi)r + P_{0i}(\pi) \tag{9}$$

where the functions $P_{0i}(\pi)$, $P_2(\pi)$ *and* $P_3(\pi)$ *are given in equations (22) - (24) in the appendix.*

Proof: See appendix. ¥

In order to discuss the implications of this proposition, we must discuss the properties of the solutions to the ordinary differential equations (9). However, as in the risk-neutral case, we see immediately that the presence of π_t in the pricing functions (8) will yield a cross-covariance of returns that depends on whether π_t is close to 0.5 or close to its boundaries 0 or 1. The time-variation in π_t, dictated by the process for rational Bayesian updating (2), will then generate time-varying covariances and correlations across the international stock markets.

3.1.3 The Function $S_i(\pi_t)$

This section describes the behavior of the function $S_i(\pi)$. Consider the case of cyclical countries first, for which $\theta_P^i > \theta_T^i$. In this case, from the results in Veronesi (1999), the function $S_i(\pi)$ is a negative, U-shaped function of π. As a consequence, the price function in equation (8) shows that the value of the stock market index for country i is given by its risk-neutral value, given in Proposition 2, plus a (negative) discount, given by $p_{0i} + S_i(\pi)$. The latter depends on the probability of being in a global boom π. As hinted already earlier, and explained in more detail below, the reason why π affects the discount required to hold the market indices of cyclical countries is the following: when $\pi \approx 0.5$, the global investor can rationally anticipate that expected future cash flows across assets would be very sensitive to news about the global economy (see discussion after Lemma 1). This in turn generates a high covariance between the returns of this country and the global market portfolio, which imply poor diversification opportunities for the global investor. Hence, the latter will require a high discount to hold the stock of that country, over the risk-neutral value (4). When π is close to 1, by contrast, the lack of reaction of expectations to global news leads the rational global investors to anticipate a low correlation across stock-market indices, which then induce them to require a low discount.

For a countercyclical country, the opposite holds: In this case one can show that $S_i(\pi)$ is a positive inverse U shaped function of π. That is to say, investors are ready to pay a premium to hold the asset of a country i, whose cash payouts are countercyclical. As we shall see, the reason why this occurs is that during periods of higher uncertainty the covariance between the global market portfolio and the return of a countercyclical country becomes even more negative, thereby generating a natural hedge against adverse movement of the global market. However, the estimates in Section 5 show that all countries we consider are in fact cyclical, albeit with different degrees, which then makes this case less interesting for our purposes.

To gauge the size of the effects, Figure 4 shows the price functions for different magnitudes of $(\theta_P^i - \theta_T^i)$ keeping the same average growth rate.[5] The price function is steeper if the difference between the peak and trough drift rates is large, because the discrepancy between the two states is more pronounced for firms that are more cyclical. For ease of presentation, the price functions were standardized by dividing each one by its respective average price. The difference in drift rates also affects the level of the price function and the more cyclical countries tend be discounted more heavily by the global investor. For completeness, we also plot the price function of a countercyclical country, which we find, not surprisingly, inversely related to the posterior probability π_t of a global expansion.

Finally, for all cyclical countries the price function is convex π, a phenomenon that could lead to contemporaneous "over-reaction" of international stock market indices to global bad news during "good

[5] All figures in this section use the following parameters: r=0.03, γ=1, λ=0.005, μ=0.05. The average growth rate of dividend is 0.009. In the first case, we chose θ_P^i and θ_T^i with different degrees of dispersion of growth rates. These results are qualitatively insensitive to the choice of parameters within a range of plausible values. The dividend growth covariance matrix was constructed assuming that all assets have same covariances with all the other assets. The aggregate dividend variance is equal to 0.12. In one of our simulations, we chose different values for the correlations between one of the dividend processes and the aggregate dividend, keeping all others constant. In the benchmark case, the correlation is equal to 0.8.

times".

[Figure 4 about here]

The correlation between the underlying output processes mainly affects the level of the price function. Figure 5 presents the price function for different correlations between the individual output innovations. The stock markets with less correlated aggregate dividend processes have higher prices for obvious reasons, but there is no clear interaction with the posterior belief about the true state of the global economy. If we introduce stocks with different $(\theta_P^i - \theta_T^i)$ and different innovation correlations, the contribution of each of these factors is still the same.

[Figure 5 about here]

We finally notice that the price of an acyclical country is not affected by the probability of being in a boom π:

Corollary 4 *The price of an acyclical country is not affected by π. That is, its price is given by*

$$P_t^i = p_{0i} + p_D D_t^i + p_{1i} \tag{10}$$

where $p_D = \frac{1}{r}$, $p_{i1} = \frac{\theta^i}{r^2}$, and $p_{0i} = -\frac{\gamma}{r^2}\sigma_i \sigma_M \rho_{i.M}$.

Proof. Since $\theta_T^i = \theta_P^i$, it follows $p_{\pi i} = 0$. From the appendix, we have $P_{0i}(\pi) = 0$. Clearly, $S_i(\pi) = 0$ is a solution to (9). ¥

3.2 The Global Market Portfolio

We finally need to derive the value of the global market portfolio. Aggregating the total output across countries we find that $D_t^M = \sum_{i=1}^n D_t^i$ evolves according to the process

$$dD_t^M = \theta_t^M dt + \sigma_M d\xi_t$$

where $\theta_t^M \in \left\{\theta_P^M, \theta_T^M\right\} = \left\{\sum_{i=1}^n \theta_P^i,\ \sum_{i=1}^n \theta_T^i\right\}$, $\sigma_M^2 = \sum_{i=1}^n \sum_{j=1}^n \sigma_i \sigma_j \rho_{ij}$, and $d\xi_t$ is a Brownian motion defined as $d\xi_t = \sigma_M^{-\frac{1}{2}} \sum_{i=1}^n \sigma_i d\xi_t^i$. This set up is then the one studied in Veronesi (1999) and we can use Proposition 2.2 from that article to find:

Corollary 5 *The price of the global market portfolio P_t^M is given by*

$$P_t^M = p_0 + S(\pi_t) + p_D D_t^M + p_1 + p_\pi \pi_t \tag{11}$$

where $p_0 = -\frac{\gamma \sigma_M}{r^2}$, $p_1 = \theta_T^M / r^2 + \mu\left(\theta_P^M - \theta_T^M\right) / \left(r^2 (r + \lambda + \mu)\right)$, $p_\pi = \left(\theta_P^M - \theta_T^M\right) / (r(r + \lambda + \mu))$ and $S(\pi)$ satisfies the Ordinary Differential Equation (11) in Veronesi (1999). In addition, $S(\pi) = \sum_{i=1}^n S_i(\pi)$, where $S_i(\pi)$ satisfy (9), and hence $P_t^M = \sum_{i=1}^n P_t^i$.

We remark that Corollary 5 highlights an important implication of assuming D_t^i as linear Gaussian in (1) and a negative exponential utility for investors' preferences: These assumptions not only enable us to solve conveniently for the price functions of individual securities, but they also lead to a convenient *aggregation* across securities. In fact, Corollary 5 shows that the price of the global market portfolio has indeed the same functional form as the one of individual securities. As we shall see, the simplicity of the solution at the aggregate level makes it simple to analyze the cross-sectional properties of individual asset returns, such as their covariances and correlations with the global market portfolio.[6]

4 Time-Varying Covariance and Correlation of Stock Returns

Given Proposition 3, we can characterize the conditional covariances across countries' stock market indices and study how they change over time as a function of the belief π_t of being in a global expansion.

[6] In contrast, the typical alternative setting where the dividends of individual assets follow geometric Brownian motions, with regime shifts in drift, does not allow for a convenient general equilibrium solution, as the summation of log-normal processes does not aggregate to a log-normal process. Alternatives proposed recently in the literature, such as the use of an exogeneous consumption process (see e.g. Bansal et al. (2002)) or the modeling of "shares" of dividend to consumption (Santos and Veronesi (2001)), turn out to either have less appealing formulas or properties for the quantities we are interested in, or simply to be harder to work with in the framework of a general equilibrium model with regime shifts.

Recall that we denoted the market portfolio by $P_t^M = \sum_{i=1}^n P_t^i$ and the conditional covariance of asset i with the market as $\sigma_{iM}(\pi_t) = E_t\left[dQ_t^i dQ_t^M\right]$. We then have the following result:

Proposition 6 *The covariance of dollar return of asset i with the market is given by*

$$\sigma_{iM}(\pi_t) = \overline{\sigma}_{iM}(\pi_t) + \pi_t(1-\pi_t)\frac{1}{r}\left(\Delta\boldsymbol{\theta}' \cdot \mathbf{1}_n\ S_i'(\pi_t) + \Delta\theta^i\ S'(\pi_t)\right)\left(\pi_t(1-\pi_t)\frac{\Delta\boldsymbol{\theta}' \cdot \boldsymbol{\Sigma}^{-1} \cdot \Delta\boldsymbol{\theta}}{(\lambda+\mu+r)} + 1\right) \tag{12}$$

where $\overline{\sigma}_{iM}(\pi_t) = E_t\left[d\overline{Q}_t^i d\overline{Q}_t^M\right]$ is the covariance between the dollar return of asset i and the market under risk neutrality, whose formula is provided in equation (6).

