

Was there a Nasdaq bubble in the late 1990s?

Ľuboš Pástor ^{a*}, Pietro Veronesi ^a

^a*University of Chicago, 5807 S. Woodlawn Ave., Chicago, IL 60637, USA
CEPR and NBER*

Received 14 December 2004; received in revised form 25 April 2005; accepted 18 May 2005

Abstract

Not necessarily: a firm's fundamental value increases with uncertainty about average future profitability, and this uncertainty was unusually high in the late 1990s. After calibrating a stock valuation model that takes this uncertainty into account, we compute the level of uncertainty that is needed to match the observed Nasdaq valuations at their peak. The uncertainty we obtain seems plausible because it matches not only the high level but also the high volatility of Nasdaq stock prices. In general, we argue that the level and volatility of stock prices are positively linked through firm-specific uncertainty about average future profitability.

JEL classifications: G12

Keywords: bubble, Nasdaq, valuation, uncertainty, volatility

For helpful comments, we thank Dan Bens, John Campbell, George Constantinides, Doug Diamond, Darrell Duffie, Gene Fama, Ken French, Marcin Kacperczyk, Mark Kamstra, Raymond Kan, Leonid Kogan, Stefan Nagel, Robert Novy-Marx, Lasse Pedersen, Lukasz Pomorski, Dick Roll, Steve Ross, Ioanid Rosu, Mark Rubinstein, Tano Santos, Rob Stambaugh, Jeremy Stein, Dick Thaler, Stijn Van Nieuwerburgh, Dimitri Vayanos, and Tuomo Vuolteenaho. We also thank the seminar participants at the following conferences: NBER (Asset Pricing and Entrepreneurship meetings), 2005 American Finance Association Meetings, the University of Chicago Conference Honoring the Work of Eugene F. Fama, CEPR/CREI Conference on Asset Price Bubbles (Universitat Pompeu Fabra), ESSFM at Gerzensee, Society of Quantitative Analysts, The III Workshop on Dynamic Macroeconomics (Bocconi University), UC-Davis Conference on Valuation in Financial Markets, and the following universities: Berkeley, Bocconi, Chicago, Columbia, HEC, INSEAD, Iowa, LBS, Maryland, Michigan, Purdue, Rice, Stanford, San Diego, Southern California, British Columbia, UCLA, Wisconsin, and Yale.

**Corresponding author contact information: E-mail address: lubos.pastor@chicagosb.edu*

“Before we relegate a speculative event to the fundamentally inexplicable or bubble category driven by crowd psychology, however, we should exhaust the reasonable economic explanations... “bubble” characterizations should be a last resort because they are non-explanations of events, merely a name that we attach to a financial phenomenon that we have not invested sufficiently in understanding.” Garber (2000, p.124)

1. Introduction

On March 10, 2000, the Nasdaq Composite Index closed at its all-time high of 5,048.62. In comparison, the same index stood at 1,114 in August 1996 as well as in October 2002. The unusual rise and fall in the prices of technology stocks has led many academics and practitioners to describe the event as a stock price “bubble.”¹ This label seems appropriate if the term “bubble” is interpreted as an ex post description of an extended rise in prices followed by a sharp fall (e.g., Kindleberger, 1978). However, a more common interpretation is that the prices of technology stocks exceeded their fundamental values in the late 1990s. This paper analyzes whether technology stocks were indeed overvalued at that time.

An important determinant of a firm’s fundamental value is uncertainty about the firm’s average future profitability, which can also be thought of as uncertainty about the average future growth rate of the firm’s book value. This uncertainty increases fundamental value (Pástor and Veronesi, 2003). We argue that the late 1990s witnessed high uncertainty about the average growth rates of technology firms, and that this uncertainty helps us understand the observed high level and volatility of technology stock prices.

To illustrate how uncertainty about a firm’s growth rate raises the firm’s fundamental value, consider the Gordon growth model, $P/D = 1/(r - g)$, where P is the stock price, D is the next period’s dividend, r is the discount rate, and g is the mean dividend growth rate. If g is uncertain, then P/D is equal to the expectation of $1/(r - g)$, under the conditions discussed in the Appendix. This expectation increases with uncertainty about g because $1/(r - g)$ is convex in g . Intuitively, uncertainty about g makes the distribution of future dividends right-skewed. Although this uncertainty may increase or decrease r , it always increases expected future dividends, and its overall effect on P/D is positive. (We prove these statements in the Appendix, where we generalize the Gordon model in two different ways.) Loosely speaking, a firm with some probability of failing (a very low g) and some probability of becoming the next Microsoft (a very high g) is very valuable.²

¹See, for example, Thaler (1999), Shiller (2000), Ofek and Richardson (2002, 2003), Ritter and Welch (2002), Abreu and Brunnermeier (2003), Brunnermeier and Nagel (2004), Ljungqvist and Wilhelm (2003).

²Bill Miller, portfolio manager of the Legg Mason Value Trust, used similar logic to justify the valuation of Amazon.com in 1999: “...being wrong isn’t very costly, and being right has a high payoff... With Amazon, we believe the payoff for being right is high.” Amazon’s Allure..., Barron’s, November 15, 1999.

Ofek and Richardson (2002) argue that the earnings of Internet firms would have to grow at implausibly high rates to justify the Internet stock prices of the late 1990s. Their argument implicitly assumes that the earnings growth rate is known. However, when uncertainty about the growth rate is acknowledged, the observed price can be justified by a significantly lower expected growth rate. For example, consider a stock with $r = 20\%$ and $P/D = 50$. To match the observed P/D in the Gordon formula with a known value of g , the required dividend growth rate is $g = 18\%$. Suppose instead that g is unknown and that it is drawn from a uniform distribution with a standard deviation of 4% . The expected g required to match the P/D of 50 then drops to 13.06% . Mathematically, Jensen’s inequality implies that

$$\frac{P}{D} = E\left(\frac{1}{r - g}\right) > \frac{1}{r - E(g)}, \quad (1)$$

so simply plugging the expected growth rate, $E(g)$, into the Gordon formula understates the P/D ratio. This understatement is especially large when uncertainty about g is large.

Although the Gordon model illustrates our basic idea, it is not well suited for pricing technology firms because many of these firms pay no dividends. To avoid this problem, we develop a stock valuation model that focuses on the ratio of the market value to book value of equity (M/B) instead of the price-dividend ratio. In our closed-form pricing formula, M/B is an increasing function of the uncertainty about the average growth rate of the firm’s book value. The pricing formula can be inverted to compute “implied uncertainty,” i.e., the level of uncertainty that sets the firm’s model-implied M/B equal to the observed M/B.

We calibrate the valuation model and compute the implied uncertainty of the Nasdaq-traded firms on March 10, 2000. We argue that the uncertainty level we obtain is plausible because it implies a return volatility that is close to the volatility observed in the data. The Nasdaq stock prices in the late 1990s were not only high but also highly volatile, and both facts are consistent with high uncertainty about average profitability.

There are good reasons to believe that uncertainty about the average future growth rates of technology firms increased in the late 1990s. The past decade witnessed rapid technological progress, especially in the Internet and telecom industries.³ Technological revolutions are likely to be accompanied by high uncertainty about future growth. When old paradigms fade away as some investors herald the arrival of a “new era,” uncertainty increases because historical experience is discounted. Indeed, the popular press contains numerous anecdotes

³For example, the number of utility patents (i.e., patents for inventions) granted by the U.S. Patent and Trademark Office in 1999 (153,485) was 51% higher than the number granted in 1995 (101,419), which was 5% higher than the number granted in 1991 (96,513). See www.uspto.gov for more information.

of investors being unusually uncertain about future growth in the late 1990s.⁴

Empirical evidence also supports the conjecture of an increase in uncertainty about the growth rates of technology firms in the late 1990s. First, Nasdaq return volatility increased dramatically at that time; Schwert (2002) attributes the high Nasdaq volatility to the technology sector. Second, the dispersion of profitability across Nasdaq stocks increased in the late 1990s. Third, the stock price reaction to earnings announcements was unusually strong in the late 1990s (e.g., Ahmed, Schneible, and Stevens, 2003, and Landsman and Maydew, 2002), which is consistent with high uncertainty because signals elicit large revisions in beliefs when prior uncertainty is high. Fourth, technology firms went public unusually early in their lifecycles in the late 1990s (e.g., Schultz and Zaman, 2001), and uncertainty about the future growth rates of firms with short earnings track records should be especially high. Finally, Pástor and Veronesi (2005) argue that high uncertainty about the average profitability of new firms attracted many private firms to go public at the end of the past decade.

The turn of the millenium can be characterized not only by high uncertainty but also by a low equity premium. Although academics do not agree on the exact magnitude of the premium, they tend to agree that the premium was relatively low in the late 1990s (e.g., Welch, 2001). We argue that the low equity premium amplified the effect of uncertainty on stock prices in the late 1990s. In the Gordon formula, the convexity of P/D in g is strongest when r is low. The same intuition holds in our model. When the discount rate is low, a large fraction of firm value comes from earnings in the distant future, and those earnings are the most affected by uncertainty about the firm's average future growth rate.

While an asset price bubble caused by investor irrationality can burst at any time for any reason, there was a fundamental reason for Nasdaq prices to come down after the 1990s, namely, an unprecedented decline in the profitability of Nasdaq-traded firms in 2000 and 2001. In our model, low realized profitability induces investors to revise their expectations of future profitability downward, which in turn pushes prices down. We demonstrate that the model is capable of producing a post-peak Nasdaq price decline that is comparable to (in fact, larger than) that observed in the data.

Two recent studies provide different explanations for the high valuations of technology stocks in the late 1990s. According to Ofek and Richardson (2003), these valuations were high

⁴E.g., "...the projections of revenue growth were, by and large, wild guesses." New Economy, Bad Math, ... Avital Loria Hahn, Investment Dealers Digest, October 23, 2000. E.g., "The problem is that since we know so little about where the Net is headed, predicting cash flow so far into the future is largely meaningless... investing in this new technology was a bet..." You Believe? ... Fortune Magazine, June 7, 1999. Trueman (2001) discusses the Internet firms' "highly unpredictable growth rates."

due in part to short-sale constraints. According to Cochrane (2003), technology stocks were valued highly because they offered high convenience yields. Neither study demonstrates that the magnitudes of these effects could be sufficiently large to justify the observed valuations of Nasdaq firms. In contrast, our calibration shows that the effect of uncertainty can be strong enough to rationalize those valuations. Moreover, neither of the two studies explains why the prices of technology stocks were so volatile at that time. In our model, the high return volatility is a natural consequence of high uncertainty.

In another related study, Schwartz and Moon (2000) argue that the observed valuations of the Internet stocks can be rationalized by revenue growth that is both sufficiently high and sufficiently volatile. They calibrate their model to match the valuation of Amazon.com, but they report that the implied return volatility is too high and that the implied revenue distribution is unrealistic. Our model, which is substantially different from theirs, produces distributions of returns and cash flows that seem more realistic.

This paper is also related to the theoretical literature on asset bubbles, e.g., Tirole (1985), Allen and Gorton (1993), Santos and Woodford (1997), Abreu and Brunnermeier (2003), Allen, Morris, and Shin (2003), and Scheinkman and Xiong (2003). Garber (2000) proposes rational explanations (unrelated to ours) for the Dutch tulip mania of the 1630s and other historical “bubble” episodes. Donaldson and Kamstra (1996) develop a neural network model for dividends, and use it to dismiss the idea of a stock price bubble in the 1920s.

Note that we do not claim that investor behavior in the late 1990s was fully rational. Good examples of apparent irrationality are presented by Cooper, Dimitrov, and Rau (2001), Lamont and Thaler (2003), and others. Also, we do not attempt to rule out any behavioral explanations for the “bubble.” Rather, we only argue that such explanations may not be necessary because stock prices in March 2000 appear to be consistent with a rational model. Finally, we do not claim that there was no “bubble;” instead, we argue that it is not obvious that there was one. We compute the beliefs about future profitability that can rationalize the observed Nasdaq prices. As long as some readers find these beliefs plausible, as we do, the notion of a Nasdaq “bubble” caused by investor irrationality should not be held as a self-evident truth.

A broader contribution of this paper is to calibrate a rational valuation model to match the level and volatility of stock prices, both for the Nasdaq and NYSE/Amex indexes as well as for individual Nasdaq firms. The parameters we use in the calibration also match the time-series properties of firm and index profitability, and produce reasonable levels and volatilities for the equity premium and the risk-free rate. To our knowledge, a similar calibration of the

level and volatility of stock prices is not present in the literature.

The paper is organized as follows. Section 2 describes and Section 3 calibrates the model. Section 4 computes the implied uncertainty on March 10, 2000 for the Nasdaq index as well as for individual firms such as Amazon, eBay, and Yahoo. Section 5 investigates the model's performance after March 2000. Sections 6 and 7 examine the time series and cross-section of implied uncertainty, Section 8 discusses selected additional issues, and Section 9 concludes.

2. The model

The stock valuation model that we develop in this section builds on the models of Pástor and Veronesi (2003, 2005; henceforth PV). Let $\rho_t^i = Y_t^i/B_t^i$ denote firm i 's instantaneous profitability at time t , where Y_t^i is the earnings rate and B_t^i is the book value of equity. We assume that profitability follows a mean-reverting process:

$$d\rho_t^i = \phi^i (\bar{\rho}_t^i - \rho_t^i) dt + \sigma_{i,0}dW_{0,t} + \sigma_{i,i}dW_{i,t}, \quad \phi^i > 0, \quad t < T_i, \quad (2)$$

where $W_{0,t}$ and $W_{i,t}$ are uncorrelated Wiener processes that capture the systematic ($W_{0,t}$) and firm-specific ($W_{i,t}$) components of the random shocks that drive the firm's profitability. We also assume that the firm's average profitability, $\bar{\rho}_t^i$, can be decomposed as

$$\bar{\rho}_t^i = \bar{\rho}_t + \bar{\psi}_t^i. \quad (3)$$

The common component, $\bar{\rho}_t$, exhibits mean-reverting variation that reflects business cycles:

$$d\bar{\rho}_t = k_L (\bar{\rho}_L - \bar{\rho}_t) dt + \sigma_{L,0}dW_{0,t} + \sigma_{L,L}dW_{L,t}, \quad k_L > 0, \quad (4)$$

where $W_{L,t}$ is uncorrelated with both $W_{0,t}$ and $W_{i,t}$. The firm-specific component, $\bar{\psi}_t^i$, which we refer to as the firm's average excess profitability, slowly decays to zero:

$$d\bar{\psi}_t^i = -k_\psi \bar{\psi}_t^i dt, \quad k_\psi > 0, \quad t < T_i. \quad (5)$$

Positive average excess profitability can reflect market power or intangible assets. The gradual decay in $\bar{\psi}_t^i$ can be interpreted as an outcome of slow-moving competitive market forces.

Competition in the firm's product market can also arrive suddenly, at some random future time T_i . We assume that T_i is distributed exponentially with density $h(T^i; p)$, so that at any point in time, T_i arrives in the next instant dt with probability $p \cdot dt$. The sudden entry of competition at time T_i eliminates the present value of the firm's future abnormal

earnings, defined as earnings in excess of those earned at the rate equal to the cost of capital. As a result, the firm's market value of equity at time T_i equals the book value, $M_{T_i}^i = B_{T_i}^i$. This implication follows from the residual income model (e.g., Ohlson, 1995), in which the market equity equals book equity plus the present value of future abnormal earnings.

The firm pays out a constant fraction of its book equity in dividends, $D_t^i = c^i B_t^i$, where $c^i \geq 0$ is the dividend yield. The firm is financed only by equity, and it issues no new equity. These assumptions are made for simplicity; relaxing them would add complexity with no obvious new insights. (For example, debt financing has no effect as long as it does not affect the profitability process.) Given these assumptions, the clean surplus relation implies that book equity grows at the rate equal to profitability minus the dividend yield:

$$dB_t^i = (Y_t^i - D_t^i) dt = (\rho_t^i - c^i) B_t^i dt. \quad (6)$$

The market value of equity is the present value of any dividends plus the final payoff B_{T_i} :

$$M_t^i = E_t \left[\int_t^\infty \left(\int_t^{T^i} \frac{\pi_s}{\pi_t} D_s^i ds + \frac{\pi_{T^i}}{\pi_t} B_{T^i}^i \right) h(T^i; p) dT^i \right]. \quad (7)$$

The stochastic discount factor (SDF) π_t is assumed to be given by

$$\pi_t = e^{-\eta t - \gamma(s_t + \varepsilon_t)}, \quad (8)$$

where

$$s_t = a_0 + a_1 y_t + a_2 y_t^2, \quad (9)$$

$$dy_t = k_y (\bar{y} - y_t) dt + \sigma_y dW_{0,t}, \quad (10)$$

$$d\varepsilon_t = \mu_\varepsilon dt + \sigma_\varepsilon dW_{0,t}. \quad (11)$$

PV (2005) derive π_t in Eq. (8) from a habit utility model in which ε_t is the log of aggregate consumption and s_t is the log surplus consumption ratio introduced by Campbell and Cochrane (1999). Our SDF is a special case of the one in PV (2005), with $b_1 = 0$ in their Eq. (8). Similar SDFs are used in the term structure literature (e.g., Constantinides, 1992). Given this specification, the equity premium varies over time due to the time-varying risk aversion of the representative investor. High values of y_t imply a low volatility of the SDF, and thus a low equity premium. A time-varying equity premium helps us avoid the excess volatility puzzle of Shiller (1981) in our calibration in Section 3.

2.1. Valuation

We use the following function repeatedly:

$$Z^i \left(y_t, \bar{\rho}_t, \rho_t^i, \bar{\psi}_t^i, s \right) = e^{Q_0(s) + \mathbf{Q}(s)' \cdot \mathbf{N}_t + Q_5(s) y_t^2}, \quad (12)$$

where $\mathbf{N}_t = (y_t, \bar{\rho}_t, \rho_t^i, \bar{\psi}_t^i)$ and the Q functions are defined in the Appendix.

Proposition 1. Suppose that $\bar{\psi}_t^i$ is known. The firm's M/B ratio is given by

$$\frac{M_t^i}{B_t^i} = G^i \left(y_t, \bar{\rho}_t, \rho_t^i, \bar{\psi}_t^i \right) = (c^i + p) \int_0^\infty Z^i \left(y_t, \bar{\rho}_t, \rho_t^i, \bar{\psi}_t^i, s \right) ds. \quad (13)$$

This proposition serves as a benchmark for the more interesting case, analyzed next, in which $\bar{\psi}_t^i$ is unobservable. According to Proposition 1, a firm's M/B ratio is high if expected profitability is high and the discount rate is low. With respect to profitability, M/B increases with $\bar{\rho}_t$, $\bar{\psi}_t^i$, and ρ_t^i . As for the discount rate, M/B increases with y_t in the plausible parameter range: when y_t is high, the equity premium is low and M/B is high.

Suppose now that $\bar{\psi}_t^i$ is unknown, and that the investors' beliefs about $\bar{\psi}_t^i$ can be summarized by the probability density function $f_t(\bar{\psi}_t^i)$. The law of iterated expectations implies

$$\frac{M_t^i}{B_t^i} = E_t \left[G^i \left(y_t, \bar{\rho}_t, \rho_t^i, \bar{\psi}_t^i \right) \right] = \int G^i \left(y_t, \bar{\rho}_t, \rho_t^i, \bar{\psi}_t^i \right) f_t \left(\bar{\psi}_t^i \right) d\bar{\psi}_t^i. \quad (14)$$

Since $G^i(y_t, \bar{\rho}_t, \rho_t^i, \bar{\psi}_t^i)$ is a convex function of $\bar{\psi}_t^i$, more uncertainty about $\bar{\psi}_t^i$ (i.e., a mean-preserving spread in $\bar{\psi}_t^i$) implies a higher expected value of $G^i(y_t, \bar{\rho}_t, \rho_t^i, \bar{\psi}_t^i)$, and thus a higher M/B ratio. This relation holds for any distribution $f_t(\bar{\psi}_t^i)$. To obtain a closed-form solution for M/B, we assume that $f_t(\bar{\psi}_t^i)$ is normal.

