Conditional Betas[*]

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Abstract

Empirical evidence shows that conditional market betas vary substantially over time. Yet, little is known about the source of this variation, either theoretically or empirically. Within a general equilibrium model with multiple assets and a time varying aggregate equity premium, we show that conditional betas depend on (a) the level of the aggregate premium itself; (b) the level of the firm's expected dividend growth; and (c) the firm's fundamental risk, that is, the one pertaining to the covariation of the firm's cash-flows with the aggregate economy. Especially when fundamental risk (c) is strong, the model predicts that market betas should display a large time variation, that their cross-sectional dispersion should be negatively related to the aggregate premium, and that investments in physical capital should be *positively* related to changes in betas. These predictions find considerable support in the data.

I. INTRODUCTION

A firm's decision to take on a new investment project depends on whether the discounted value of future payouts from the project exceeds the direct current investment cost. To this day, the standard textbook recommendation is to appeal to the CAPM to compute the cost of equity: The rate used to discount future cash-flows should be proportional to the excess return on the market portfolio, where the proportionality factor is the market beta. The task of estimating the cost of equity though is complicated because there is substantial empirical evidence showing that both the market premium and individual assets' betas fluctuate over time.[1] There are many theoretical explanations for the time series variation in the aggregate premium but the same cannot be said of fluctuations in betas.[2] Why and how do betas move? How do they depend on the characteristics of the cash-flows that the firm promises to its investors? How do betas correlate with the aggregate premium? How do they correlate with investments in physical capital?

In this paper we answer these questions within a general equilibrium model where both the aggregate equity premium and the expected dividend growth of individual securities are time varying. We show that conditional betas depend on (a) the level of the aggregate premium itself; (b) the level of the firm's expected dividend growth; and (c) the firm's fundamental risk, that is, the one pertaining to the covariation of the firm's cash-flows with the aggregate economy. This characterization yields novel predictions for the time variation of conditional betas as well as their relation with investments in physical capital. Specifically, when the firm's cash-flow risk (c) is substantial, the model predicts that conditional betas should display a large time variation, that their cross sectional dispersion is high when the aggregate equity premium is low, and that capital investment growth should be *positively* related to changes in betas. These predictions are met with considerable support in the data

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[1] On time-varying betas see Bollerslev, Engle, and Wooldridge (1988), Braun, Nelson, and Sunier (1995), Bodhurta and Mark (1991), Campbell (1987), Chan (1988), Evans (1994), Ferson (1989), Ferson and Harvey (1991, 1993), Fama and French (1997), Harvey (1989), and more recently, Franzoni (2001), Lettau and Ludvigson (2001b), and Lewellen and Nagel (2003). On the fluctuations of market premia, see Ang and Beckaert (2002), Campbell and Shiller (1988), Fama and French (1988,1989), Goyal and Welch (2003), Hodrick (1992), Keim and Stambaugh (1986), Lamont (1998), Lettau and Ludvigson (2001a), Menzly, Santos and Veronesi (2004), and Santos and Veronesi (2003).

[2] On time varying equity premium, see Campbell and Cochrane (1999), Barberis, Huang, and Santos (2001), Veronesi (2000) and Santos and Veronesi (2003). On time varying betas, see Berk, Green and Naik (1999) and Gomes, Kogan and Zhang (2003).

To grasp intuitively the results in this paper, consider first an asset that has little cash-flow risk, that is, an asset for which cash-flows have little correlation with the "ups and downs" of the economy, see (c) above. In this case, the risk-return trade-off is only determined by the timing of cash-flows, that is by the duration of the asset. As in the case of fixed income securities, the price of an asset that pays far in the future is more sensitive to fluctuations in the aggregate discount rate than an otherwise identical asset paying relatively more today. Clearly, return volatility due to shocks to the aggregate discount rate is systematic. As a consequence, the asset is riskier and thus its beta is higher the longer its duration.

This intuition though does not hold if the asset has substantial cash-flow risk. Indeed, consider now the case of an asset whose cash-flow growth is highly correlated with the growth rate of the aggregate economy. Furthermore, assume as well that the asset has a low duration, that is, it pays relatively more today than in the future. In this case, the total value of this asset is mainly determined by the current level of cash-flows, rather those in the future. The price of the asset is then mostly driven by cash-flow shocks and the fundamental risk embedded in these cash-flows drives also the risk of the asset. Thus, when cash-flows display substantial fundamental risk, the conditional market beta is higher when the duration is lower. If instead the asset has high duration, current cash-flows matter less and the asset becomes less risky.

These findings highlight a tension between "discount effects" (high risk when the asset has a high duration) and "cash-flow risk effects" (high risk when the asset has low duration.) This tension has deep implications for the behavior of the cross section of risk as a function of fluctuations in the aggregate equity premium. Assume first that cash-flow risk effects are negligible compared to discount effects. Then the cross sectional dispersion of conditional betas moves together with the aggregate equity premium: It is low (high) when the aggregate equity premium is low (high). Intuitively, when the aggregate equity premium is low, individual asset prices are determined by the average growth rate of its cash flows over the long run. Given some mean reversion in expected dividend growth – a necessary condition if no asset is to dominate the economy – this implies that the current level of expected dividend growth is not important in determining prices. In this case, assets' prices have similar sensitivities to changes in the stochastic discount factor and hence have similar market betas as well. Thus when the equity premium is low so is the dispersion in betas. Instead, when the market premium is high, differences in current expected dividend growth matter more in determining the differences in value of the asset. This results in a wide dispersion of price sensitivity to changes in the stochastic discount factor and hence more dispersed betas.

In contrast if cash-flow risk is a key determinant of the dynamics of conditional betas low discount rates lead to an increase in the dispersion of betas. Assets with high cash-flow risk have a component of their systematic volatility that is rather insensitive to changes in the discount rate. However, since a low aggregate discount rate (i.e. good times) tends to yield a low volatility of the market portfolio itself, the relative risk of the individual asset with high cash-flow risk increases, and therefore so does its beta.

Finally we link the fluctuations in market betas to fluctuations in investment. To do so we propose a simple model of firm investment behavior where the standard textbook NPV rule holds. According to this rule, investments occur whenever market valuations are high, which happens when the aggregate risk premium is low, or when the industry is paying relatively high dividends compared to the future, or both. The relation between investment growth and changes in betas is now clear. If cash flow risk is negligible, a decrease in the aggregate equity premium or an increase in current dividend payouts result in a lower conditional beta, as already discussed. Thus a negative relation between changes in betas and investment growth obtains. Instead, when cash-flow risk dominates the risk return trade-off of the asset, there is a positive relation between changes in betas and investment growth. The reason is that now the beta of a low duration asset increases as the aggregate discount decreases.

These observations produce simple empirical tests to gauge the size of discount effects relative to cash-flow effects in determining the dynamics of conditional betas. Empirically, we find that the dispersion of industry conditional betas is high when the market price dividend ratio is high, which in turn occurs when the aggregate market premium is low (e.g. Campbell and Shiller (1989)), confirming that cross sectional differences in cash-flow risk must be large. Similarly, we find that investments growth is higher for industries that experienced increases in their market betas, as well as declines in their expected dividend growth, consistently again with the model and the presence of a significant cross sectional differences in cash-flow risk. Monte Carlo simulations of our theoretical model yield the same conclusion: When cash-flow risk is small and only discount effects matter, the model-implied conditional betas show little variation over time, unlike what is observed in the data. In contrast, when we allow for substantial cash-flow risk our simulations produce fluctuations in conditional betas and investment growth that match well their empirical counterparts.

We obtain our results within the convenient general equilibrium model of Menzly, Santos and Veronesi (2004) – henceforth MSV. This paper, however, differs substantially from MSV, which focused exclusively on the time series predictability of dividend growth and stock returns

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for both the market and individual portfolios. The present paper is instead concerned with the equilibrium dynamic properties of the conditional risk embedded in individual securities, a key variable for the computation of the cost of equity and thus for the decisions to raise capital for new investments. As discussed, we fully characterize conditional betas as a function of fundamentals and the aggregate market premium, and obtain numerous novel predictions about their dynamics and their relation to investments in physical capital. This paper is also related to Campbell and Mei (1993), Vuolteenaho (2002), and Campbell and Vuolteenaho (2002), who also investigate the relative importance of shocks to cash flows and shocks to the aggregate discount in determining the cross-section of stock returns and market betas. These papers though focus on unconditional betas while we emphasize the dynamic aspect of betas.

This paper relates as well to the recent literature on the ability of the conditional CAPM to address the asset pricing puzzles in the cross section.[3] Typically, researchers assume ad-hoc formulations of betas and, in addition, little effort is taken to quantify the magnitude of the variation in betas needed to resolve the puzzles.[4] In contrast, in this paper we obtain the market betas within an equilibrium model that successfully reproduces the variation of the aggregate risk premium, as well as the variation in expected dividend growth of individual assets. Our characterization of betas allows us to quantify the magnitude of their variation at the industry level and yields several interesting insights about expected returns: For instance, it is not surprising that industry portfolios have little differences in unconditional expected returns, notwithstanding large differences in conditional betas. In fact, consistently with the model, our empirical results show that the dispersion of betas is high when the aggregate equity premium is low, and viceversa, which imply a little dispersion in expected returns in average.

The paper develops as follows. Section II contains a brief summary of the MSV model. Section III contains the theoretical results. In Section IV we propose a simple model of investment and link the fluctuations in betas to changes in investments. Section V offers empirical tests as well as simulations of the many implications of the model. Section VI concludes. All proofs are contained in the Appendix.

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[3]See e.g. Jagannathan and Wang (1996), Lettau and Ludvigson (2001b), Santos and Veronesi (2001), Franzoni (2001).

[4]Lewellen and Nagel (2003) is a noteworthy exception.

II. THE MODEL

II.A Preferences

There is a representative investor who maximizes

$$E\left[\int_0^\infty u\left(C_t, X_t, t\right) dt\right] = E\left[\int_0^\infty e^{-\rho t} \log\left(C_t - X_t\right) dt\right], \tag{1}$$

where X_t denotes an *external* habit level and ρ denotes the subjective discount rate.[5] In this framework, as advanced by Campbell and Cochrane (1999), the fundamental state variable driving the attitudes towards risk is the *surplus consumption ratio*, $S_t = \left(C_t - X_t\right)/C_t$. Movements of this surplus produce fluctuations of the local curvature of the utility function,

$$Y_t = -\frac{u_{CC}}{u_C} C_t = \frac{1}{S_t} = \frac{C_t}{C_t - X_t} = \frac{1}{1 - \left(\frac{X_t}{C_t}\right)} > 1, \tag{2}$$

which translate into the corresponding variation on the prices and returns of financial assets. MSV assume that the *inverse* of the surplus consumption ratio, or *inverse surplus* for short, Y_t, follows a mean reverting process, perfectly negatively correlated with innovations in consumption growth

$$dY_t = k\left(\overline{Y} - Y_t\right) dt - \alpha\left(Y_t - \lambda\right)\left(dc_t - E_t\left[dc_t\right]\right), \tag{3}$$

where $\lambda \geq 1$ is a lower bound for the inverse surplus, and an upper bound for the surplus itself, $\overline{Y} > \lambda$ is the long run mean of the inverse surplus and k is the speed of the mean reversion. Here $c_t = \log\left(C_t\right)$ and we assume that it can be well approximated by the process:

$$dc_t = \mu_c dt + \sigma_c dB_t^1, \tag{4}$$

where μ_c is the mean consumption growth, possibly time varying, $\sigma_c > 0$ is a scalar, and B_t^1 is a standard Brownian motion. Given (3) and (4) then, we assume that the parameter α in (3) is positive ($\alpha > 0$), so that a negative innovation in consumption growth, for example, results in an increase in the inverse surplus, or, equivalently, a decrease in the surplus level, capturing the intuition that the consumption level C_t moves further away from a slow moving habit X_t.[6]

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[5]On habit persistence and asset pricing see Sundaresan (1989), Constantinides (1990), Abel (1990), Ferson and Constantinides (1991), Detemple and Zapatero (1991), Daniel and Marshall (1997), Campbell and Cochrane (1999), Li (2001), and Wachter (2000). These papers though only deal with the time series properties of the market portfolio and have no implications for the risk and return properties of individual securities.

[6]MSV show that $\alpha \leq \overline{\alpha}\left(\lambda\right) = \left(2\lambda - 1\right) + 2\sqrt{\lambda\left(\lambda - 1\right)}$ is needed in order to ensure that $cov_t\left(dC_t, dX_t\right) > 0$ for all S_t, as economic intuition would have it.

II.B The cash-flow model

There are n risky financial assets paying a dividend rate, $\{D_t^i\}_{i=1}^n$, in units of a homogeneous and perishable consumption good. Agents total income is made up of these n cash-flows, plus other proceeds such as labor income and government transfers. Denoting by D_t^0 the aggregate income flow that is not financial in nature, standard equilibrium restrictions require $C_t = \sum_{i=0}^n D_t^i$. Define the *share* of consumption that each asset produces,

$$s_t^i = \frac{D_t^i}{C_t}. \tag{5}$$

Then MSV assume that s_t^i evolves according to a mean reverting process of the form

$$ds_t^i = \phi^i \left(\bar{s}^i - s_t^i \right) dt + s_t^i \boldsymbol{\sigma}^i \left(\mathbf{s}_t \right) d\mathbf{B}_t', \qquad \text{for each } i = 1, ..., n. \tag{6}$$

In (6) $\mathbf{B}_t = \left(B_t^1, ..., B_t^N \right)$ is a N-dimensional row vector of standard Brownian motions, $\bar{s}^i \in [0, 1)$ is the average long-term consumption share, ϕ^i is the speed of mean reversion, and

$$\boldsymbol{\sigma}^i \left(\mathbf{s}_t \right) = \mathbf{v}^i - \sum_{j=0}^n s_t^j \mathbf{v}^j = [\sigma_1^i \left(\mathbf{s}_t \right), \sigma_2^i \left(\mathbf{s}_t \right), \cdots, \sigma_N^i \left(\mathbf{s}_t \right)] \tag{7}$$

is a N dimensional row vector of volatilities, with $\mathbf{v}^i$ for $i = 0, 1, \cdots, n$ a row vector of constants with $N \leq n + 1$.[7]

The share process described in (6) has a number of reasonable properties. First, the functional form of the volatility term (7) arises for *any* homoskedastic dividend growth model. That is, denoting by $\delta_t^i = \log \left(D_t^i \right)$, (7) is consistent with any model of the form, $d\delta_t^i = \mu^i \left(\mathbf{D}_t \right) dt + \mathbf{v}^i d\mathbf{B}_t'$, as it is immediate to verify by applying Ito's Lemma to the quantity $s_t^i = D_t^i / (\sum_{j=0}^n D_t^j)$. Second, the assumption that the share s_t^i is mean reverting ensures that no asset will ever dominate the whole economy, as it appears ex-ante reasonable. Third, under the conditions $\sum_{i=1}^n \bar{s}^i < 1$ and $\phi^i > \sum_{j=1}^n \bar{s}^j \phi^j$, dividends are positive and total income equals total consumption at all times.