Proof: Immediate from applying Ito's Lemma to (8) and (11), and then computing $E_t\left[dQ_t^i dQ_t^M\right]$

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Although equation (12) is not "self-evident," it allows us to obtain a full characterization of its properties. For instance, notice that if $\Delta\boldsymbol{\theta}' \cdot \mathbf{1}_n$ and $\Delta\theta^i$ are positive, then the second term in (12) is negative for π_t close to zero and positive for π_t close to one. Hence, an immediate corollary is the following

Corollary 7 *Let $\Delta\boldsymbol{\theta}' \cdot \mathbf{1}_n > 0$ and $\Delta\theta^i > 0$ and let $\widehat{\pi}^i$ be such that the second term in (12) is zero. Then*

$$\sigma_{iM}(\pi_t) > \overline{\sigma}_{iM}(\pi_t) \text{ if and only if } \pi_t > \widehat{\pi}^i$$

That is to say, the presence of risk aversion increases the covariance between the return on country i and the global market portfolio when the probability π_t of being in the high state is high. This is an effect of the "overreaction" to bad news in good times: During good times, a bad news has the effect of decreasing the value of all the (cyclical) national market indices, implying a high local covariance of *dollar* returns between them.

In addition, formula (12) entails a number of properties of the covariance between (dollar) returns for asset i and the global market, summarized in the next corollary:

Corollary 8 *(a) A cyclical country i with zero covariance in fundamentals (i.e. $E_t\left[dD_t^i dD_t^j\right]=0$ for $j=1,..,n$) displays nonetheless a time-varying covariance in stock returns;*

(b) An acyclical country displays a scaled covariance of dollar returns with the market as for fundamentals, given by

$$E\left[dQ_t^i dQ_t^M\right]=p_D^2\sigma_i\sigma_M\rho_{i,M}=p_D^2 E\left[dD_t^i dD_t^M\right] \tag{13}$$

Proof*: (a) It is immediate from (12) and (6) that even by setting $E\left[dD_t^i dD_t^M\right]=\sigma_i\sigma_M\rho_{i,M}=0$, all the other terms do not vanish. (b) Just set $\theta_P^i=\theta_T^i$, $p_{\pi i}=0$ and $S_i(\pi)=0$ in (12).* ¥

Notice that the result in part (b) implies that a model without global expansions and recessions would imply that for all countries $i=1,..,n$, the covariance of each asset returns with the global market portfolio is given by $E_t\left[dQ_t^i dQ_t^M\right]=\frac{1}{r^2}\sigma_i\sigma_M\rho_{i.M}$, which is constant. The explicit modeling of global booms and recessions together with the unobservability of the underlying state of the global economy leads to time-varying covariances and correlations.

Equation (12) and Corollary 8 apply to dollar returns. An additional interesting quantity is the percent return, which is given by

$$E_t\left[\frac{dQ_t^i}{P_t^i}\frac{dQ_t^M}{P_t^M}\right]=\frac{E_t\left[dQ_t^i dQ_t^M\right]}{\left(p_{0i}+S^i(\pi_t)+p_D D_t^i+p_{1i}+p_{\pi i}\pi_t\right)\left(p_0+S(\pi_t)+p_D D_t^M+p_1+p_\pi\pi_t\right)}$$

This implies the following:

Corollary 9 *An acyclical country has a time varying covariance of percent returns with the market:*

$$E_t\left[\frac{dQ_t^i}{P_t^i}\frac{dQ_t^M}{P_t^M}\right]=\frac{p_D^2\sigma_i\sigma_M\rho_{i,M}}{\left(p_{0i}+p_D D_t^i+p_{1i}\right)\left(p_0+S(\pi_t)+p_D D_t+p_1+p_\pi\pi_t\right)}$$

Clearly, the effect described in this result is only stemming from the changes in the global market portfolio's discount $S(\pi_t)$ as a function of the probability π_t.

To gauge whether the size of these effects is economically significant, Figures 6 to 8 plot the covariances and correlations implied by our model, as a function of π_t, when the model is calibrated to the set of countries we use in our empirical section. These countries are US, Canada, Japan, France, Germany, UK and Switzerland.[7]

[Figure 6 about here]

Figure 6 shows that cyclical countries tend to have higher covariance with the market dollar returns. The covariance is larger in periods of high uncertainty about the true state. Whenever the posterior probability declines to intermediate levels, the covariance among all international stock markets become significantly higher and the effect is more accentuated for highly cyclical countries. The interesting implication is that the covariance between asset returns will increase after the market becomes uncertain about the true state even when the uncertainty is not followed by a recession.

[Figure 7 about here]

Figure 7 shows that the same pattern is present when percent returns are considered. The percent returns covary relatively more when the probability of a boom is close to zero if compared to the case where this probability goes to one. The covariance of counter-cyclical countries with the global market returns would decrease when the economy becomes more uncertain about its true state[8].

[Figure 8 about here]

Figure 8 shows that an inverted U-shape can also be found for correlations with the global market portfolio's returns. Nevertheless this outcome depends heavily on the estimated parameters but holds in the data we analyze later on. For example, the correlation with the world market is U-shaped in

[7] See section 5 for details on data and estimation.

[8] This result actully holds with simulated data when we assume the one of the firms has a negative $(\theta^{i}_{P} - \theta^{i}_{T})$. However, the data we use in the next section does not include any country with counter-cyclical aggregate dividends.

the case of Japan, implying that the correlation is actually lower in periods of higher uncertainty. The fact that the Japanese stock market index turns out to be indeed the less correlated with the global portfolio in the data shows an interesting parallel with our model.

We end this section by noticing that these results are also in line with the empirical finding of Longin and Solnik (1995), who document a positive relationship between international stock market correlations and return volatility. Since from Corollary 5, the global market portfolio has the same form as the pricing function in Veronesi (1999), we can conclude that high uncertainty (π_t close to 0.5) leads to high volatility of the returns of the global market portfolio. The contemporaneous increase in correlations displayed in Figure 8 for the same values of π_t yields the result.

5 The Co-movement of International Stock Markets over the Business Cycle

We now estimate the model discussed in the previous sections and apply the parameter estimates to the functional forms we uncovered in Section 4 for the covariance functions. The estimation procedure will also provide us with a time-series of beliefs π_t. We will then be able to check whether our model is able to generate the pattern of time-variation in cross-sectional covariances and correlations observed in the data.

More explicitly, we estimate a joint state-dependent Markov switching model for the growth rates of all countries' gross real products. As discussed in the introduction, this procedure implicitly assumes that there is only a international business cycle and ignores the possibility of country-specific cycles. The latter would then be captured by the residual term. The sample consists of seven countries with available quarterly data from 1970 to 2000: Canada, United States, Japan, France, Germany, Switzerland and United Kingdom. Table 2 reports the summary statistics of the GDP data and stock market return data. All the entries are in local currencies, deflated using the local inflation indices. We performed the same exercise by converting the currency units in US dollars and obtained very similar results. In addition, we also performed the estimation using monthly industrial production (again, both in US dollar or local currency) and the results are qualitatively the same. Finally, whenever we calculate the average covariances and correlations with the world market returns, we assume an equally-weighted world index in order to minimize the effect of each country's size. Note that correlations and standard deviations based on GDP tend to be much lower than the same statistics for aggregate market returns.

[Table 2 about here]

One difficulty in estimating the model in Section 2 is that in order to obtain a tractable model with multiple securities, and thus to be able to solve for prices and returns, the output processes (1) were assumed to evolve *linearly* according to Gaussian diffusion processes. Instead, data realizations tend to display a geometric growth. We overcome this problem by using the technique developed in Campbell and Kyle (1993), that is, by deflating the individual time series according to a constant long-run growth rate. To check for robustness, we also use GDP growth which, we shall see, leads to essentially the same results.

Table 3 reports the estimates of the joint-state regime switching model. The estimated probabilities of switching from a global expansion to a contraction is 0.05 (quarterly), which implies an average length of a global expansion of about 5 years. The probability of switching from a contraction to an expansion is much higher, 0.27. This implies an average length of a global contraction of slightly more than 11 months. These values match well the average lengths of the NBER-dates expansions and contractions in the same period, which are 4.7 years and slightly more than 10 months, respectively.

[Table 3 about here]

The point estimates of the drift rates in expansion and in contraction show that all countries are indeed cyclical, albeit with different degrees. We also tested whether the cyclicality is statistically significant and found that United States, Canada, United Kingdom and Switzerland have statistically significant drift differentials $(\theta_P - \theta_T)$.[9] Indeed, the drift differential $(\theta_P - \theta_T)$ is considerably smaller for Japan and France, which is the reason for the insignificance of their statistics. Table 4 shows similar results using the GDP growth, rather than changes in detrended GDP. The probabilities of regime switching are almost identical and again all countries resulted as cyclical. Also in this case, Japan and France show the lowest (and insignificant) drift differential $(\theta_P - \theta_T)$.

[Table 4 about here]

The last two rows of Table 4 also report the average covariances and correlations of each stock market with the other countries. We see that Japan is the country with the lowest average correlation and covariance with the others. Interestingly, as mentioned above, it is also exactly the one with the least economically (and statistically) significant drift differential $(\theta_P - \theta_T)$. The same relation is not as clear for the other countries, but it illustrates the possible connection between the international business cycle and the level, but not the variation, of the international stock returns covariances and correlations, which is also investigated in Dumas et al. (2001).

5.1 Explaining the Excess Comovement in Bad Times

The estimated joint process is then applied to the model, which generates the implied covariances and correlations, among other outputs. As already previewed in Figures 6 to 8, the model produces substantial changes in covariances and correlations of individual returns with the global market portfolio. In this section, we concentrate on "averages" to better compare the results with Figures 1 and 2 in the introduction. Figure 9 shows that for intermediate levels of the probability π, the average correlation attains its largest values, while Figure 10 shows that the same pattern holds for the average covariance.

[Figures 9 and 10 about here]

[9] The t-statistics were calculated using the asymptotic standard errors from the maximum likelihood estimation.

Figures 11 and 12 report the time-series of the fitted probability to be in a boom as estimated from the regime switching model, along with plots of the model-implied average correlation and average covariance across markets. A few remarks are in order: First, we see that indeed the probability of global boom dips at the times of the NBER recessions, which are denoted by the bold lines in the bottom panel. Given the sheer size of the US economy with respect to the other countries in the sample, this finding is indeed reasonable and consistent with the empirical results of Hamilton (1989), who investigated a regime switching model for the US economy alone.