Proposition 2. Suppose that $\bar{\psi}_t^i$ is unknown, and that the market perceives a normal distribution for $\bar{\psi}_t^i$, $f_t(\bar{\psi}_t^i) = N(\hat{\psi}_t^i, \hat{\sigma}_{i,t}^2)$. The firm's M/B ratio is given by

$$\frac{M_t^i}{B_t^i} = (c^i + p) \int_0^\infty Z^i \left(y_t, \bar{\rho}_t, \rho_t^i, \hat{\psi}_t^i, s \right) e^{\frac{1}{2}Q_4(s)^2 \hat{\sigma}_{i,t}^2} ds. \quad (15)$$

The fact that M/B increases with $\hat{\sigma}_{i,t}$, uncertainty about $\bar{\psi}_t^i$, is the key relation in the paper. The proofs of Propositions 1 and 2 are in the Appendix, along with a lemma that describes learning about $\bar{\psi}_t^i$ and the formulas for the expected return and return volatility of firm i .

3. Calibration

In this section, we calibrate the model to match some key features of the data on asset returns and profitability. We summarize the parameters in Table 1.

We divide firms into “new economy” and “old economy” groups. For simplicity, the new economy includes firms traded on Nasdaq, and the old economy includes firms traded on the

NYSE and Amex. The new economy firms are described in Section 2. The old economy’s aggregate profitability is given by $\bar{\rho}_t$ in Eq. (4). The old economy pays aggregate dividends forever at the rate of $D_t^O = c^O B_t^O$, where B_t^O is the old economy’s aggregate book value. We compute $c^O = 5.67\%$ as the time-series average of the old economy’s annual dividend yields, each of which is computed as the sum of the current-year dividends across all NYSE/Amex firms, divided by the sum of the book values of equity at the end of the previous year.⁵ The old economy’s aggregate market value is given by $M_t^O = E_t \left[\int_t^\infty \frac{\pi_s}{\pi_t} D_s^O ds \right]$, and its M/B ratio is a function of its profitability and the market-wide discount rate:

$$M_t^O / B_t^O = \Phi(\bar{\rho}_t, y_t). \quad (16)$$

An explicit formula for the function Φ is provided in the Appendix.

To estimate the process for $\bar{\rho}_t$ in Eq. (4), we compute the old economy’s profitability as the sum of the current-year earnings across all NYSE/Amex firms, divided by the sum of the book values of equity at the previous year-end. This time series is adjusted for inflation by using the GDP deflator, obtained from the Bureau of Economic Analysis. Eq. (4) implies a normal likelihood function for $\bar{\rho}_t$, as described in the Appendix (Lemma 4). Maximizing this likelihood yields $k_L = 0.3574$, $\bar{\rho}_L = 12.17\%$ per year, and an estimate of the total volatility, $\sigma_L = \sqrt{\sigma_{L,0}^2 + \sigma_{L,L}^2}$. We split this estimate into its components, $\sigma_{L,0} = 1.47\%$ and $\sigma_{L,L} = 1.31\%$ per year, by using the covariance of $\bar{\rho}_t$ with the SDF, which we discuss next.

We choose the parameters of the SDF to produce reasonable properties for the returns and M/B of the old economy, as well as for the risk-free rate. First, we set the values of μ_ϵ and σ_ϵ to match the moments of consumption growth in the data (recall that ϵ_t in Eq. (11) represents log consumption). Second, we construct the 1962 to 2002 annual time series of the old economy’s M/B by computing the ratio of the sums of the market values and the most recent book values of equity across all NYSE/Amex firms. We then invert the pricing formula in Eq. (16) to obtain the time series of y_t as a function of the old economy’s M/B ratio, $\bar{\rho}_t$, and the parameters $\Theta = (\eta, \gamma, \bar{y}, k_y, \sigma_y, \sigma_{L,0}, \sigma_{L,L})$. To help us choose reasonable parameters, we estimate Θ by a simple minimum distance procedure. We construct the moment conditions from the stationary distribution of $(\bar{\rho}_t, y_t)$, obtained by substituting $(\bar{\rho}_t, y_t)$ for \mathbf{z}_t in Lemma 4 in the Appendix. We also impose additional moment

⁵Throughout the paper, firms are defined as ordinary common shares (i.e., Center for Research in Security Prices (CRSP) sharecodes 10 and 11). Gebhardt, Lee, and Swaminathan (2001) argue that “In theory, share repurchases and new equity issues that can be anticipated in advance should also be included in the dividend payout estimate. However, we know of no reliable technique for making these forecasts.” Like Gebhardt et al., we do not attempt to forecast share repurchases and new equity issues for the purpose of computing the expected dividend yield.

conditions to ensure that the average values of the estimated equity premium $\mu_{R,t}$, market return volatility $\sigma_{R,t}$, and the real interest rate $r_{f,t}$ are close to the values observed in the data. The estimated parameters, listed in Table 1, imply the average equity premium of 5.06% per year, the average old-economy volatility of 14.47%, the average real risk-free rate of 6.25%, and the average risk-free rate volatility of 1.55% (all annual).

In the process for $\bar{\psi}_t^i$ in Eq. (5), we set $k_\psi = 0.0139$, which implies a half-life of 50 years. We choose such slow mean reversion for $\bar{\psi}_t^i$ so that $\bar{\psi}_t^i$ can be thought of as virtually constant before time T_i ; for simplicity, we drop the t subscript from $\bar{\psi}_t^i$ in the rest of the paper. If we were to assume that $\bar{\psi}^i$ is literally constant, we would need to impose a finite upper bound on the range of possible values of $\bar{\psi}^i$ to ensure that prices are well defined. This upper bound would prevent us from using a normal distribution for $\bar{\psi}^i$, which would complicate the analysis and make the pricing formulas less elegant. However, in unreported analysis, we solve the model with constant $\bar{\psi}^i$ under the assumption of a truncated normal distribution for $\bar{\psi}^i$, and obtain the same conclusions throughout the paper.

4. Matching Nasdaq prices on March 10, 2000

In this section, we examine the ability of our valuation model to match the prices of Nasdaq stocks on March 10, 2000, the day when the Nasdaq index peaked.

4.1. Matching Nasdaq's valuation

In this subsection, we view Nasdaq as one large firm, whose profitability ρ_t^N follows the process in Eq. (2). We estimate the parameters of this process by maximum likelihood, where the likelihood function is obtained by substituting $(\rho_t^N, \bar{\rho}_t, y_t)$ for \mathbf{z}_t in Lemma 4 in the Appendix. In this substitution, we take as given the parameters of the $\bar{\rho}_t$ and y_t processes described in Section 3, without imposing any restrictions on the covariances between ρ_t^N and $(\bar{\rho}_t, y_t)$. This procedure yields $\phi^N = 0.3667$, $\sigma_{N,0} = 2.46\%$, and $\sigma_{N,N} = 4.90\%$ per year.

For each Nasdaq-traded firm, we compute the market value of equity on March 10, 2000 by multiplying the share price by the number of shares outstanding, both of which we obtain from CRSP. Nasdaq's M/B ratio is the sum of the market values of all Nasdaq firms on March 10, 2000, divided by the sum of the 1999 year-end book values of equity. This ratio is equal to 8.55. Nasdaq's dividend yield, $c = 1.35\%$, is the sum of the dividends of all Nasdaq firms in 1999, divided by the sum of the 1998 year-end book values. We measure profitability as the accounting return on equity (ROE), following the definition of ρ_t^i . Nasdaq's current profitability, $\rho_t^N = 9.96\%$ per year, is computed as the 1999Q4 annualized value of Nasdaq's

aggregate profitability, i.e., the sum of 1999Q4 earnings across all Nasdaq firms, divided by the sum of the most recent pre-1999Q4 book values of equity. Analogously, $\bar{\rho}_t = 18.62\%$ per year is computed as the 1999Q4 annualized value of NYSE/Amex's aggregate profitability. It is not surprising that this value is higher than $\bar{\rho}_t$'s central tendency of 12.17%, because the U.S. economy was near the peak of a ten-year expansion at the end of 1999.

We obtain data for our profitability calculations from Compustat. Earnings are computed as income before extraordinary items available for common, plus deferred taxes and the investment tax credit. Book equity is stockholders' equity plus deferred taxes and the investment tax credit, minus the book value of preferred stock. Throughout the paper, we eliminate the values of market equity and book equity that are below \$1 million, as well as the values of ROE (earnings over book equity) that are above 1,000% in absolute value.

We assume that the competition that wipes out the present value of Nasdaq's future abnormal earnings can arrive in any instant dt with probability $p \cdot dt$, with $p = 1/20$. As a result, the expected time over which Nasdaq can earn abnormal profits ($\bar{\psi}^N$) is $E(T) = 1/p = 20$ years. Later, we also review the results for $E(T) = 15$ and 25 years.

Panel A of Table 2 reports the model-implied M/B for Nasdaq on March 10, 2000 under zero uncertainty about $\bar{\psi}^N$, for different values of the equity premium and $\hat{\psi}^N$. (Recall that $\hat{\psi}^N$ is Nasdaq's expected profitability in excess of the NYSE/Amex profitability.) The model-implied M/B increases with $\hat{\psi}^N$ and decreases with the equity premium, as expected. With $\hat{\psi}^N \leq 3\%$ per year, not even the equity premium of 1% per year can match Nasdaq's M/B of 8.55. With $\hat{\psi}^N = 4\%$ per year, the premium needed to match Nasdaq's M/B is about 1.4% per year, and with $\hat{\psi}^N = 5\%$, the required premium is about 2.8% per year.

Panel B of Table 2 reports the model-implied return volatility for Nasdaq on March 10, 2000 under zero uncertainty. This volatility ranges mostly between 20% and 30% per year. For comparison, we compute Nasdaq's actual return volatility in March 2000 as the standard deviation of the daily Nasdaq returns in that month and obtain 41.49% per year.⁶ Since this estimate is noisy, we also compute the average of the monthly volatilities in 2000, all computed from daily returns within the month, and obtain 47.03% per year. Both 41.49% and 47.03% are far above the model-implied volatility values in Panel B. Our model is clearly unable to match Nasdaq's return volatility under the assumption of zero uncertainty.

⁶We perform the annualization by multiplying the daily standard deviation by $\sqrt{252}$. Using data obtained from Optionmetrics, we also compute Nasdaq's return volatility expected by the option market on March 10, 2000. The implied volatility of the nearest-to-the-money call (put) option on QQQ with one month to expiration is 49.4% (51.4%), which is slightly higher than the realized volatility.

Next, we acknowledge that $\overline{\psi}^N$ is unknown. Table 3 provides the equivalent of Table 2 under the assumption that the standard deviation of the perceived distribution of $\overline{\psi}^N$ is 3% per year. The M/B ratios in Table 3 are higher than in Table 2, as we expect from Proposition 2. For example, with the equity premium of 3% and $\widehat{\psi}^N = 3\%$, the model-implied M/B ratio is 7.41, whereas the corresponding ratio in Table 2 is only 4.87. The return volatilities are also higher in Table 3: under the same parameters, the model-implied return volatility is 40.37%, compared to 24.52% in Table 2. Thus, acknowledging uncertainty about $\overline{\psi}^N$ leads to values of M/B and volatility that are closer to the observed values.

The differences between the values in Tables 2 and 3 are the greatest for the lowest values of the equity premium. For example, for $\widehat{\psi}^N = 0$ and the equity premium of 1%, M/B in Table 3 is 1.41 times larger than M/B in Table 2, and the return volatility is 1.86 times larger. Under the 8% equity premium, the M/B in Table 3 is only 1.08 times larger, and the volatility is only 1.20 times larger. When the equity premium is lower, future cash flow is discounted at a lower rate when valuing a firm. As a result, a larger fraction of the firm's value comes from earnings in the distant future, which are more affected by uncertainty about $\overline{\psi}^N$ than earnings in the near future, due to compounding. Thus, uncertainty about $\overline{\psi}^N$ has the greatest effect on prices and volatilities when the equity premium is low.

Panel A of Table 4 reports implied uncertainty, defined as the uncertainty that equates the model-implied M/B to the observed M/B.⁷ Implied uncertainty is listed as zero for all pairs of $\widehat{\psi}^N$ and the equity premium that deliver $M/B \geq 8.55$ in Panel A of Table 2. When $\widehat{\psi}^N = 0$ and the equity premium is 3%, matching an M/B of 8.55 requires an uncertainty of 5.06% per year. Raising $\widehat{\psi}^N$ to 3% per year, implied uncertainty drops to 3.38%. What values of implied uncertainty are plausible? We address this question next.

4.1.1. Plausible values of implied uncertainty

To judge the plausibility of a given value of uncertainty, we need to find a measurable quantity that is closely related to uncertainty. One natural candidate is return volatility, which is strongly positively associated with uncertainty in the model. For any value of uncertainty, we can compute the model-implied return volatility (Eq. (20)). The plausibility of a given value of uncertainty can then be assessed by comparing the corresponding return

⁷This calculation is similar in spirit to computing the implied volatility of an option. The idea of backing out the prior uncertainty needed to match the observed evidence is not new. For example, in a mean-variance framework in which investors can invest in U.S. as well as non-U.S. stocks, Pástor (2000) computes the prior uncertainty about mispricing that is necessary to explain the observed degree of home bias. Similarly, in a regime-switching model for the drift rate of earnings, David and Veronesi (2002) use options data to back out the implied uncertainty about the future earnings growth of the S&P 500 index.

volatility with the volatility observed in the data. We judge uncertainty to be implausibly high if it produces a return volatility that is significantly higher than the observed volatility.

The model-implied return volatilities are reported in Panel B of Table 4. The implied uncertainty of 3.38%, discussed above, produces a return volatility of 46.66% per year. This value is close to Nasdaq’s observed volatility computed above (about 41.49% to 47.03% per year). This result suggests that the implied uncertainty of 3.38%, which we obtain for the combination of $\widehat{\psi}^N = 3\%$ and the equity premium of 3%, is plausible.

More generally, Table 4 identifies the pairs of values of the equity premium and $\widehat{\psi}^N$ for which implied uncertainty matches not only Nasdaq’s M/B but also its return volatility. One such pair is $\widehat{\psi}^N = 2\%$ and the equity premium of 2%, which leads to implied uncertainty of 3.54%, which in turn produces a return volatility of 47.54%. Another pair, discussed above, is $\widehat{\psi}^N = 3\%$ and the equity premium of 3%. Yet another pair is $\widehat{\psi}^N = 4\%$ and the equity premium of 4%, which leads to implied uncertainty of 3.32% and a return volatility of 47.81%. These equity premium and $\widehat{\psi}^N$ combinations seem plausible, as we discuss next.

4.1.2. Plausible values of the discount rate

The equity premium in March 2000 was probably low. According to Cochrane (2003), “The top of the largest economic boom in postwar U.S. history is exactly when you’d expect a risk premium to be low and stock prices to be high.” Lettau, Ludvigson, and Wachter (2004) argue that the equity premium declined significantly in the 1990s due to a decline in the volatility of aggregate consumption. Based on their dividend and earnings growth models, Fama and French (2002) estimate the equity premium for the 1951 to 2000 period to be 2.6% and 4.3% per year, respectively, but some of their premium estimates for 1991 to 2000 are as low as 0.32% per year. Using the residual income model and analyst forecasts, Claus and Thomas (2001) estimate the equity premium in 1998 at about 2.5%. Welch (2001) surveys 510 academics in 2001 and reports a median equity premium forecast of 3%. Pástor and Stambaugh (2001) estimate the premium of 4.8% at the end of 1999. Ilmanen (2003) estimates the premium of 2% in March 2000. Based on this evidence, we regard 1% to 5% as the most plausible range of values of the equity premium for the old economy.

Given its exposure to the SDF, Nasdaq commands a higher risk premium than the old economy; in fact, more than twice as high. For example, when the equity premium is 3%, Nasdaq’s expected excess return equals 6.62% for $\widehat{\psi}^N = 0$ and 6.24% for $\widehat{\psi}^N = 3\%$. Also note that the risk-free rate in Tables 2 through 4 is not specified exogenously. Both the risk-free rate and the equity premium are driven by the same variable, y_t , so by specifying

the equity premium, we are implicitly choosing the risk-free rate as well. The implied risk-free rates for Tables 2 through 4 appear to be reasonable. For example, the real rates that correspond to the equity premiums between 1% and 6% are all between 4.79% (for the equity premium of 3%) and 6.32% per year (for the premium of 6%). In comparison, on March 10, 2000, the annual nominal yields on Treasury bonds with maturities between one and 20 years were between 6.21% and 6.55%. With 2% to 3% inflation,⁸ the real rates in our model slightly exceed the observed real rates, which induces a mild conservative bias in the valuation procedure. Note that when the (old economy's) equity premium is 3% per year, Nasdaq's total expected real return in the model exceeds 11% per year, which translates into a nominal discount rate of almost 14% per year.

4.1.3. Plausible values of expected profitability

To match Nasdaq's M/B and return volatility in Table 4, we require relatively small positive values of $\hat{\psi}^N$ such as 2% or 3% per year. Indeed, the market appears to have expected higher average profitability from Nasdaq firms than from NYSE/Amex firms in March 2000. For example, consider equity analyst forecasts provided by I/B/E/S. For each firm, we compute the average forecast of long-term earnings growth by averaging forecasts across all analysts covering the firm in March 2000. The average of these average forecasts is 15.1% when computed across NYSE/Amex firms, but the same average computed across Nasdaq firms is substantially higher, at 28.8%. When these forecasts are combined with the data on current profitability (9.96% for Nasdaq, 18.62% for NYSE/Amex) and dividend yield (1.35% for Nasdaq, 5.67% for NYSE/Amex), they imply that Nasdaq's profitability should begin exceeding the NYSE/Amex profitability in year 2005. The long-term growth forecasts are generally considered valid for up to five years. Since we do not know of any forecast data with a longer horizon, we project these forecasts further into the future, for illustration. The 2010 forecast of Nasdaq's ROE exceeds that of NYSE/Amex's ROE by 3.73%, and in the limit, the ROE difference grows to 5.36%. While this example is only illustrative, it suggests that $\hat{\psi}^N > 0$ might be reasonable. There is also abundant anecdotal evidence that cash flow expectations for Nasdaq were relatively high at that time.⁹

The assumption $\hat{\psi}^N > 0$ receives some support from historical data as well. Consider two portfolios formed in 1972, one year after Nasdaq was created. The first (second) portfolio

⁸The GDP deflator increased by 2.2% in 2000, and the core inflation rate in 2000 was 2.6%. We obtain both the inflation and interest data from the website of the Federal Reserve Bank of St. Louis.

⁹E.g., "Applegate: This business cycle is extraordinary... Today tech earnings are growing 24%, which is significantly better than the rest of the market. Their prices are accordingly richer... I'm comfortable right here, sticking with the Ciscos, Microsofts and Intels." David Henry, USA Today, December 16, 1999.

includes all Nasdaq (non-Nasdaq) firms with valid book values in Compustat at the end of 1972. We compute the annual profitability (i.e., the sum of earnings divided by the sum of the most recent book values) of these two portfolios going forward, without accounting for any migration of firms between the exchanges and without including any newly listed firms. Between 1973 and 1999, the average profitability of the Nasdaq portfolio (14.63%) exceeds the profitability of the non-Nasdaq portfolio (13.28%) by 1.35% per year.