In this framework the *relative share, $\bar{s}^i / s_t^i$,* stands as a proxy for the asset's duration. When the relative share is high (low) the assets pays relatively more (less) as a fraction of total consumption in the future than it does presently and then we say that the asset has a high (low) duration. Clearly, high duration assets are also those that experience high dividend growth. Indeed, an application of Ito's Lemma to $\delta_t^i = \log \left(D_t^i \right)$ yields

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[7]The process for the alternative source of income, s_t^0, follows immediately from the fact that $1 - \sum_{i=1}^n s_t^i$.

$$d\delta_t^i = \mu_D^i(\mathbf{s}_t) \, dt + \boldsymbol{\sigma}_D^i(\mathbf{s}_t) \, d\mathbf{B}_t',$$

where

$$\mu_D^i(\mathbf{s}_t) = \mu_c + \phi^i \left(\frac{\overline{s}^i}{s_t^i} - 1 \right) - \frac{1}{2} \boldsymbol{\sigma}^i(\mathbf{s}_t) \boldsymbol{\sigma}^i(\mathbf{s}_t)', \tag{8}$$

$$\boldsymbol{\sigma}_D^i(\mathbf{s}_t) = \boldsymbol{\sigma}_c + \boldsymbol{\sigma}^i(\mathbf{s}_t). \tag{9}$$

and $\boldsymbol{\sigma}_c = (\sigma_c, 0, ..., 0)$.[8] Notice that the volatility of the share process, $\boldsymbol{\sigma}^i(\mathbf{s}_t)$, is parametrically indeterminate, that is, adding a constant vector to all $\mathbf{v}^i$'s leaves the share processes unaltered. A convenient parametrization is then to rescale the vector of constants $\mathbf{v}^i$'s, for $i = 0, 1, ..., n$ so that

$$\sum_{j=0}^n \overline{s}^j \mathbf{v}^j = \mathbf{0}. \tag{10}$$

Finally the model offers a simple characterization of the fundamental measure of an asset's risk, the covariation of the growth rate of its cash-flows with consumption growth,

$$cov_t\left(d\delta_t^i, dc_t\right) = \sigma_c^2 + \theta_{CF}^i - \sum_{j=0}^n s_t^j \theta_{CF}^j \qquad \text{where} \qquad \theta_{CF}^i = v_1^i \sigma_c. \tag{11}$$

The normalization in (10) implies that the unconditionally $E\left[cov_t\left(d\delta_t^i, dc_t\right)\right] = \sigma_c^2 + \theta_{CF}^i$, as $\sum_{j=0}^n \overline{s}^j \theta_{CF}^i = 0$. Thus, the parameter θ_{CF}^i determines the unconditional cross sectional differences of *cash-flow risks* across the various assets.[9]

III. CONDITIONAL BETAS

III.A Preliminaries

In the absence of any frictions the price of asset i is given by:

$$P_t^i = E_t\left[\int_t^\infty e^{-\rho(\tau-t)} \left(\frac{u_c(C_\tau - X_\tau)}{u_c(C_t - X_t)}\right) D_\tau^i d\tau\right] = \frac{C_t}{Y_t} E_t\left[\int_t^\infty e^{-\rho(\tau-t)} s_\tau^i Y_\tau d\tau\right], \tag{12}$$

where $D_\tau^i = s_\tau^i C_\tau$. Notice that for the total wealth portfolio, the claim to total consumption, $s_\tau^i = 1$ for all τ. In this case a complete characterization of the price and return process is possible and they are given by

$$\frac{P_t^{TW}}{C_t} = \Phi^{TW}(S_t) = \frac{1}{\rho + k}\left[1 + \frac{k\overline{Y}}{\rho} S_t\right] \tag{13}$$

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[8] MSV find substantial empirical support for both the fact that dividends and consumption are cointegrated, and that the relative share $\overline{s}^i/s_t^i$ predicts future dividend growth, as (8) implies.

[9] $1 + \theta_{CF}^i/\sigma_c^2$ can then be taken to be the unconditional cash-flow beta of asset i, the covariance of dividend growth with consumption growth divided by the variance of consumption growth.

and $dR_t^{TW} = \mu_R^{TW}(S_t)\,dt + \sigma_R^{TW}(S_t)\,dB_{1,t}$, where

$$\mu_R^{TW}(S_t) = (1 + \alpha(1 - \lambda S_t))\,\sigma_R^{TW}(S_t)\,\sigma_c \tag{14}$$

$$\sigma_R^{TW}(S_t) = \left[1 + \frac{k\overline{Y}S_t(1 - \lambda S_t)\,\alpha}{k\overline{Y}S_t + \rho}\right]\sigma_c. \tag{15}$$

Equation (13) shows that price of the total wealth portfolio is increasing in the surplus consumption ratio. Roughly, if the surplus consumption ratio is high the degree of risk aversion is low and thus the high price of the total wealth portfolio. As for $\mu_R^{TW}(S_t)$ and $\sigma_R^{TW}(S_t)$ they are both decreasing in S_t for high values of S_t, as the intuition would have it. However, they are increasing in S_t for very low values of S_t. The reason is that since $S_t \in (0, 1/\lambda)$, the volatility of S_t must vanish as $S_t \to 0$. This translates in a lower volatility of returns, and, hence, in a decrease in expected returns as well.[10]

As for individual securities, assume first that their prices can be written as:

$$\frac{P_t^i}{D_t^i} = \Phi^i\left(S_t, \frac{\overline{s}^i}{s_t^i}\right) \tag{16}$$

Equation (16) can be intuitively understood appealing to the traditional Gordon model. Here S_t is the main variable determining movements in the aggregate discount rate, whereas $\overline{s}^i/s_t^i$, stands for the dividend growth of asset i, as shown in equation (8). In other words, we expect $\Phi^i\left(S_t, \overline{s}^i/s_t^i\right)$ to be increasing in both S_t and $\overline{s}^i/s_t^i$. Below we provide closed form solutions for $\Phi^i\left(S_t, \overline{s}^i/s_t^i\right)$ and confirm these intuitions. However, much can be said about conditional betas without making any additional assumptions once we assume that the price dividend ratio can be written as in (16).

Proposition 1: Let the price function be given by (16). Then, (a) the process for returns is given by

$$dR_t^i = \mu_{R,t}^i dt + \sigma_{1,R,t}^i dB_{1,t} + \sum_{j=2}^{n} \sigma_{j,R,t}^i dB_{j,t} \tag{17}$$

where the loadings to the systematic and idiosyncratic shocks are, respectively,

$$\sigma_{1,R,t}^i\left(S_t, s_t^i\right) = \sigma_c + \left(\frac{\partial P_t^i/P_t^i}{\partial S_t/S_t}\right)\sigma_S(S_t)\,\sigma_c + \left(\frac{\partial P_t^i/P_t^i}{\partial s_t^i/s_t^i}\right)\sigma_1^i(\mathbf{s}_t); \tag{18}$$

$$\sigma_{j,R,t}^i\left(S_t, s_t^i\right) = \left[1 + \left(\frac{\partial P_t^i/P_t^i}{\partial s_t^i/s_t^i}\right)\right]\sigma_j^i(\mathbf{s}_t);$$

and $\sigma_1^i(\mathbf{s}_t)$ and $\sigma_j^i(\mathbf{s}_t)$ are given in (7) and $\sigma_S(S_t) = \alpha(1 - \lambda S_t)$ is the time varying component of the volatility of the surplus consumption ratio dS_t/S_t.

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[10]For a plot of $\mu_R^{TW}(S_t)$ and $\sigma_R^{TW}(S_t)$ the reader can turn to Figure 1 in MSV.

(b) The CAPM beta with respect to the total wealth portfolio can be written as,

$$\beta^i\left(S_t, \overline{s}^i/s_t^i, \mathbf{s}_t\right) = \frac{cov_t\left(dR_t^i, dR_t^{TW}\right)}{var_t\left(dR_t^{TW}\right)} = \beta_{DISC}^i\left(S_t, \overline{s}^i/s_t^i\right) + \beta_{CF}^i\left(S_t, \overline{s}^i/s_t^i, \mathbf{s}_t\right) \qquad (19)$$

where

$$\beta_{DISC}^i\left(S_t, \overline{s}^i/s_t^i\right) = \frac{1 + \left(\frac{\partial P^i/P^i}{\partial S_t/S_t}\right)\sigma_S\left(S_t\right)}{1 + \left(\frac{\partial P^{TW}/P^{TW}}{\partial S_t/S_t}\right)\sigma_S\left(S_t\right)}; \qquad (20)$$

$$\beta_{CF}^i\left(S_t, \overline{s}^i/s_t^i, \mathbf{s}_t\right) = \frac{\left(\frac{\partial P^i/P^i}{\partial s_t^i/s_t^i}\right)\left(\theta_{CF}^i - \sum_{j=0}^n s_t^j\theta_{CF}^j\right)}{1 + \left(\frac{\partial P^{TW}/P^{TW}}{\partial S_t/S_t}\right)\sigma_S\left(S_t\right)}\left(\frac{1}{\sigma_c^2}\right) \qquad (21)$$

Consider first part (a) of the proposition. As it intuitively follows from (16), consumption shocks, dB_t^1, affect returns through three channels: (i) the impact on the dividend of the asset $D_t^i = s_t^i C_t$; (ii) the impact on the surplus consumption ratio S_t, which only loads on dB_t^1; and (iii) the impact on the share s_t^i, that is, the relative share $\overline{s}^i/s_t^i$.

Part (b) of Proposition 2 now follows naturally from part (a). The CAPM beta has two components to it. The first one captures the component of the covariance that is driven by shocks to the discount factor, and, logically, we refer to it as the "discount beta." It depends on the sensitivity of the price of the asset to shocks in the surplus consumption ratio, $\frac{\partial P^i/P^i}{\partial S_t/S_t}$. If this elasticity is higher than that of the total wealth portfolio, $\frac{\partial P^{TW}/P^{TW}}{\partial S_t/S_t}$, the asset is riskier on this account than the total wealth portfolio and thus it has a higher discount beta.

The second component of the return beta is driven by asset's cash-flow shocks and for this reason we refer to it as the "cash-flow beta." It depends on the elasticity of prices to shocks in shares, $\frac{\partial P^i/P^i}{\partial s_t^i/s_t^i}$. Of course, only the component of the shock that covaries with consumption is relevant for pricing and for this reason the expression for the cash-flow beta includes the covariance of ds_t^i/s_t^i with consumption growth itself:

$$cov_t\left(ds_t^i/s_t^i, dc_t\right) = \theta_{CF}^i - \sum_{j=0}^n s_t^j\theta_{CF}^j, \qquad (22)$$

where we recall that θ_{CF}^i is the parameter that regulates the unconditional covariance between consumption growth and dividend growth, as defined in (11). This component then is driven by the covariance of the cash-flows of asset i with consumption, and hence with the stochastic discount factor.[11]

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[11]Campbell and Vuolteenaho (2002) refer to the "cash flow beta" as bad beta and the discount beta as "good." Our terminology is closer to that of Campbell and Mei (1996)

The results in Proposition 1 are generic. They rest on assuming that the price dividend ratio can be written as in (16). We show next that this is indeed the case for the two polar cases where either cash-flow effects or discount effects are assumed away. For the general case we show that equation (16) is a very accurate approximation so that the intuitions built in Proposition 1 remain.

III.B The discount beta

To asses the impact of the variation in the discount factor on the cross section of stock prices and returns, we shut down the cross sectional differences in unconditional cash-flow risk, that is, we set $\theta^i_{CF} = 0$ for all $i = 1, .., n$ in (11). The next proposition characterizes prices and betas in this case. Part (a) is shown in MSV, and it is reported for completeness:

Proposition 2. Let $\theta^i_{CF} = 0$ for all $i = 1, .., n$. Then, (a) the price dividend ratio of asset i, is given by

$$\frac{P^i_t}{D^i_t} = \Phi^i\left(S_t, \overline{s}^i/s^i_t\right) \equiv \left(a^i_0 + a^i_1 \overline{Y} k S_t\right)\left(\frac{\overline{s}^i}{s^i_t}\right) + \left(a^i + a^i_2 \overline{Y} k S_t\right) \tag{23}$$

where $a^i = \left(\rho + k + \phi^i\right)^{-1}$, $a^i_0 = a^i \phi^i \left(\rho + k\right)^{-1}$, $a^i_1 = a^i_0 \left(2\rho + k + \phi^i\right) / \left(\rho \left(\rho + \phi^i\right)\right)$ and $a^i_2 = a^i \left(\rho + \phi^i\right)^{-1}$.

(b) The CAPM beta is given by

$$\beta^i_{DISC}\left(S_t, \overline{s}^i/s^i_t\right) = \frac{1 + \frac{k\overline{Y}S_t}{k\overline{Y}S_t + \rho f\left(\overline{s}^i/s^i_t\right)}\sigma_S\left(S_t\right)}{1 + \frac{k\overline{Y}S_t}{k\overline{Y}S_t + \rho}\sigma_S\left(S_t\right)}, \tag{24}$$

where $f\left(\cdot\right)$ is such that $f' < 0$ and $f\left(1\right) = 1$ and it is given explicitly by equation (37) in the Appendix.

Equation (23) shows that asset i's price dividend ratio is increasing in both $\overline{s}^i/s^i_t$ and S_t. This is intuitive: As shown in (8), $\overline{s}^i/s^i_t$ is positively associated with the asset's dividend growth, whereas S_t is negatively associated with the aggregate discount (see equation (14)).

Part (b) of Proposition 2 characterizes the CAPM beta in the case where there are only discount effects. Since $f\left(1\right) = 1$ and $f'\left(\overline{s}^i/s^i_t\right) < 0$, for any level of the surplus consumption ratio, high duration assets, that is, those with $\overline{s}^i/s^i_t > 1$, have a $\beta^i_{DISC}\left(S_t, \overline{s}^i/s^i_t\right) > 1$, while the opposite is true for low duration assets. The reason is that high duration assets deliver dividends in the distant future, and thus their prices are particularly sensitive to changes in

the aggregate discount, which is regulated by S_t. These assets are then riskier than otherwise identical assets with lower duration.

An additional characterization of the CAPM beta is provided in the following corollary:

Corollary 3. Let $\theta^i_{CF} = 0$ for all $i = 1, .., n$. Then, for any given level of $\overline{s}^i/s^i_t > (<)1$, there exists a S^*_t such that $\beta_{DISC}\left(S_t, \overline{s}^i/s^i_t\right)$ is decreasing (increasing) in the surplus consumption ratio, S_t for $S_t > S^*_t$.