[Figures 11 and 12 about here]

Second, there is an asymmetry between booms and recessions: During booms, the probability π_t hovers around .95 and it is relatively stable at that level. During recessions, although π_t does dip to a lower value, it remains nonetheless substantially above zero – the minimum is about 0.3. The reason of the asymmetry is simply that from Table 3 the probability to switch from a boom to a recession is much lower than the probability to switch from a recession to a boom (.05 versus .27). This has two implications: First, recessions are shorter, which in turn entails that beliefs do not have a sufficient amount of time to reach the lower boundary. Second, from the formula of rational Bayesian updating, good news during bad times are more likely to be interpreted as a switch back to the high state, than bad news during good times to be interpreted as a switch to a low state. From Figures 9 and 10, this asymmetry between booms and recessions in the level of the probability π_t implies that booms are characterized by a lower level of cross-correlations and covariances than recession.

A final remark about Figures 11 and 12 is that when the probability of a boom dips, both the average correlation and the average covariance across assets increases. The relationship between π_t and the average covariance is more clear-cut than the one between π_t and the average correlation. In other words, the average correlation is high at times when π_t is only slightly below 1. In contrast, the average covariance increases basically only when the probability π_t dips. This result justifies in part the empirical finding in Figures 1 and 2 that the average correlation seems to be less correlated with the business cycle than the average covariance is.

In order to quantitatively assess the ability of the model to reproduce the changes in covariances and correlations observed in the data, Table 5 reports the average model-implied covariances and correlations of returns during booms and recessions, along with their empirical counterparts.

[Table 5 about here]

We notice that our fitted model enables us to determine dates of global recessions, defined as those period when $\pi_t \leq 0.5$. It is natural to expect that these dates will not coincide exactly with the NBER-dated recessions, which only interest the USA. Table 5 shows that the model tends to match well the relation between average covariances and correlations and the business cycle. The model is relatively more successful to replicate the level and time-variation of the average correlations. We should note that the level of average covariances implied by the model is slightly lower than the one in the data, probably partly due to the fact that GDP growth is smoother than actual market dividends. Yet, looking at relative terms, the difference in covariances between booms and recessions implied by the model is similar to the one in the data. We conclude that the model identifies the degree of variability over time of covariances and correlations, and their relation to business cycle fluctuations.

6 Conclusion

This article proposes a rational expectations model to justify the time variation in the cross-covariance and correlation of asset returns, as observed empirically in international asset markets. The main intuition of the model is that the value of each market index depends both the current level of the corresponding dividend (or output level) and on the probability that global investors assign to the world economy being in a high state. The latter probability enters in valuation of all national stock markets as it determines the expected level of future dividend payouts from holding each market index in the portfolio. Rational Bayesian updating implies that when agents believe that the global economy is in a boom, the probability of the high state does not react much to news, thereby making stock returns mainly affected by changes in local dividends. Hence, the cross-covariance and correlations across stock market indices are mainly due to the fundamentals. However, after a series of bad news are observed, the probability of being in high state moves closer to 0.5, where rational Bayesian updating implies a higher sensitivity to news. Hence, now changes in prices (and hence returns) are strongly affected by the probability of being in an expansion, which affects all the assets at the same time. This increases the cross-covariances of asset returns as well as their cross-correlations. As a consequence, the model predicts that during periods of relative more uncertainty about the prospects of the global economy, asset returns should become more correlated, even if at the fundamental levels their instantaneous correlation is unchanged.

A model similar to the one presented in this paper could potentially partly explain the contagion phenomenon in emerging markets. Contagion is usually defined as a substantial increase in correlations without changes in the fundamental linkages. However, we just showed that changes in uncertainty about factors that affect simultaneously different countries may drive correlations and covariances upwards, even without an actual change in these factors. Our model focus on the uncertainty about the growth rates of these economies, but other factors could also be important in the case of the emerging markets, such as exchange rate and political regimes (e.g. Drazen (1998)).

An additional avenue of research pertains the empirical exploration of the correlated "overreaction" to global bad news in good times, as the non-linear form of the pricing function obtained in equation (8) suggests. For instance, Conrad et al. (2002) find empirically that at the firm level, bad earnings news in good times have a higher impact on stock prices than bad earnings news in bad times. The model presented in this paper provides partial rationale to that finding, and suggests that a similar effect may exists across international markets.

Finally, the model described in this paper has also implications for the cross-section of stock returns. In this case, the belief to be in a boom or recession becomes itself a state-variable so that the covariance of each asset returns with this state variable would also determine the conditional expected return of the asset. Empirical evidence indeed suggests that stock returns on individual portfolios are indeed affected by variables linked to the business cycle (see e.g. Fama and French (1989), Lettau and Ludvigson (2001)). We leave the investigation of these implications to future research.

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7 Appendix

In this appendix we use the following notation, which is more convenient for the algrebric derivations: define a vector of independent Wiener process $\left(d\widehat{\xi}_t^1, ..., d\widehat{\xi}_t^n\right)$ and a matrix $\mathbf{\Phi}$ so as to redefine the process for dividends in vector form as

$$d\mathbf{D}_t = \boldsymbol{\theta}_t dt + \mathbf{\Phi} d\widehat{\boldsymbol{\xi}}_t$$

where the vector $\boldsymbol{\theta}_t = \left(\theta_t^1, ..., \theta_t^n\right)'$ follows a two-state, continuous time Markov chain with regimes $\boldsymbol{\theta}_T = \left(\theta_T^1, ..., \theta_T^n\right)$ and $\boldsymbol{\theta}_P = \left(\theta_P^1, ..., \theta_P^n\right)$. Also, we shall denote by $\Delta\theta^i = \theta_P^i - \theta_T^i$ and by $\Delta\boldsymbol{\theta} = \boldsymbol{\theta}_P - \boldsymbol{\theta}_T$. In addition, denote the orthogonalized expectetaion error in (2) by

$$d\mathbf{v}_t = \mathbf{\Phi}^{-1}\left(d\mathbf{D}_t - E\left(d\mathbf{D}_t|\mathcal{F}_t\right)\right)$$

Then the conditional posterior probability process (2) can be written as

$$d\pi_t = (\lambda + \mu)(\pi^s - \pi_t)dt + \mathbf{h}(\pi_t)d\mathbf{v}_t$$

where

$$\mathbf{h}(\pi_t) = \pi_t\left(1 - \pi_t\right)\Delta\boldsymbol{\theta}'\left(\mathbf{\Phi}'\right)^{-1}$$

Finally, it is convenient to rewrite the dividend processes on the filtered space $(\Omega, P, \mathcal{F}_t)$, where $\{\mathcal{F}_t\}$ is the filtration generated by $d\mathbf{D}$, as

$$d\mathbf{D}_t = \mathbf{m}_t dt + \mathbf{\Phi} d\mathbf{v}_t$$

where $\mathbf{m}_t = \pi_t\boldsymbol{\theta}_P + (1 - \pi_t)\boldsymbol{\theta}_T$.

Proof of Proposition 3: We conjecture that

$$P_i\left(D_i, \pi\right) = p_{0i} + S_i(\pi) + p_D D_i + p_{1i} + p_{\pi i}\pi$$

Using Ito's Lemma

$$dP_i = \alpha_{Pi}\left(\pi\right)dt + \boldsymbol{\sigma}_{Pi}\left(\pi\right)d\mathbf{v}$$

where

$$\alpha_{Pi}(\pi) = \frac{1}{r}m_i + (p_{\pi i} + S_i'(\pi))(\pi^s - \pi)(\lambda + \mu) + \frac{1}{2}S_i''(\pi)\,\mathbf{hh}'$$
$$\boldsymbol{\sigma}_{Pi} = (p_{\pi i} + S_i'(\pi))\,\mathbf{h} + p_D\boldsymbol{\phi}_i$$

Hence, defining $\boldsymbol{\Sigma}(\pi) = (\boldsymbol{\sigma}_{P1}(\pi), ..., \boldsymbol{\sigma}_{Pn}(\pi))'$, we can write

$$dW = \left((rW - c) + \sum_{i=1}^{n} X_i\left[(D_i - rP_i) + \alpha_{Pi}(\pi)\right]\right)dt + \mathbf{X\Sigma}(\pi)\,d\mathbf{v}$$

and therefore $E_t[dP_i dP_j] = \boldsymbol{\sigma}_{Pi}\boldsymbol{\sigma}'_{Pj}$. We now solve for the Bellman equation, given by

$$0 = \max_{c,X_1,...,X_n} U(c,t) + \frac{E_t[dJ]}{dt}$$

First, by using Ito's lemma we find:

$$E_t[dJ] = J_t + J_W E_t[dW] + J_\pi E_t[d\pi] + \frac{1}{2}J_{WW}E_t\left[dW^2\right] + \frac{1}{2}J_{\pi\pi}E_t\left[d\pi^2\right] + J_{W\pi}E_t[dW d\pi]$$

Since we have

$$E_t[dW] = \left((rW - c) + \sum_{i=1}^{n} X_i\left[(D_i - rP_i) + \alpha_{Pi}(\pi)\right]\right)dt$$
$$E_t\left[d\pi^2\right] = \mathbf{hh}'dt;\; E_t\left[dW^2\right] = \mathbf{X\Sigma\Sigma'X'}dt;\; E_t[dW d\pi] = \mathbf{X\Sigma h}'dt$$

we can substitute in the Bellman equation to obtain

$$0 = \max_{c,X} U(c) + J_t + J_W\left((rW - c) + \sum_{i=1}^{n} X_i\left[D_i - rP_i + \alpha_{Pi}(\pi)\right]\right) +$$
$$+ J_\pi(\pi^s - \pi)(\lambda + \mu) + \frac{1}{2}J_{WW}\mathbf{X\Sigma\Sigma'X'} + \frac{1}{2}J'_{\pi\pi}\mathbf{hh}' + J_{W\pi}\mathbf{X\Sigma h}'$$