These results may seem surprising, since Nasdaq’s ROE has generally been lower than NYSE/Amex’s ROE. This apparent contradiction can be explained by two facts. First, newly listed firms tend to be less profitable than incumbent firms, and most new lists appear on Nasdaq (Fama and French, 2003). Second, firms that switch exchanges induce a “migration bias” into the profitability of the Nasdaq and NYSE/Amex indexes. Since the listing requirements are less strict on Nasdaq than on NYSE/Amex, firms that migrate from NYSE/Amex to Nasdaq typically perform poorly (they no longer meet NYSE/Amex’s listing requirements), and firms that migrate from Nasdaq to NYSE/Amex typically perform well.¹⁰ Due to the persistence in ROE, this migration reduces the ROE of Nasdaq relative to NYSE/Amex. However, the ROE earned by an investor who buys a portfolio of Nasdaq stocks is obviously unaffected by any post-purchase migration, nor by any post-purchase new listings on Nasdaq. Therefore, $\hat{\psi}^N$ should exceed the historical difference between the ROEs on the regularly rebalanced Nasdaq and NYSE/Amex indexes.

Another reason to be optimistic about $\hat{\psi}^N$ is that in 2000, Nasdaq firms had already expensed large investments in intangible assets. While intangible assets are often not included in the book value of the firm, they do contribute to the firm’s earnings, and thus they increase the earnings-to-book ratio, or ROE. To the extent that Nasdaq firms had more intangible capital than NYSE/Amex firms in March 2000, $\hat{\psi}^N > 0$ seems reasonable.

One example of beliefs that justify Nasdaq’s valuation in March 2000 is $\hat{\psi}^N = 3\%$, the equity premium of 3%, and $E(T) = 20$ years. To better understand these beliefs, we use Eq. (2) to compute the distribution of Nasdaq’s profitability over the following 20 years. As Panel A of Fig. 1 demonstrates, Nasdaq’s profitability is expected to improve over the first three years before declining slowly until time T . Twenty years ahead, in year 2019, the 1st, 50th, and 99th percentiles of the predictive distribution for Nasdaq’s ROE (ρ_{t+20}^i) are

¹⁰We verify the existence of the migration bias empirically. For each firm that switched exchanges between 1973 and 1999, we compute the firm’s ROE in the year immediately preceding the year in which the migration took place. We then compute the firm’s excess ROE as the firm’s ROE minus the aggregate ROE of all firms on the same original exchange (NYSE/Amex or Nasdaq) in the same year. We find that the excess ROE of the firms that migrated from NYSE/Amex to Nasdaq is -42.6% per year, on average. In contrast, the average excess ROE of the firms that migrated from Nasdaq to NYSE/Amex is +4.9% per year.

-3.07%, 14.56%, and 32.20% per year, respectively. The same percentiles for the average ROE between 1999 and 2019 are 4.90%, 15.07%, and 25.25%, respectively (Panel B).

While an average ROE of 25.25% per year over a 20-year period has not been observed in Nasdaq’s short history, such a possibility cannot be dismissed. Certain sectors of the economy have delivered comparable average profitability for much longer periods of time. Suppose that, in 1954, you formed the portfolio of all firms in the pharmaceutical industry that had valid book values in Compustat at the end of 1954. This portfolio earned an average ROE of 25.20% over the 45-year period between 1955 and 1999. A similarly constructed candy-and-soda industry portfolio, formed in 1963, earned an average ROE of 24.34% over the 36-year period between 1964 and 1999. Over the same 36-year period, the tobacco products industry portfolio formed in 1963 earned an average ROE of 22.12%. We do not wish to push this anecdotal evidence too far; after all, it is easier to observe high average ROE at the level of an industry than at the level of an index. The purpose of our examples is simply to illustrate the fact that sustained high profitability is possible, even at the sectoral level.¹¹

To provide an additional perspective on the beliefs that justify Nasdaq’s M/B in March 2000, in Fig. 2 we plot the model-predicted distribution of the future ratio of Nasdaq’s book value to the NYSE/Amex/Nasdaq book value.¹² At the end of 1999, the value of this ratio was 0.18. The 1st, 50th, and 99th percentiles for this ratio after 10 years are 0.12, 0.27, and 0.50. The 1st, 5th, 50th, 95th, and 99th percentiles after 20 years are 0.11, 0.17, 0.43, 0.73, and 0.82. These distributions do not seem implausible to us. Note that these are predictions for firms that trade on Nasdaq vs. NYSE/Amex today (as of March 2000), not for firms that will be trading on these exchanges in the future. Given the migration bias discussed earlier, the future Nasdaq may well be smaller than what is suggested by Fig. 2.

In Table 4, Nasdaq is expected to earn abnormal profits over 20 years after March 2000. In the NBER version of this paper, we report a table (Table 5, omitted here to save space) that is an equivalent of Table 4 for $E(T) = 15$ and 25 years. For $E(T) = 15$, matching Nasdaq’s M/B requires more optimism than for $E(T) = 20$. For example, with a 3% equity premium, we require $\hat{\psi}^N = 5\%$ to match Nasdaq’s M/B and also obtain realistic return volatility (41.76%). In contrast, matching Nasdaq’s M/B is easier for $E(T) = 25$. With a 3% equity premium, we need a $\hat{\psi}^N$ of only 2% to match both M/B and return volatility.

¹¹We define industries based on the 48-industry classification scheme from Ken French’s website. A portfolio’s ROE in a given year is computed as the sum of earnings in that year divided by the sum of book values at the end of the previous year, where both sums are computed across all firms in the original portfolio. That is, the industry portfolios are not rebalanced to include new firms in the industry.

¹²Plotting future market values would be less informative because they depend on future stock returns. All stocks in our model earn fair expected returns determined by the covariances of returns with the SDF.

4.2. Matching the valuations of individual firms

In this subsection, we use the model to value 12 high-profile technology firms: Akamai, Amazon, Ciena, Cisco, Dell, eBay, Immunex, Intel, Microsoft, Priceline, Red Hat, and Yahoo. The parameters for the process governing profitability ρ_t^i in Eq. (2) are chosen to match the median Nasdaq firm in the data. For each year and each firm, we compute the firm's profitability (ROE) as the ratio of the firm's current-year earnings and its book value of equity at the end of the previous year. For each firm, we construct the longest possible uninterrupted time series of valid ROEs (i.e., ROEs smaller than 1,000% in absolute value). If this series is at least 10 years long, we estimate an AR(1) model for ROE. The slope coefficients are adjusted for the small-sample bias (e.g., Stambaugh, 1999). The median value of ϕ^i across all Nasdaq firms that satisfy $0 < \phi^i < 1$ is 0.3891. The median residual volatility of ROE is 10.46% per year. We decompose this volatility into $\sigma_{i,0} = 6.65\%$ and $\sigma_{i,i} = 8.07\%$ per year, which implies a M/B ratio of 1.7 for a firm with zero uncertainty, zero $\overline{\psi}^i$, and $\rho_t^i = \overline{\rho}_t = \overline{\rho}_L$.

Each firm's M/B is computed by dividing the March 10, 2000 market value of equity by the 1999 year-end book equity. Firm profitability, ρ_t^i , is the 1999 earnings divided by the 1998 year-end book equity. If the 1998 year-end book value is unavailable on Compustat, which is the case for Akamai, Priceline, and Red Hat, we replace it by the 1999 year-end value. Amazon's book value fluctuates dramatically in 1998 through 2000; in fact, it turns negative in 2000Q2. We use the 1999Q2 quarter-end book equity to compute Amazon's profitability in 1999. The dividend yield, c^i , equals the 1999 dividends divided by the 1998 year-end book equity. Only one firm in our set, Intel, paid dividends in 1999.

Table 5 reports the implied uncertainty and the associated return volatility on March 10, 2000 for all 12 firms. Throughout the table, the expected horizon until competition eliminates the present value of future abnormal earnings is 15 years, $E(T) = 15$. This choice seems fairly conservative: Schwartz and Moon (2000) use a 25-year horizon when valuing Amazon, and Ofek and Richardson (2002) consider horizons of 10 to 30 years. If we were to use instead the same horizon as we did for Nasdaq, $E(T) = 20$, the firms' valuations would be easier to match. However, it seems reasonable to assume a shorter horizon for individual firms than for Nasdaq. For any given firm, competition can arrive in the form of a single firm that develops a superior product. In contrast, competition that wipes out the future abnormal earnings of Nasdaq as a whole is likely to arrive in the form of new technology that the Nasdaq incumbents will fail to implement (e.g., Hobijn and Jovanovic, 2001). Such competition is likely to arrive later for the index as a whole than for any given firm.

When describing our results, we use the equity premium of 3% per year for the old economy. Table 5, however, reports the results for all values of the equity premium between 1% and 6%. The lower the equity premium, the easier it is for our model to match the observed M/B ratios. Importantly, our results are not overly sensitive to the equity premium, and the discussion below would be very similar under the equity premiums of 2% or 4%.

First, we consider some of the largest technology firms: Microsoft (market capitalization \$516bn on March 10, 2000), Cisco (\$456bn), Intel (\$395bn), and Dell (\$130bn). The M/B ratios of these firms were high on March 10, 2000: 18.79 for Microsoft, 39.02 for Cisco, 11.09 for Intel, and 24.47 for Dell. All of these M/B ratios can be matched with reasonable levels of uncertainty about $\bar{\psi}^i$. As before, we determine the plausibility of a given value of implied uncertainty by comparing the model-implied return volatility with the observed volatility.

Consider $\hat{\psi}^i = 0$. The implied uncertainty for Microsoft is 3.84%, which implies a return volatility of 59.44%. This value is close to Microsoft's actual volatility, which is estimated to be 57.44% (56.10%), based on the March 2000 returns (all returns in 2000), as before. Intel's implied uncertainty is 4.86%, which yields a return volatility of 69.90%, which in turn is close to Intel's actual volatility of 45.81% (68.71%). Dell's implied uncertainty is 3.85%, which leads to a return volatility of 59.71%, which is close to Dell's actual volatility of 51.75% (69.50%). In other words, under the assumption that the average future profitabilities of Microsoft, Intel, and Dell are equal to the profitability of the old economy, the implied uncertainty in our model matches not only the firms' observed M/B ratios but approximately also their return volatilities.

Among our four largest firms, only Cisco has a M/B and volatility that cannot be matched with $\hat{\psi}^i = 0$. However, Cisco's implied uncertainty under $\hat{\psi}^i = 4\%$ is 3.33%, which implies a return volatility (59.90%) that is close to Cisco's estimated volatility of 51.75% (69.50%). Assuming that Cisco can deliver an average profitability of 4% in excess of the old economy's profitability over the expected horizon of 15 years does not seem implausible. In comparison, the average of all of Cisco's valid ROEs at the time (1992 to 1999) was 47.13%, which is significantly higher than the old economy's average ROE of 13.03% over the same period.

The assumption of $\hat{\psi}^i = 4\%$ can also rationalize Yahoo's M/B of 78.41. Yahoo's implied uncertainty of 4.40% yields a return volatility of 81.78%, which is close to Yahoo's observed volatility of 75.41% (90.61%). Compared to Cisco, Yahoo has a higher implied uncertainty, which seems reasonable because its M/B and return volatility are both higher than Cisco's.

Next, consider the M/B ratios of eBay (27.87), Red Hat (26.50), and Immunex (105.70).

One might expect that matching these high M/Bs requires large values of $\widehat{\psi}^i$, but that is not the case; in fact, the value that works best for all three firms is $\widehat{\psi}^i = -2\%$. This surprising finding is due to the fact that all three firms have highly volatile returns. Under $\widehat{\psi}^i = -2\%$, the firms' implied uncertainties are 5.83%, 6.35%, and 6.04%, respectively, and the model-implied return volatilities are close to the observed volatilities of 129.24% (113.64%) for eBay, 121.00% (122.33%) for Red Hat, and 155.94% (117.71%) for Immunex. For these firms' stock returns to be so volatile, uncertainty about the firms' growth rates must be so large that the expected growth rates can be below the growth rate of the old economy.

The firms whose M/Bs are the most difficult to match are Amazon (88.07) and Priceline (39.58). This difficulty stems from the firms' extremely poor profitability: Amazon's 1999 ROE is -126%, and Priceline's ROE is -264%. Investors holding Amazon and Priceline must have expected these firms to become highly profitable in the future. If investors expected $\widehat{\psi}^i = 4\%$, which works well for Yahoo and Cisco, the implied uncertainties for Amazon and Priceline would be implausibly large, implying return volatilities of 147.12% for Amazon and 191.66% for Priceline. (The actual return volatilities are smaller: 71.67% (103.33%) for Amazon, and 128.17% (133.65%) for Priceline.) Matching the observed volatilities (as well as M/B) requires a $\widehat{\psi}^i$ of about 10% for both Amazon and Priceline.

If $\widehat{\psi}^i = 10\%$ seems large, note that in the absence of uncertainty about $\overline{\psi}^i$, justifying Amazon's M/B ratio would require $\overline{\psi}^i > 16\%$, and justifying Priceline's M/B would require $\overline{\psi}^i > 20\%$. Moreover, the implied return volatilities for both firms would be counterfactually low. Uncertainty about $\overline{\psi}^i$ helps us understand these firms' high valuations and volatilities.

To assess the plausibility of the beliefs that justify Amazon's valuation in March 2000 ($\widehat{\psi}^i = 10\%$ and 3% equity premium), we use Eq. (2) to compute the distribution of Amazon's future profitability, $\rho_{t+\tau}^i$, over the next 15 years. As we show in Panel A of Fig. 3, Amazon's profitability is expected to improve sharply, but it is expected to remain negative for about four years. Median profitability turns positive in 2004, and it reaches 20.30% in 2014. (In reality, Amazon's earnings turned positive in 2003Q3 and Amazon's first profitable year was 2003.) Five years ahead (in 2004), the 1st and 99th percentiles of the predictive distribution for Amazon's ROE are -27.34% and 32.44%. In 2014, the same percentiles are -10.11% and 50.71%. These quantities are well within the range of the ROEs observed in the data.

Panel B of Fig. 3 plots the model-predicted distribution of Amazon's future book value as a fraction of the 1999 book value. Due to the large current losses, the median forecast of Amazon's book value is below its 1999 value even after 15 years. However, M/B depends on the expectation of the future book value, not its median. This expectation exceeds the

median, as Fig. 3 shows, because the distribution of the future book value is right-skewed. For example, in 2014, the 10th and 90th percentiles of the distribution of B_{t+15}^i/B_t^i are 0.16 and 2.76, and the 1st and 99th percentiles are 0.05 and 8.75. The logic behind Amazon’s high M/B ratio in March 2000 thus seems clear: high uncertainty about Amazon’s future growth rate leads to a right-skewed distribution of Amazon’s future book value, which in turn leads to a high expected book value, which then gives Amazon a high M/B ratio.

Fig. 4 illustrates the same logic for the example of Yahoo. The distribution of Yahoo’s ROE in 2014 has a median of 15.66%, with the 1st and 99th percentiles of -14.66% and 46.01%. Yahoo’s book value is expected to grow more quickly than Amazon’s, as Yahoo’s 1999 ROE (10.52%) is higher than Amazon’s. In 2014, the 10th and 90th percentiles of the distribution of Yahoo’s B_{t+15}^i/B_t^i are 2.73 and 45.79, and the 1st and 99th percentiles are 0.87 and 144.46. As a result of this large skewness, Yahoo’s 2014 expected book value is more than 20 times its 1999 book value, which implies a large current M/B ratio.

Is this growth rate in Yahoo’s book value realistic? From Eq. (6), book value grows at the rate equal to profitability, assuming no dividends. Yahoo’s average profitability over the next 15 years (2000 to 2014) has a distribution whose 1st, 50th, and 99th percentiles are -0.97%, 16.11%, and 33.18%, respectively. This distribution seems plausible. While the 99th percentile of 33.18% is large, it is far from unprecedented. Consider all 2,969 firms whose longest continuous series of valid annual ROEs between 1950 and 2002 are at least 15 years long. In this universe, there are 35 firms (1.2% of the total) whose average ROE over the previous 15 or more years exceeds 33.18%. (These numbers should be viewed as merely illustrative, since they are subject to an obvious survival bias.) The 99.9th percentile of Yahoo’s average ROE distribution, 38.79%, seems plausible as well, because 17 firms (0.6% of the total) achieved higher average ROEs over periods of 15 years or more. Microsoft’s and Oracle’s average annual ROEs between 1988 (first year available) and 1999 are 44.46% and 47.19%, respectively.

Given our assumptions, we can infer the probability that the market assigned at the end of 1999 to the event that “Yahoo will become the next Microsoft.” Specifically, we can compute the probability that Yahoo’s average ROE over the 12 years following 1999 will exceed Microsoft’s average ROE of 44.46% over the previous 12 years. This probability is 0.0066, or one in 152, which does not strike us as implausibly large. Yahoo’s valuation in March 2000 seems consistent with plausibly high uncertainty about average excess profitability.

We make some strong simplifying assumptions in this section. For example, all firms face an expected horizon of 15 years over which abnormal profits can be earned, and the

parameters governing the mean reversion and volatility of the firm’s ROE (ϕ^i , $\sigma_{i,0}$, and $\sigma_{i,i}$) are also assumed to be equal across firms. If this model is to be applied in practice, all of these parameters can be tailored to the specific firm or industry.

5. Why did the “bubble” burst?

After reaching a peak on March 10, 2000, the Nasdaq index fell in 2000, 2001, and 2002. This section analyzes the extent to which our valuation model can explain this decline.

Fig. 5 plots the time series of the aggregate M/B ratios for Nasdaq and NYSE/Amex. The aggregate M/B for each index is the ratio of the sums of the year-end market values and the most recent book values of equity across all firms in the index. The M/B of Nasdaq rises and falls dramatically around year 2000, but NYSE/Amex’s M/B exhibits a substantially less pronounced pattern. Based on this figure, we concur with Cochrane (2003), who observes that “if there was a ‘bubble,’ it was concentrated in Nasdaq stocks.”

In our model, investors update their beliefs about Nasdaq’s average excess profitability, $\bar{\psi}^N$, by observing the realized profitability of Nasdaq and NYSE/Amex (see the Appendix for more on learning). Fig. 6 plots the time series of ROE for both indexes. NYSE/Amex’s ROE was around 15% per year in the 1990s, but it fell to about 10% after 2000. Nasdaq’s ROE experienced a substantially larger decrease, from 9% in 1999, to -3% in 2000, -20% in 2001, and -3% in 2002. Given this fall in Nasdaq’s ROE, $\hat{\psi}^N$ must have been revised downward. Moreover, this revision is likely to have been substantial, given the high uncertainty at that time and the properties of Bayesian updating (a given signal elicits a larger revision in beliefs when prior uncertainty is high). Therefore, we attribute the “bursting of the bubble” to unexpected negative news about Nasdaq’s average future profitability.

To analyze the quantitative predictions of our argument, we assume that investors’ prior beliefs about $\bar{\psi}^N$ in March 2000 are summarized by the normal distribution with mean 3% and standard deviation of 3.38% per year. (Recall that these beliefs, along with the equity premium of 3%, justify Nasdaq’s M/B and volatility in March 2000 in Table 4.) We then examine how these beliefs are revised by Bayesian investors who observe realized profitability subsequent to March 2000. Given the resulting posterior beliefs about $\bar{\psi}^N$, we compute the model-implied M/B and return volatility for Nasdaq at the year-ends of 2000, 2001, and 2002, and we compare them with the M/B and volatility observed in the data.