Corollary 3 says that for a given relative share $\overline{s}^i/s^i_t$, the CAPM betas are more dispersed for low, but not too low, levels of S_t.[12] To gain some intuition it is useful to turn to Panel A of Figure 1, where we plot the beta as a function of S_t and $\overline{s}^i/s^i_t$. First, during booms, when S_t is high, the aggregate equity premium is low and thus the prices of all assets are mainly driven by the expected dividends in the far future. Mean reversion in expected dividend growth then implies that the variation in the aggregate discount rate has a similar impact on the prices of the different assets, and thus that they all have similar risk: All betas are close to each other and around 1. In contrast, when S_t is low and the aggregate discount rate is high, agents discount future dividends considerably, and thus the level of current dividend growth matters more. In this case then, whether the asset has high or low duration is a key determinant of its riskiness and this yields a high cross sectional dispersion of betas when S_t is low and the aggregate premium is high.

III.C The cash-flow beta

How do cross sectional differences in unconditional cash-flow risk affect the main conclusions obtained in the previous section? In order to obtain sharp implications about cash-flow risk in the context of our cash-flow model (6), we focus in this section on the case with no discount effects, and leave for the next section the more general case. To shut down discount effects, we must ensure that $X_t = 0$ for all t, and thus we assume $\alpha = 0$ and $Y_t = \overline{Y} = \lambda = 1$. We then obtain the standard log utility representation with multiple assets. The next proposition characterizes the prices and returns of individual securities in this case. Again, part (a) is shown in MSV.

Proposition 4. Let $\alpha = 0$ and $Y_t = \overline{Y} = \lambda = 1$. Then:

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[12]Recall that for low levels of the surplus consumption ratio, its volatility has to go down in order to keep S_t above zero. This effect decreases the volatility of the total wealth portfolio. From the stationary density of S_t, a low value of S_t has a very small probability of occurring, however. See Figure 1 in MSV.

(a) The price dividend ratio of asset i, is given by

$$\frac{P_t^i}{D_t^i} = \Phi^i\left(\bar{s}^i/s_t^i\right) \equiv \left(\frac{1}{\rho + \phi^i}\right) + \left(\frac{1}{\rho + \phi^i}\right)\left(\frac{\phi^i}{\rho}\right)\frac{\bar{s}^i}{s_t^i} \tag{25}$$

(b) The CAPM beta is given by

$$\beta_{CF}^i\left(\bar{s}^i/s_t^i, \mathbf{s}_t\right) = 1 + \frac{1}{1 + \left(\frac{\phi^i}{\rho}\right)\frac{\bar{s}^i}{s_t^i}}\left(\theta_{CF}^i - \sum_{j=0}^n s_t^j\theta_{CF}^j\right)\left(\frac{1}{\sigma_c^2}\right) \tag{26}$$

Equation (25) shows that, as before, the price dividend ratio is increasing in $\bar{s}^i/s_t^i$. Part (b) of Proposition 4 provides the CAPM beta with respect to the total wealth portfolio, which is the specialization of the cash-flow beta in equation (21) to this case. In particular, recall that under condition (10), $\sum_{j=0}^n s_t^j\theta_{CF}^j \approx 0$, and thus (26) simply shows that, intuitively, assets with a high unconditional cash-flow risk θ_{CF}^i have a high market beta.

Notice that now if $\theta_{CF}^i > 0$, the premium is higher the lower the relative share, $\bar{s}^i/s_t^i$, that is the lower the assets i's duration. This is also intuitive: assets with low $\bar{s}^i/s_t^i$ have prices that are mainly determined by the current cash-flows. Thus, naturally, the covariance of cash-flows with consumption growth, regulated by θ_{CF}^i, has substantial impact on the riskiness of the asset. This results in a relatively higher risk for low duration assets. This implication is in stark contrast with the behavior of $\beta_{DISC}^i\left(S_t, \bar{s}^i/s_t^i\right)$ obtained in the previous section, where we found that high duration assets had a higher risk. As we will see, this implication about the cash-flow beta, β_{CF}^i, carries over in the general case, yielding a tension between discount betas and cash-flow betas.

III.D Betas in the general case

The general model, where the cash-flow and discount effects are combined, is more complex than either one of the cases discussed so far. For this reason, an exact closed form solution for prices and the corresponding CAPM representation is not available. However, there is a very accurate analytical approximate solution of the same form as (16), where the nature of the approximation is contained in the Appendix of MSV. As in equations (23) and (25), we find

$$P_t^i/D_t^i \approx \Phi^i\left(S_t, s_t^i/\bar{s}^i\right) = \Phi_0^i\left(S_t\right) + \Phi_1^i\left(S_t\right)\left(\frac{\bar{s}^i}{s_t^i}\right) \tag{27}$$

where $\Phi_j^i\left(S_t\right)$, $j = 1, 2$, are linear functions of S_t given explicitly in (34) and (35), respectively. The important additional feature of this pricing formula is that it now depends on the

parameter θ^i_{CF}, that is, the parameter defined in equation (11) that regulates the long-term unconditional cash-flow risk. Generically speaking, a high θ^i_{CF} tends to decrease the price of the asset.

Given $\Phi\left(S_t, \overline{s}^i/s^i_t\right)$ in (27), we can apply the general result in Proposition 2 (b), and thus obtain the beta representation (19). The formulas are explicitly given in (36) and (38) in the Appendix. Briefly, $\beta^i_{DISC}\left(S_t, \overline{s}^i/s^i_t\right)$ is essentially identical to the one obtained in equation (24), with the only additional feature that a high unconditional cash-flow risk θ^i_{CF} is associated with a higher discount beta.

The most interesting effect of the general model, instead, pertains to the cash-flow beta $\beta^i_{CF}\left(S_t, \overline{s}^i/s^i_t, \mathbf{s}_t\right)$. As in the case with no discount effects, $\beta^i_{CF}\left(S_t, \overline{s}^i/s^i_t, \mathbf{s}_t\right)$ is still decreasing in the relative share $\overline{s}^i/s^i_t$ when the unconditional cash-flow risk $\theta^i_{CF} > 0$ (see discussion in Section III.C). In addition, however, it now depends also on the surplus consumption ratio S_t. That is, how important cash-flow risk is also depends on the aggregate state of the economy.

Panels B and C of Figure 1 plot the $\beta^i_{CF}\left(S_t, \overline{s}^i/s^i_t\right)$ for the cases where $\theta^i_{CF} > 0$ and $\theta^i_{CF} < 0$, respectively.[13] In contrast to the discount beta $\beta^i_{DISC}\left(S_t, \overline{s}^i/s^i_t\right)$, we can see that $\beta^i_{CF}\left(S_t, \overline{s}^i/s^i_t\right)$ tends to display a higher relative cross sectional dispersion during good times, that is, when S_t is high. Intuitively, as we discussed in Proposition 4 (b), a low duration asset with a positive unconditional cash-flow risk $\theta^i_{CF} > 0$ tends to have a high beta, as its price is mainly determined by current dividends rather than the future ones. This component of the systematic volatility of the asset price is relatively insensitive to the fluctuations in the discount rate, as it stems from cash-flow fluctuations. However, during good times the volatility of the total wealth portfolio is lower than in bad times, as shown in equation (15). Thus, the low duration asset tends to become relatively riskier – compared to the total wealth portfolio – during good times, that is, when S_t is high. A similar argument holds for $\theta^i_{CF} < 0$, although in this case the source of the difference stems from the hedging properties of the asset. In this case, we obtain that the cash-flow beta, which is negative, is lower when S_t is high when assets have low duration. In summary, independently of whether θ^i_{CF} is positive or negative the cross sectional dispersion of cash-flow betas increases when the aggregate premium decreases.

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[13]We make use of the normalization (10) and thus set $\Sigma^n_{i=1} s^i_t \theta^i_{CF} \approx 0$. The plots are for values of the parameters of the underlying cash flow process that are of the same magnitude as the ones found in the estimation procedure below for the set of industry portfolios we use.

IV. CONDITIONAL BETAS AND INVESTMENTS

The cost of equity is a key determinant of the firm's decision to invest. To address the relation between investment decisions and time-varying betas we propose next a simple model of the firm's investment behavior. In this section, we interpret the n risky assets introduced in Section II as industries, and the betas derived in Section III as industry portfolio betas. We then link the investment decisions of a small firm with its corresponding industry beta, a relation that is taken to the data in the empirical section. MSV indeed show that the cash-flow model (6) offers a reasonable description of the cash-flows associated with industry portfolios.

IV.A A simple model of investment

Consider a small firm in industry i faced with the decision of whether to undertake an investment project at time t. We assume this project can only be undertaken at time t, as it vanishes afterwards, has a fixed scale, and requires an exogenous initial investment amount I_t. We also assume that projects arrive independently of the firm's previous investment decisions.[14] All these assumptions imply that the textbook NPV rule holds and the firm chooses to invest by simply comparing the value of the discounted cash flows to the investment needed to attain them, I_t. If the investment does take place, the project produces a continuum random cash flow CF_τ up to some random time $t + T$, where T is a random variable exponentially distributed with parameter $p > 0$. We assume that the cash flow process is given by

$$CF_\tau = aD_\tau^i \varepsilon_\tau.$$

where a is a constant. Here D_τ^i is the aggregate dividend of industry i and ε_τ is an idiosyncratic component that follows a mean reverting process

$$d\varepsilon_t = k_\varepsilon \left(1 - \varepsilon_t\right) dt + \sqrt{\varepsilon_t}\sigma_\varepsilon dB_t,$$

where dB_t is uncorrelated with the Brownian motions introduced in Section II. This setting ensures that the cash flows produced by the new investment inherits the cash-flow risk characteristics of industry i, although the idiosyncratic component may drift these cash flow far away from the industry mean.[15]

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[14]Berk, Green, and Naik (1999) and Gomes, Kogan, and Zhang (2003) have recently proposed similar models of investments though to answer different questions.

[15]We do not attempt here to offer a general equilibrium model of investments, as doing so is outside the scope of the simple investment model offered in this section. However, note that if there are N investment projects

The discounted value of the project's cash-flows, V_t, is now easy to calculate. Assuming that investors are well diversified the value of the project at time t is

$$V_t = E_t \left[\int_t^{t+T} e^{-\rho(s-t)} \frac{u_c\left(C_s - X_s\right)}{u_c\left(C_t - X_t\right)} CF_s ds \right] \tag{28}$$

and investment occurs according to the textbook NPV rule, that is, if $V_t > I_t$.

To understand the relation between betas and investments, it is convenient to rewrite (28), the value of the specific project at hand,[16] in the more familiar form (see Appendix):

$$V_t = E_t \left[\int_t^{t+T} e^{-\int_t^s r_\tau + \beta_\tau \times \mu_\tau^{TW} d\tau} CF_s ds \right], \tag{29}$$

where r_τ is the risk free rate at τ, μ_τ^{TW} is the expected excess return on the total wealth portfolio, and

$$\beta_\tau = \beta\left(S_\tau, \overline{s}^i/s_\tau^i, \varepsilon_\tau\right) = \frac{Cov_\tau\left(dV_\tau/V_\tau, dR_\tau^{TW}\right)}{Var_\tau\left(dR_t^{TW}\right)}$$

is the beta with respect to the total wealth portfolio. Equation (43) in the Appendix shows that β_τ has a representation similar to the one in (19).

It is clear now that even when the standard positive NPV rule applies and the conditional CAPM holds, as they do in this simple framework, the prescription of computing separately the cost of capital and expected future cash flows is misleading as

$$E_t \left[e^{-\int_t^s r_\tau + \beta_\tau \times \mu_s^{TW} d\tau} CF_s \right] \neq E_t \left[e^{-\int_t^s r_\tau + \beta_\tau \times \mu_\tau^{TW} d\tau} \right] E_t\left[CF_s\right].$$

Even when the expected excess returns on the market portfolio μ_s^{TW} is constant, the presence of predictable components in dividend growth induce time varying betas that naturally correlate with the future cash flows of new projects.[17] Variation in the aggregate premium only complicates the problem further.

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alive at any time t in industry i, and $a = 1/N$, an application of the central limit theorem shows that the total cash flows from these projects approaches D_t^i as $N \to \infty$. The model can then potentially be closed by a simple assumption that the industry produces a total output rate given by $K_t^i = D_t^i + I_t^i$, where I_t^i is the aggregate investment defined by the optimal investment rule below.

[16]This should not be confused with the value of the firm, which includes the portfolio of current projects plus the options to invest in all future projects that arise.

[17]And there are predictable components in dividend growth. MSV show that the relative share $\overline{s}^i/s_t^i$ forecasts dividend growth for the majority of industries in our sample (see their Table III.) Ang and Liu (2004) also emphasize that the cost of capital cannot be computed separately from the expected cash-flows in a setting where the beta dynamics are assumed exogenously.

Given that the decision to invest has to be taken before ε_t is known and that $E[\varepsilon_t] = 1$, the Appendix shows that NPV rule is given by

$$V_t = aD_t^i \Phi^V\left(S_t, \overline{s}^i/s_t^i\right) > I_t \tag{30}$$

where $\Phi^V\left(S_t, \overline{s}^i/s_t^i\right)$ is as in (27) but where the parameter ρ is substituted for $\rho + p$. That is, investments occur when prices are high, which occur when either the surplus consumption ratio S_t is high, D_t^i is high or $\overline{s}^i/s_t^i$ is high. In our setting, however, $D_t^i = s_t^i C_t$. From the formula of $\Phi^V\left(S_t, \overline{s}^i/s_t^i\right)$ in (27), and assuming that the size of investment grows with the economy, $I_t = bC_t$, we find that investment occurs whenever

$$V_t^N = \frac{s_t^i}{\overline{s}^i}\Phi_0^V\left(S_t\right) + \Phi_1^V\left(S_t\right) > I^* = \frac{b}{a\overline{s}^i}, \tag{31}$$

where $V_t^N = V_t/C_t$, and $\Phi_0^V\left(S_t\right)$ and $\Phi_1^V\left(S_t\right)$ are as in (34) and (35) in the Appendix with the only exception that ρ is substituted for $\rho + p$, as already mentioned. The implications for the firm's investment rule are now clear and intuitive. Given that $\Phi_0^V\left(S_t\right)$ and $\Phi_1^V\left(S_t\right)$ are positive, increasing functions of S_t, investments occur when the surplus consumption ratio, S_t, is high, that is whenever the aggregate premium is low.[18] It also occurs whenever $s_t^i/\overline{s}^i$ is high, that is, when the industry expected dividend growth is low. The reason is that an industry with high dividend today relative to those in the future is one with high valuations as well, as measured for instance by the price consumption ratio.

IV.B Changes in betas and changes in investments

Equation (31) offers a complete characterization of the firm' investment policy. Our purpose next is to link this behavior to the variation in betas. After all, cross sectional differences in the discount can only arise due to cross sectional differences in betas. Here turning to Figure 1 is helpful to offer intuitive predictions about the relation between investments and betas. The question is whether β is high when prices are high, or, to put it differently, whether β increases or decreases when prices increase, since the decision to invest is related to changes in prices that push V_t^N above I^*. The classical CAPM setting would intuitively suggest that a high beta implies a high cost of capital, and thus lower prices discouraging the firm to invest. The endogenous time variation in betas offers a more subtle picture of the cross sectional differences in the cost of equity firms may face depending on the industry they belong to.