The first order conditions are $U'(c) = J_W$ and

$$J_W\left[\mathbf{D} - r\mathbf{P} + \boldsymbol{\alpha}_P(\pi)\right] + J_{WW}\mathbf{\Sigma\Sigma'X'} + J_{W\pi}\mathbf{\Sigma h}' = 0 \qquad (14)$$

where

$$\left[\mathbf{D} - r\mathbf{P} + \boldsymbol{\alpha}_P(\pi)\right] = \begin{pmatrix} \left[D_1 - rP_1 + \alpha_{P1}(\pi)\right] \\ \\ \left[D_n - rP_n + \alpha_{Pn}(\pi)\right] \end{pmatrix}$$

Hence:

$$\mathbf{X}' = -\frac{J_W}{J_{WW}}\left(\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\right)^{-1}\left[\mathbf{D}-r\mathbf{P}+\boldsymbol{\alpha}_P\left(\pi\right)\right] - \frac{J_{W\pi}}{J_{WW}}\left(\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\right)^{-1}\boldsymbol{\Sigma}\mathbf{h}'$$

Now, we conjecture that the value function is given by $J(W,\pi) = -e^{-\rho t - r\gamma W - g(\pi) - \beta}$ for some function $g(\pi)$ to be defined in equilibrium. Then we obtain

$$J_W = -r\gamma J; J_\pi = -g'\left(\pi\right)J; J_{WW} = \left(r\gamma\right)^2 J; J_{\pi\pi}\left(g'(\pi)^2 - g''(\pi)\right)J;\ \ J_{W\pi} = r\gamma g'(\pi)J$$

By substituting in the formula for consumption and portfolio allocation

$$c(W,\pi) = rW + \frac{1}{\gamma}\left(g\left(\pi\right) + \beta - \log\left(r\right)\right)$$

$$X' = \frac{1}{r\gamma}\left(\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\right)^{-1}\left[\mathbf{D}-r\mathbf{P}+\boldsymbol{\alpha}_P\left(\pi\right)\right] - \frac{g'\left(\pi\right)}{\gamma r}\left(\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{\theta}\right)^{-1}\boldsymbol{\Sigma}\mathbf{h}'$$

In equilibrium $\mathbf{X}' = \mathbf{1}_n$ implying

$$\mathbf{D}-r\mathbf{P}+\boldsymbol{\alpha}_P\left(\pi\right) - g'\left(\pi\right)\boldsymbol{\Sigma}\mathbf{h}' = r\gamma\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{\theta}1_n$$

Since

$$\mathbf{P}\left(D,\pi\right) = \mathbf{p}_0 + \mathbf{S}\left(\pi\right) + \frac{1}{r}\mathbf{D} + \mathbf{p}_1 + \mathbf{p}_\pi\pi$$

we have

$$\mathbf{D}-r\mathbf{P}+\boldsymbol{\alpha}_P\left(\pi\right) = -r\mathbf{p}_0 - r\mathbf{S}(\pi) + \mathbf{S}'(\pi)(\pi^s-\pi)(\lambda+\mu) + \frac{1}{2}\mathbf{S}''(\pi)\mathbf{h}\mathbf{h}' \tag{15}$$

$$\begin{aligned}\alpha_{Pi}(\pi) &= \frac{1}{r}m_i + \left(p_{\pi i} + S_i'\left(\pi\right)\right)\left(\pi^s - \pi\right)\left(\lambda+\mu\right) + \frac{1}{2}S_i''\left(\pi\right)\mathbf{h}\mathbf{h}' \\ \boldsymbol{\sigma}_{Pi} &= \left(p_{\pi i} + S_i'\left(\pi\right)\right)\mathbf{h} + p_D\boldsymbol{\phi}_i\end{aligned}$$

as we can make

$$-r\mathbf{p}_1 - r\mathbf{p}_\pi\pi + \frac{1}{r}\mathbf{m} + \mathbf{p}_\pi\left(\pi^s - \pi\right)\left(\lambda+\mu\right) = \mathbf{0}$$

because for every i we can choose p_{1i} and $p_{\pi i}$ so that

$$-rp_{1i} - rp_{\pi i}\pi + \frac{1}{r}\left(\pi\theta_i^P + \left(1-\pi\right)\theta_i^T\right) + p_{\pi i}\left(\pi^s - \pi\right)\left(\lambda+\mu\right) = 0$$

Indeed, collecting terms, we have that the equation

$$\pi\left(-\left(r+\lambda+\mu\right)p_{\pi i}+\frac{1}{r}\left(\theta_i^P-\theta_i^T\right)\right)-rp_{1i}+\frac{1}{r}\theta_i^T+p_{\pi i}\mu=0$$

implies

$$p_{\pi i}=\frac{\left(\theta_i^P-\theta_i^T\right)}{r\left(r+\lambda+\mu\right)},\ p_{i1}=\frac{\theta_i^T}{r^2}+\frac{\left(\theta_i^P-\theta_i^T\right)}{r^2\left(r+\lambda+\mu\right)}\mu$$

With these parameters for $p_{\pi i}$ and p_{i1}, we then obtain the system of n differential equations:

$$-r\mathbf{p}_0-r\mathbf{S}(\pi)+\mathbf{S}'(\pi)(\pi^s-\pi)(\lambda+\mu)+\frac{1}{2}\mathbf{S}''(\pi)\mathbf{hh}'-g'\left(\pi\right)\mathbf{\Sigma h}'=r\gamma\mathbf{\Sigma\Sigma}'\mathbf{1}_n$$

or, for every $i=1,...,n$:

$$-rp_{i0}-rS_i(\pi)+S_i'(\pi)(\pi^s-\pi)(\lambda+\mu)+\frac{1}{2}S_i''(\pi)\mathbf{hh}'-g'\left(\pi\right)\boldsymbol{\sigma}_{Pi}\mathbf{h}'=r\gamma\boldsymbol{\sigma}_{Pi}\sum_{k=1}^{n}\boldsymbol{\sigma}'_{Pk} \qquad (16)$$

Use now the market clearing condition and substitute $X_i=1$ in the Bellman equation again to obtain

$$\begin{aligned} 0 &= \max_{c,X}U(c)+J_t+J_W\left((rW-c)+\sum_{i=1}^{n}\left[D_i-rP_i+\alpha_{Pi}\left(\pi\right)\right]\right)+ \\ &\quad +J_\pi\left(\pi^s-\pi\right)\left(\lambda+\mu\right)+\frac{1}{2}J_{WW}\mathbf{X\Sigma\Sigma}'\mathbf{X}'+\frac{1}{2}J'_{\pi\pi}\mathbf{hh}'+J_{W\pi}\mathbf{X\Sigma h}'\end{aligned}$$

Substitute for $[D_i-rP_i+\alpha_{Pi}\left(\pi\right)]$ to obtain

$$\begin{aligned} 0 &= U(c)+J_t+J_W\left(rW-c-rp_0-rS(\pi)+S'(\pi)(\pi^s-\pi)(\lambda+\mu)+\frac{1}{2}S''(\pi)\mathbf{hh}'\right) \\ &\quad +J_\pi\left(\pi^s-\pi\right)\left(\lambda+\mu\right)+\frac{1}{2}J_{WW}\mathbf{1}_n'\mathbf{\Sigma\Sigma}'\mathbf{1}_n+\frac{1}{2}J'_{\pi\pi}\mathbf{hh}'+J_{W\pi}\mathbf{1}_n'\mathbf{\Sigma h}'\end{aligned}$$

where $p_0=\sum_{i=1}^n p_{i0}$, $S\left(\pi\right)=\sum_{i=1}^n S_i\left(\pi\right)$. Hence, the Bellman equation depends only on the aggregate discount $S\left(\pi\right)$. Notice that

$$\mathbf{1}_n'\mathbf{\Sigma h}'=\sum_{i=1}^{n}\boldsymbol{\sigma}_{Pi}\mathbf{h}' \text{ and } \mathbf{1}_n'\mathbf{\Sigma\Sigma}'\mathbf{1}_n=\sum_{i=1}^{n}\sum_{j=1}^{n}\boldsymbol{\sigma}_{Pi}\boldsymbol{\sigma}'_{Pj}$$

From the definitions, we have

$$\begin{aligned} \boldsymbol{\sigma}_{Pi}\mathbf{h}' &= \left(p_{\pi i}+S_i'\left(\pi\right)\right)\mathbf{hh}'+p_D\boldsymbol{\phi}_i\mathbf{h}' \\ \boldsymbol{\sigma}_{Pi}\boldsymbol{\sigma}'_{Pj} &= \left(p_{\pi i}+S_i'\left(\pi\right)\right)\left(p_{\pi j}+S_j'\left(\pi\right)\right)\mathbf{hh}'+p_D^2\boldsymbol{\phi}_i\boldsymbol{\phi}_j' \\ &\quad +\left(p_{\pi i}+S_i'\left(\pi\right)\right)\boldsymbol{\phi}_j\mathbf{h}'+\left(p_{\pi j}+S_j'\left(\pi\right)\right)\boldsymbol{\phi}_i\mathbf{h}'\end{aligned}$$

Hence, algebric manipulations show:

$$\begin{aligned}
\mathbf{1}_n'\mathbf{\Sigma}\mathbf{h}' &= \left(p_\pi + S'(\pi)\right)\mathbf{h}\mathbf{h}' + p_D\mathbf{1}_n\mathbf{\Phi}\mathbf{h}' \\
\mathbf{1}_n'\mathbf{\Sigma}\mathbf{\Sigma}'\mathbf{1}_n &= \left(p_\pi + S'(\pi)\right)^2\mathbf{h}\mathbf{h}' + p_D^2\mathbf{1}_n\mathbf{\Phi}\mathbf{\Phi}'\mathbf{1}_n' + 2\left(p_\pi + S'(\pi)\right)p_D\mathbf{1}_n\mathbf{\Phi}\mathbf{h}'
\end{aligned}$$

Hence, we find that the Bellman equation depends *only* on the aggregate $S(\pi)$ and not the singular ones.