The evolution of beliefs about $\bar{\psi}^N$ is fairly dramatic. Given the poor realized profitability and high uncertainty, the posterior mean of $\bar{\psi}^N$ drops from 3% at the end of 1999 to 0.4%

at the end of 2000, to -3.0% in 2001 and -2.0% in 2002. Posterior uncertainty about $\bar{\psi}^N$ declines slowly due to learning, from 3.38% in March 2000 to 2.98% in 2002.

Given the large negative revision to $\bar{\psi}^N$ in 2000 (from 3% to 0.4%), the model-implied M/B of Nasdaq drops from 8.6 to 3.0. Over the same period, the actual M/B exhibits a comparable decline, from 8.6 to 3.5. While our model matches the observed price decline in 2000 reasonably well, it implies an unrealistically large price decline in 2001. In that year, Nasdaq's model-implied M/B drops to 0.9, while the actual M/B dropped only to 3.3.

The main reason behind the unrealistically large decline in the model-implied M/B is the large revision in $\bar{\psi}^N$ (from 0.4% to -3.0%) induced by Nasdaq's unprecedented -20% ROE in 2001. In reality, the revision in $\bar{\psi}^N$ may not have been so large because Nasdaq's -20% ROE reflects not only poor operating performance but also massive write-offs. For example, JDS Uniphase lost over \$50bn in 2001, mostly as a result of write-offs from bad acquisitions.¹³ A write-off taken today instead of tomorrow increases the profits expected tomorrow. This mechanism is absent from our model, in which low earnings today signal low earnings tomorrow. While it might be useful to model the occasional write-offs separately from the rest of the firm's earnings, such an extension is beyond the scope of the paper.

Nasdaq's model-implied return volatility declines from 46.8% in March 2000 to 39.3% at the end of 2000 and 27.7% at the end of 2001, before rising to 29.4% at the end of 2002. This pattern is not driven by uncertainty, which declines slowly, but rather by the perception of $\bar{\psi}^N$. In our model, volatility increases with $\bar{\psi}^N$; higher $\bar{\psi}^N$ implies that firm profits lie farther in the future, which makes the stock price more sensitive to revisions in $\bar{\psi}^N$. The downward revisions in $\bar{\psi}^N$ in 2000 and 2001 thus reduce the model-implied volatility.

We estimate Nasdaq's actual return volatility at each year-end by averaging the realized monthly volatilities across the 12-month centered moving window. (As before, realized volatility in a given month is computed from daily returns within the month.) Nasdaq's year-end volatility falls from 47.2% in 2000 to 32.0% in 2001, before rising to 37.4% in 2002. This pattern is similar to the pattern produced by the model. Volatility is higher in the data than in the model, by 4% to 8% per year, but this difference is on the margin of statistical significance. One possible reason behind this difference is that the post-peak downward revision in $\bar{\psi}^N$ was not as large as predicted by the model, for reasons discussed above. Higher perceived $\bar{\psi}^N$ would imply higher volatility in the model.

¹³Most of the 2001 write-offs did not qualify as extraordinary items under the U.S. accounting rules, so they appear in Compustat's earnings before extraordinary items, which we use throughout the paper.

Overall, our model does a fairly good job explaining the post-peak behavior of Nasdaq return volatility. The model also delivers a realistically large price decline in 2000, but it produces a price decline in 2001 that is too large relative to the data. We argue that this discrepancy could be due to investors treating the accounting write-offs in 2001 differently from operating earnings when updating their beliefs about Nasdaq’s future profitability.

6. The time series of implied uncertainty

In this section, we use our model to match the whole time series of the aggregate M/B ratios in the new and old economy, plotted in Fig. 5, and we analyze the time series of the implied uncertainty about Nasdaq’s average excess profitability. We obtain this time series in two steps. First, we extract the time series of the equity premium from the observed M/B and profitability of the old economy. Second, we compute the uncertainty that equates the observed M/B of the new economy to its model-implied value, given the new economy’s observed profitability and the equity premium computed in the first step.

In the first step, we compute the time series of y_t , which is the key determinant of the equity premium, by inverting the pricing formula in Eq. (16). In this formula, the M/B ratio of the old economy is a function of y_t and $\bar{\rho}_t$, so y_t can be computed conditional on M/B and $\bar{\rho}_t$. Fig. 7 plots the time series of the implied equity premium. The premium increases from about 5% to about 7.5% per year in the mid-1970s, and then it gradually declines into the late 1990s, to about 5% in 1994, 4% in 1996, and 1% in 1998 (year-ends). The premium then rises to 1.8% in 1999 and 3.2% in 2002.

In the second step, we rely on Proposition 2 to relate Nasdaq’s M/B to uncertainty about $\bar{\psi}^N$, as well as to y_t , the old-economy profitability $\bar{\rho}_t$, the new-economy profitability ρ_t^N , and the new-economy expected excess profitability $\hat{\psi}^N$. For a given value of $\hat{\psi}^N$, we invert the formula (15) for every t to obtain the time series of implied uncertainty $\{\hat{\sigma}_{N,t}\}$.

The outcome of this procedure must be interpreted with caution. Given the deterministic process for $\bar{\psi}^N$ in Eq. (5), $\hat{\sigma}_{N,t}$ in our model can only decrease as more information becomes available, whereas $\hat{\sigma}_{N,t}$ inferred from the observed prices may actually increase over time. This dynamic inconsistency has an analogy in option valuation, where it is customary to invert the Black-Scholes pricing formula at various points in time to obtain the time series of implied volatility. This time series is generally considered informative about time-varying volatility, even though it is inconsistent with the constant volatility assumption of the Black-Scholes model. The Black-Scholes model can be extended to remove this inconsistency,

but only at the cost of added complexity. Similarly, our model can be extended to allow for increases in $\hat{\sigma}_{N,t}$ by adding random shocks to $\bar{\psi}^N$ and by allowing investors to learn about $\bar{\psi}^N$ from signals with time-varying precision. We have examined such a (significantly more complicated) extension, and found that random fluctuation in $\bar{\psi}^N$ increases prices, for the same reason that uncertainty about $\bar{\psi}^N$ increases prices. Therefore, our simplifying assumption of a deterministic $\bar{\psi}^N$ can be viewed as conservative in that it makes it more difficult for our model to match the observed prices.

Panel A of Fig. 8 plots the time series of implied uncertainty, $\hat{\sigma}_{N,t}$, computed under three different assumptions about $\hat{\psi}^N$. For $\hat{\psi}^N = 2\%$, implied uncertainty is zero until 1980; it then rises to about 3% per year in the early 1980s, and then falls back to zero. The uncertainty rises again in the second half of the 1990s, to about 3% at the ends of 1999 and 2000 and to about 4% at the end of 2001, before falling in 2002. This pattern underlines the message of this paper: we argue that the runup in technology stock prices in the late 1990s was partly due to an increase in uncertainty about average profitability.

We also examine the time variation in return volatility implied by this variation in uncertainty. In the model, the new economy's volatility is positively related to uncertainty, but the old economy's volatility is not. Therefore, increases in uncertainty should increase the difference between the model-implied return volatilities in the new and old economies. We compute the year-end time series of this difference, and plot it in Panel B of Fig. 8. The difference is high in the early 1980s, but it is especially high in the late 1990s.

Interestingly, while Nasdaq's implied uncertainty in Panel A of Fig. 8 is only slightly higher in the late 1990s than in the early 1980s, the difference between the return volatilities in Panel B is substantially higher in the late 1990s. The reason is that the equity premium in the late 1990s is substantially lower than in the early 1980s (Fig. 7). Uncertainty has the greatest effect on prices when the equity premium is low, as we argue earlier.

How does the model-implied pattern in Panel B of Fig. 8 compare with data? For each month, we compute the return volatility of the Nasdaq and NYSE/Amex indexes as the standard deviation of the daily index returns in that month. To obtain smoother annual series, we average the monthly values within the year. Nasdaq's volatility increases from 12% per year in 1994 to 47% in 2000, before it declines to 34% in 2002. Panel C of Fig. 8 plots the difference between the Nasdaq and NYSE/Amex return volatilities. This difference rises from 3% per year in 1994 to almost 30% in 2000, after which it falls to 13% in 2002. This pattern is similar to the model-implied pattern in Panel B, consistent with an increase in uncertainty about $\bar{\psi}^N$ in the late 1990s.

In addition to return volatility, we also analyze cash flow volatility. Uncertainty about average future ROE is likely to be high when the cross-sectional variance of ROE is high. By variance decomposition, this variance is the sum of the cross-sectional variance of expected ROE and the cross-sectional expectation of the variance of ROE, both of which make the average future ROE less certain. We compute the cross-sectional standard deviation of ROE for Nasdaq stocks as well as for NYSE/Amex stocks. Fig. 9 shows that the dispersion in the Nasdaq ROEs increases relative to the dispersion in the NYSE/Amex ROEs in the late 1990s, consistent with an increase in uncertainty in the late 1990s.

To summarize, we find that the volatilities of Nasdaq returns and profits increased sharply at the end of the past decade, both in absolute terms and relative to NYSE/Amex. This evidence supports our premise that the uncertainty about the average profitability of Nasdaq firms was unusually high in the late 1990s. The effect of this uncertainty on stock prices was further amplified by a relatively low equity premium at that time.

7. The cross section of implied uncertainty

We argue that Nasdaq valuations in the late 1990s were high in part due to high uncertainty about average profitability. According to this argument, stocks with the highest uncertainty should have not only some of the highest M/B ratios, but also some of the highest return volatilities. We provide anecdotal evidence consistent with this argument in Section 4. In this section, we examine the whole cross-section of Nasdaq firms and find a strong positive relation between implied uncertainty (computed from M/B) and return volatility.

We compute implied uncertainty on March 10, 2000 for all 2,691 Nasdaq firms with valid M/B and ROE data at the end of 1999. For each firm, we choose $\hat{\psi}^i = 0$, $E(T) = 15$ years, and $c = 1.35\%$ (Nasdaq's dividend yield in 1999), for simplicity. The variables y_t and \bar{p}_t are the 1999 year-end values computed in Section 6. We find substantial differences in implied uncertainty across firms. For 66.6% of firms, implied uncertainty is zero (i.e., the model-implied M/B matches or exceeds the actual M/B under zero uncertainty). The 90th percentile of the cross-sectional distribution of implied uncertainty is 5.79%, and the 99th percentile is 9.03% per year. The highest implied uncertainty is observed in the Internet, biotechnology, and telecommunications sectors. In the subsequent analysis, we winsorize the top 1% of the observations of implied uncertainty, i.e, we set their values equal to the 99th percentile of the cross-sectional distribution of implied uncertainty.

According to the model, stocks with high implied uncertainty should have highly volatile

returns. Specifically, Eq. (20) predicts a linear positive relation between squared implied uncertainty and idiosyncratic return volatility. We compute idiosyncratic volatility for a given stock in a given month as the residual volatility from the regression of the stock’s daily returns within the month (if at least ten daily returns are available) on the contemporaneous and lagged market returns. Market returns lagged by one and two days are included to mitigate potential concerns about nonsynchronous trading. We compute idiosyncratic volatility in a given year as the average of the 12 monthly volatilities. Note that the exact definition of idiosyncratic volatility does not appear to be crucial because all results in this section are also highly significant when residual variance is replaced by total variance.

The model’s prediction is strongly supported by the data. The cross-sectional correlation between squared implied uncertainty on March 10, 2000 and idiosyncratic return volatility in 2000 is 53%. When the year-2000 values of idiosyncratic volatility are replaced by the noisier March 2000 values, the correlation remains high, at 38%. To assess the significance of the correlation, we regress squared implied uncertainty on March 10, 2000 on idiosyncratic volatility in 2000. Since many observations of implied uncertainty are censored at zero, we estimate a censored regression model by using the maximum likelihood procedure. The estimated slope coefficient implies that a 10% per year difference in return volatility translates into a difference of 0.00056 in squared implied uncertainty, which is the difference between the implied uncertainties of zero and 2.37% per year, or between 5% and 5.53%. The asymptotic t -statistic for the slope coefficient is 19.69, which indicates a highly significant relation, assuming that the residuals are cross-sectionally independent.

To provide additional evidence on the relation between implied uncertainty and return volatility, we compute implied uncertainty and idiosyncratic volatility for all Nasdaq firms at each year-end between 1973 and 2002, and we run the same censored cross-sectional regression for each year. The slope coefficient is positive in every single year, and the Fama and MacBeth (1973) t -statistic computed from the time series of the 30 estimated coefficients is 4.79. This t -statistic is adjusted for any significant serial correlation in the time series of the estimated coefficients (the t -statistic that assumes serial independence is 8.14). The cross-sectional correlation between squared implied uncertainty and idiosyncratic return volatility varies from the low of 4% at the end of 1990 to the high of 53% on March 10, 2000, and the time-series average of the year-end values is 25%. High correlations obtain not only in the late 1990s but also in the early 1980s. Because the early 1980s witnessed a technology boom that has been characterized as a “biotech revolution” (Malkiel, 1999), it seems plausible for uncertainty to play a role in that period. Note that Panel A of Fig. 8 indicates high uncertainty in the early 1980s, without using any information about return volatility.

The above discussion focuses on the univariate relation between implied uncertainty and return volatility, but this relation survives controls for various firm characteristics such as market capitalization and the dividend yield. PV (2003) report a significant positive cross-sectional relation between M/B (the key determinant of implied uncertainty) and idiosyncratic return volatility, after controlling for a larger set of firm characteristics. To summarize, we find that implied uncertainty is positively cross-sectionally related to idiosyncratic return volatility, as predicted by the model.

8. Discussion

The first part of this section discusses the effect of uncertainty about $\bar{\psi}$ on the discount rate. The second part discusses several issues – the firm’s investment and dividend policies, the dynamics of learning, the convergence of market value to book value, uncertainty about T , extreme values of $\bar{\psi}$, and employee stock options – that help us understand the extent to which our valuation procedure can be viewed as conservative.

8.1. The effect of uncertainty on the discount rate

First, we dispel the popular myth that uncertainty about average profitability always increases the risk premium. In a general equilibrium model in which dividends equal consumption, uncertainty about average dividend growth can increase or decrease the risk premium, depending on the representative agent’s elasticity of intertemporal substitution (Veronesi, 2000). In the presence of uncertainty, observing realized dividends leads investors to revise their expectations of future consumption growth, which leads to intertemporal consumption smoothing, which in turn pushes prices in the direction opposite to that obtained in the absence of uncertainty. In a power utility setting, Veronesi shows that more uncertainty translates into a lower risk premium if the elasticity of intertemporal substitution is less than one. We present a more general version of the same result in the Appendix.

In a model similar to ours, Johnson (2004) shows that higher uncertainty translates into a lower expected return in the presence of leverage. Johnson also confirms the result of PV (2003) that uncertainty has no effect on expected return when there is no leverage and no dividends, and when T is fixed. Berk, Green, and Naik (2004) show that higher uncertainty about R&D productivity can lead to a lower expected return and a higher valuation.

The SDF in Eq. (8) is independent of $\bar{\psi}$. As a result, uncertainty about $\bar{\psi}$ has no effect on expected return in the special case in which the firm pays no dividends and T is known, consistent with PV (2003) and Johnson (2004). In our model, however, T is random, and

some firms (e.g., Nasdaq as a whole) pay dividends, which makes it possible for uncertainty to affect the risk premium. Indeed, it turns out that uncertainty about $\bar{\psi}$ increases the risk premium in our model for any reasonable set of parameter values because it increases the sensitivity of prices to the systematic shocks (W_0). For example, under the beliefs used in Fig. 1 ($\hat{\psi}^N = 3\%$ and the equity premium of 3%), Nasdaq's risk premium increases from 5.36% under zero uncertainty to 6.62% under 4% uncertainty.

One might think that uncertainty would have a stronger effect on the risk premium if the SDF were allowed to depend on $\bar{\psi}^N$. To evaluate this conjecture, we extend our model to allow the drift of ϵ_t in Eq. (11) to depend on Nasdaq's average profitability:

$$d\epsilon_t = \left(\alpha_0 + \alpha_1(\bar{p}_t + \bar{\psi}_t^N) \right) dt + \sigma_\epsilon dW_{0,t}. \quad (17)$$

In our model, $\alpha_1 = 0$ in Eq. (17); however, $\alpha_1 > 0$ also seems plausible. In the consumption-based interpretation of Eq. (11), ϵ_t is log aggregate consumption, and it seems plausible for expected consumption growth to be positively related to Nasdaq's expected profitability. We show in an earlier draft that an especially reasonable value of α_1 is the fraction of total consumption that is financed by Nasdaq dividends. In 1999, this fraction was $\alpha_1 = 0.00138$, i.e., Nasdaq dividends accounted for less than one-seventh of 1% of total consumption.

To assess the effect of uncertainty on the risk premium, consider the difference between the Nasdaq risk premium obtained under 3% uncertainty and the premium obtained under zero uncertainty (using $\hat{\psi}^N = 3\%$ and the equity premium of 3%, as before). This difference is 0.67% per year for $\alpha_1 = 0$. For $\alpha_1 = 0.00138$, the difference grows, but only to 0.73%. A 0.06% per year difference in the risk premium has a small effect on M/B, since it is tiny compared to Nasdaq's expected total real return, which exceeds 11%. We also consider larger values of α_1 , because Nasdaq dividends could potentially become larger relative to total consumption in the future. For $\alpha_1 = 5 \times 0.00138$, the risk premium difference grows to 0.84%. For $\alpha_1 = 10 \times 0.00138$, the difference declines to 0.83%, and for $\alpha_1 = 20 \times 0.00138$, the difference declines further to 0.76%. That is, as α_1 increases, the positive effect of uncertainty on the risk premium becomes slightly stronger at first, but then it weakens for larger values of α_1 due to intertemporal consumption smoothing discussed earlier in this section. To summarize, the effect of uncertainty on the risk premium does not change much when we allow Nasdaq's expected profitability to enter the drift of consumption growth.

While the above discussion focuses on the drift of consumption growth, the volatility of consumption growth could in principle also be affected by Nasdaq's average profitability or by uncertainty about it. Such an effect does not obtain in a pure exchange economy, but it

might perhaps obtain in a production economy in which consumption decisions are linked to investment decisions. This conjecture can be investigated in future work.

8.2. Is our valuation procedure conservative?

The firm's investment policy. The profitability process in Eq. (2) is a reduced-form model for the firm's investment policy. We assume that the firm makes optimal investment decisions and that the resulting profitability process is mean-reverting, consistent with empirical evidence (e.g., Beaver, 1970). For tractability, we also assume that the firm cannot shut down its operations, even if its $\bar{\psi}$ turns out to be low. This assumption makes our approach conservative because the shut-down option would make the firm more valuable.

The firm's dividend policy. The assumption that dividends are a constant fraction of book value also makes our approach conservative. PV (2003) explain that if the firm issues more equity when expected profits are high and pays higher dividends when expected profits are low, then the firm's market value becomes even more convex in $\bar{\psi}$. As a result, uncertainty about $\bar{\psi}$ has an even greater positive effect on firm value, and the observed valuations can be matched with more conservative beliefs about future profitability.

Bayesian learning. In our model, investors learn about average excess profitability ($\bar{\psi}$) by observing realized profits. Given the deterministic process for $\bar{\psi}$ in Eq. (5), uncertainty about $\bar{\psi}$ declines deterministically over time (Eq. 19). As we explain in Section 6, extending our model to allow for increases in uncertainty would make it easier to match the observed prices in Section 4, which makes our approach conservative.

The convergence of market value to book value. We assume that a firm's M/B ratio equals one when competition arrives and wipes out the firm's future abnormal profits. This assumption might be too conservative. The absence of intangible assets from the accounting books implies that M/B is likely to exceed one even after profits are competed away. The convention of conservative accounting (that profits are booked when earned but losses when anticipated) has the same implication. In practice, therefore, it might be reasonable to assume $M/B > 1$ when competition arrives. Of course, such an assumption makes it easier for our model to match the observed Nasdaq prices.