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[18]This proposition has, of course, received considerable attention. See, for example, Barro (1990), Lamont (2000), Baker, Stein, and Wurgler (2003), and Porter (2003).

Assume first that there are no cash flow effects ($\theta_{CF} = 0$) so that $\beta_t = \beta_{DISC}(.)$, which is plotted in the top panel in Figure 1. Equation (31) shows that investment occurs when the surplus consumption ratio S_t is high or the relative share $\overline{s}^i/s_t^i$ is low. As shown in the top panel of Figure 1, the combination of a high S_t and a low $\overline{s}^i/s_t^i$ results in a low discount beta. Thus, if discount effects dominate the risk-return characteristics of projects, investment occurs when betas decrease.

The opposite conclusion obtains in the presence of substantial cash-flow risk. In this case, the total beta is the sum of the discount beta and the cash flow beta. Consider first the case where $\theta_{CF} > 0$ (the middle panel in Figure 1.) The cash-flow beta is high whenever the surplus consumption ratio S_t is high or the relative share $\overline{s}^i/s_t^i$ is low, the conditions that lead to higher investment according to (31). In addition, a positive θ_{CF} implies that, on average, an increase in the surplus S_t is correlated with an increase in the share s_t^i and thus negatively correlated with the relative share $\overline{s}^i/s_t^i$. Thus, on average, the cash-flow beta of assets with a high $\theta_{CF} > 0$ moves along the ray of low surplus−high duration to high surplus−low duration. This implies that if θ_{CF} is positive and sufficiently large, a positive relation between investment growth and change in betas should occur.

The case where $\theta_{CF} < 0$, plotted in the bottom panel of Figure 1, leads to the same conclusion, although the intuition is slightly more involved. First of all, a negative $\theta_{CF} < 0$ implies on average cash-flow betas move along the ray of low surplus−low duration area to the high surplus−high duration. Moreover β_{CF} is increasing along this diagonal. Since the effect of changes in S_t on prices is intuitively the most important one – all prices are high in good times – it follows that, on average, a positive relation between investment growth and the cash-flow beta obtains as well.

V. EMPIRICAL ANALYSIS

V.A Data

Our data and estimation of parameters can be found in MSV. Briefly, quarterly dividends, returns, market equity and other financial series are obtained from the CRSP database, for the sample period 1946-2001. We use the Shiller (1989) annual data for the period 1927-1945, where we interpolate the consumption data to obtain quarterly quantities. We focus our empirical exercises on a set of twenty value-weighted industry portfolios for which summary statistics are provided in Table AI. There are two reasons to focus on this set of portfolios: The first is that they enable us to obtain relatively smooth cash-flow data that are a-priory consistent with

the underlying model for cash-flows put forward in this paper (equation (6)). We concentrate our analysis on a coarse definition of industries – the first two SIC codes – which are likely to generate cash-flows for a very long time. A second reason to focus on industry portfolios is that, as shown by Fama and French (1997), they display a large time series variation in their betas, precisely the object of interest in this paper.[19] Moreover, industry portfolios show little, if any, cross sectional dispersion in average returns. This may suggest that there is little cross sectional dispersion in cash-flow risk across these portfolios. We show how testing whether the cross section of betas is positively or negatively related to the aggregate equity premium uncovers instead important cash-flow effects. This set of test portfolios then seems an ideal laboratory to test many of the implications of the model.

The cash-flow series includes both dividends as well as share repurchases (constructed as in Jagannathan, Stephens, and Weisbach (2000)) a detailed description is included in the Appendix in MSV. With some abuse of terminology we use the expressions "cash-flow" and "dividend" interchangeably throughout the empirical section. Finally consumption is defined as real per capita consumption of non durables plus services, seasonally adjusted and is obtained from the NIPA tables. All nominal quantities are deflated using the personal consumption expenditure deflator, also obtained from NIPA.

MSV contain a number of tests showing that $\log(D_t^i)$ and $\log(C_t)$ are cointegrated series for most industries (twelve out of twenty), and that indeed the relative share $\bar{s}_t^i / s_t^i$ is the strongest predictor of future dividend growth, as the model implies. Finally, they show that the cross-sectional and time variation in price dividend ratios implied by the model nicely line up with the empirical data.

As for the definition of investments, we define them as Capital Expenditures (Compustat Item 128) over Property, Plants, and Equipment (PPE, Compustat, Item 8). Individual firm investments are aggregated to industry investments in three different ways: Total Capital Expenditure over Total PPE, referred to as Total Investments, or as a value-weighted or equally weighted average of firm investments. Data are available from 1951 - 2001, at the annual frequency.

Finally, Table I reports the estimates of the parameters used for the simulations below.

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[19]Braun, Nelson, and Sunier (1995, page 1584-5) also find that "the evidence for time-varying betas is somewhat strong" for their set of industry and decile portfolios. In addition these authors compare the rolling regression estimate of the five year window beta with the estimate obtained from an EGARCH model and show that these two estimates track each other rather well (see their Figure 1.) Ferson and Harvey (1991) also find substantial variation in the betas of the industry portfolios in their sample.

These parameters are as in MSV and the reader is referred to Appendix B in that paper for details.

Estimation of θ^i_{CF}

As repeatedly emphasized, θ^i_{CF} is the key parameter in evaluating many of the asset pricing implications of the model. We estimate this parameter using two alternative procedures. Our first estimate relies exclusively on cash-flow data. Specifically we make use of expressions (11) and (10) which yield $\theta^i_{CF} = E\left[cov_t\left(d\delta^i_t, dc_t\right)\right] - var\left(dc_t\right)$. Given that $E_t\left[dc_t\right]$ is constant, we simply have $\theta^i_{CF} = cov\left(d\delta^i_t, dc_t\right) - var\left(dc_t\right)$ and estimate it accordingly. These estimates are reported in Table I in the column denoted θ^i_{CF}-Cash-flow.

Our second estimation procedure uses stock return data to back out the cash-flow parameter θ^i_{CF}. This estimation procedure is motivated by the fact that, as we show below, when we estimate θ^i_{CF} using only cash-flow data, the cash-flow beta β^i_{CF} fluctuates too little. As noted by Campbell and Mei (1993, page 575) cash-flow betas are only imprecisely estimated and thus it is natural to ask whether the lack of variation in betas is due to a downward bias in our estimates of θ^i_{CF}. Specifically, we estimate θ^i_{CF} and $\mathbf{v}^i$ using a GMM procedure where the moment conditions are constructed as follows. First define,

$$
\begin{aligned}
u^i_{1,t} &= R^i_t - \beta^i\left(S_t, \overline{s}^i/s^i_t, \mathbf{s}_t\right) R^M_t \\
u^i_{2,t} &= \left(R^i_t\right)^2 - \sigma^2_{R^i_t}\left(S_t, \overline{s}^i/s^i_t, \mathbf{s}_t\right)
\end{aligned}
$$

where $\beta^i\left(S_t, \overline{s}^i/s^i_t, \mathbf{s}_t\right)$ is the theoretical beta as given in expression (19) and $\sigma^2_{R^i_t}\left(S_t, \overline{s}^i/s^i_t, \mathbf{s}_t\right)$ is the theoretical variance of returns implied by expression (17). The moment conditions are then given by

$$
E\left[\left(u^i_{1,t}, u^i_{1,t}R^M_t, u^i_{2,t}\right)'\right] = \mathbf{0}
$$

To make sure that the system is not underidentified we assume, for simplicity, that the vector of constants governing the diffusion component of the share process (see expression (6)) is such that

$$
\mathbf{v}^i = \left(\frac{\theta^i_{CF}}{\sigma_c}, 0, \ldots, 0, v^i, 0, \ldots, 0\right),
$$

where the only non-zero element besides θ^i_{CF}/σ_c, the systematic component, occurs in entry $i + 1$.

The results of the estimation are contained in Table I under the heading $\theta^i_{CF}-$Return. As can be readily noted, there is a remarkable difference in the estimates across these two alternative procedures. First notice that the estimates in, absolute terms, are off by a factor of

ten! Estimating θ^i_{CF} using returns emphasizes the point that resorting only to cash-flow data may seriously underestimate the amount of cash-flow risk present in the data. Second, notice as well that many of the estimates flip signs, and whereas negative signs dominate when only cash-flow data is used, positive ones do when returns data is used.

V.B Can the model generate substantial variation in betas?

Fama and French (1997) provide a simple estimator of the time variation in betas: Under the assumption that the sampling error associated with the market betas is uncorrelated with the true value of the beta, the variance of the rolling regression beta is the sum of the variance of the true market beta and the variance of the estimation error, or in symbols,

$$\sigma^2\left(\widehat{\beta}_t^{\text{rolling-regress.}}\right) = \sigma^2\left(\beta_t\right) + \sigma^2\left(\varepsilon_t\right),\tag{32}$$

where $\widehat{\beta}_t^{\text{rolling-regress.}}$ is the estimated rolling regression beta, β_t stands for the true beta and ε_t is the estimation error.[20]

Table II reports the estimates for $\sigma^2\left(\beta_t\right)$ for our set of industry portfolios. The average standard deviation of betas is .14, which, incidentally, is only slightly higher than the one obtained by Fama and French (1997) for a set of 48 industry portfolios. Thus if the beta of an average industry were to be one, a two standard deviation of beta yields variation between .74 and 1.28, which is rather substantial. Some of them, like Retail, Petroleum, Mining, Department Stores, Fabrication Metals, and Primary Metals display standard deviation of betas that are above .20. Thus if the *average* beta of retail is around one, a two standard deviation around the mean yields betas that fluctuate between .46 and 1.54!

Can our model yield comparable variation in betas? The next two columns in Table II report the standard deviation of the betas in our model in 40,000 quarters of artificial data. The column under the heading "θ^i_{CF}−Cash-flow" reports the standard deviation of theoretical betas when θ^i_{CF} is estimated using only cash-flow data, that is as the covariance of dividend and consumption growth. The variation of betas in this case does not match the one observed in the data and hovers around .02. The only exception is Primary Metals, where the variation of the theoretical beta reaches 0.10.

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[20]Clearly, when the variance of the true beta is estimated as the difference of the variance of the rolling regression beta and the variance of the estimation error there is no guarantee that the variance of the true beta is greater than zero. In this case we follow Fama and French (1997) and set the variance equal to 0. This occurs in our sample for only two industries, Electrical Equipment and Manufacturing.

The results are rather different when we estimate the cash-flow parameters using returns data, as described in the previous section. These results are reported in the column under the heading "θ^i_{CF}–Returns." In this case the average standard deviation is given by .10, which is close to the average standard deviation obtained through the Fama and French (1997) procedure, see equation (32) above, which was .14. Also notice that in the case of θ^i_{CF}–Cash-flow only one industry out of twenty had a standard deviation of beta above .10, Primary Metals. Now the number has increased up to ten. For instance, the model can generate a substantial variation in the betas of Primary Metals, Utilities and Food, which also had a large variation in the betas as estimated by Fama and French (1997). There are clearly some shortcomings as, for example, Electrical Equipment where the data suggests a very low variation in the market loading whereas the model attributes a standard deviation .22. However, small sample accounts for a large part of these differences. In fact, Figure II reports the results of a different simulation exercise: we obtain 1,000 samples of artificial data, each 54 years long. On each sample we estimate the standard deviation of beta as described in (32). The top panel in Figure II reports the 95% simulation bands of $\sigma(\beta_t)$ (solid lines) along with the point estimates in the data (stars) for the case where θ^i_{CF} is estimated using cash flows. The bottom panel reports the same quantities for the case where θ^i_{CF} is estimated using stock returns. In this latter case, it is indeed the case that the majority of point estimates of $\sigma(\beta_t)$ from the data (stars) fall in the simulated bands (thirteen out of twenty). When θ^i_{CF} is instead estimated from cash flow data, the empirical estimate of $\sigma(\beta_t)$ fall in the bands for only five industries, a result that is in line with those reported in Table II.

In summary then, the estimate of θ^i_{CF} turns out to have a rather substantial impact on the behavior of the conditional beta, not only the unconditional one, as one may suppose at first. The reason is that the duration effect associated with cash-flow risk, the fact that assets with high cash-flow risk have higher risk the shorter their duration, is a key determinant of risk. But if this is the case, this observation has strong implications for the time series behavior of the cross sectional dispersion of risk over time, to which we now turn.

V.C The cross sectional dispersion of betas

We now investigate the time series properties of the cross sectional dispersion of betas. We run the following time series regressions

$$R^i_{t+1} = \alpha^i + \beta^i_{Up}\left(Idx^{Up}_t R^M_{t+1}\right) + \beta^i_{Do}\left(Idx^{Do}_t R^M_{t+1}\right) + \varepsilon^i_{t+1}$$

where R^i_{t+1} and R^M_{t+1} are the excess return on industry i and the market between t and $t+1$,

respectively, and Idx_t^{Up} and Idx_t^{Do} are indicator functions of whether times are good (Up) or bad (Do), that is, whether the aggregate equity premium is low or high. We consider two different proxies for good and bad times: (i) the market price dividend ratio, with $Idx_t^{Up} = 1$ if the price dividend ratio of the market is above its historical 70 percentile, and $Idx_t^{Do} = 1$ if price dividend ratio is below its historical 30 percentile; and (ii) the surplus-consumption ratio S_t itself, where again $Idx_t^{Up} = 1$ or $Idx_t^{Do} = 1$, if the surplus is above its 70 percentile, or below its 30 percentile.[21] How can we formally test whether the cross sectional variance of β_{Up}^i is higher or lower than the cross sectional variance of β_{Do}^i? Assuming that β_{Up}^i and β_{Do}^i are drawn from a normal distribution with two different variances, σ_{Up}^2 and σ_{Do}^2, we can use the statistics $Var^{CS}\left(\beta_{Up}^i\right)/Var^{CS}\left(\beta_{Do}^i\right)$, which has an F-distribution, with 19 degrees of freedom. The results are in Table III.

Panel A of Table III shows that for both samples, 1927 - 2001 and 1947 - 2001 the dispersion of betas is significantly higher when the aggregate equity premium is low, with the exception of the long sample when the surplus consumption ratio is used a sorting variable, in which case the difference is not statistically significant. In particular, there is no evidence that dispersion of betas is higher during bad times.

These findings have a clear interpretation in light of our model. Essentially, cash-flow effects have to be strong in order to undo the positive relation between the cross sectional dispersion of betas and the aggregate equity premium that discount betas induce (see Corollary 3). That is, these findings can be explained by either a strong time variation in the cross-sectional dispersion of expected dividend growth, proxied by $STD^{CS}\left(\overline{s}^i/s_t^i\right)$, and/or substantial unconditional cash-flow risk $\theta_{CF}^i \neq 0$. Indeed, the effect of the time variation in $\overline{s}^i/s_t^i$ can also be seen in the last line of Panel A, where it shows that the dispersion of betas is higher when also the dispersion of relative shares is high, especially in the postwar period.