From the definition of $h_j(\pi) = \pi(1-\pi)\left[\Delta\boldsymbol{\theta}'(\mathbf{\Phi}')^{-1}\right]_j$ we find $\mathbf{h}(\pi) = \pi(1-\pi)\Delta\boldsymbol{\theta}'(\mathbf{\Phi}')^{-1}$ which implies

$$\begin{aligned}
\mathbf{h}\mathbf{h}' &= \pi^2(1-\pi)^2\Delta\boldsymbol{\theta}'\left(\mathbf{\Phi}\mathbf{\Phi}'\right)^{-1}\Delta\boldsymbol{\theta} \\
\mathbf{1}_n\mathbf{\Phi}\mathbf{h}' &= \pi(1-\pi)\mathbf{1}_n\Delta\boldsymbol{\theta}
\end{aligned}$$

Define for convenience:

$$\underline{h}(\pi) = \pi(1-\pi);\ A = \Delta\boldsymbol{\theta}'\left(\mathbf{\Phi}\mathbf{\Phi}'\right)^{-1}\Delta\boldsymbol{\theta};\ B = \mathbf{1}_n\Delta\boldsymbol{\theta};\ \sigma_W^2 = \mathbf{1}_n\mathbf{\Phi}\mathbf{\Phi}'\mathbf{1}_n'$$

so that $\mathbf{h}\mathbf{h}' = \underline{h}^2A$, $\mathbf{1}_n'\mathbf{\Sigma}\mathbf{h}' = \left(p_\pi + S'(\pi)\right)\underline{h}^2A + \frac{1}{r}\underline{h}B$ and

$$\mathbf{1}_n'\mathbf{\Sigma}\mathbf{\Sigma}'\mathbf{1}_n = \left(p_\pi + S'(\pi)\right)^2\underline{h}^2A + \frac{1}{r^2}\sigma_W^2 + 2\frac{1}{r}\underline{h}B\left(p_\pi + S'(\pi)\right)$$

In the Bellman equation, substitute for c, J_t and all these quantities and divide by J to obtain

$$\begin{aligned}
0 &= r - \rho - r\gamma\left(-\frac{1}{\gamma}\left(g(\pi) + \beta - \log(r)\right) - rp_0 - rS(\pi) + S'(\pi)(\pi^s - \pi)(\lambda+\mu) + \frac{1}{2}S''(\pi)\underline{h}^2A\right) + \\
&\quad + J_\pi(\pi^s - \pi)(\lambda+\mu) + \frac{1}{2}J_{WW}\left(\left(p_\pi + S'(\pi)\right)^2\underline{h}^2A + \frac{1}{r^2}\sigma_W^2 + 2\frac{1}{r}\underline{h}B\left(p_\pi + S'(\pi)\right)\right) \\
&\quad + \frac{1}{2}J_{\pi\pi}\underline{h}^2A + J_{W\pi}\left(\left(p_\pi + S'(\pi)\right)\underline{h}^2A + \frac{1}{r}\underline{h}B\right)
\end{aligned}$$

Substitute now for J_π , J_W, J_{WW}, $J_{\pi\pi}$, $J_{W\pi}$ to obtain

$$\begin{aligned}
0 &= r - \rho - r\gamma\left(-\frac{1}{\gamma}\left(g(\pi) + \beta - \log(r)\right) - rp_0 - rS(\pi) + S'(\pi)(\pi^s - \pi)(\lambda+\mu) + \frac{1}{2}S''(\pi)\underline{h}^2A\right) + \\
&\quad - g'(\pi)(\pi^s - \pi)(\lambda+\mu) + \frac{1}{2}(r\gamma)^2\left(\left(p_\pi + S'(\pi)\right)^2\underline{h}^2A + \frac{1}{r^2}\sigma_W^2 + 2\frac{1}{r}\underline{h}B\left(p_\pi + S'(\pi)\right)\right) \\
&\quad + \frac{1}{2}\left(g'(\pi)^2 - g''(\pi)\right)\underline{h}^2A + r\gamma g'(\pi)\left(\left(p_\pi + S'(\pi)\right)\underline{h}^2A + \frac{1}{r}\underline{h}B\right)
\end{aligned}$$

Let

$$\beta = \frac{\gamma^2\sigma_W^2}{2r^2} + \frac{\rho}{r} + \log(r) - 1 \text{ and } p_0 = -\frac{\gamma\sigma_W^2}{r^2}$$

to obtain, after deleting common terms

$$\begin{aligned} 0 &= rg(\pi) - r\gamma\left(-rS(\pi) + S'(\pi)(\pi^s - \pi)(\lambda+\mu) + \frac{1}{2}S''(\pi)\underline{h}^2A\right) + \\ &\quad -g'(\pi)(\pi^s-\pi)(\lambda+\mu) + \frac{1}{2}(r\gamma)^2\left(\left(p_\pi + S'(\pi)\right)^2\underline{h}^2A + 2\frac{1}{r}\underline{h}B\left(p_\pi + S'(\pi)\right)\right) \\ &\quad +\frac{1}{2}\left(g'(\pi)^2 - g''(\pi)\right)\underline{h}^2A + r\gamma g'(\pi)\left(\left(p_\pi + S'(\pi)\right)\underline{h}^2A + \frac{1}{r}\underline{h}B\right) \end{aligned}$$

Now, let $g(\pi) = -r\gamma S(\pi) + f(\pi)$ for a function $f(\pi)$ to be determined. Substituting, algebric manipulations show that the following ODE has to be satisfied:

$$-f''(\pi)Q_3(\pi) + f'(\pi)^2Q_3(\pi) + f'Q_2(\pi) + f(\pi)r + Q_0(\pi) = 0$$

where

$$\begin{aligned} Q_3(\pi) &= \frac{1}{2}\underline{h}(\pi)^2A \\ Q_2(\pi) &= \gamma\underline{h}(\pi)B - (\pi^s - \pi)(\lambda+\mu) + r\gamma p_\pi\underline{h}^2A \\ Q_0(\pi) &= \frac{(r\gamma)^2}{2}p_\pi^2\underline{h}^2A + r\gamma^2\underline{h}Bp_\pi \end{aligned}$$

Veronesi (1999) shows that a solution to this equation exists and it is U shaped and convex. We can now rewrite (16) as

$$-rp_{i0} - rS_i(\pi) + S_i'(\pi)(\pi^s - \pi)(\lambda+\mu) + \frac{1}{2}S_i''(\pi)\mathbf{hh}' - g'(\pi)\boldsymbol{\sigma}_{iP}\mathbf{h}' = r\gamma\boldsymbol{\sigma}_{Pi}\sum_{k=1}^{n}\boldsymbol{\sigma}'_{Pk} \tag{17}$$

Again, notice that from the previous definitions, $\boldsymbol{\sigma}_{iP}\mathbf{h}' = \left(p_{\pi i} + S_i'(\pi)\right)\underline{h}^2A + \frac{h}{r}B_i$ with $B_i = \boldsymbol{\phi}_i\boldsymbol{\Phi}^{-1}\Delta\boldsymbol{\theta} = e_i\boldsymbol{\Phi}\boldsymbol{\Phi}^{-1}\Delta\boldsymbol{\theta} = e_i\Delta\boldsymbol{\theta} = \theta_P^i - \theta_T^i$. From

$$\begin{aligned} \boldsymbol{\sigma}_{Pi}\boldsymbol{\sigma}'_{Pj} &= \left(p_{\pi i} + S_i'(\pi)\right)\left(p_{\pi j} + S_j'(\pi)\right)\mathbf{hh}' + p_D^2\boldsymbol{\phi}_i\boldsymbol{\phi}_j' \\ &\quad + \left(p_{\pi i} + S_i'(\pi)\right)\boldsymbol{\phi}_j\mathbf{h}' + \left(p_{\pi j} + S_j'(\pi)\right)\boldsymbol{\phi}_i\mathbf{h}' \end{aligned}$$

we have

$$\boldsymbol{\sigma}_{Pi}\sum_{k=1}^{n}\boldsymbol{\sigma}'_{Pk} = \left(p_{\pi i} + S_i'(\pi)\right)\left(p_\pi + S'(\pi)\right)\underline{h}^2A + \left(p_\pi + S'(\pi)\right)\frac{h}{r}B_i$$

$$+\left(p_{\pi i}+S_i'(\pi)\right)\underline{h}B+\frac{1}{r^2}C_i$$

with $C_i=\boldsymbol{\phi}_i\boldsymbol{\Phi}'\mathbf{1}_n$. Hence, we obtain

$$-rp_{i0}-rS_i(\pi)+S_i'(\pi)(\pi^s-\pi)(\lambda+\mu)+\frac{1}{2}S_i''(\pi)\underline{h}^2A-g'(\pi)\left(\left(p_{\pi i}+S_i'(\pi)\right)\underline{h}^2A+\frac{h}{r}B_i\right) \quad (18)$$

$$= r\gamma\left(\left(p_\pi+S'(\pi)\right)\left(p_{i\pi}+S_i'(\pi)\right)\underline{h}^2A+\left(p_\pi+S'(\pi)\right)\frac{h}{r}B_i+\frac{1}{r}\left(p_{i\pi}+S_i'(\pi)\right)\underline{h}B+\frac{1}{r^2}C_i\right) \quad (19)$$