To illustrate this point, suppose that the assumption of $M/B \rightarrow 1$ is replaced by $M/B \rightarrow 1.77$, which is the average M/B ratio of the old economy (Table 1). Also assume a 3% equity premium. In the equivalent of Table 4, Nasdaq's M/B and volatility can be matched with $\hat{\psi}^N = 1\%$ instead of $\hat{\psi}^N = 3\%$. (With $\hat{\psi}^N = 1\%$, the uncertainty that matches Nasdaq's

M/B of 8.55 is 3.67%, which implies a return volatility of 46.50%, which is close to the observed volatility.) As in Fig. 2, we compute the model-predicted distribution of the ratio of Nasdaq’s book value to the NYSE/Amex/Nasdaq book value after 20 years. The 5th, 50th, and 95th percentiles of this distribution are 0.13, 0.36, and 0.68; all of these values are smaller than under $M/B \rightarrow 1$. In short, our assumption of $M/B \rightarrow 1$ is conservative, and its realistic modification makes the observed Nasdaq valuations easier to match.

Uncertainty about T . Firm i can earn abnormal profits until competition arrives at time T_i (or T , for short). We assume that competition can arrive in any instant with intensity p . Thus, T is exponentially distributed, the expected value of T is $E(T) = 1/p$, and the median is $-\log(0.5)E(T) \approx 0.69E(T)$. For $p = 1/20$ (used for Nasdaq), the median value of T is 13.9 years, and for $p = 1/15$ (used for individual firms), the median T is 10.4 years.

Uncertainty about T increases the firm’s M/B ratio. Just like uncertainty about $\bar{\psi}$ increases M/B because M/B is convex in $\bar{\psi}$, uncertainty about T increases M/B because M/B is convex in T . The reason in both cases is compounding; loosely speaking, $\exp(\bar{\psi}T)$ is convex in both $\bar{\psi}$ and T . To illustrate the effect of uncertainty about T on M/B, we reconstruct Table 4 under the assumption that $T = 20$ years with certainty. With the 3% equity premium, we require a $\hat{\psi}^N$ of about 5.5% (along with an uncertainty of about 5.3%) to match Nasdaq’s M/B as well as volatility, as opposed to $\hat{\psi}^N = 3\%$ when T is uncertain. Of course, T is unknown in practice. Acknowledging uncertainty about T makes it easier for us to match the observed Nasdaq valuations. (Note that uncertainty about T should not affect the SDF. The arrival of competition transfers wealth between firms without affecting aggregate consumption because the representative investor owns not only the incumbent firm but also its competitor.)

For tractability, we assume that p is constant, but one might argue that competition is more likely to arrive when $\hat{\psi}$ is high. Incorporating a positive correlation between p and $\hat{\psi}$ would make it more difficult to match high M/Bs. On the other hand, highly profitable firms often possess some monopoly power, which allows them to create barriers to entry and keep the competition away. It is not clear which of the two effects prevails.

Our analysis focuses on the implied distribution of $\bar{\psi}$, holding the distribution of T constant. In principle, we could also reverse the focus and compute the implied distribution of T , holding the distribution of $\bar{\psi}$ constant. To implement such an exercise, it would be useful to replace the single-parameter exponential distribution for T by a two-parameter distribution, so that the mean and variance of T can be governed by different parameters. In the interest of tractability, we leave such an exercise for future research.

Extreme values of $\bar{\psi}$. A normal distribution for $\bar{\psi}$ assigns a positive probability to extremely large values of $\bar{\psi}$. While our results are somewhat sensitive to such values, our conclusions are not, because the prices and volatilities of technology stocks can also be matched when the normal distribution is truncated.

Consider Yahoo, whose M/B and volatility in March 2000 can be matched with $\hat{\psi} = 4\%$, 3% equity premium, and 4.40% uncertainty. When the normal distribution of Yahoo's $\bar{\psi}$ is truncated at the mean ± 3 standard deviations (i.e., at -9.2% and 17.2%), Yahoo's M/B drops from 78.4 to 56.4. The drop is not surprising due to the convexity of M/B in $\bar{\psi}$. Importantly, the truncation reduces not only Yahoo's M/B, but also its return volatility, from 81.78% to 72.46%. To bring Yahoo's M/B back to 78.4, uncertainty must increase from 4.40% (under no truncation) to 4.72% (under truncation). This higher level of uncertainty seems plausible, because it implies return volatility of 77.65%, which is close to Yahoo's observed volatility. Even when the truncation of $\bar{\psi}$ is tighter, at ± 2 standard deviations, an increase in uncertainty from 4.40% to 5.76% matches Yahoo's M/B as well as volatility under the truncated beliefs. If the same two-standard-deviation truncation is applied to the beliefs that rationalize Nasdaq's pricing in Fig. 1, an increase in uncertainty from 3.38% to 4.04% is sufficient to match both Nasdaq's M/B and volatility under the truncated beliefs. In short, our conclusions are not driven by extreme values of $\bar{\psi}$.

Employee stock options. The earnings reported in Compustat are not adjusted for the employee stock option expense. If stock options were expensed, the reported earnings of the S&P 500 firms would be reduced by 8% in 1999, and by almost 10% in 2000 (The Wall Street Journal, July 16, 2002). Botosan and Plumlee (2001) report median reductions between 9.8% and 14% in 1996 to 1999 based on the sample of the 100 fastest-growing firms in the U.S., as identified by Fortune magazine in September 1999. As a simple though imperfect robustness check, we repeat our analysis with all Nasdaq earnings reduced by one-seventh, and we find that the observed valuations and volatilities can still be matched with reasonable parameter values. For example, to match Nasdaq's M/B with $\hat{\psi}^N = 3\%$ and a 3% equity premium, the implied uncertainty increases from 3.38% to 3.47%, which implies a return volatility of 48.33%, which is still close to the volatility observed in the data.

Reducing current earnings is an accounting adjustment for employee stock options. This adjustment reduces future book value, but the actual reduction in book value is more complicated because call options tend to be exercised after favorable stock performance. The optimal exercise is likely to reduce the skewness of the distribution of the future book equity per share, and this reduction could be larger than the reduction that obtains under the

accounting approach. Alas, quantifying the effect of the optimal option exercise on future book value seems too difficult to be attempted here. Our accounting adjustment provides only an approximation to the effect of employee options on firm value.

9. Conclusions

Some academics and practitioners believe it is self-evident that Nasdaq stocks were overvalued in the late 1990s. We argue that the Nasdaq valuations were not necessarily irrational *ex ante* because uncertainty about average profitability, which increases the fundamental value of a firm, was unusually high in the late 1990s. After calibrating a stock valuation model that incorporates such uncertainty, we compute the level of uncertainty that rationalizes the observed Nasdaq valuations at the peak of the “bubble.” The key question is whether this uncertainty, which we call implied uncertainty, is plausible. While we make it possible for readers to draw their own conclusions, we also offer our own assessment. We find the implied uncertainty plausible because it matches not only the high level but also the high volatility of Nasdaq stock prices, both for the index as a whole and for individual firms. Stocks with the highest M/B ratios in the late 1990s also had highly volatile returns, consistent with our premise that these stocks had the most uncertain future growth rates. We argue that the level and volatility of stock prices are linked through uncertainty.

The Nasdaq “bubble” was accompanied not only by high return volatility but also by a high volume of trading in technology stocks (e.g., Ofek and Richardson, 2003). The high trading volume is broadly consistent with high uncertainty about the average profitability of technology firms. There is no trading in our single-agent model, but consider an extension in which some agents observe different signals about average future profitability and some agents trade for liquidity reasons. In this extension, agents will trade because different signals imply different perceptions of the fundamental value. Moreover, the amount of trading is likely to increase with uncertainty. When uncertainty is high, signals are drawn from a wider distribution, which implies perceptions of value that are more dispersed across agents, and hence more trading. This model can be explored in future work.

Why did the “bubble” burst? Although we argue that Nasdaq prices rose in the late 1990s partly due to an increase in uncertainty, we do not claim that prices fell in 2000 due to a decline in uncertainty. Instead, we argue that Nasdaq’s expected profitability was revised downward when Nasdaq’s profitability plummeted in 2000 and 2001. This revision must have been substantial, given the high uncertainty at that time. We quantify the Bayesian updating process and show that our model is capable of explaining large price declines

on Nasdaq after March 2000. Starting with prior beliefs that match Nasdaq's M/B and volatility in March 2000, the model predicts a realistic price drop in 2000, and the predicted price decline in 2001 is even larger than the decline observed in the data. The model also produces a post-peak pattern in return volatility that is comparable to the empirical pattern.

The effect of uncertainty on stock prices is especially strong when the equity premium is low. For example, our analysis suggests that uncertainty was high not only in the late 1990s but also during the biotech boom in the early 1980s. The M/B ratios were higher in the 1990s because the equity premium declined between the early 1980s and the late 1990s, according to our model. A decline in the equity premium boosts prices in two ways: by reducing the discount rate, and by amplifying the positive effect of uncertainty on prices.

A decline in the equity premium also strengthens the impact of uncertainty on idiosyncratic return volatility. As a result, holding uncertainty constant, idiosyncratic volatility increases in our model when the equity premium declines. Therefore, the gradual increase in average idiosyncratic return volatility documented by Campbell, Lettau, Malkiel, and Xu (2001) might be related to the apparent gradual decline in the equity premium over the past few decades. This conjecture can be examined further in future work.

Future research can also test our model against alternatives that involve behavioral biases. To construct a fair horserace, it would be useful to develop a behavioral model that can be calibrated to match the observed prices and volatilities of Nasdaq firms in the late 1990s, as our model does. Until our model is rejected in favor of such an alternative, the existence of a Nasdaq "bubble" in the late 1990s should not be taken for granted.

Appendix.

Learning. Let $\bar{\psi}_t^i$ follow Eq. (5), $\mathbf{Z}_t = (\rho_t^i, \bar{\rho}_t, y_t)'$, and the prior distribution of $\bar{\psi}_t^i$ at $t = 0$ be normal, $N(\widehat{\psi}_0^i, \widehat{\sigma}_{i,0}^2)$. The posterior of $\bar{\psi}_t^i$ conditional on $\mathcal{F}_t = \{\mathbf{Z}_\tau : 0 \leq \tau \leq t\}$ is also normal, and the posterior moments $\widehat{\psi}_t^i = E_t[\bar{\psi}_t^i]$ and $\widehat{\sigma}_{i,t}^2 = E_t\left[\left(\bar{\psi}_t^i - \widehat{\psi}_t^i\right)^2\right]$ at $t > 0$ follow

$$d\widehat{\psi}_t^i = -k_\psi \widehat{\psi}_t^i dt + \widehat{\sigma}_{i,t}^2 \left(\frac{\phi^i}{\sigma_{ii}}\right) d\widehat{W}_{i,t} \quad (18)$$

$$\frac{d\widehat{\sigma}_{i,t}^2}{dt} = -2k_\psi \widehat{\sigma}_{i,t}^2 - (\widehat{\sigma}_{i,t}^2)^2 \left(\frac{\phi^i}{\sigma_{ii}}\right)^2. \quad (19)$$

Above, $\widehat{W}_{i,t}$ is the third entry in the vector of expectation errors, $\widehat{\mathbf{W}}_t = [\widehat{W}_{0,t}, \widehat{W}_{L,t}, \widehat{W}_{i,t}]$, which follows $d\widehat{\mathbf{W}}_t = \boldsymbol{\Sigma}_Z^{-1} [d\mathbf{Z}_t - E_t(d\mathbf{Z}_t)]$. To obtain the dynamics of \mathbf{Z}_t , we can define matrices \mathbf{A}_Z , \mathbf{B}_Z , \mathbf{C}_Z , and $\boldsymbol{\Sigma}_Z$ such that Eqs. (2), (4), and (10) can be combined into one as

$$d\mathbf{Z}_t = \left(\mathbf{A}_Z + \mathbf{B}_Z \mathbf{Z}_t + \mathbf{C}_Z \bar{\psi}_t^i\right) dt + \boldsymbol{\Sigma}_Z d\mathbf{W}_t,$$

where $\mathbf{W}_t = [W_{0,t}, W_{L,t}, W_{i,t}]$. These results follow from Liptser and Shirayev (1977).

Expected Return and Volatility. Let $M_t^i/B_t^i \equiv \Phi^i(\rho_t^i, \bar{\rho}_t, y_t, \widehat{\psi}_t^i, \widehat{\sigma}_{i,t}^2)$, following Proposition 2.

Itô's Lemma implies that firm i 's return volatility is given by $\sqrt{\sigma_R^i \sigma_R^{i'}}$, where

$$\sigma_R^i = \frac{1}{\Phi^i} \left(\frac{\partial \Phi^i}{\partial y_t} \bar{\sigma}_y + \frac{\partial \Phi^i}{\partial \bar{\rho}_t} \sigma_L + \frac{\partial \Phi^i}{\partial \rho_t^i} \sigma_i + \frac{\partial \Phi^i}{\partial \widehat{\psi}_t^i} \sigma_{\widehat{\psi}_t^i} \right), \quad (20)$$

$\bar{\sigma}_y = (\sigma_y, 0, 0)$, $\sigma_L = (\sigma_{L,0}, \sigma_{L,L}, 0)$, $\sigma_i = (\sigma_{i,0}, 0, \sigma_{i,i})$, and $\sigma_{\widehat{\psi}_t^i} = \left(0, 0, \frac{\phi^i}{\sigma_{ii}} \widehat{\sigma}_{i,t}^2\right)$. We also have

$$E[dR_t^i] = \sigma_{R,1}^i \sigma_{\pi,t},$$

for expected excess return, where $\sigma_{R,1}^i$ is the first element in σ_R^i and $\sigma_{\pi,t}$ is given in Appendix A.

M/B of the Old Economy: The M/B ratio of the old economy is given by

$$M_t^O/B_t^O = \Phi(\bar{\rho}_t, y_t) = c^O \int_0^\infty Z(y_t, \bar{\rho}_t, s) ds, \quad (21)$$

where

$$Z(y_t, \bar{\rho}_t, s) = e^{Q_0^O(s) + Q_1^O(s)y_t + Q_2^O(s)\bar{\rho}_t + Q_3^O(s)y_t^2},$$

and $Q_0^O(s) = K_0(0; s)$, $Q_1^O(s) = K_2(0; s) + \gamma a_1$, $Q_2^O(s) = K_3(0; s)$, and $Q_3^O(s) = K_6(0; s) + \gamma a_2$, where $K_i(\cdot; s)$ are in Lemma 1 in Appendix B under $\zeta_0 = \zeta_2 = v = 1$ and $\zeta_1 = 0$.

The rest of the Appendix is available as the Technical Appendix on the journal's website as well as on the authors' websites. Part A of the Technical Appendix contains the description of the stochastic discount factor. Part B contains the proofs of Propositions 1 and 2, along with Lemmas 1 and 4 that we refer to here. Part C develops the Gordon growth model under the assumption that the drift rate g of dividend growth is uncertain.

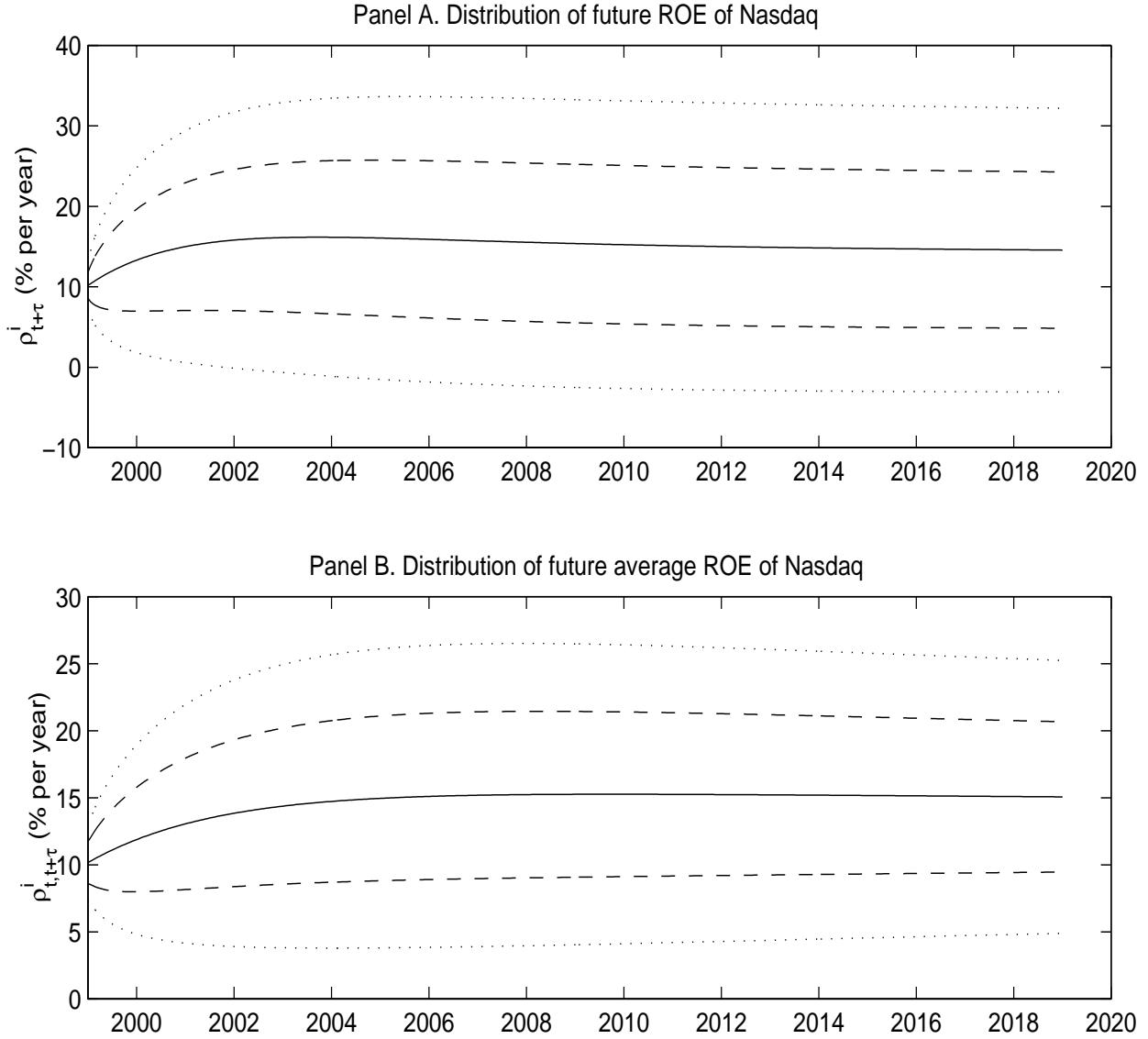


Fig. 1. Model-predicted distributions of future profitability and average future profitability for Nasdaq. Panel A plots the selected percentiles (1st, 10th, 50th, 90th, and 99th) of the model-predicted distribution of Nasdaq's future profitability (measured as return on equity, ROE). Panel B plots the selected percentiles of the distribution of Nasdaq's average future profitability, computed by averaging the ROE values between 1999 and the year given on the horizontal axis. In both panels, the dotted lines denote the 1st and 99th percentiles, the dashed lines denote the 10th and 90th percentiles, and the solid line denotes the 50th percentile. In both panels, the market's expectation of Nasdaq's average excess profitability is 3% per year, the associated uncertainty is 3.38%, the equity premium is 3% per year, and the expected horizon is 20 years. Under these assumptions, the model-implied M/B ratio and return volatility correspond to Nasdaq's actual M/B ratio and return volatility observed on March 10, 2000.