To disentangle the effects of the dynamics of the dispersion of relative shares $STD^{CS}\left(\overline{s}^i/s_t^i\right)$ from the unconditional cash-flow risk, we decompose in Panel B of Table III the variation in return betas in its two basic sources, variation in aggregate discounts (S_t) and variation in dispersion in cash-flow growth $\left(\overline{s}^i/s_t^i\right)$. In this case, in addition to Up and Down periods as defined in Panel A, we also define an index of whether the cross sectional dispersion of relative shares $STD^{CS}\left(\overline{s}^i/s_t^i\right)$ is high or low, where we set the cutoff levels to the median in all cases now in order to have a sufficient number of observations for each of the four categories (Up-Hi,

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[21]We obtain the surplus consumption ratio S_t by computing a sequence of consumption shocks $d\widehat{B}_t = dc_t - E_t\left[dc_t\right]$ and then applying recursively formula (3).

Do-Hi, Up-Lo, Do-Lo). As before, we run the time series regressions

$$R_{t+1}^i = \alpha^i + \sum_{k=Up,Do} \sum_{h=Hi,Lo} \beta_{kh}^i \left(Idx_t^{kh} R_{t+1}^M \right) + \varepsilon_{t+1}^i$$

and test whether the ratios $Var^{CS} \left(\beta_{kh}^i \right) / Var^{CS} \left(\beta_{k',h'}^i \right)$ are statistically different from 1.

Panel B of Table III reports the results for the case where Up and Down periods are defined either with the log price dividend ratio of the market or the surplus consumption ratio. There is a strong difference in the dispersion of market betas between the Up-High period and Down-Low period for both the 1927 - 2001 and the 1947 - 2001 sample. Indeed, the difference in the cross sectional standard deviation of market betas is not only strongly statistical but also economically significant, as it equals 0.27 and 0.39 for the Up-High period in the 1927-2001 and 1947-2001 sample respectively, while it is less than half those numbers during the Do-Low period. The second finding is that even after controlling for the dispersion of relative shares, Up periods are characterized by a higher dispersion of betas than Down periods. The only exception to this is again in the full sample when the surplus consumption ratio is our proxy for the aggregate state of the economy and the cross sectional dispersion of relative shares is high. However, the difference is again not statistically significant.

These results are also important because they help to bring together two statements that may seem difficult to reconcile at first. On the one hand the cross sectional dispersion of unconditional returns in our set of industry portfolio is low whereas as Fama and French (1997) demonstrate, and the results in Section V.B confirm, there is considerable variation in the loadings on the market portfolio. Table III shows why: The main cross sectional variation in betas occurs during good times, that is periods when aggregate expected returns are low. But this implies that when beta are dispersed, they are multiplied by a low aggregate market premium, and thus the dispersion of industry average returns is low. In contrast, when the dispersion of betas is low, the aggregate expected excess return is high, and thus the variation in conditional expected returns of industry portfolio is still low. Unconditionally, then, we should observe relatively little cross sectional dispersion in average returns, precisely what we see in the data for the set of industry portfolios.

To summarize, the evidence in Table II and III supports the view that cash flow effects have to be relatively strong to induce both a substantial variation in the market betas and, in addition, generate a dispersion in betas that is inversely related to the aggregate market premium.[22]

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[22]Our empirical results are robust to alternative methodologies and sample periods: for instance, we also

V.C.1 Simulations

We now turn to our artificial data to verify whether the model can reproduce the magnitudes of the empirical results in Table III. Table IV reports the results. The headings θ_{CF}−Cash-flows shows the simulation results for the case where θ_{CF}^i are estimated from cash flow data (see Section V.B). The magnitudes are puny compared to what is observed in the empirical data though there is a slightly higher dispersion of the betas in "Up" periods versus "Down" periods. This result extends to the case where the dispersion of betas is also conditional on the cross sectional dispersion of the relative share. That is, using the parameters θ_{CF}^i−Cash-flow, the model cannot yield the magnitudes of the cross sectional dispersion of betas that is observed in the data. This is the same observation made in Section V.B – Table II – regarding the little time series variation in betas that result when using θ_{CF}^i−Cash-flow.

Results are quite different when the cash-flow risk parameters are estimated from returns, reported under the heading θ_{CF}^i−Returns. Now the overall magnitudes are much closer to the corresponding ones in the empirical data. The cross sectional dispersion of betas is higher when both the price dividend ratio of the market portfolio and the surplus consumption ratio are high, which matches the empirical results in Table III. When we condition on both the aggregate state of the economy and $\text{STD}^{CS}\left(\overline{s}^i/s_t^i\right)$ though, the model cannot generate the differences in the cross sectional dispersion in betas due to variation in cross-sectional dispersion in relative share. Still, the very strong difference in the cross sectional dispersion of betas across the Up-High and Down-Low states observed in the long and short samples and for both P_t^M/D_t^M and S_t is nicely born in simulations almost to the point.

V.D Conditional Betas and Investments

The negative relation between the cross sectional dispersion of betas and the aggregate equity premium established in Section V.C confirms that the cash-flow component of market betas dominates the risk-return trade-off. As Section IV shows this also implies that changes in beta should be positively correlated with changes in investments. To test this proposition Tables V and VI report the results of annual panel regressions of industry real investment growth on changes in the price consumption ratio of the industry portfolio, normalized by its average price consumption ratio, $\left(P_t^i/C_t\right)/\overline{PC}$, changes in relative share, $\overline{s}^i/s_t^i$, and changes

computed conditional betas both through simple rolling regressions and by using standard conditioning variables, such as the market dividend yield, the term spread, the corporate bond spread, and Lettau and Ludvigson (2001) *cay*. In either case we find a strong positive relation between the cross-sectional dispersion of conditional betas and the market P/D ratio.

in conditional betas, β_t^i, and their lags. Specifically we run

$$g_t^i = \alpha_{0,i} + \alpha_{0,t} + \boldsymbol{\alpha_1} \cdot \boldsymbol{\Delta X_t} + \boldsymbol{\alpha_2} \cdot \boldsymbol{\Delta X_{t-1}} + \varepsilon_t^i$$

where g_t^i denotes the investment growth at time t in industry i, as defined earlier, $\alpha_{0,i}$ denotes an industry fixed effect, $\alpha_{0,t}$ denotes a year dummy, and $\boldsymbol{\Delta X_t^i}$ denotes the changes in explanatory variables. Lags are included in the regression to control for possible lags on investments growth (see Lamont (2000)). Panels A, B and C report the results when industry investment is measured as industry total investments, or as the value or equal weighted average investment, respectively, as defined in Section V.A. Table V does not include year dummies whereas Table VI does in order to control for market wide factors. Finally $t-$statistics are computed using robust standard errors clustered by year.

Start with Line 1 of Table V across the three different Panels, which only includes contemporaneous and lagged changes in prices. Lagged changes in prices are always positive and statistically significant at the 5% level independently of the definition of investment growth used. Instead contemporaneous changes in prices are never significant. These results are consistent with previous literature (see e.g. Barro (1990) and Lamont (2000)).[23] As for the changes in the relative share (Line 2) notice that this variable always enter with the negative sign, as predicted by the model, but it is only strongly significant when investments are measured as total investments. Instead it is not significant at the 5% when investment is measured as equally or value weighted average investment.

Lines 3 and 4 test the proposition that if cash-flow risk is determinant in the risk return trade-off of assets prices, a positive correlation should obtain between contemporaneous changes in betas and investment growth. We estimate β_t^i of industry i at time t by using a rolling regression of industry i returns in excess of the one month t-bill rate on the market portfolio excess return for the 24 months preceding t.[24] Recall that when cash-flow effects are

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[23]To follow standard practice in the investment literature we also ran the panel regression using changes in market-to-book as our measure of changes in valuation and find, consistent with the unsatisfactory performance of $q-$models, much weaker results for M/B. The results regarding betas where instead very similar.

[24]The results in Tables V and VI are robust to alternative definitions of beta. For instance, we find that the alternative timing convention where β_t is estimated using a rolling regression in the 24 months around t, rather than preceeding t, yields in fact stronger results. Similarly, since it can be argued that the relevant beta for a firm's investment decision is the asset beta rather than the levered equity beta, we also run the panel regressions by using the standard correction $\beta_{asset,t} = E_t/(L_t + E_t)\beta_{equity,t}$,where E_t is total industry equity and L_t is total industry debt. Again, the results are very similar and typically stronger, since $E_t/(L_t + E_t)$ tends to be high in good times. These results are available upon request.

strong, betas should correlate negatively with the aggregate premium. As discussed in Sections III and IV, prices (and valuations) increase as the aggregate premium falls and thus so does investment. As a consequence, in the presence of strong cash-flow risk, a positive relation between investment growth and betas results. This implication is met with considerable support in the data across the different specifications. The coefficient has always the sign predicted by theory and it is statistically significant throughout. Lagged values of changes in betas are also significant. This result confirms the evidence presented in section V.C concerning the importance of cash-flow effects in determining the risk-return characteristics of asset prices.

Finally, Table VI redoes the exercise in Table V but now year dummies are added to remove period specific effects. Briefly, notice that now lagged changes in valuations are no longer significant whereas the contemporaneous changes in betas are still significant throughout all different specifications. As for changes in the relative share, as before, they are only significant when investment growth is measured as total investments.

V.D.1 Simulations

To gauge the magnitude of the cash-flow effects, we reproduce in Table VII the results of panel regressions equivalent to those in Tables V and VI but now in artificial data. According to the model, investments occur when the industry price consumption ratio is above a cut off, which we assume to be equal to the long term average price consumption ratio. The cost of each investment project is assumed to be proportional to consumption. The normalized investment rate in a given quarter is then just simply a constant. We aggregate quarterly investments to annual to have comparable figures to those of Table V and VI. Finally, to deal with a dimensionality problem that arises in inserting year dummies in 10,000 years of artificial data, we divide our long sample in 20 time series of 500 years each. As in previous sections, results are reported for the parameter choices θ_{CF}^i-Cash-flow and θ_{CF}^i-Returns. For each panel regression we report the mean, median, 5 and 95 percentiles of the estimated coefficients across the 20 samples. In these simulations, we only run the multivariate regression, corresponding to line 4 in each panel of Tables V and VI, and we did not include any lags, as the simple model proposed in Section IV does not account for any adjustment costs or time differences between investment decision and actual investments.

Start with Panel A which reports panel regressions without year dummies and thus should be compared with Table V. The sign of the coefficient on $\Delta\left(P_t^i/C_t\right)/\overline{PC}$ is positive and close in magnitude to the corresponding one on the lagged coefficient in the empirical data, especially for the case where the cash-flow risk parameter θ_{CF}^i is calibrated using returns

(Panel A.2). Recall that from Section V.B and V.C, this calibration is also the most effective to match the magnitude of the time variation in asset betas. As for the changes in the relative share $\overline{s}^i/s_t^i$, they are negative, as expected, but their magnitude is smaller in absolute value than the corresponding empirical estimates in Table V. Still, in most cases the estimates in Table V are imprecise and thus the numbers are not statistically different from each other.

As for the impact of changes in betas $\Delta\beta_t^i$, when the θ_{CF}^i is measured using cash-flows alone (Panel A.1) the sign of the mean and median estimates of the coefficient is negative, which is consistent with the fact that discount effects dominate the risk return trade-off. When θ_{CF}^i-Returns is used instead (Panel A.2), the magnitudes are large enough to, once again, induce sufficiently strong variation in the cash-flow beta and yield the positive correlation between investment growth and changes in betas. The magnitude of the coefficient in simulated data is smaller though than the corresponding point estimates in Table V, showing that the cash flow effect in the data may be even stronger than what the calibrated model implies.

Finally, similar results obtain when year dummies are included. In both data and simulations, especially in the case θ_{CF}^i-Return (Panel A.2), the coefficients on changes in prices decrease, while the coefficients on $\Delta\beta_t^i$ increase. The effect on relative share is instead unchanged between the cases with and without year dummies, as one would expect because relative shares are industry specific.

VI. CONCLUSIONS

Betas, the classic measure of an asset's risk, is a fundamental input in any valuation problem, whether it be an investment project or a financial asset. This paper is concerned with the determinants of this fundamental measure of risk, a challenging problem given that there is substantial evidence that these betas fluctuate over time. This paper uses a general equilibrium asset pricing model to show that conditional betas depend on the level of the aggregate premium itself; the level of the firm's expected dividend growth; and the firm's fundamental risk, that is, the one pertaining to the covariation of the firm's cash-flows with the aggregate economy.

We investigate the interaction between these three elements by decomposing the conditional beta into a discount beta and a cash-flow beta. The first reflects the sensitivity of prices to shocks in the aggregate discount whereas the second captures the sensitivity of the price to shocks to cash flows. We show that the time series properties of the cross section of betas is driven by whether the discount beta or the cash-flow beta is a more important determinant

of the overall conditional beta. In particular, if the cash-flow beta dominates the risk-return trade-off we show that the cross sectional dispersion of betas correlates negatively with the aggregate discount. Moreover, we show that strong cash-flow effects are needed to match the observed time series variation in betas. We also propose a model of firm behavior that links investment decisions to changes in betas and find that, in the presence of strong cash-flow effects, changes in investments should correlate positively with changes in betas.

Our empirical exercises reveal the consistent pattern that cash-flow risk must play a dominant role in shaping the conditional risk-return characteristics of asset prices. Indeed, we find substantial evidence that industry market betas display a large time variation, that their cross-sectional dispersion is high when the aggregate equity premium is low and, finally, that investment growth in physical capital is high when market betas increase. The magnitudes of these empirical facts can only be explained in our model when industries are characterized by cross-sectional differences in cash flow risk, and the aggregate equity premium is time varying.

An important message of this paper then is that the properties of the underlying cash-flow process, both the asset's duration as well as the covariation of the asset's cash-flow with the aggregate state of the economy, are key if one is to understand the role of conditioning information in asset pricing tests. For instance, one way researchers typically capture conditioning information is by instrumenting betas with observable state variables. Differences on how assets' betas load on these state variables can only be due to differences in their cash-flow processes. Thus, for example, the way the conditional cross section of returns varies with the relevant conditioning variables depends on the cash-flow properties of the set of test portfolios. An important direction for future research then is to link the observed cross sectional dispersion in average returns to the dispersion in the fundamental cash-flow parameters in order to obtain a more economically based view of what determines the dispersion in the risk-return trade-off across different assets and different time periods. This paper makes progress in this direction by offering an explicit characterization of the betas as a function of the relevant cash-flow parameters.