Substituting for $g'=-r\gamma S'+f'$ and setting $p_{i0}=-\frac{\gamma}{r^2}C_i$ we finally obtain

$$-rS_i(\pi)+S_i'(\pi)(\pi^s-\pi)(\lambda+\mu)+\frac{1}{2}S_i''(\pi)\underline{h}^2A-f'(\pi)\left(\left(p_{\pi i}+S_i'(\pi)\right)\underline{h}^2A+\frac{h}{r}B_i\right) \quad (20)$$

$$= r\gamma p_\pi\left(p_{i\pi}+S_i'(\pi)\right)\underline{h}^2A+\gamma p_\pi\underline{h}B_i+\gamma\left(p_{i\pi}+S_i'(\pi)\right)\underline{h}B \quad (21)$$

or

$$S_i''(\pi)\,P_3(\pi)=S_i'(\pi)P_2(\pi)+S_i(\pi)r+P_{0i}(\pi)$$

where

$$P_3(\pi) = \frac{1}{2}\underline{h}^2A \quad (22)$$

$$P_2(\pi) = \gamma\underline{h}\Delta\boldsymbol{\theta}'\mathbf{1}_n-(\pi^s-\pi)(\lambda+\mu)+r\gamma p_\pi\underline{h}^2A+f'(\pi)\underline{h}^2A \quad (23)$$

$$P_{0i}(\pi) = r\gamma p_\pi p_{i\pi}\underline{h}(\pi)^2A+\gamma(p_\pi\Delta\theta^i+p_{\pi i}\Delta\boldsymbol{\theta}'\mathbf{1}_n)\underline{h}(\pi)+f'(\pi)\left(p_{\pi i}\underline{h}(\pi)^2A+\frac{\underline{h}(\pi)}{r}\Delta\theta^i\right) \quad (24)$$

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Table 1. Average Covariances and Correlations

This table reports average covariances and correlations for aggregate stock market returns, gross domestic product and industrial production growth across seven international markets (United States, Canada, Japan, France, Germany, Switzerland and United Kingdom). The covariances and correlations are averages of the covariances and correlations with an equally-weighted index of the aggregate stock market returns, gross domestic product and industrial production growth rates. Results are based on monthly or quarterly data and reported in annualized terms. The recessions are based on the NBER definitions and the covariance matrices were calculated dividind the sample in two sub-periods. All variables are in local currency, but similar results hold when defined is US dollars terms

		Panel A- Monthly Data	
	Stock Returns	Industrial Production	Domestic Product
		Average Covariance	
Recession	0.0344	0.0006	-
Boom	0.0152	0.0005	-
		Average Correlation	
Recession	0.7847	0.5295	-
Boom	0.7229	0.5207	-
		Panel B- Quarterly Data	
	Stock Returns	Industrial Production	Domestic Product
		Average Covariance	
Recession	0.0560	0.0008	0.0003
Boom	0.0163	0.0003	0.0001
		Average Correlation	
Recession	0.8546	0.6439	0.5807
Boom	0.7263	0.5558	0.4801

Table 2. Summary Statistics

This table reports the summary statistics for GDP growth rates and aggregate market returns. All statistics are reported in annualized terms.

	US	*CA*	*JA*	*FR*	*GE*	*UK*	*SW*
				Gross Domestic Product			
Average	0.0256	0.0307	0.0277	0.0261	0.0284	0.0239	0.0183
Std.Dev.	0.0202	0.0221	0.0292	0.0159	0.0360	0.0294	0.0317
				Correlation			
US	1.0000						
CA	0.3883	1.0000					
JA	0.2558	0.0595	1.0000				
FR	0.2830	0.2302	0.2478	1.0000			
GR	0.0661	-0.1625	0.1096	0.2173	1.0000		
UK	0.2317	0.0981	0.4160	0.2344	0.1922	1.0000	
SW	0.0529	0.0280	0.1341	0.1231	0.1733	0.0347	1.0000
				Returns			
Average	0.0926	0.0888	0.0718	0.1051	0.0926	0.0929	0.0855
Std.Dev.	0.1539	0.1726	0.1896	0.2089	0.1839	0.2108	0.1744
				Correlation			
US	1						
CA	0.7286	1.0000					
JP	0.3390	0.3214	1.0000				
FR	0.4926	0.4814	0.3313	1.0000			
GE	0.4207	0.3738	0.3211	0.5916	1.0000		
UK	0.5715	0.5324	0.3062	0.5104	0.3994	1.0000	
SW	0.5908	0.5141	0.3524	0.5649	0.6298	0.5626	1.0000

Table 3. Regime Switching Model Estimates - Changes in Detrended GDP

This table reports all the estimates of the Markov swithing model with detrended GDP. The data was detrended following the method in Campbel and Kyle (1993). T-stat refers to the t-statistics of the coeffecients using the asymptotic standard errors. Correlation and Standard Deviation refer to the statistics of the residuals. All results are reported in quarterly terms. The probability of switching from the "Peak " to "Trough " is 0.0499, while the reverse happens with probability 0.2725. The standard deviation of these probabilities are respectively, 0.0237 and 0.1067.

	US	*CA*	*JA*	*FR*	*GE*	*UK*	*SW*
θ_P	0.0019	0.0029	0.0004	0.0003	0.0008	0.0014	0.0024
t-stat	2.0948	2.8435	0.2279	0.4008	0.6382	0.9753	1.5234
θ_T	-0.0101	-0.0150	-0.0019	-0.0018	-0.0043	-0.0076	-0.0124
t-stat	-5.8920	-4.3700	-5.5394	-0.4830	-0.8760	-1.4193	-2.0222
$\theta_P - \theta_T$	0.0120	0.0178	0.0023	0.0021	0.0051	0.0090	0.0148
t-stat	4.8552	6.3803	0.5578	0.9568	1.5406	2.0632	3.5057
				Correlation			
US	1.0000						
CA	0.1885	1.0000					
JA	0.2673	0.0321	1.0000				
FR	0.2815	0.2143	0.2530	1.0000			
GR	0.1248	0.0005	0.1315	0.3394	1.0000		
UK	0.1584	-0.0310	0.4284	0.2306	0.2251	1.0000	
SW	-0.0959	-0.1940	0.1379	0.0992	0.3144	-0.0411	1.0000
Std.Dev.	0.0090	0.0101	0.0161	0.0085	0.0128	0.0145	0.0156

Table 4. Regime Switching Model Estimates - GDP Growth Rate

This table reports all the estimates of the Markov swithing model. T-stat refers to the t-statistics of the coeffecients using the asymptotic standard errors. All results are reported in quarterly terms. The probability of switching from the "Peak " to "Trough " is 0.0501, while the reverse happens with probability 0.2716. The standard deviation of these probabilities are respectively, 0.0259 and 0.1150. We also report the equally-weighted average correlation and covariance of the stock returns.

	US	CA	JA	FR	GE	UK	SW
	Regime Switching Estimates						
θ_P	0.0084	0.0103	0.0073	0.0069	0.0067	0.0074	0.0068
t-stat	9.0767	10.8782	4.9053	8.7668	5.6289	4.7306	4.3857
θ_T	-0.0040	-0.0059	0.0049	0.0048	0.0016	-0.0017	-0.0069
t-stat	-1.6346	-2.2649	1.4417	2.3757	0.5442	-0.4403	-1.7781
$\theta_P - \theta_T$	0.0124	0.0162	0.0025	0.0021	0.0052	0.0091	0.0136
t-stat	4.7575	6.1755	0.6552	0.9800	1.6142	2.1261	3.2149
	Returns Data - Ave.Covariances and Correlations						
Ave. Cov.	0.0117	0.0119	0.0108	0.0168	0.0138	0.0147	0.0139
Ave. Corr.	0.7114	0.6610	0.4983	0.7451	0.6613	0.7128	0.7321

Table 5. Implied and Actual Average Covariances and Correlations in Booms and Recessions

This table reports the average covariance and correlation with the world market equally-weighted index during recessions and booms for the actual datal and the model. Two different definitions of recession are considered: NBER recession dates and the estimated global recession determined by the regime switching model. A certain period is said to be a recession if the probability of global boom is less than 0.5.