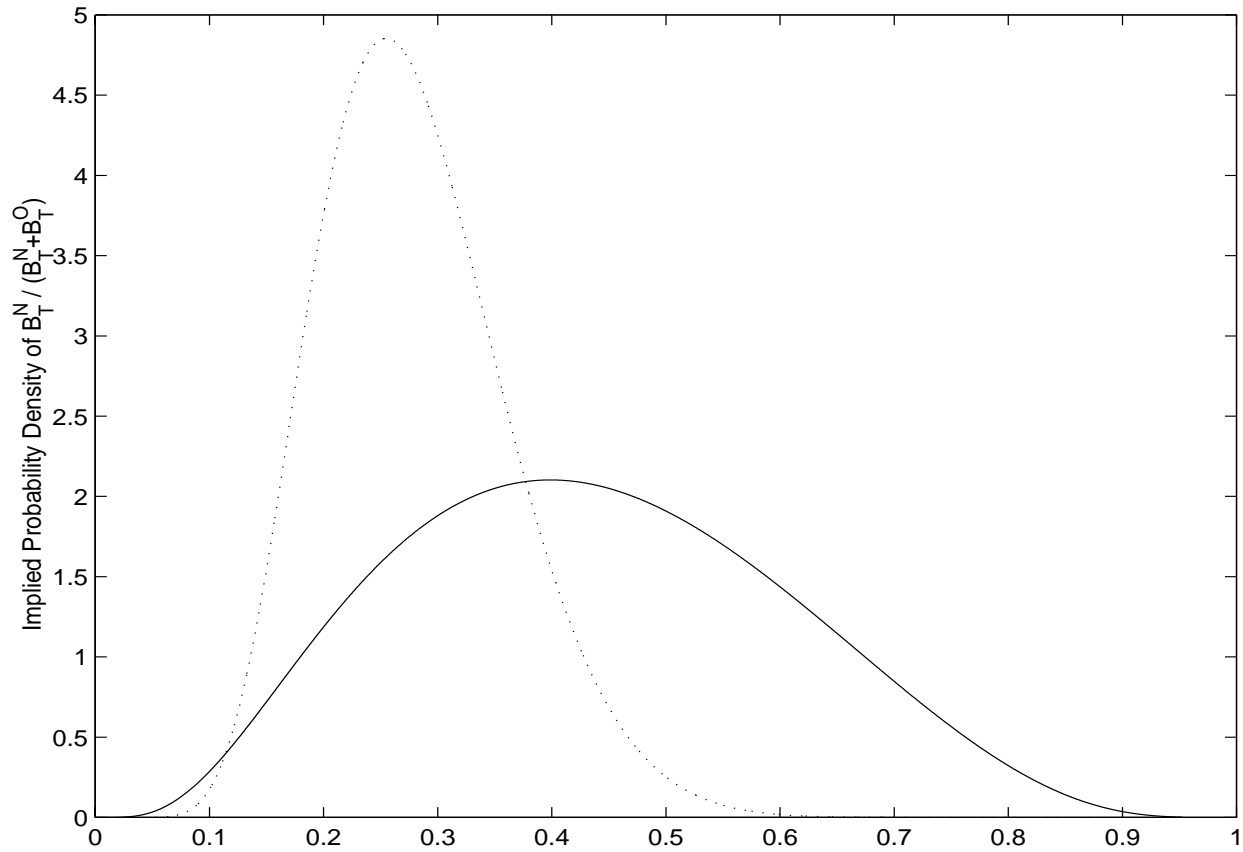


Fig. 2. Model-predicted distribution of the future ratio of Nasdaq book value to NYSE/Amex/Nasdaq book value. The figure plots the model-predicted distribution of the ratio of Nasdaq book value to NYSE/Amex/Nasdaq book value $T = 10$ (dotted line) and $T = 20$ (solid line) years in the future. The market's expectation of Nasdaq's average excess profitability is 3% per year, the associated uncertainty is 3.38%, the equity premium is 3% per year, and the expected horizon is 20 years. Under these assumptions, the model-implied M/B ratio and return volatility correspond to Nasdaq's actual M/B ratio and return volatility observed on March 10, 2000.

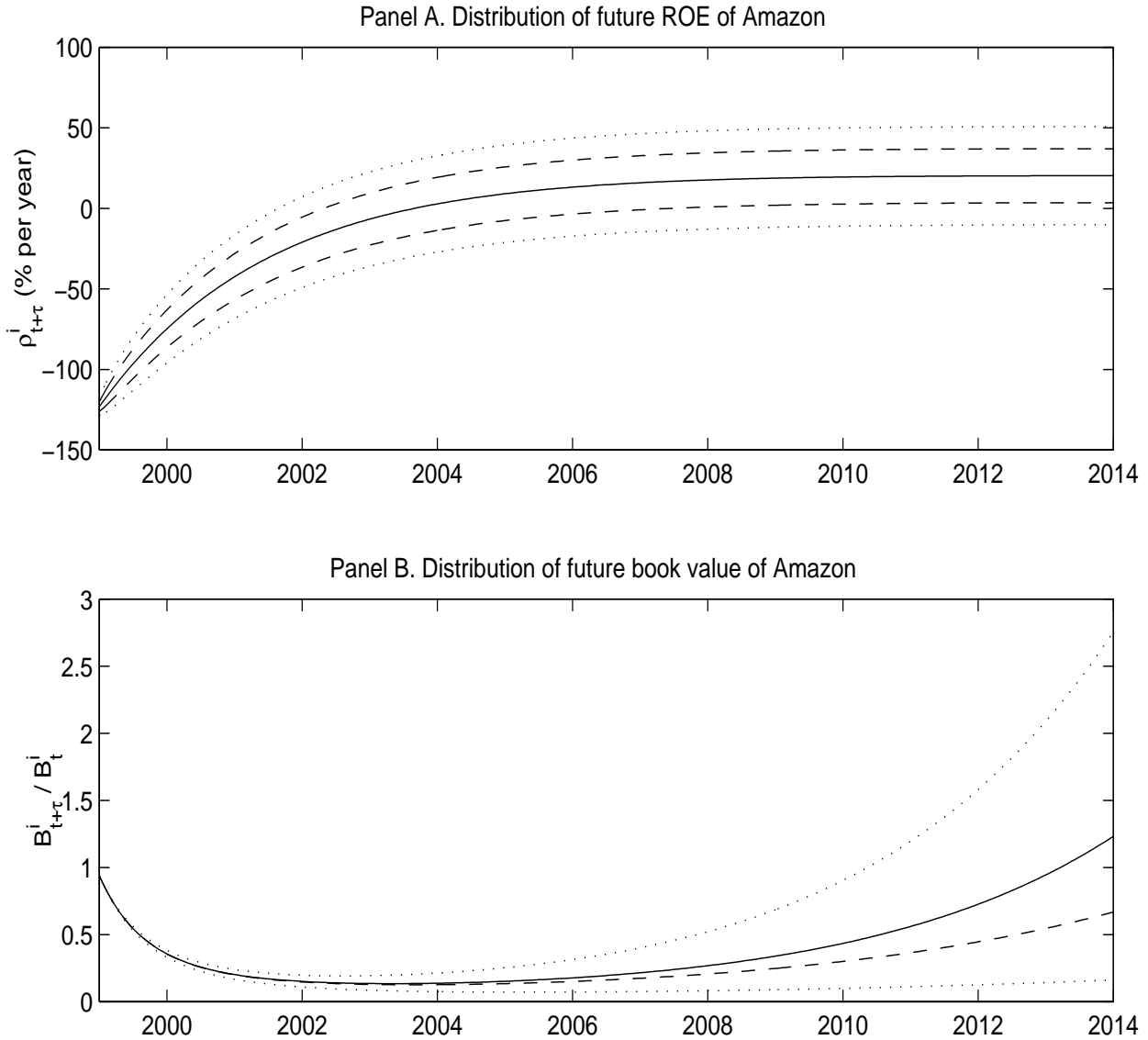


Fig. 3. Model-predicted distributions of future profitability and book value for Amazon. Panel A plots the selected percentiles (1st: dotted line, 10th: dashed line, 50th: solid line, 90th: dashed line, and 99th: dotted line) of the model-predicted distribution of Amazon's future profitability (measured as return on equity, ROE). Panel B plots Amazon's expected future book value (solid line), along with the 10th (dotted line), 50th (dashed line), and 90th (dotted line) percentiles of the book value's model-predicted distribution. The future book values are normalized by the 1999 book value. In both panels, the market's expectation of Amazon's average excess profitability is 10% per year, the associated uncertainty is 4.51%, the equity premium is 3% per year, and the expected horizon is 15 years. Under these assumptions, the model-implied M/B ratio and return volatility correspond to Amazon's actual M/B ratio and return volatility observed on March 10, 2000.

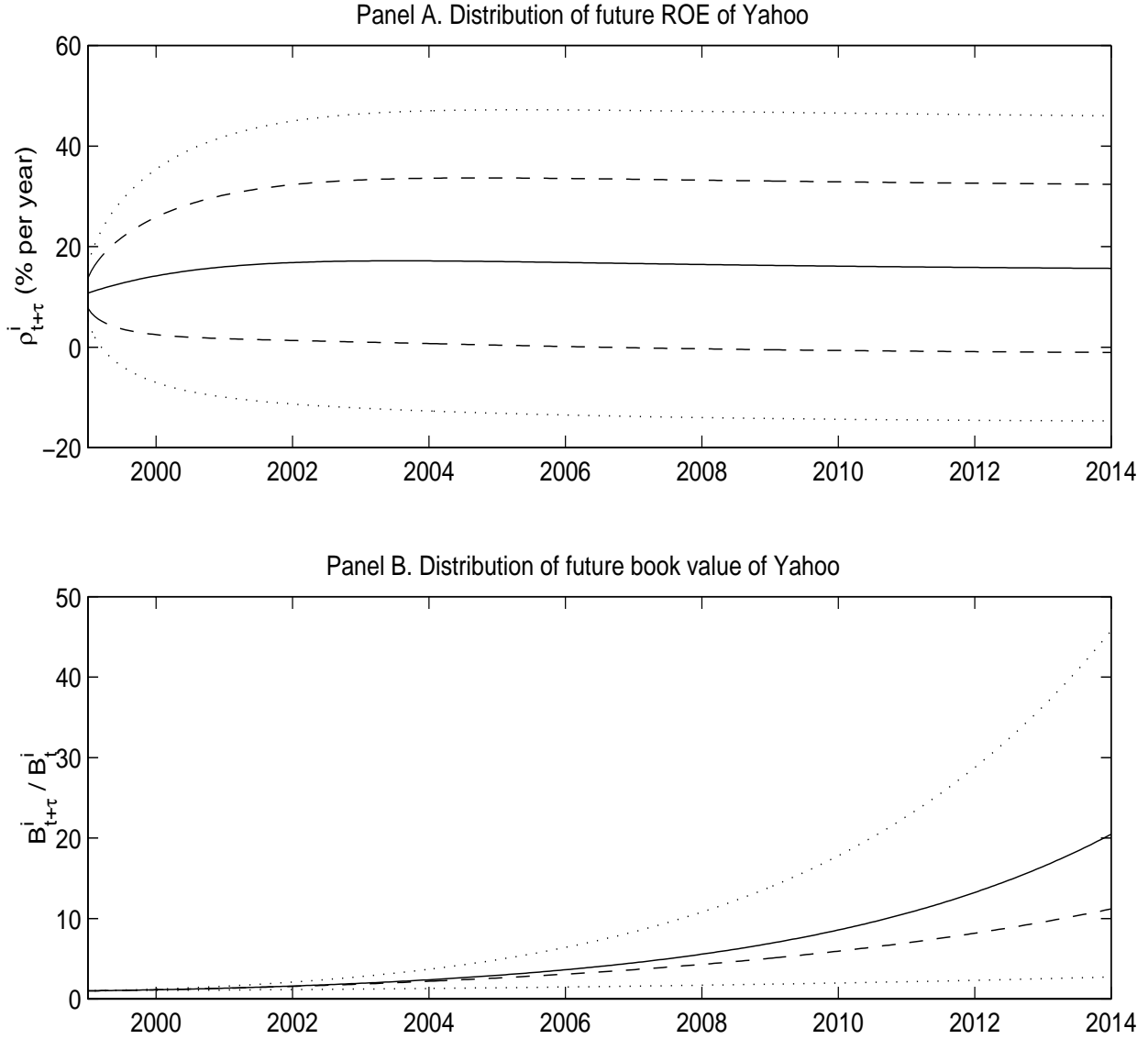


Fig. 4. Model-predicted distributions of future profitability and book value for Yahoo. Panel A plots the selected percentiles (1st: dotted line, 10th: dashed line, 50th: solid line, 90th: dashed line, and 99th: dotted line) of the model-predicted distribution of Yahoo's future profitability (measured as return on equity, ROE). Panel B plots Yahoo's expected future book value (solid line), along with the 10th (dotted line), 50th (dashed line), and 90th (dotted line) percentiles of the book value's model-predicted distribution. The future book values are normalized by the 1999 book value. In both panels, the dotted lines denote the 1st and 99th percentiles, the dashed lines denote the 10th and 90th percentiles, and the solid line denotes the 50th percentile. In both panels, the market's expectation of Yahoo's average excess profitability is 4% per year, the associated uncertainty is 4.40%, the equity premium is 3% per year, and the expected horizon is 15 years. Under these assumptions, the model-implied M/B ratio and return volatility correspond to Yahoo's actual M/B ratio and return volatility observed on March 10, 2000.

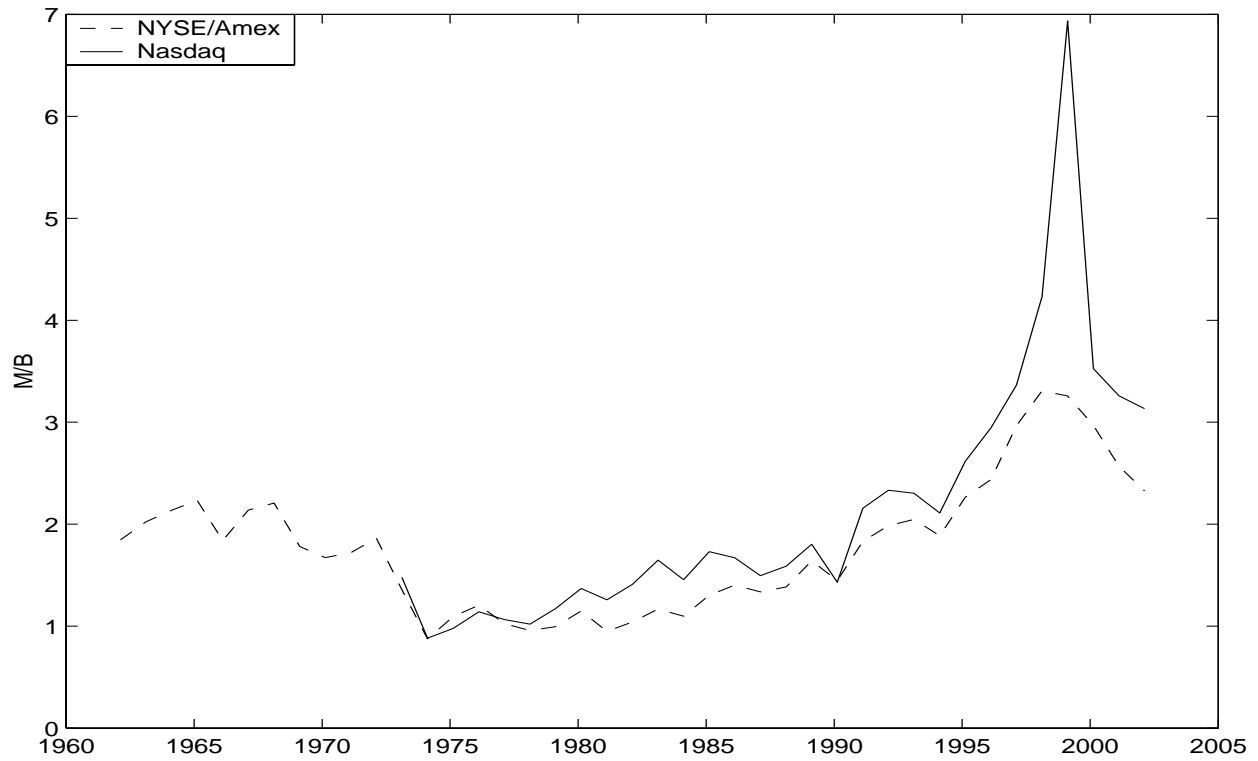


Fig. 5. M/B ratios. This figure plots the annual time series of the market-to-book ratios (M/B) of the Nasdaq index and the combined NYSE/Amex index. The M/B ratio of each index is computed as the sum of the market values of equity across firms in the index at the end of the current year, divided by the sum of the book values of equity at the end of the previous year.

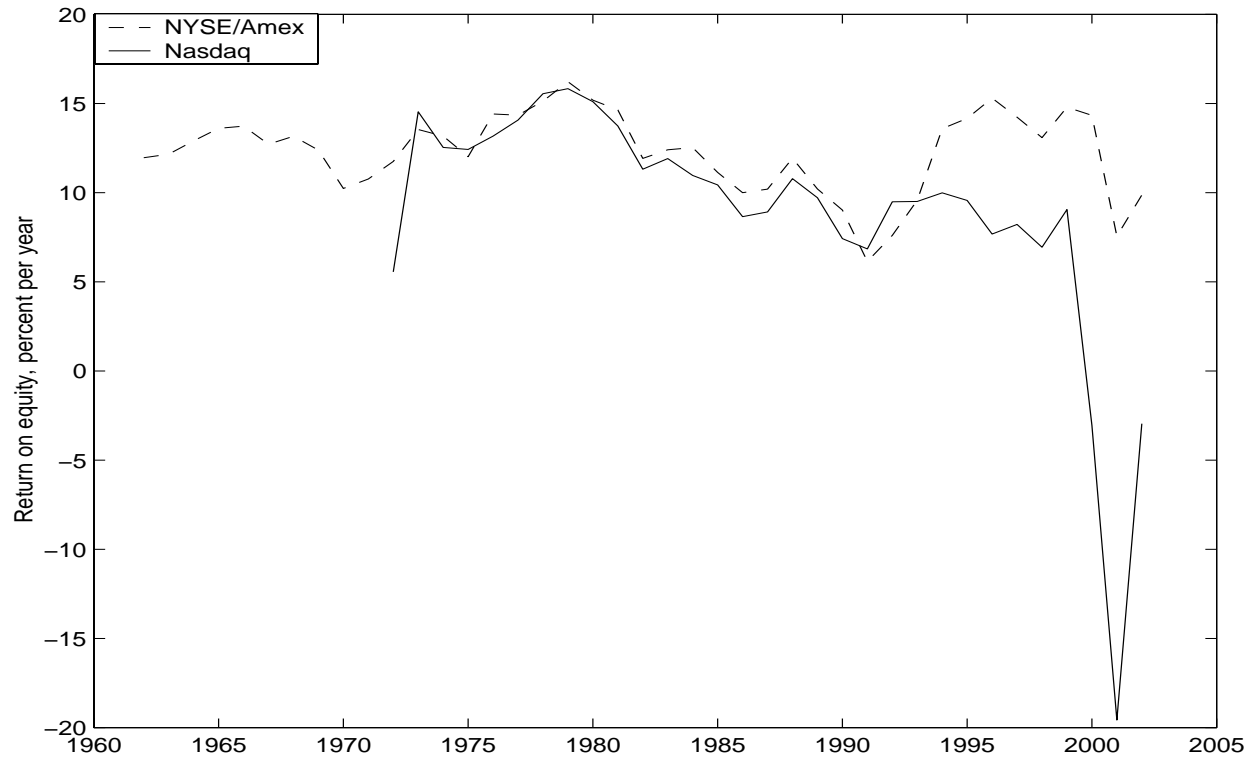


Fig. 6. Realized profitability. This figure plots the annual time series of the real realized profitability (return on equity, ROE) of the Nasdaq index and the combined NYSE/Amex index. The ROE of each index is computed as the sum of current-year earnings across firms in the index, divided by the sum of the book values of equity at the end of the previous year.