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APPENDIX

(A) The Approximate Pricing Functions and Betas

(I) The approximate pricing formula is given by

$$\frac{P_t^i}{D_t^i} \approx \Phi_0(S_t) + \Phi_1^i(S_t) \frac{\overline{s}^i}{s_t^i}; \tag{33}$$

where

$$\Phi_0^i(S_t) = \frac{1}{(\rho + k + \phi^i + \alpha\theta_{CF}^i)} \left(1 + \frac{k\overline{Y} + \lambda\alpha\,\theta_{CF}^i}{\rho + \phi^i} S_t\right) \tag{34}$$

$$\Phi_1^i(S_t) = \frac{\phi^i/\rho}{(\rho + k + \phi^i + \alpha\theta_{CF}^i)} \left(\frac{\rho + k\overline{Y}S_t}{\rho + k} + \frac{(k\overline{Y} + \lambda\alpha\,\theta_{CF}^i)\,S_t}{\rho + \phi^i}\right) \tag{35}$$

It is easy to see that if $\overline{Y} = 1 = S_t = \lambda$ and $\alpha = 0$, the formula (33) is the same as the one in Proposition 4 (Model B), while if $\theta_{CF}^i = 0$, the formula is the same as the one shown in the proof of Proposition 3 (Model A). The derivation below also shows that in these cases the approximation is in fact exact.

(II) The betas corresponding to (33) are as follows:

$$\beta^i\left(S_t, \frac{\overline{s}^i}{s_t^i}\right) = \beta_{DISC}^i\left(S_t, \frac{\overline{s}^i}{s_t^i}\right) + \beta_{CF}^i\left(S_t, \frac{\overline{s}^i}{s_t^i}\right)$$

(a) The Discount Beta is given by

$$\beta_{DISC}^i\left(S_t, \frac{\overline{s}^i}{s_t^i}\right) = \frac{1 + \frac{\overline{Y}kS_t}{\overline{Y}kS_t + \rho f(\overline{s}^i/s_t^i;\theta_{CF}^i)}\sigma_S(S_t)}{1 + \frac{k\overline{Y}S}{k\overline{Y}S_t + \rho}\sigma_S(S_t)} \tag{36}$$

where

$$f\left(\overline{s}^i/s_t^i;\theta_{CF}^i\right) = \frac{\phi^i\,(\rho + \phi^i)\,(\overline{s}^i/s_t^i) + (\rho + k)\,(\rho + \phi^i)}{\phi^i\,(\rho + \phi^i)\,(\overline{s}^i/s_t^i) + (\rho + k)\left(1 + \lambda\,\alpha\theta_i\,(\overline{Y}k)^{-1}\right)(\phi^i\,(\overline{s}^i/s_t^i) + \rho)}$$

In addition, $f\left(\overline{s}^i/s_t^i;\theta_{CF}^i\right)$ has the properties (i): $f'\left(\overline{s}^i/s_t^i;\theta_{Cf}^i\right) < 0$ if and only if $\theta_{CF}^i > -\frac{(\rho + \phi^i + k)\overline{Y}}{\lambda\alpha}$; (ii) $f\left(1;\theta_{CF}^i\right) > 1$ if and only if $\theta_{CF}^i < 0$; (iii) $\partial f\left(\overline{s}^i/s_t^i;\theta_{Cf}^i\right)/\partial\theta_{CF}^i < 0$; and (iv) $\theta_{CF}^i = 0$ implies

$$f\left(\overline{s}^i/s_t^i\right) = \frac{\phi^i\,(\rho + \phi^i)\,(\overline{s}^i/s_t^i) + (\rho + k)\,(\rho + \phi^i)}{\phi^i\,(\rho + \phi^i)\,(\overline{s}^i/s_t^i) + (\rho + k)\,(\phi^i\,(\overline{s}^i/s_t^i) + \rho)} \tag{37}$$

which, in turn, has $f(1) = 1$.

(b) The Cash-Flow Beta is given by

$$\beta_{CF}^i\left(S_t, \frac{\overline{s}^i}{s_t^i}, \mathbf{s}_t\right) = H(S)\left(\frac{1}{1 + \frac{\phi^i}{\rho}G(S)\,(\overline{s}^i/s_t^i)}\right)\left(\theta_{CF}^i - \sum_{j=0}^{n} s_t^j\theta_{CF}^j\right)\left(\frac{1}{\sigma_{c,1}^2}\right) \tag{38}$$

where

$$H(S) = \frac{\sigma_{c,1}}{\sigma^{TW}(S_t)} = \frac{k\overline{Y}S_t + \rho}{k\overline{Y}S_t + \rho + k\overline{Y}S_t\sigma_S(S_t)}$$

$$G(S_t) = \frac{(\rho + \phi^i)\,(\rho + k\overline{Y}S_t) + (\rho + k)\,(\overline{Y}k + \lambda\,\alpha\theta_{CF}^i)\,S_t}{(\rho + \phi^i)\,(\rho + k) + (\rho + k)\,(\overline{Y}k + \lambda\,\alpha\theta_{CF}^i)\,S_t}$$

In addition,

(i) $H'(S) >$ if and only if $S_t > \frac{-\rho\lambda + \sqrt{\rho^2\lambda^2 + k\overline{Y}\lambda\rho}}{k\overline{Y}\lambda}$;

(ii) $G\left(\overline{Y}^{-1}\right) = 1$

(iii) $G'(S) > 0$ if and only if $\theta_i > -\frac{\overline{Y}\left(\rho + \phi^i + k\right)}{\lambda\,\alpha}$.

(B) Proofs

In this appendix, define for convenience $\mathbf{v}_Y = -\alpha\sigma_c$. The inverse surplus process can be rewritten as

$$dY_t = k\left(\overline{Y} - Y_t\right)dt + \left(Y_t - \lambda\right)\mathbf{v}_Y\,d\mathbf{B}_t. \tag{39}$$

By Ito's Lemma, the process for surplus $S_t = 1/Y_t$ is then

$$dS_t/S_t = \left(k\left(1 - \overline{Y}S_t\right) + \left(1 - \lambda S_t\right)^2\alpha^2\sigma_c^2\right)dt - \left(1 - \lambda S_t\right)\mathbf{v}_Y\,d\mathbf{B}_t'.$$

Since the diffusion part can be written as $-\left(1 - \lambda S_t\right)\mathbf{v}_Y = \alpha\left(1 - \lambda S_t\right)\sigma_c$, it is convenient to denote $\sigma\left(S_t\right) = \alpha\left(1 - \lambda S_t\right).\;.$ Finally, the pricing kernel $m_t = u_c\left(C_t, X_t, t\right) = e^{-\phi t}Y_t/C_t$ follows the dynamics

$$dm_t/m_t = -r_t dt + \sigma_m d\mathbf{B}_t'$$

where

$$
\begin{aligned}
r_t &= \phi + \mu_{c,t} - \sigma_c^2 + k\left(1 - \overline{Y}S_t\right) - \alpha\left(1 - \lambda S_t\right)\sigma_c^2, \\
\sigma_m &= -\left(1 + \alpha\left(1 - S_t\lambda\right)\right)\sigma_c,
\end{aligned}
$$

Proof of Proposition 1: From $P_t^i/D_t^i = \Phi\left(S_t, \overline{s}^i/s_t^i\right)$, we can generically write (with a slight abuse of notation) $P_t^i = P^i\left(C_t, S_t, s_t^i\right) = C_t\Psi\left(S_t, s_t^i\right)$, where $\Psi\left(S_t, s_t^i\right) = s_t^i\Phi\left(S_t, \overline{s}^i/s_t^i\right)$. An application of Ito's Lemma yields equation (17). In fact,

$$\frac{dP_t^i}{P_t^i} = \frac{\partial P^i/\partial C_t}{P^i\left(C_t, S_t, s_t^i\right)}dC_t + \frac{\partial P^i/\partial S_t}{P^i\left(C_t, S_t, s_t^i\right)}dS_t + \frac{\partial P^i/\partial s_t^i}{P^i\left(C_t, S_t, s_t^i\right)}ds_t^i$$

Since $\partial P^i/\partial C_t = \Psi\left(S_t, s_t^i\right)$ and $P^i\left(C_t, S_t, s_t^i\right) = C_t\Psi\left(S_t, s_t^i\right)$, the diffusion component of dP_t^i/P_t^i is

$$\sigma_P^i = \sigma_c + \frac{\partial P^i/P^i}{\partial S_t}S_t\sigma\left(S_t\right)\sigma_c + \frac{\partial P^i/P^i}{\partial s_t^i}s_t^i\sigma^i\left(\mathbf{s}_t\right)$$

where $\sigma\left(S\right) = \alpha\left(1 - \lambda S_t\right)$ and $\sigma^i\left(\mathbf{s}_t\right)$ is the diffusion of ds_t^i/s_t^i. Since the diffusion part of the price process σ_P^i must equal the one of excess returns $dR_t^i = dP_t^i/P_t^i + D_t^i/P_t^i dt - r_t dt$, equation (17) follows.

As for part (b), the price of the total wealth portfolio is $P_t^{TW} = C_t\Psi^{TW}\left(S_t\right)$. A similar derivation as above implies that we can write $\sigma_R^{TW} = \sigma_c + \frac{\partial P^{TW}/P^{TW}}{\partial S_t/S_t}\sigma\left(S_t\right)\sigma_c$. Since the total wealth portfolio is perfectly correlated with the stochastic discount factor, a beta representation exists for the expected returns of individual securities (see e.g. Duffie (1996, page 229)). Thus,

$$\beta^i\left(S_t, s_t^i\right) = \frac{cov_t\left(dR_t^i, dR_t^{TW}\right)}{var_t\left(dR_t^{Tw}\right)} = \frac{\sigma_R^i\left(S_t, \mathbf{s}_t\right)\sigma_R^{TW}\left(S_t\right)'}{\sigma_R^{TW}\left(S_t\right)\sigma_R^{TW}\left(S_t\right)'}$$

Since by definition $\sigma_c = \left(\sigma_{c,1}, 0..., 0\right)$, we find

$$\sigma_R^{TW}\left(S_t\right)\sigma_R^{TW}\left(S_t\right)' = \left(1 + \frac{\partial P^{TW}/P^{TW}}{\partial S_t/S_t}\sigma\left(S_t\right)\right)^2\sigma_c\sigma_c'$$

and

$$
\begin{aligned}
\sigma_R^i\left(S_t, \mathbf{s}_t\right)\sigma_R^{TW}\left(S_t\right)' &= \left(1 + \frac{\partial P^i/P^i}{\partial S_t}S_t\sigma\left(S_t\right)\right)\left(1 + \frac{\partial P^{TW}/P^{TW}}{\partial S_t/S_t}\sigma\left(S_t\right)\right)\sigma_c\sigma_c' \\
&+ \left(\frac{\partial P^i/P^i}{\partial s_t^i/s_t^i}\right)\left(1 + \frac{\partial P^{TW}/P^{TW}}{\partial S_t/S_t}\sigma\left(S_t\right)\right)\sigma^i\left(\mathbf{s}_t\right)\sigma_c'.
\end{aligned}
$$

Substitution yield equations (20) and (21).∎

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Proof of Proposition 2: Part (a) is shown in MSV. Using their results, one obtains expression (23) where $\Phi_0\left(S_t\right)$ and $\Phi_1\left(S_t\right)$ are identical to equations (34) and (35) for $\theta^i_{CF}=0$. Part (b) can be obtained by following the same steps as in the proof for the general case, obtaining $\beta^i_{DISC}\left(S_t,\overline{s}^i/s^i_t\right)$ as in (36) with $f\left(\overline{s}^i/s^i_t\right)$ given in (37).$\blacksquare$

Proof of Proposition 3: Part (a) is shown in MSV. Part (b) follows from the general result in equation (21) with the pricing function in (25), where we must set $\frac{\partial P^{TW}/P^{TW}}{\partial S_t}=0$. $\blacksquare$

Derivation of Beta Formulas in Appendix A

(a) Discount Beta: The pricing function (33) is in the form discussed in Proposition 1. Thus, representation (19) applies. From $P^{TW}_t=P^{TW}\left(C_t,S_t\right)=C_t\frac{1}{\rho+k}\left[1+\frac{k\overline{Y}}{\rho}S_t\right]$, we have $\partial P^{TW}/\partial S_t=C_t\frac{k\overline{Y}}{(\rho+k)\rho}$ yielding

$$\frac{\left(\partial P^{TW}/P^{TW}\right)}{\left(\partial S_t/S_t\right)}=\frac{k\overline{Y}S}{\rho\left[1+\frac{k\overline{Y}}{\rho}S_t\right]}$$

Similarly, as in the proof of proposition 2, from the general pricing function $P^i_t=P^i\left(C_t,S_t,\overline{s}^i/s^i_t\right)=C_t\Psi\left(S_t,s^i_t\right)$. For convenience, let me rewrite

$$\Psi^i\left(S_t,\overline{s}^i/s^i_t\right)=a^i_0\overline{s}^i+a^i_1\overline{Y}k\overline{s}^iS_t+a^is^i_t+a^i_2\overline{Y}ks^i_tS_t$$

where a little algebra shows that given $a^i=\left(\rho+k+\phi^i+\alpha\theta^i_{CF}\right)^{-1}$ and

$$a^i_0 = a^i\frac{\phi^i}{(\rho+k)};\ a^i_1=a^i\left(\frac{\phi^i\left(2\rho+k+\phi^i\right)+\phi^i\left(\rho+k\right)\lambda\alpha\theta_i\left(\overline{Y}k\right)^{-1}}{\rho\left(\rho+k\right)\left(\rho+\phi^i\right)}\right)$$

$$a^i_2 = a^i\frac{\left(1+\lambda\alpha\theta_i\left(k\overline{Y}\right)^1\right)}{\left(\rho+\phi^i\right)}$$

This implies

$$\frac{\partial P^i/P^i}{\partial S_t/S_t}=\frac{\left(a^i_1\overline{s}^i+a^i_2s^i_t\right)\overline{Y}kS_t}{a^i_0\overline{s}^i+a^is^i_t+\left(a_1\overline{s}^i+a^i_2s^i_t\right)\overline{Y}kS_t}=\frac{\overline{Y}kS_t}{\frac{a^i_0\overline{s}^i+a^is^i_t}{a^i_1\overline{s}^i+a^i_2s^i_t}+\overline{Y}kS_t}$$

Finally, notice that

$$\widetilde{f}\left(\overline{s}/s^i_t\right) = \frac{a^i_0\overline{s}^i+a^is^i_t}{a^i_1\overline{s}^i+a^i_2s^i_t}=\frac{a^i_0\left(\overline{s}^i/s^i_t\right)+a^i}{a^i_1\left(\overline{s}^i/s^i_t\right)+a^i_2}$$

$$= \frac{\rho\left[\phi^i\left(\rho+\phi^i\right)\left(\overline{s}^i/s^i_t\right)+\left(\rho+k\right)\left(\rho+\phi^i\right)\right]}{\left(\phi^i\left(2\rho+k+\phi^i\right)+\phi^i\left(\rho+k\right)\lambda\alpha\theta_i\left(\overline{Y}k\right)^{-1}\right)\left(\overline{s}^i/s^i_t\right)+\rho\left(\rho+k\right)\left(1+\lambda\alpha\theta_i\left(k\overline{Y}\right)^1\right)}$$

$$= \frac{\rho\left[\phi^i\left(\rho+\phi^i\right)\left(\overline{s}^i/s^i_t\right)+\left(\rho+k\right)\left(\rho+\phi^i\right)\right]}{\phi^i\left(\rho+\phi^i\right)\left(\overline{s}^i/s^i_t\right)+\left(\rho+k\right)\left(1+\lambda\alpha\theta_i\left(\overline{Y}k\right)^{-1}\right)\left(\phi^i\left(\overline{s}^i/s^i_t\right)+\rho\right)}$$