	Returns - Data	Returns - Model	Returns - Data	Returns - Model
	NBER Recessions		Estimated Global Recessions	
	Average Covariance		Average Covariance	
Recession	0.0344	0.0167	0.0363	0.0272
Boom	0.0152	0.0096	0.0205	0.0072
	Average Correlation		Average Correlation	
Recession	0.7847	0.7382	0.8108	0.7913
Boom	0.7229	0.6926	0.7589	0.6815

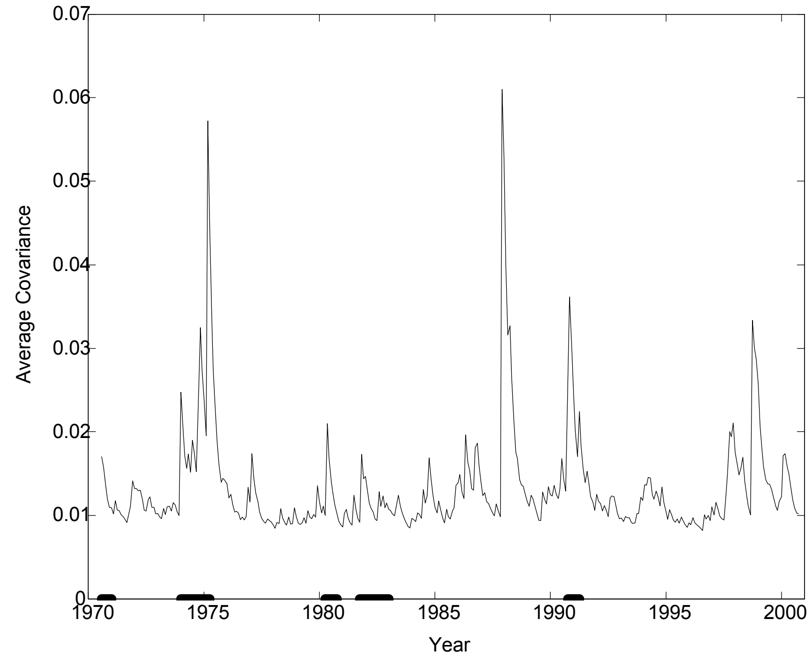

Figure 1: Conditional Average Covariance across International Stock Markets

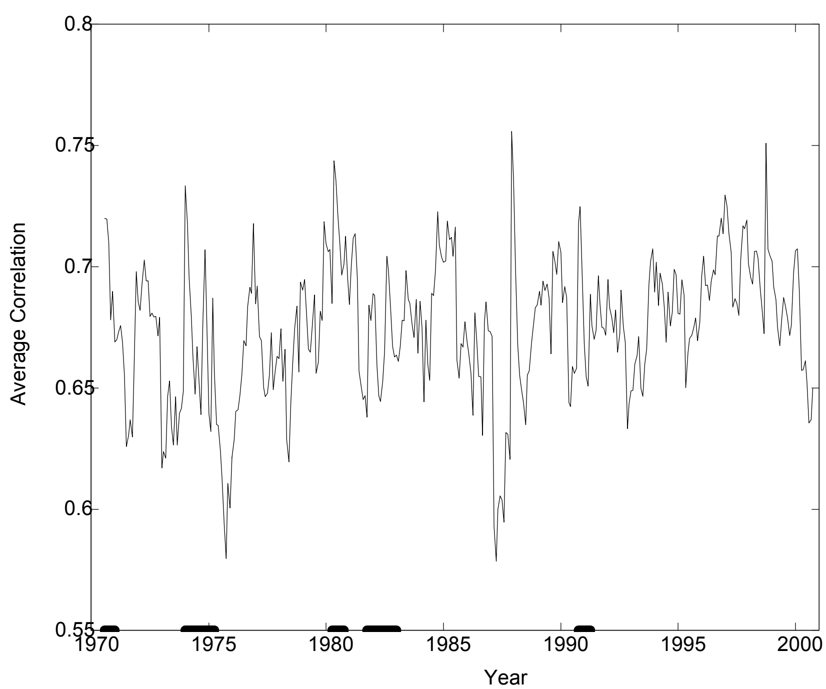

Figure 2: Conditional Average Correlation across International Stock Markets

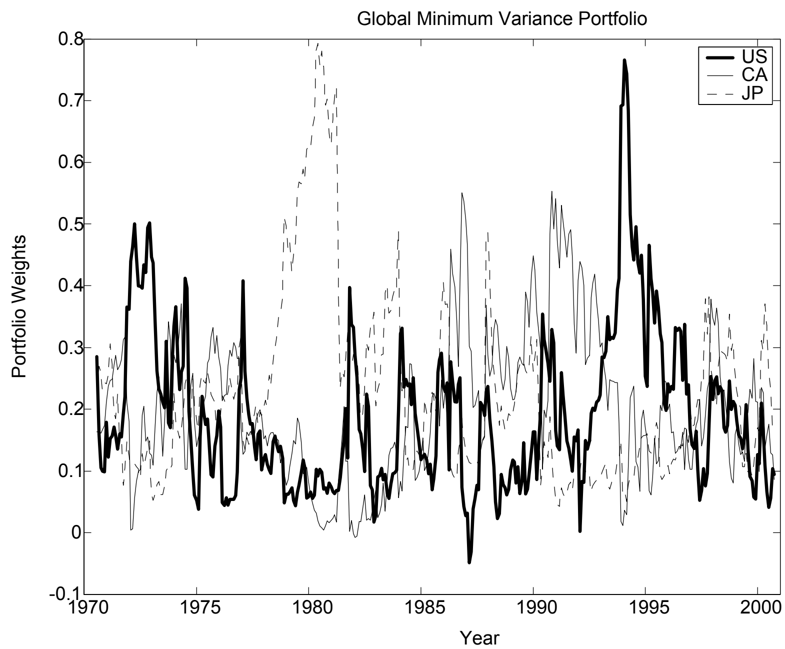

Figure 3: Global Minimun Variance Portfolio - Selected Countries

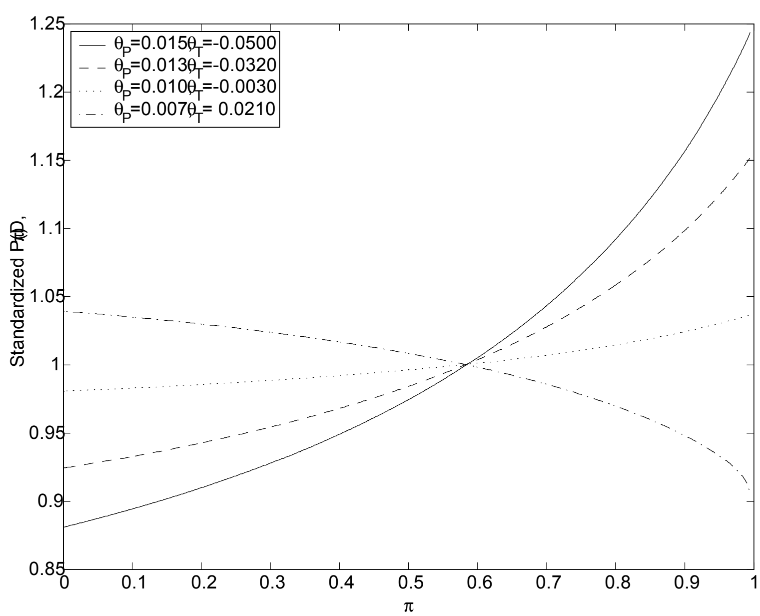

Figure 4: Standardized Price Functions for Different $\left(\theta_P^i - \theta_T^i\right)$

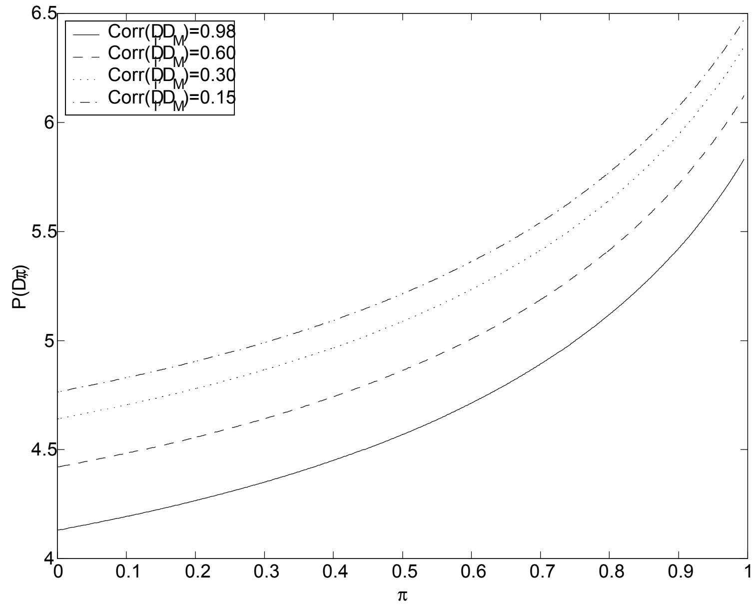

Figure 5: Price Function for Different Correlation between Individual Stock and Market Dividends Innovations