Fig. 7. Implied equity premium. This figure plots the annual time series of the equity premium that sets the actual M/B ratio of the NYSE/Amex index at the end of the current year equal to its model-implied value.

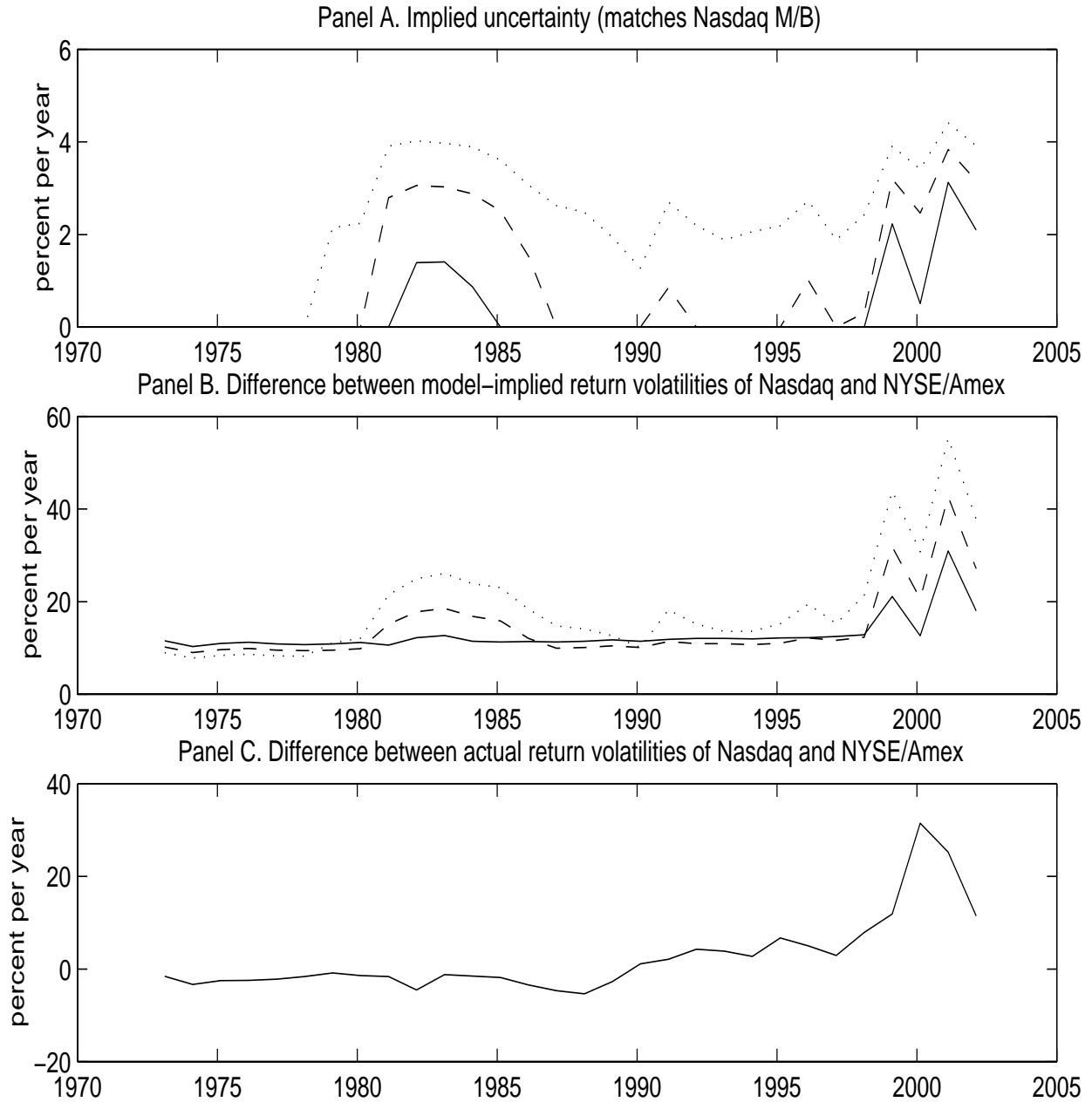


Fig. 8. Implied uncertainty and return volatility. Panel A plots the time series of the implied uncertainty about Nasdaq’s average excess profitability. Implied uncertainty sets the actual M/B ratio of the Nasdaq index at the end of the current year equal to its model-implied value. This uncertainty is plotted for three different values of expected excess profitability: 1% (dotted line), 2% (dashed line), and 3% (solid line). Panel B plots the difference between the model-implied return volatilities of the Nasdaq and NYSE/Amex indices. Panel C plots the difference between the actual return volatilities of Nasdaq and NYSE/Amex. The actual volatility in each month is computed as the standard deviation of the daily index returns within the month. The annual volatility values are then computed by averaging the monthly values within the year.

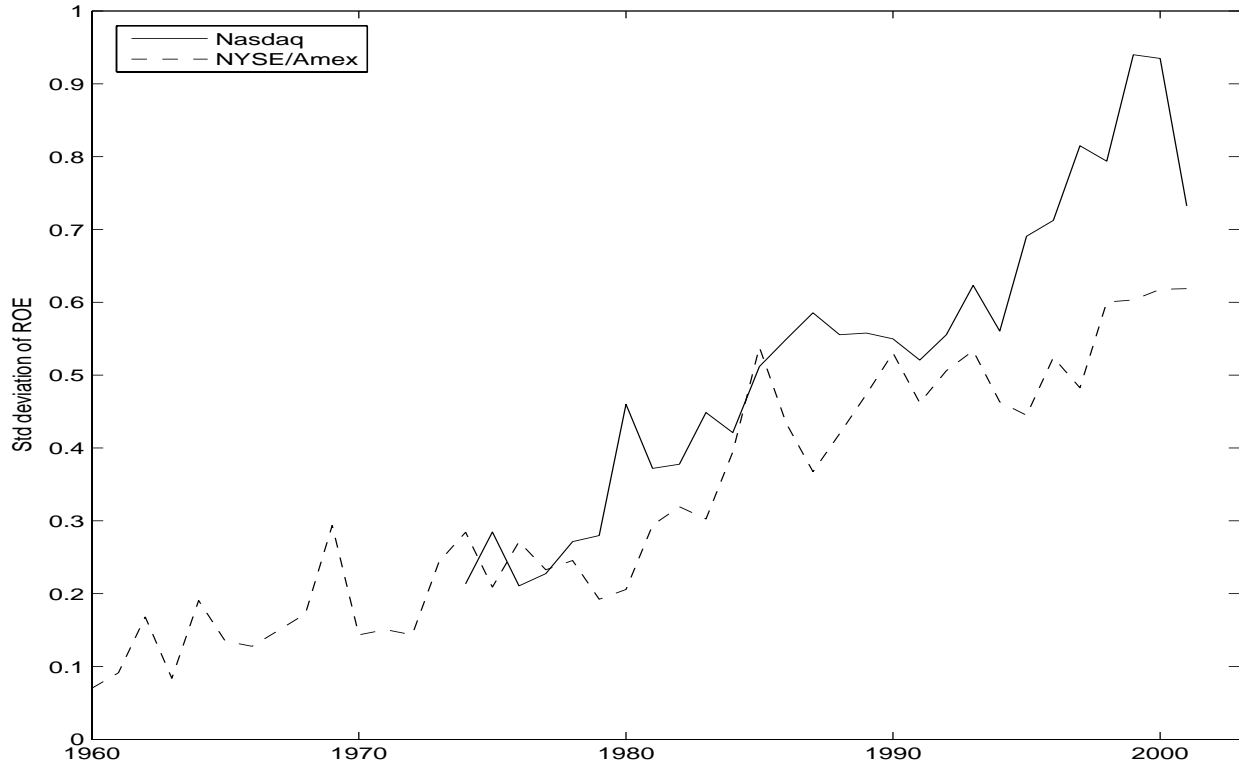


Fig. 9. Cross-sectional standard deviation of profitability for Nasdaq firms and for NYSE/Amex firms. Profitability (return on equity, ROE) of each firm in each year is computed as the firm's earnings in the given year divided by the firm's book equity at the end of the previous year. ROEs larger than 1,000% per year in absolute value are excluded.

Table 1
Parameter Values in the Calibrated Model.

The table reports the parameter values used to calibrate our model. The parameters of the processes for the new-economy and old-economy aggregate profitability are estimated by maximum likelihood from the data on the aggregate profitability of Nasdaq and NYSE/Amex firms. The parameters of the individual firm profitability process are calibrated to the median Nasdaq firm in our sample. The utility parameters (η and γ), the parameters defining the log surplus consumption ratio $s(y) = a_0 + a_1 y_t + a_2 y_t^2$, and those characterizing the state variable y_t are calibrated to match the observed levels of the equity premium, market volatility, aggregate M/B , and the interest rate. The means and standard deviations of the fitted quantities are computed from the time series of the fitted values of the old economy's M/B ratio, conditional expected excess return $\mu_{R,t}^{mkt}$, conditional standard deviation of excess returns $\sigma_{R,t}^{mkt}$, and the real risk-free rate $r_{f,t}$ over the period 1962-2002. All entries are annualized.

Old Economy Profitability				New Economy Profitability			Individ. Firm Profitability		
k_L	$\bar{\rho}_L$	$\sigma_{L,0}$	$\sigma_{L,L}$	ϕ^N	$\sigma_{0,N}$	$\sigma_{N,N}$	ϕ^i	$\sigma_{i,0}$	$\sigma_{i,i}$
0.3574	12.17%	1.47%	1.31%	0.3667	2.46%	4.90%	0.3891	6.65%	8.07%
Stochastic Discount Factor									
η	γ	k_y	\bar{y}	σ_y	a_0	a_1	a_2	μ_ε	σ_ε
0.0471	3.9474	0.0367	-0.08%	25.30%	-2.8780	0.3084	-0.0413	2%	1%
Means of Fitted Quantities				Std Deviations of Fitted Quantities					
$E[M/B]$	$E[\mu_{R,t}^{mkt}]$	$E[\sigma_{R,t}^{mkt}]$	$E[r_{f,t}]$	$\sigma[M/B]$	$\sigma[\mu_{R,t}^{mkt}]$	$\sigma[\sigma_{R,t}^{mkt}]$	$\sigma[r_{f,t}]$		
1.77	5.06%	14.47%	6.25%	0.6477	1.72%	2.24%	1.55%		

Table 2
Nasdaq's Valuation on March 10, 2000 Assuming Zero Uncertainty

Panel A reports the model-implied M/B for the Nasdaq Composite Index on March 10, 2000, assuming zero uncertainty about average excess profitability $\overline{\psi}^N$. Panel B reports the model-implied return volatility for Nasdaq under zero uncertainty. The observed M/B for Nasdaq on March 10, 2000 is 8.55. Nasdaq's annualized standard deviation of daily returns in March 2000 is 41.49%, and its average monthly volatility in 2000 is 47.03% per year. Nasdaq's most recent annualized profitability (ROE in 1999Q4) is $\rho_t^N = 9.96\%$ per year, and its most recent dividend yield (dividends over book equity in 1999) is $c = 1.35\%$ per year. The expected time period over which the Nasdaq index can earn abnormal profits is $E(T) = 20$ years. All variables (equity premium, $\widehat{\psi}^N$, and return volatility) are expressed in percent per year.

Excess ROE $\widehat{\psi}^N$ (% per year)	Equity Premium (% per year)							
	1	2	3	4	5	6	7	8
Panel A: Model-implied M/B with zero uncertainty (Actual M/B: 8.55)								
-5	1.46	1.41	1.30	1.18	1.04	0.89	0.74	0.56
0	3.33	3.02	2.63	2.23	1.84	1.47	1.12	0.76
1	4.15	3.70	3.17	2.64	2.14	1.68	1.25	0.83
2	5.27	4.62	3.89	3.19	2.53	1.95	1.41	0.90
3	6.83	5.89	4.87	3.92	3.05	2.29	1.62	1.00
4	9.06	7.68	6.23	4.92	3.75	2.74	1.88	1.11
5	12.28	10.22	8.15	6.31	4.71	3.36	2.23	1.26
6	17.02	13.92	10.90	8.28	6.04	4.19	2.69	1.45
7	24.09	19.38	14.91	11.12	7.93	5.36	3.32	1.69
8	34.80	27.55	20.85	15.28	10.67	7.02	4.20	2.02
Panel B: Model-implied return volatility with zero uncertainty (Actual volatility: 41.49% in March 2000, 47.03% in 2000)								
-5	15.47	16.94	18.07	18.90	19.50	19.83	19.83	19.25
0	18.09	20.17	21.76	22.93	23.76	24.18	24.10	23.04
1	18.69	20.93	22.65	23.92	24.83	25.31	25.22	24.05
2	19.31	21.71	23.57	24.97	25.97	26.52	26.46	25.18
3	19.93	22.50	24.52	26.05	27.18	27.83	27.81	26.45
4	20.54	23.30	25.47	27.16	28.44	29.21	29.27	27.85
5	21.14	24.07	26.42	28.27	29.72	30.66	30.85	29.42
6	21.71	24.82	27.34	29.37	31.01	32.15	32.52	31.15
7	22.25	25.53	28.23	30.44	32.28	33.65	34.27	33.05
8	22.76	26.20	29.06	31.45	33.51	35.13	36.05	35.10

Table 3
Nasdaq's Valuation on March 10, 2000 Assuming Uncertainty of 3% Per Year

Panel A reports the model-implied M/B for the Nasdaq Composite Index on March 10, 2000, assuming that uncertainty about average excess profitability $\overline{\psi}^N$ is 3% per year. Panel B reports the model-implied return volatility for Nasdaq under 3% uncertainty. The observed M/B for Nasdaq on March 10, 2000 is 8.55. Nasdaq's annualized standard deviation of daily returns in March 2000 is 41.49%, and its average monthly volatility in 2000 is 47.03% per year. Nasdaq's most recent annualized profitability (ROE in 1999Q4) is $\rho_t^N = 9.96\%$ per year, and its most recent dividend yield (dividends over book equity in 1999) is $c = 1.35\%$ per year. The expected time period over which the Nasdaq index can earn abnormal profits is $E(T) = 20$ years. All variables (equity premium, $\widehat{\psi}^N$, and return volatility) are expressed in percent per year.

Excess ROE $\widehat{\psi}^N$ (% per year)	Equity Premium (% per year)							
	1	2	3	4	5	6	7	8
Panel A: Model-implied M/B with 3% uncertainty (Actual M/B: 8.55)								
-5	1.70	1.61	1.46	1.30	1.13	0.95	0.78	0.58
0	4.70	4.09	3.43	2.81	2.23	1.72	1.26	0.82
1	6.16	5.27	4.33	3.47	2.69	2.02	1.44	0.90
2	8.29	6.95	5.59	4.39	3.33	2.43	1.67	1.00
3	11.44	9.40	7.41	5.69	4.21	2.99	1.98	1.13
4	16.17	13.03	10.07	7.57	5.46	3.76	2.40	1.30
5	23.39	18.53	14.05	10.35	7.29	4.87	2.99	1.52
6	34.59	26.96	20.10	14.53	9.99	6.49	3.83	1.82
7	52.23	40.10	29.44	20.91	14.08	8.89	5.04	2.24
8	80.36	60.90	44.08	30.83	20.36	12.54	6.86	2.85
Panel B: Model-implied return volatility with 3% uncertainty (Actual volatility: 41.49% in March 2000, 47.03% in 2000)								
-5	24.90	25.23	25.36	25.29	25.01	24.49	23.60	21.95
0	33.66	33.93	33.97	33.76	33.22	32.25	30.62	27.60
1	35.63	35.96	36.06	35.89	35.37	34.36	32.60	29.23
2	37.59	38.02	38.21	38.12	37.66	36.66	34.81	31.11
3	39.51	40.05	40.37	40.41	40.05	39.13	37.26	33.25
4	41.33	42.03	42.50	42.70	42.51	41.73	39.93	35.70
5	43.05	43.91	44.56	44.94	44.96	44.40	42.77	38.46
6	44.65	45.67	46.50	47.09	47.35	47.07	45.73	41.52
7	46.11	47.29	48.31	49.10	49.63	49.67	48.72	44.85
8	47.45	48.77	49.96	50.96	51.75	52.14	51.65	48.36

Table 4
Matching Nasdaq's Valuation on March 10, 2000

Panel A reports the implied uncertainty for the Nasdaq Composite Index on March 10, 2000, i.e., the uncertainty about average excess profitability $\widehat{\psi}^N$ that equates Nasdaq's model-implied M/B to Nasdaq's observed M/B of 8.55. Panel B reports the model-implied return volatility for Nasdaq computed under implied uncertainty. Nasdaq's annualized standard deviation of daily returns in March 2000 is 41.49%, and its average monthly volatility in 2000 is 47.03% per year. Nasdaq's most recent annualized profitability (ROE in 1999Q4) is $\rho_t^N = 9.96\%$ per year, and its most recent dividend yield (dividends over book equity in 1999) is $c = 1.35\%$ per year. The expected time period over which the Nasdaq index can earn abnormal profits is $E(T) = 20$ years. All variables (equity premium, expected excess profitability $\widehat{\psi}^N$, implied uncertainty, and return volatility) are expressed in percent per year.

Excess ROE	Equity Premium (% per year)							
$\widehat{\psi}^N$ (% per year)	1	2	3	4	5	6	7	8
Panel A. Uncertainty needed to match the observed M/B								
-5	6.41	6.60	6.81	7.04	7.29	7.57	7.89	8.36
0	4.39	4.71	5.06	5.43	5.81	6.22	6.67	7.27
1	3.81	4.17	4.59	5.01	5.44	5.89	6.38	7.03
2	3.08	3.54	4.04	4.53	5.03	5.54	6.08	6.77
3	2.08	2.73	3.38	3.98	4.57	5.15	5.75	6.50
4	0.00	1.45	2.51	3.32	4.04	4.71	5.39	6.22
5	0.00	0.00	0.97	2.43	3.40	4.22	5.00	5.91
6	0.00	0.00	0.00	0.78	2.56	3.63	4.56	5.58
7	0.00	0.00	0.00	0.00	1.18	2.90	4.06	5.23
8	0.00	0.00	0.00	0.00	0.00	1.86	3.47	4.84
Panel B. Model-implied return volatility under implied uncertainty (Actual volatility: 41.49% in March 2000, 47.03% in 2000)								
-5	141.49	151.69	165.51	182.07	202.27	226.99	258.78	307.51
0	64.69	73.70	85.81	100.56	119.11	142.51	173.80	223.37
1	51.35	60.15	71.80	85.98	103.93	126.70	157.49	206.85
2	38.94	47.54	58.69	72.23	89.41	111.45	141.54	190.50
3	27.79	36.12	46.66	59.43	75.73	96.84	126.12	174.41
4	20.54	26.53	36.07	47.81	63.03	83.03	111.22	158.63
5	21.14	24.07	27.70	37.78	51.53	70.16	96.98	143.20
6	21.71	24.82	27.34	30.14	41.66	58.44	83.59	128.22
7	22.25	25.53	28.23	30.44	34.09	48.24	71.18	113.79
8	22.76	26.20	29.06	31.45	33.51	40.08	60.04	100.05

Table 5
Matching the Valuations of Selected Technology Firms on March 10, 2000

The table reports the implied uncertainty for selected technology firms on March 10, 2000, i.e., the uncertainty about average excess profitability $\bar{\psi}^i$ that equates the firm's model-implied M/B to its observed M/B. The table also reports the model-implied return volatility computed under the corresponding value of implied uncertainty. Each firm's name is accompanied by the firm's market capitalization on March 10, 2000, the firm's observed M/B on the same day, the 1999 values of the firm's realized profitability ρ_t^i and dividend yield c^i , as well as two estimates of the firm's actual return volatility: the standard deviation of the stock's daily returns in March 2000, and the average monthly volatility in 2000, in that order. The expected time period over which the firms can earn abnormal profits is $E(T) = 15$ years. All variables (equity premium, expected excess profitability $\hat{\psi}^i$, and implied uncertainty) are expressed in percent per year.