Thus, we can write

$$\beta^i_{DISC}\left(S_t,s^i_t\right)=\frac{1+\left(\frac{\partial P^i/P^i}{\partial S_t/S_t}\right)\sigma_S\left(S_t\right)}{1+\left(\frac{\partial P^{TW}/P^{TW}}{\partial S_t/S_t}\right)\sigma_S\left(S_t\right)}=\frac{1+\frac{\overline{Y}kS_t}{\overline{Y}kS_t+\rho f\left(\overline{s}^i/s^i_t;\theta^i_{CF}\right)}\sigma_S\left(S_t\right)}{1+\frac{k\overline{Y}S}{k\overline{Y}S_t+\rho}\sigma_S\left(S_t\right)}$$

where

$$f\left(\overline{s}^i/s^i_t;\theta^i_{CF}\right)=\frac{\phi^i\left(\rho+\phi^i\right)\left(\overline{s}^i/s^i_t\right)+\left(\rho+k\right)\left(\rho+\phi^i\right)}{\phi^i\left(\rho+\phi^i\right)\left(\overline{s}^i/s^i_t\right)+\left(\rho+k\right)\left(1+\lambda\alpha\theta_i\left(\overline{Y}k\right)^{-1}\right)\left(\phi^i\left(\overline{s}^i/s^i_t\right)+\rho\right)}$$

Proof of properties (i) - (iv): (i) after taking the first derivative with respect to $\overline{s}^i/s_t^i$ and canceling common terms, we find $f'\left(\overline{s}^i/s_t^i;\theta_{CF}^i\right)<0$ if and only if

$$-\left(\rho+k\right)\phi^i\left(\rho+\phi^i\right)\left(\left(\rho+\phi^i\right)+k\left(1+\lambda\,\alpha\theta_i\left(\overline{Y}k\right)^{-1}\right)\right)<0,$$

which yields the condition. (ii) is immediate, as

$$f\left(1;\theta_{CF}^i\right)=\frac{\phi^i+\left(\rho+k\right)}{\phi^i+\left(\rho+k\right)\left(1+\lambda\,\alpha\theta_i\left(\overline{Y}k\right)^{-1}\right)}>1\,\text{if and only if }\theta_{CF}^i<0$$

(iii) and (iv) are also immediate.

(b) Cash-Flow Beta.

In this case, we must compute $\partial P^i/\partial s_t^i$. From $P^i\left(C_t,S_t,\overline{s}^i/s_t^i\right)=C_t\Psi\left(S_t,s_t^i\right)$ with

$$\Psi^i\left(S_t,\overline{s}^i/s_t^i\right)=a_0^i\overline{s}^i+a_1^i\overline{Y}k\overline{s}^iS_t+a^is_t^i+a_2^i\overline{Y}ks_t^iS_t$$

we find

$$\frac{\partial P^i/P^i}{\partial s_t^i/s_t^i}=\frac{1}{1+\frac{a_0^i+a_1^i\overline{Y}kS_t}{a^i+a_2^i\overline{Y}kS_t}\left(\overline{s}^i/s_t^i\right)}.$$

Define

$$\widetilde{G}\left(S_t\right)=\frac{a_0^i+a_1^i\overline{Y}kS_t}{a^i+a_2^i\overline{Y}kS_t}=\frac{\phi^i}{\rho}\frac{\left(\rho+\phi^i\right)\left(\rho+k\overline{Y}S_t\right)+\left(\rho+k\right)\left(\overline{Y}k+\lambda\,\alpha\theta_i\right)S_t}{\left(\rho+\phi^i\right)\left(\rho+k\right)+\left(\rho+k\right)\left(\overline{Y}k+\lambda\,\alpha\theta_i\right)S_t},$$

and thus

$$G\left(S_t\right)=\frac{\left(\rho+\phi^i\right)\left(\rho+k\overline{Y}S_t\right)+\left(\rho+k\right)\left(\overline{Y}k+\lambda\,\alpha\theta_i\right)S_t}{\left(\rho+\phi^i\right)\left(\rho+k\right)+\left(\rho+k\right)\left(\overline{Y}k+\lambda\,\alpha\theta_i\right)S_t}$$

Notice that $G'\left(S\right)>0$ if and only if

$$0<\left(\rho+\phi^i\right)k\overline{Y}\left(\rho+\phi^i\right)\left(\rho+k\right)+\left(\rho+k\right)\left(\overline{Y}k+\lambda\,\alpha\theta_i\right)\left(\rho+\phi^i\right)k,$$

which yields the condition $\theta_i>-\frac{\overline{Y}\left(\rho+\phi^i+k\right)}{\lambda\,\alpha}$. We can then write

$$
\begin{aligned}
\beta_{CF}^i\left(S_t,s_t^i\right) &= \frac{\left(\frac{\partial P^i/P^i}{\partial s_t^i/s_t^i}\right)\left(\theta_{CF}^i-\sum_{j=1}^n s_t^j\theta_{CF}^j\right)}{1+\left(\frac{\partial P^{TW}/P^{TW}}{\partial S_t}\right)\sigma_S\left(S_t\right)}\left(\frac{1}{\sigma_{c,1}^2}\right)\\
&= \frac{\frac{1}{1+\frac{\phi^i}{\rho}G(S)\left(\overline{s}^i/s_t^i\right)}}{1+\frac{k\overline{Y}S}{k\overline{Y}S_t+\rho}\sigma_S\left(S_t\right)}\left(\theta_{CF}^i-\sum_{j=1}^n s_t^j\theta_{CF}^j\right)\left(\frac{1}{\sigma_{c,1}^2}\right)\\
&= \left(\frac{k\overline{Y}S_t+\rho}{k\overline{Y}S_t+\rho+k\overline{Y}S_t\sigma_S\left(S_t\right)}\right)\left(\frac{1}{1+\frac{\phi^i}{\rho}G\left(S\right)\left(\overline{s}^i/s_t^i\right)}\right)\left(\theta_{CF}^i-\sum_{j=1}^n s_t^j\theta_{CF}^j\right)\left(\frac{1}{\sigma_{c,1}^2}\right).
\end{aligned}
$$

Thus, formula (38) follows. Finally,

$$H\left(S\right)=\frac{k\overline{Y}S_t+\rho}{k\overline{Y}S_t+\rho+k\overline{Y}S_t\left(1-\lambda S\right)\alpha}$$

is such that $H'\left(S\right)>0$ if and only if $0<-\rho+k\overline{Y}\left(S_t\right)^2\lambda+\rho 2S_t\lambda$. Since the two roots of the equation $k\overline{Y}\lambda\left(S_t\right)^2+2\rho\lambda S_t-\rho=0$ are

$$S_1=\frac{-\rho\lambda-\sqrt{\rho^2\lambda^2+k\overline{Y}\lambda\rho}}{k\overline{Y}\lambda}<0<\frac{-\rho\lambda+\sqrt{\rho^2\lambda^2+k\overline{Y}\lambda\rho}}{k\overline{Y}\lambda}=S_2$$

we find the condition

$$H'\left(S\right)>0\text{ if and only if }S_t>\frac{-\rho\lambda+\sqrt{\rho^2\lambda^2+k\overline{Y}\lambda\rho}}{k\overline{Y}\lambda}.$$

Proof of expression (29) and (30): From (28) and the notation $\pi_t = e^{-\rho t} u_c (C_t - X_t)$, we can apply the law of iterated expectations and write

$$V_t = E\left[E\left[\int_t^{t+T} \frac{\pi_\tau}{\pi_t} CF_\tau d\tau | T \right] \right] \tag{40}$$

Since the stochastic discount factor does not depend explicitly on the random arrival of T, the inner expectation

$$V_t(T) = E_t\left[\int_t^{t+T} \frac{\pi_\tau}{\pi_t} CF_\tau d\tau | T \right]$$

satisfies the Euler equation

$$E_t\left[d\left(V_t(T) \pi_t \right) \right] + E_t\left[\pi_t CF_t \right] = 0$$

Let $\mathbf{X}_t$ be the set of state variables affecting all random processes in this economy and let them satisfy the stochastic differential equation $d\mathbf{X}_t = \boldsymbol{\mu}(\mathbf{X}_t) dt + \boldsymbol{\sigma}(\mathbf{X}) d\mathbf{B}_t$. Rewriting $V(\mathbf{X}_t, t; T) = V_t(T)$, an application of Ito's Lemma yields

$$V r_t = \frac{\partial V}{\partial t} + \sum_i \frac{\partial V}{\partial X_i} \cdot (\mu_i(\mathbf{X}) + \sigma_{i,1}(\mathbf{X}_t) \sigma_\pi) + \frac{1}{2} \sum_i \sum_j \frac{\partial^2 V}{\partial X_i \partial X_j} \cdot \boldsymbol{\sigma}_i(\mathbf{X}) \boldsymbol{\sigma}_j(\mathbf{X})' + CF_t$$

where $r_t = r(\mathbf{X}_t)$ is the riskless rate. The excess expected return is given by

$$\mu_R(\mathbf{X}_t) = -cov\left(\frac{dV}{V}, \frac{d\pi}{\pi} \right) = -\frac{1}{V} \sum_i \frac{\partial V}{\partial X_i} \sigma_{i,1}(\mathbf{X}_t) \sigma_\pi$$

Thus, we can rewrite

$$V\left(r_t + \mu_R(\mathbf{X}_t) \right) = \frac{\partial V}{\partial t} + \sum_i \frac{\partial V}{\partial X_i} \cdot \mu_i(\mathbf{X}) + \frac{1}{2} \sum_i \sum_j \frac{\partial^2 V}{\partial X_i \partial X_j} \cdot \boldsymbol{\sigma}_i(\mathbf{X}) \boldsymbol{\sigma}_j(\mathbf{X})' + CF_t$$

Feynman Kac theorem then yields

$$V_t(T) = E\left[\int_t^T e^{-\int_t^\tau r_s + \mu_R(\mathbf{X}_s) ds} CF_\tau d\tau | T \right]$$

Since the return on the total wealth portfolio dR_t^{TW} is perfectly correlated with the stochastic discount factor, it is immediate to see that we can also write $\mu_R(\mathbf{X}_t) = \beta_t \times E_t\left[dR_t^{TW} \right]$ where

$$\beta_t = \frac{cov\left(\frac{dV}{V}, dR_t^{TW} \right)}{var\left(dR_t^{TW} \right)}$$

yielding the representation (29).

We can finally obtain an expression for V_t: The random time T has an the exponential distribution with $f(T) = pe^{-pT}$. From (40) we obtain

$$V_t = \int_t^\infty E_t\left[\int_t^{t+T} \frac{\pi_\tau}{\pi_t} CF_\tau d\tau | T \right] pe^{-pT} dT = \int_t^\infty \left(\int_t^{t+T} E\left[\frac{\pi_\tau}{\pi_t} CF_\tau \right] d\tau \right) pe^{-pT} dT \tag{41}$$

Using the integration by parts rule

$$\int G(x) F(x) dx = \left(\int G(x) dx \right) F(x) - \int \left(\int G(x) dx \right) F'(x) dx$$

and recalling that $\int pe^{-pT} = -e^{-pT}$, we obtain

$$\int_t^\infty \left(\int_t^{t+T} E\left[\frac{\pi_\tau}{\pi_t} CF_\tau \right] d\tau \right) pe^{-pT} dT = \left[\left(\int_t^{t+T} E\left[\frac{\pi_\tau}{\pi_t} CF_\tau \right] d\tau \right) pe^{-pT} \right]_{T=0}^{T=\infty} - \int_t^\infty E\left[\frac{\pi_\tau}{\pi_t} CF_\tau \right] \left(-e^{-p(\tau-t)} \right) d\tau$$

Assuming that $\left(\int_t^{t+T} E \left[\frac{\pi_\tau}{\pi_t} CF_\tau \right] d\tau \right)$ does not diverge to infinity faster than e^{-pT}, we obtain that the first term is zero and thus

$$V_t = \int_t^\infty e^{-p(\tau-t)} E_t \left[\frac{\pi_\tau}{\pi_t} CF_\tau \right] d\tau$$

From the definition of CF_τ and the fact that ε_τ is independent of D_τ^i we find

$$V_t = a \int_t^\infty e^{-p(\tau-t)} E_t \left[\frac{\pi_\tau}{\pi_t} D_\tau^i \varepsilon_\tau \right] d\tau = a \int_t^\infty e^{-p(\tau-t)} E_t \left[\frac{\pi_\tau}{\pi_t} D_\tau^i \right] E_t \left[\varepsilon_\tau \right] d\tau$$

Substituting $E_t \left[\varepsilon_\tau \right] = 1 + (\varepsilon_t - 1) e^{-k_\varepsilon (\tau - t)}$

$$V_t = a \left\{ \int_t^\infty E_t \left[e^{-(\rho+p)(t-t)} \frac{u_c (C_\tau - X_\tau)}{u_c (C_t - X_t)} D_\tau^i \right] d\tau + (\varepsilon_t - 1) \int_t^\infty E_t \left[e^{-(\rho+p+k_\varepsilon)(t-t)} \frac{u_c (C_\tau - X_\tau)}{u_c (C_t - X_t)} D_\tau^i \right] d\tau \right\}$$

A proof identical to the one in the Appendix of MSV then shows that for every t after the investment takes place, V_t has the approximate solution

$$V_t \approx aD_t^i \left\{ \Phi^V \left(S_t, \overline{s}^i / s_t^i \right) + (\varepsilon_t - 1) \widehat{\Phi}^V \left(S_t, \overline{s}^i / s_t^i \right) \right\}, \tag{42}$$

where $\Phi^V \left(S_t, \overline{s}^i / s_t^i \right)$ and $\widehat{\Phi}^V \left(S_t, \overline{s}^i / s_t^i \right)$ are as in (27), but where the parameter ρ is substituted for $\rho + p$ and $\rho + p + k_\varepsilon$, respectively. At the time of the investment, however, ε_t is not known, and thus the value equals the unconditional expectation of (42). Since unconditionally $E[\varepsilon_t] = 1$ we obtain (30).