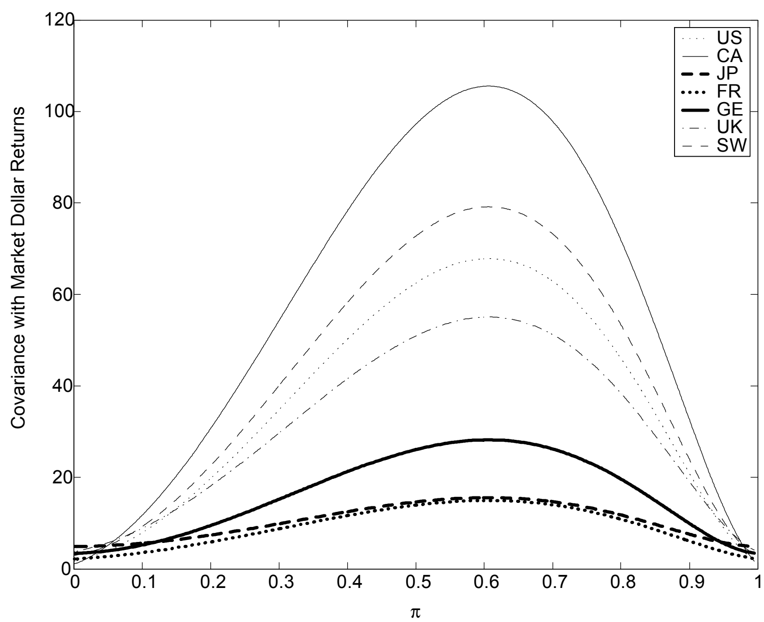

Figure 6: Covariance between Individual Stock and Market Dollar Returns

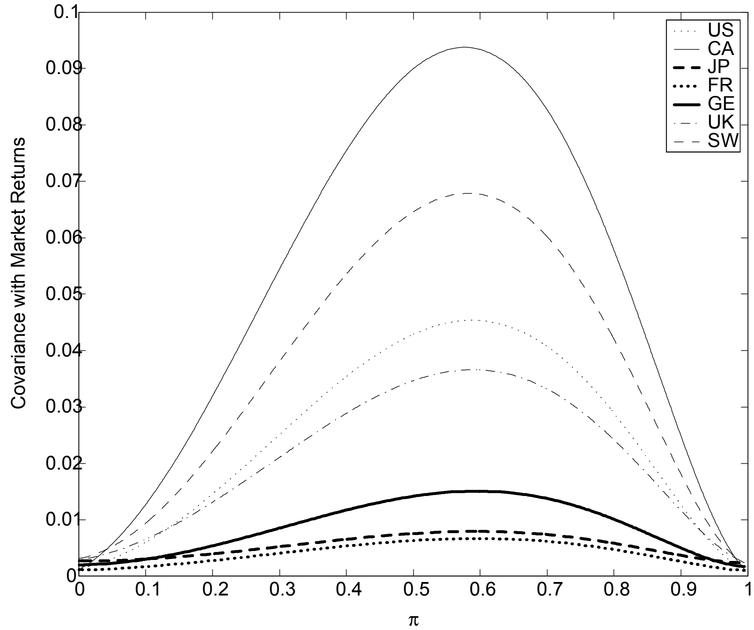

Figure 7: Covariance between Individual Stock and Market Returns

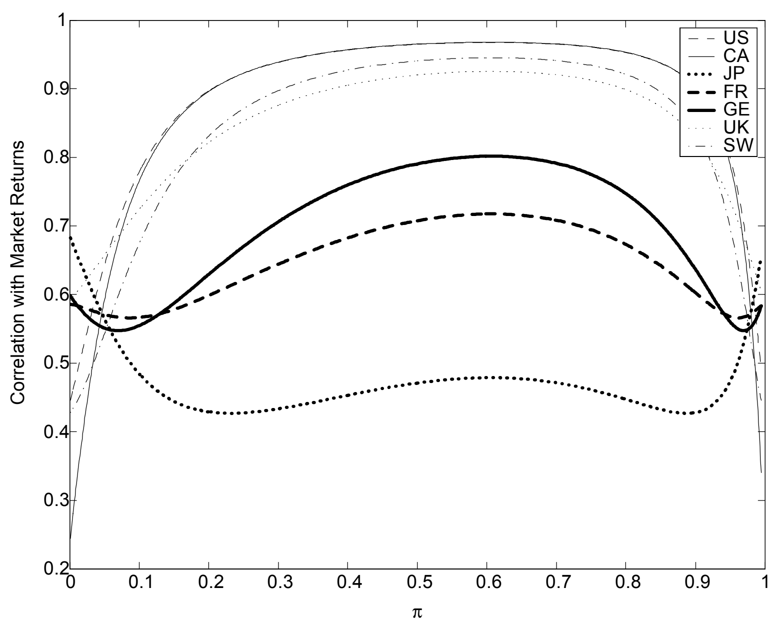

Figure 8: Correlation between Individual Stock and Market Returns

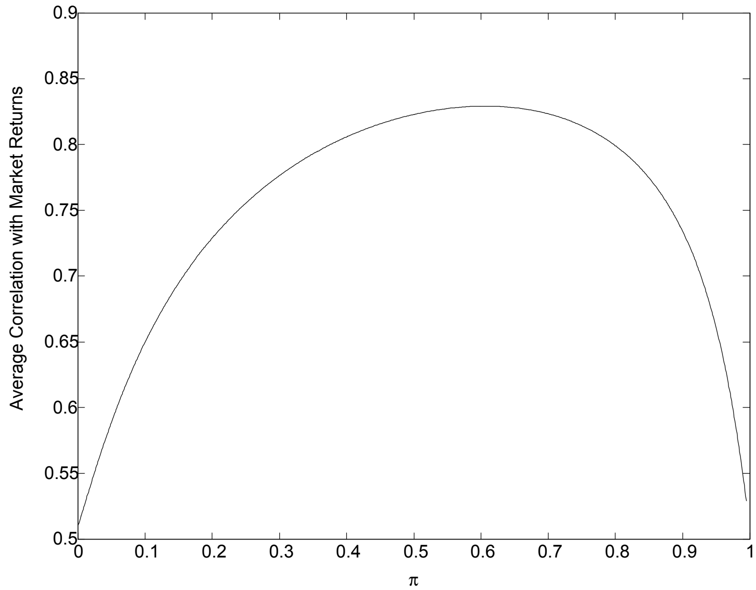

Figure 9: Average Correlation across International Markets for Different Probabilities of Boom

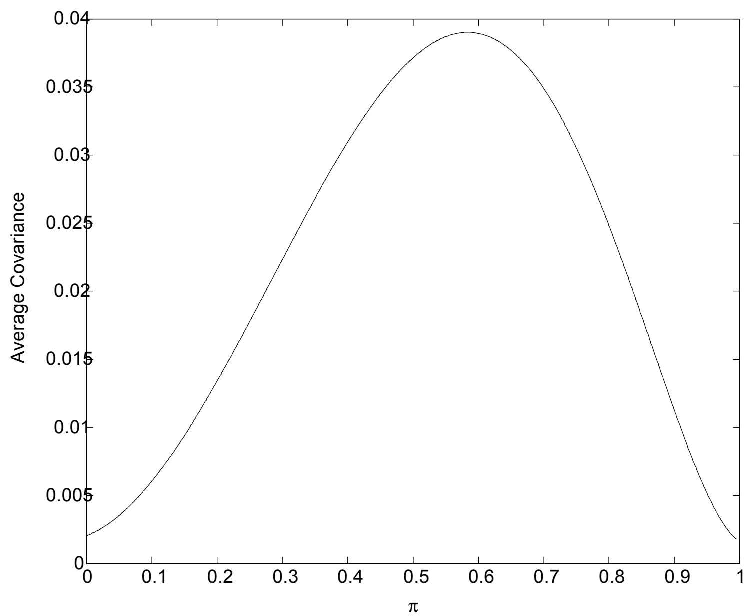

Figure 10: Average Covariance across International Markets for Different Probabilities of Boom

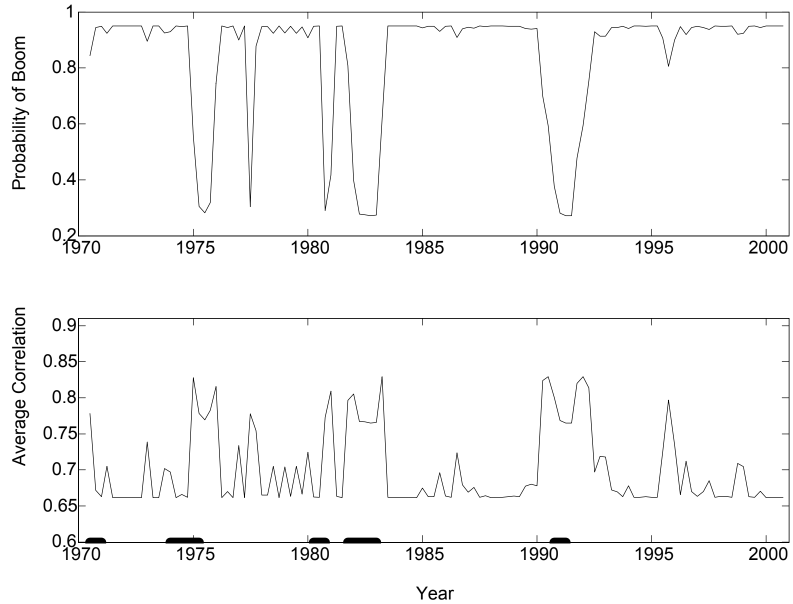

Figure 11: Implied Average Correlation among International Markets

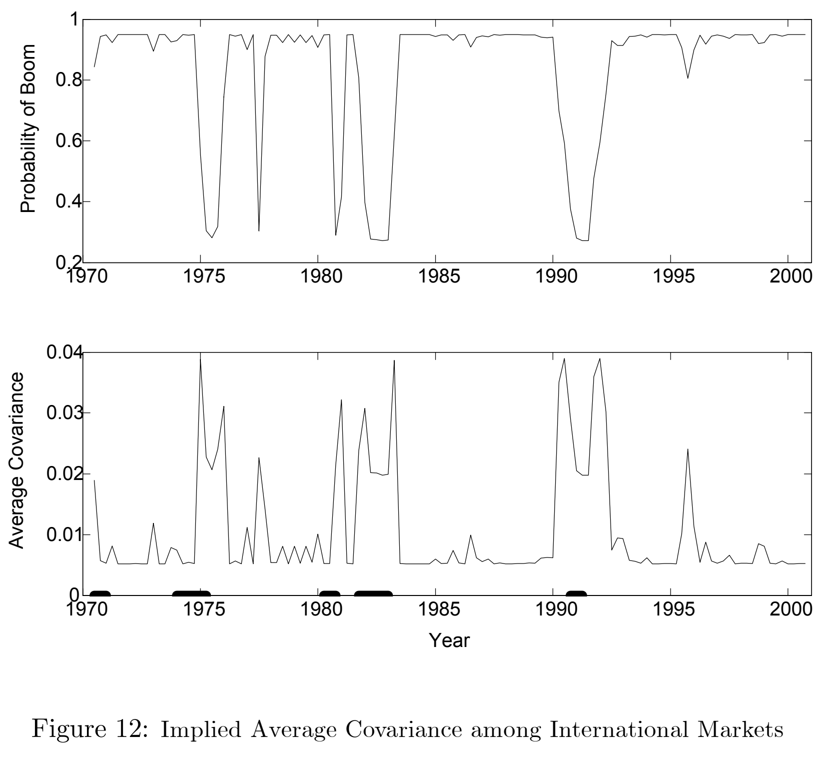

Figure 12: Implied Average Covariance among International Markets