Excess ROE $\hat{\psi}^i$ (% per year)	Equity Premium (% per year)						Equity Premium (% per year)					
	1	2	3	4	5	6	1	2	3	4	5	6
	Implied Uncertainty (% per year)						Implied Return Volatility (% per year)					
	AKAMAI (\$26.15bn): $M/B = 92.92$, $\rho_t = -20.15\%$, $c = 0$, Return volatility: (88.98%, 141.91%)											
-2	6.39	6.54	6.70	6.87	7.06	7.28	142.70	150.38	158.85	167.95	178.30	190.34
0	5.86	6.02	6.20	6.39	6.60	6.83	122.39	130.18	138.76	147.95	158.42	170.57
2	5.26	5.45	5.65	5.86	6.10	6.36	102.55	110.45	119.09	128.35	138.87	151.09
4	4.57	4.79	5.03	5.27	5.54	5.83	83.44	91.43	100.09	109.33	119.85	132.06
6	3.73	4.01	4.30	4.60	4.91	5.25	65.51	73.54	82.14	91.26	101.64	113.71
8	2.59	3.00	3.40	3.78	4.17	4.58	49.56	57.53	65.86	74.65	84.67	96.40
10	0.00	1.32	2.10	2.70	3.25	3.78	38.18	44.72	52.34	60.40	69.67	80.66
	AMAZON (\$23.45bn): $M/B = 88.07$, $\rho_t = -126.08\%$, $c = 0$, Return volatility: (71.67%, 103.33%)											
0	6.94	7.07	7.21	7.35	7.52	7.72	171.63	178.75	186.55	194.90	204.44	215.59
4	5.96	6.12	6.28	6.46	6.65	6.88	131.76	139.13	147.12	155.65	165.36	176.67
8	4.77	4.97	5.18	5.40	5.64	5.91	93.21	100.84	109.01	117.66	127.45	138.83
10	4.02	4.26	4.51	4.77	5.05	5.35	74.95	82.72	90.93	99.57	109.32	120.64
12	3.09	3.40	3.72	4.03	4.37	4.72	58.02	65.89	74.04	82.55	92.14	103.25
16	0.00	0.00	0.67	1.74	2.45	3.07	39.16	43.49	47.87	54.99	63.18	72.94
	CIENA (\$23.40bn): $M/B = 41.24$, $\rho_t = -2.25\%$, $c = 0$, Return volatility: (116.68%, 121.45%)											
-2	5.79	5.97	6.16	6.36	6.57	6.82	115.02	123.03	132.00	141.70	152.78	165.66
0	5.16	5.37	5.58	5.81	6.06	6.33	95.07	103.13	112.13	121.85	132.98	145.92
2	4.42	4.67	4.93	5.20	5.48	5.79	76.12	84.20	93.12	102.76	113.81	126.70
4	3.49	3.82	4.15	4.48	4.82	5.19	58.74	66.76	75.48	84.86	95.64	108.29
6	2.14	2.66	3.14	3.59	4.04	4.49	43.90	51.68	59.93	68.78	79.00	91.11
8	0.00	0.00	1.51	2.34	3.02	3.63	37.53	41.40	47.69	55.51	64.69	75.81
	CISCO (\$455.72bn): $M/B = 39.02$, $\rho_t = 26.58\%$, $c = 0$, Return volatility: (49.81%, 68.88%)											
0	4.59	4.84	5.10	5.37	5.65	5.97	76.33	84.39	93.52	103.50	115.05	128.59
2	3.67	3.99	4.33	4.66	5.00	5.37	58.84	66.78	75.62	85.30	96.53	109.80
4	2.34	2.85	3.33	3.78	4.23	4.68	43.92	51.59	59.90	68.96	79.57	92.25
6	0.00	0.00	1.77	2.57	3.24	3.85	36.69	40.28	47.41	55.42	64.93	76.56
8	0.00	0.00	0.00	1.65	2.72	3.75	37.57	41.46	44.74	47.57	53.68	63.61
	DELL (\$129.88bn): $M/B = 24.47$, $\rho_t = 58.55\%$, $c = 0$, Return volatility: (51.75%, 69.50%)											
-4	5.05	5.29	5.55	5.82	6.10	6.41	76.88	84.69	94.10	104.81	117.48	132.54
-2	4.15	4.47	4.80	5.13	5.48	5.84	59.28	66.80	75.76	85.97	98.19	112.93
0	2.90	3.37	3.85	4.30	4.75	5.19	44.30	51.44	59.71	69.14	80.56	94.57
2	0.00	1.42	2.43	3.17	3.82	4.41	34.31	39.50	46.77	55.06	65.25	78.05
4	0.00	0.00	0.00	0.97	2.46	3.40	35.64	38.92	41.52	44.60	53.07	64.11

Table 5 (continued)

Excess ROE $\hat{\psi}^i$ (% per year)	Equity Premium (% per year)						Equity Premium (% per year)					
	1	2	3	4	5	6	1	2	3	4	5	6
	Implied Uncertainty (% per year)						Implied Return Volatility (% per year)					
	EBAY (\$23.76bn): $M/B = 27.87$, $\rho_t = 7.79\%$, $c = 0$, Return volatility: (129.24%, 113.64%)											
-4	6.03	6.20	6.38	6.58	6.79	7.03	119.33	127.40	136.60	146.59	158.06	171.40
-2	5.42	5.62	5.83	6.05	6.29	6.56	99.01	107.11	116.29	126.33	137.86	151.29
0	4.71	4.94	5.20	5.45	5.73	6.04	79.70	87.76	96.87	106.81	118.26	131.68
2	3.83	4.13	4.45	4.77	5.10	5.45	61.89	69.87	78.74	88.42	99.62	112.81
4	2.61	3.06	3.51	3.93	4.35	4.78	46.43	54.18	62.59	71.74	82.40	95.09
6	0.00	1.15	2.12	2.80	3.41	3.98	36.63	41.92	49.45	57.65	67.34	79.09
8	0.00	0.00	0.00	0.00	2.00	2.92	37.55	41.43	44.68	47.48	55.45	65.65
	IMMUNEX (\$37.56bn): $M/B = 105.70$, $\rho_t = 17.91\%$, $c = 0$, Return volatility: (155.94%, 117.71%)											
-4	6.54	6.68	6.84	7.01	7.20	7.42	145.55	153.37	162.07	171.46	182.16	194.58
-2	6.01	6.17	6.35	6.54	6.75	6.98	124.88	132.81	141.62	151.12	161.95	174.52
0	5.42	5.60	5.81	6.02	6.25	6.51	104.71	112.71	121.59	131.15	142.06	154.72
2	4.73	4.96	5.19	5.44	5.70	5.99	85.30	93.36	102.24	111.78	122.68	135.35
4	3.91	4.19	4.48	4.77	5.08	5.42	67.10	75.17	83.94	93.35	104.09	116.64
6	2.82	3.21	3.60	3.98	4.36	4.77	50.84	58.82	67.30	76.36	86.74	98.93
8	0.56	1.68	2.38	2.94	3.47	3.99	38.00	45.52	53.32	61.65	71.28	82.74
10	0.00	0.00	0.00	1.12	2.20	2.99	38.20	42.29	45.81	50.50	58.74	68.90
	INTEL (\$395.49bn): $M/B = 11.09$, $\rho_t = 28.65\%$, $c = 0.0148$, Return volatility: (45.81%, 68.71%)											
-2	5.11	5.38	5.68	5.98	6.30	6.64	70.70	78.20	87.56	98.52	111.75	127.82
0	4.11	4.47	4.86	5.25	5.64	6.05	54.00	61.14	69.90	80.13	92.66	108.13
2	2.64	3.22	3.80	4.34	4.86	5.37	40.09	46.82	54.75	64.01	75.48	89.94
4	0.00	0.00	2.08	3.05	3.84	4.54	33.53	36.26	42.85	50.81	60.83	73.79
6	0.00	0.00	0.00	0.00	2.27	3.44	34.86	37.98	40.40	42.26	49.42	60.38
	MICROSOFT (\$516.00bn): $M/B = 18.79$, $\rho_t = 48.28\%$, $c = 0$, Return volatility: (57.44%, 56.10%)											
-4	5.04	5.28	5.55	5.82	6.10	6.41	76.54	84.32	93.70	104.37	117.01	132.05
-2	4.15	4.46	4.80	5.13	5.48	5.84	59.01	66.50	75.42	85.60	97.80	112.50
0	2.89	3.36	3.84	4.30	4.74	5.19	44.10	51.21	59.44	68.84	80.22	94.20
2	0.00	1.40	2.42	3.16	3.81	4.41	34.24	39.33	46.58	54.83	64.97	77.74
4	0.00	0.00	0.00	0.93	2.45	3.39	35.61	38.87	41.44	44.42	52.86	63.86
	PRICELINE (\$15.94bn): $M/B = 39.58$, $\rho_t = -264.12\%$, $c = 0$, Return volatility: (128.17%, 133.65%)											
0	7.82	7.93	8.05	8.17	8.32	8.49	217.10	223.77	231.08	238.95	247.98	258.58
2	7.42	7.54	7.66	7.80	7.95	8.13	197.18	203.93	211.33	219.25	228.36	239.02
4	7.00	7.12	7.25	7.40	7.56	7.75	177.34	184.19	191.66	199.68	208.84	219.56
6	6.55	6.68	6.82	6.98	7.15	7.35	157.62	164.57	172.13	180.22	189.44	200.25
8	6.06	6.20	6.36	6.52	6.71	6.93	138.07	145.13	152.78	160.94	170.24	181.09
10	5.52	5.68	5.85	6.03	6.24	6.47	118.76	125.96	133.70	141.92	151.27	162.18
12	4.92	5.10	5.29	5.50	5.73	5.98	99.83	107.17	115.01	123.29	132.68	143.59
	RED HAT (\$10.43bn): $M/B = 26.50$, $\rho_t = -10.15\%$, $c = 0$, Return volatility: (121.00%, 122.33%)											
-4	6.23	6.39	6.57	6.75	6.96	7.18	129.65	137.61	146.60	156.34	167.48	180.41
-2	5.66	5.84	6.04	6.25	6.47	6.73	109.13	117.17	126.23	136.05	147.28	160.34
0	5.00	5.22	5.45	5.69	5.94	6.23	89.38	97.47	106.50	116.31	127.54	140.64
2	4.22	4.49	4.76	5.05	5.35	5.67	70.80	78.86	87.78	97.44	108.56	121.56
4	3.21	3.57	3.94	4.29	4.66	5.05	54.04	61.98	70.61	79.94	90.70	103.38
6	1.57	2.25	2.83	3.34	3.83	4.31	40.23	47.82	55.85	64.51	74.58	86.59
8	0.00	0.00	0.42	1.88	2.70	3.40	37.52	41.38	44.83	52.25	61.08	71.91
	YAHOO (\$98.90bn): $M/B = 78.41$, $\rho_t = 10.52\%$, $c = 0$, Return volatility: (75.41%, 90.61%)											
-2	5.95	6.12	6.30	6.49	6.70	6.94	122.40	130.35	139.20	148.76	159.64	172.29
0	5.35	5.55	5.75	5.97	6.20	6.46	102.28	110.29	119.20	128.80	139.77	152.50
2	4.66	4.89	5.13	5.38	5.65	5.94	82.96	91.03	99.93	109.50	120.44	133.17
4	3.81	4.10	4.40	4.70	5.02	5.36	64.94	73.01	81.78	91.18	101.95	114.52
6	2.67	3.08	3.49	3.89	4.28	4.70	49.01	56.95	65.39	74.40	84.75	96.93
8	0.00	1.39	2.19	2.81	3.36	3.90	37.55	44.13	51.81	60.03	69.56	80.96
10	0.00	0.00	0.00	0.63	2.02	2.86	38.20	42.29	45.80	49.39	57.44	67.43

REFERENCES

- Abreu, D., Brunnermeier, M., 2003. Bubbles and Crashes. *Econometrica* 71, 173–204.
- Ahmed, A. S., Schneible, R. A. Jr., Stevens, D. E., 2003. An empirical analysis of the impact of online trading on investor reactions to quarterly earnings announcements. *Contemporary Accounting Research*, Fall, 413–39.
- Allen, F., Gorton G., 1993. Churning bubbles. *Review of Economic Studies* 60, 813–836.
- Allen, F., Morris, S., Shin, H. S., 2003. Beauty contests, bubbles, and iterated expectations in asset markets. *Review of Financial Studies*, forthcoming.
- Beaver, W. H., 1970. The time series behavior of earnings. *Journal of Accounting Research* (Supplement), 62–89.
- Berk, J. B., Green, R. C., Naik, V., 2004. Valuation and return dynamics of new ventures. *Review of Financial Studies* 17, 1–35.
- Botosan, C. A., Plumlee, M. A., 2001. Stock option expense: The sword of Damocles revealed. *Accounting Horizons* 15, 311–327.
- Brunnermeier, M., Nagel, S., 2004. Hedge funds and the technology bubble. *Journal of Finance* 59, 2013–2040.
- Campbell, J. Y., Cochrane, J. H., 1999. By force of habit: A consumption-based explanation of aggregate stock market behavior. *Journal of Political Economy* 107, 205–251.
- Campbell, J. Y., Lettau, M., Malkiel, B. G., Xu, Y., 2001. Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk. *Journal of Finance* 56, 1–43.
- Claus, J., Thomas, J., 2001. Equity premia as low as three percent? Evidence from analysts' earnings forecasts for domestic and international stock markets. *Journal of Finance* 56, 1629–1666.
- Cochrane, J. H., 2003. Stocks as money: Convenience yield and the tech-stock bubble. In: Hunter, W. C., Kaufman, G. G., Pomerleano, M. (Eds.), *Asset Price Bubbles*. MIT Press, Cambridge.
- Constantinides, G. M., 1992. A theory of the nominal term structure of interest rates. *Review of Financial Studies* 5, 531–552.
- Cooper, M. J., Dimitrov, O., Rau, P. R., 2001. A rose.com by any other name. *Journal of Finance* 56, 2371–2388.
- David, A., Veronesi, P., 2002. Option prices with uncertain fundamentals: Theory and evidence on the dynamics of implied volatilities. Unpublished working paper. University of Chicago.

- Donaldson, R. G., Kamstra, M., 1996. A new dividend forecasting procedure that rejects bubbles in asset prices: The case of 1929s stock crash. *Review of Financial Studies* 9, 333–383.
- Fama, E. F., French, K. R., 2002. The equity premium. *Journal of Finance* 57, 637–659.
- Fama, E. F., French, K. R., 2003. New lists: Fundamentals and survival rates. *Journal of Financial Economics* 73, 229–269.
- Fama, E. F., MacBeth, J. D., 1973. Risk, return, and equilibrium: Empirical tests. *Journal of Political Economy* 81, 607–636.
- Garber, P. M., 2000. *Famous first bubbles: The fundamentals of early manias*. MIT Press, Cambridge, MA.
- Gebhardt, W. R., Lee, C. M. C., Swaminathan, B., 2001. Toward an implied cost of capital. *Journal of Accounting Research* 39, 135–176.
- Hobijn, B., Jovanovic, B., 2001. The information-technology revolution and the stock market: Evidence. *American Economic Review* 91, 1203–1220.
- Ilmanen, A., 2003. Expected returns on stocks and bonds. *Journal of Portfolio Management* 29, 7–27.
- Johnson, T. C., 2004. Forecast dispersion and the cross section of expected returns. *Journal of Finance* 59, 1957–1978.
- Kindleberger, C., 1978. *Manias, Panics, and Crashes*. Basic, New York.
- Lamont, O. A., Thaler, R. H., 2003. Can the market add and subtract? Mispricing in tech stock carve-outs. *Journal of Political Economy* 111, 227–268.
- Landsman, W. R., Maydew, E. L., 2002. Has the information content of quarterly earnings announcements declined in the past three decades? *Journal of Accounting Research* 40, 797–808.
- Lettau, M., Ludvigson, S. C., Wachter, J. A., 2004. The declining equity premium: What role does macroeconomic risk play? Unpublished working paper, New York University.
- Liptser, R. S., Shirayayev, A. N., 1977. *Statistics of Random Processes: I, II*. Springer-Verlag, New York.
- Ljungqvist, A., Wilhelm, W. J. Jr., 2003. IPO pricing in the dot-com bubble. *Journal of Finance* 58, 723–752.
- Malkiel, B. G., 1999. *A Random Walk Down Wall Street*. Norton, New York.
- Ofek, E., Richardson, M., 2003. DotCom mania: The rise and fall of internet stock prices. *Journal of Finance* 58, 1113–1137.
- Ofek, E., Richardson, M., 2002. The valuation and market rationality of internet stock prices. *Oxford Review of Economic Policy* 18, 265–287.

- Ohlson, J. A., 1995. Earnings, book values, and dividends in equity valuation. *Contemporary Accounting Research* 11, 661–687.
- Pástor, L., 2000. Portfolio selection and asset pricing models. *Journal of Finance* 55, 179–223.
- Pástor, L., Stambaugh, R. F., 2001. The equity premium and structural breaks. *Journal of Finance* 56, 1207–1239.
- Pástor, L., Veronesi, P., 2003. Stock valuation and learning about profitability. *Journal of Finance* 58, 1749–1789.
- Pástor, L., Veronesi, P., 2005. Rational IPO waves. *Journal of Finance* 60, 1713–1757.
- Ritter, J. R., Welch, I., 2002. A review of IPO activity, pricing, and allocations. *Journal of Finance* 57, 1795–1828.
- Santos, M. S., Woodford, M., 1997. Rational asset pricing bubbles. *Econometrica* 65, 19–57.
- Scheinkman, J. A., Xiong, W., 2003. Overconfidence and speculative bubbles. *Journal of Political Economy* 111, 1183–1219.
- Schultz, P., Zaman, M., 2001. Do the individuals closest to internet firms believe they are overvalued? *Journal of Financial Economics* 59, 347–381.
- Schwartz, E. S., Moon, M., 2000. Rational pricing of internet companies. *Financial Analysts Journal* 56:3, 62–75.
- Schwert, G. W., 2002. Stock volatility in the new millenium: How wacky is Nasdaq? *Journal of Monetary Economics* 49, 3–26.
- Shiller, R. J., 1981. Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review* 71, 421–436.
- Shiller, R. J., 2000. *Irrational exuberance*. Princeton University Press, Princeton, NJ.
- Stambaugh, R. F., 1999. Predictive regressions. *Journal of Financial Economics* 54, 375–421.
- Thaler, R. H., 1999. The end of behavioral finance. *Financial Analysts Journal*, November, 12–17.
- Tirole, J., 1985. Asset bubbles and overlapping generations. *Econometrica* 53, 1071–1100.
- Trueman, B., 2001. Analyzing Internet market value. TIAA-CREF Investment Forum.
- Veronesi, P., 2000. How does information quality affect stock returns? *Journal of Finance* 55, 807–837.
- Welch, I., 2001. The equity premium consensus forecast revisited. Unpublished working paper. Yale University.