Finally, since ε_t is idiosyncratic, its variation does not command a premium, and thus a similar proof as in Proposition 1 shows that for every t after the investment takes place:

$$\beta_\tau = \beta_{DISC} \left(S_\tau, \frac{\overline{s}^i}{s_\tau^i}, \varepsilon_t \right) + \beta_{CF} \left(S_\tau, \frac{\overline{s}^i}{s_\tau^i}, \varepsilon_t \right) \tag{43}$$

where the formulas for β_{DISC} and β_{CF} are given in (20) and (21).[25]

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[25]Note that although ε_t does not command a premium on its own, its level does affect the project beta, as it changes the relative weight of the two components of V_t, $\Phi^V \left(S_t, \overline{s}^i / s_t^i \right)$ and $\widehat{\Phi}^V \left(S_t, \overline{s}^i / s_t^i \right)$.

TABLE AI
Description and Summary Statistics of Industries

	Industry Description	SIC	Avg. No. of Stocks	Min. No. of Stocks	Avg. Market Cap. (%)
1.	Mining	10-14	145.2	30	2.656
2.	Food	20	98	48	4.943
3.	Apparel	22-23	74	18	0.609
4.	Paper	26	37.8	5	1.904
5.	Chemical	28	150.5	25	10.394
6.	Petroleum	29	35.4	23	10.610
7.	Construction	32	36.2	5	1.273
8.	Prim. Metals	33	75.7	44	4.269
9.	Fab. Metals	34	73.6	9	1.415
10.	Machinery	35	185.7	25	5.760
11.	Electric Eq.	36	198.9	14	6.064
12.	Transport Eq.	37	91.4	46	7.646
13.	Manufacturing	38-39	153.6	10	2.902
14.	Railroads	40	34	8	3.049
15.	Other Transport.	41-47	61.4	15	0.875
16.	Utilities	49	127.7	21	7.856
17.	Dept. Stores	53	42.4	20	3.743
18.	Retail	50-52 54-59	254.2	22	2.313
19.	Financial	60-69	441.6	15	6.927
20.	Other		619.2	57	14.788

TABLE I

TABLE I
Model parameters and moments of aggregate quantities

Panel A: Preference parameters and consumption parameters

ρ	$\bar{Y}$	k	λ	α	μ_C	σ_C
0.04	33.97	0.16	20.00	79.39	0.02	0.01

Panel B: Aggregate Moments

	$E(R)$	$Vol(R)$	$E(r_f)$	$Vol(r_f)$	Ave(PC/100)	SR
Data	0.07	0.16	0.01	0.01	0.30	0.46
Model	0.07	0.23	0.01	0.04	0.30	0.31

Panel C: Share Process

Industry	$\bar{s}^i$	ϕ^i	θ^i_{CF}-Cash-flow (x1000)	θ^i_{CF}-Return (x100)
Constr.	0.04	0.52	-0.12	0.23
Railroads	0.09	0.20	-0.47	0.04
Retail	0.04	0.20	-0.09	0.07
Petroleum	0.52	0.16	-0.20	-0.18
Mining	0.05	0.16	-0.33	-0.11
Elect.Eq.	0.09	0.14	-0.21	0.23
Apparel	0.01	0.12	-0.16	0.02
Machinery	0.12	0.11	-0.10	0.14
Paper	0.05	0.11	-0.19	-0.01
Other Transp.	0.01	0.09	-0.06	0.09
Dept.Stores	0.09	0.09	-0.03	0.08
Transp.Eq.	0.25	0.08	0.27	0.03
Manufact.	0.05	0.06	-0.13	0.04
Other	0.17	0.06	-0.08	-0.07
Fab.Metals	0.03	0.05	-0.17	-0.03
Financial	0.05	0.04	-0.02	-0.02
Chemical	0.29	0.03	-0.14	-0.06
Prim.Metals	0.12	0.01	-0.32	-0.05
Utilities	0.10	0.00	-0.06	-0.11
Food	0.15	0.00	-0.09	-0.05
Mkt.Ptfl.	2.22	0.07	-0.10	-

Notes to Table I: This is Table 1 in Menzly, Santos, and Veronesi (2003) with the only exception of the estimate of θ^i_{CF} obtained using returns data, which is under the heading "Returns". *Panel A*: Annualized preference and consumption process parameters chosen to calibrate the mean average excess returns, the average price consumption ratio, the average risk free rate and its volatility, and the Sharpe ratio of the market portfolio. *Panel B*: Expected excess return of the market portfolio, $E(R)$, standard deviation of returns of the market portfolio, $Vol(R)$, expected risk free rate, $E(r_f)$, standard deviation of the risk free rate, $Vol(r_f)$, average price consumption ratio, Ave(PC/100), and Sharpe ratio of the market portfolio, SR. *Panel C:* Estimates of the long run mean, $\bar{s}^i$, and the speed of mean reversion ϕ^i, cash flow risk, θ^i_{CF}, and covariance between dividend growth and consumption growth, $cov(d\delta^i_t, dc_t)$ for each industry. Industries are ordered, in this and subsequent tables, according to the parameter ϕ^i. All entries in the table are in annual units.

TABLE II

TABLE II
The standard deviation of market betas

Industry	Fama and French (1997)	θ^i_{CF}-Cash-flow	θ^i_{CF}-Return
Constr.	.11	.02	.11
Railroads	.19	.05	.04
Retail	.27	.02	.04
Petroleum	.24	.02	.10
Mining	.27	.05	.16
Elect.Eq.	.00	.02	.22
Apparel	.10	.03	.06
Machinery	.05	.03	.14
Paper	.14	.02	.01
Other Transp.	.05	.04	.10
Dept.Stores	.24	.02	.09
Transp.Eq.	.07	.04	.03
Manufact.	.00	.03	.05
Other	.08	.02	.08
Fab.Metals	.21	.04	.04
Financial	.08	.02	.03
Chemical	.05	.04	.10
Prim.Metals	.11	.10	.14
Utilities	.30	.02	.33
Food	.12	.03	.15

Notes to Table II: This table reports the standard deviation of betas. The column under the heading Fama and French (1997) provides an estimate of the standard deviation of the "true" beta using the procedure used by these authors. Under the assumption that the sampling error associated with the market betas is uncorrelated with the true value of the beta, the variance of the rolling regression beta, $\hat{\beta}_t^{\text{rolling-regress.}}$, is the sum of the variance of the true market beta and the variance of the estimation error and thus

$$\sigma^2\left(\hat{\beta}_t^{\text{rolling-regress.}}\right) = \sigma^2\left(\beta_t\right) + \sigma^2\left(\varepsilon_t\right).$$

The column under the heading θ^i_{CF}-Cash-flow provides an estimate of the standard deviation of the theoretical betas in 40,000 quarters of artificial data when θ^i_{CF} is estimated using only cash-flow data. The column under the heading θ^i_{CF}-Returns provides an estimate of the standard deviation of the theoretical betas in 40,000 quarters of artificial data when θ^i_{CF} is estimated using only returns data.

TABLE III
The cross sectional dispersion of market betas

Panel A: Dispersion in Betas

Sorting Variable	Sample: 1927 - 2001			Sample: 1947 - 2001		
	Up	Down	p-Value	Up	Down	p-Value
P_t^M/D_t^M	.25	.19	.04	.32	.17	.00
Surplus	.17	.22	.85	.22	.13	.01
$\text{STD}^{CS}\left(\frac{\bar{s}^i}{s_t^i}\right)$	.25	.18	.09	.27	.17	.02

Panel B: Dispersion of Betas: Interaction with dispersion of Shares

Sample: 1927 - 2001   Sample: 1947 - 2001

Sorting variable $= P_t^M/D_t^M$

		Up	Down	p-Value	Up	Down	p-Value
	High	.27	.22	.15	.39	.16	.00
$\text{STD}^{CS}\left(\frac{\bar{s}^i}{s_t^i}\right)$	Low	.19	.12	.02	.20	.14	.05
	p-value	.06	.00	.00	.00	.26	.00

Sorting variable $=$ Surplus

		Up	Down	p-Value	Up	Down	p-Value
	High	.20	.25	.85	.27	.20	.07
$\text{STD}^{CS}\left(\frac{\bar{s}^i}{s_t^i}\right)$	Low	.20	.11	.01	.24	.14	.01
	p-value	.53	.00	.01	.27	.07	.00

Notes to Table III: *Panel A*: Cross sectional dispersion of return betas in good or bad times as measured by $\text{STD}^{CS}\left(\beta^i\right)$ and p values of the difference. Betas are estimated from the regression

$$R_{t+1}^i = \alpha^i + \beta_{Up}^i\left(Idx_t^{Up}R_{t+1}^M\right) + \beta_{Do}^i\left(Idx_t^{Do}R_{t+1}^M\right) + \varepsilon_{t+1}^i$$

where R_{t+1}^i and R_{t+1}^M are the excess return on industry i and the market between t and $t+1$, respectively, and Idx_t^{Up} and Idx_t^{Do} are indicator functions of whether the aggregate equity premium is low or high, that is whether times are good (Up) or bad (Do). As proxies for the aggregate state of the economy we consider *(i)* the market price dividend ratio, with $Idx_t^{Up} = 1$ if the price dividend ratio of the market is above its historical 70 percentile, and $Idx_t^{Do} = 1$ if price dividend ratio is below its historical 30 percentile; *(ii)* the surplus-consumption ratio S_t itself, where again $Idx_t^{Up} = 1$ or $Idx_t^{Do} = 1$, if the surplus is above its 70 percentile, or below its 30 percentile; and *(iii)* the dispersion of relative shares. *Panel B*: Cross sectional dispersion of return betas in good versus bad times and periods of large dispersion of relative shares versus low dispersion of relative shares. Betas are computed from the regression

$$R_{t+1}^i = \alpha^i + \sum_{k=Up,Do}\sum_{h=Hi,Lo} \beta_{kh}^i\left(Idx_t^{kh}R_{t+1}^M\right) + \varepsilon_{t+1}^i,$$

where Idx_t^{kh} is an indicator function of whether the economy is in a high or low state and the cross sectional dispersion of relative share is high (Hi) or low (Lo). The high dispersion of relative shares as well as the good state periods are defined using the 50% percentile cutoff. The results are reported for the long sample, 1927-2001, and the short sample, 1947-2001.

TABLE IV
Simulations - The Cross Sectional Dispersion of market betas

Panel A: Dispersion of Betas

Sorting variable	θ^i_{CF}-Cash-flow			θ^i_{CF}-Return		
	Up	Down	p-Value	Up	Down	p-Value
P_t^M/D_t^M	.03	.02	.01	.27	.14	.00
Surplus	.04	.02	.00	.29	.14	.00
$\mathrm{STD}^{CS}\left(\frac{\overline{s}^i}{s_t^i}\right)$	.02	.02	.53	.13	.15	.68

Panel B: Dispersion of Betas: Interaction with dispersion of shares

Sorting variable $= P_t^M/D_t^M$

		Up	Down	p-Value	Up	Down	p-Value
	High	.03	.02	.03	.26	.12	.00
$\mathrm{STD}^{CS}\left(\frac{\overline{s}^i}{s_t^i}\right)$	Low	.04	.02	.01	.28	.14	.00
	p-Value	.78	.55	.04	.64	.77	.00

Sorting variable $=$ Surplus

		Up	Down	p-Value	Up	Down	p-Value
	High	.04	.02	.00	.27	.12	.00
$\mathrm{STD}^{CS}\left(\frac{\overline{s}^i}{s_t^i}\right)$	Low	.04	.02	.00	.30	.14	.00
	p-Value	.54	.57	.00	.65	.76	.00

Notes to Table IV: This table replicates Table III in 40,000 quarters of artificial data. Specifically, in simulated return data, the table reports the cross sectional dispersion of return betas in good or bad times as measured by $\mathrm{STD}^{CS}\left(\beta^i\right)$ and p values of the difference. Betas are estimated from the regression

$$R_{t+1}^i = \alpha^i + \beta_{Up}^i\left(Idx_t^{Up}R_{t+1}^M\right) + \beta_{Do}^i\left(Idx_t^{Do}R_{t+1}^M\right) + \varepsilon_{t+1}^i$$

where R_{t+1}^i and R_{t+1}^M are the excess return on industry i and the market between t and $t+1$, respectively, and Idx_t^{Up} and Idx_t^{Do} are indicator functions of whether the aggregate equity premium is low or high, that is whether times are good (Up) or bad (Do). As proxies for the aggregate state of the economy we consider *(i)* the market price dividend ratio, with $Idx_t^{Up} = 1$ if the price dividend ratio of the market is above its historical 70 percentile, and $Idx_t^{Do} = 1$ if price dividend ratio is below its historical 30 percentile; *(ii)* the surplus-consumption ratio S_t itself, where again $Idx_t^{Up} = 1$ or $Idx_t^{Do} = 1$, if the surplus is above its 70 percentile, or below its 30 percentile; and *(iii)* the dispersion of relative shares. *Panel B*: Cross sectional dispersion of return betas in good versus bad times and periods of large dispersion of relative shares versus low dispersion of relative shares. Betas are computed from the regression

$$R_{t+1}^i = \alpha^i + \sum_{k=Up,Do}\sum_{h=Hi,Lo} \beta_{kh}^i\left(Idx_t^{kh}R_{t+1}^M\right) + \varepsilon_{t+1}^i,$$

where Idx_t^{kh} is an indicator function of whether the economy is in a high or low state and the cross sectional dispersion of relative share is high (Hi) or low (Lo). The high dispersion of relative shares as well as the good state periods are defined using the 50% percentile cutoff. The heading θ_{CF}-Cash-flow reports results where simulated returns are generated using θ^i_{CF} estimated from only cash-flow data. The heading θ_{CF}-Return reports results where simulated returns are generated using θ^i_{CF} estimated from stock return data.

TABLE V
Changes in betas and investment growth

	$\Delta\left(\frac{P_t^i/C_t}{PC}\right)$	$\Delta\left(\frac{P_{t-1}^i/C_{t-1}}{PC}\right)$	$\Delta\left(\frac{\overline{s}^i}{s_t^i}\right)$	$\Delta\left(\frac{\overline{s}^i}{s_{t-1}^i}\right)$	$\Delta\beta_t^i$	$\Delta\beta_{t-1}^i$	Adj. R^2
Panel A: Total Investment							
1.	-0.14	0.18^*					0.06
	(0.98)	(3.06)					
2.			-0.09^*	-0.05^{**}			0.03
			(-3.83)	(-1.94)			
3.					0.06^{**}	0.11^*	0.02
					(1.79)	(2.52)	
4.	-0.04	0.17^*	-0.08^*	-0.03	0.07^*	0.08^*	0.09
	(-0.63)	(2.76)	(-3.01)	(-1.05)	(2.13)	(2.11)	
Panel B: Value-weighted Investments							
1.	-0.05	0.20^*					0.06
	(-0.76)	(3.17)					
2.			-0.06^{**}	-0.02			0.01
			(-1.91)	(-0.76)			
3.					0.09^*	0.13^*	0.02
					(2.11)	(2.62)	
4.	-0.07	0.20^*	-0.04	-0.00	0.10^*	0.11^*	0.08
	(-1.11)	(3.01)	(-1.48)	(0.14)	(2.35)	(2.31)	
Panel C: Equal-weighted Investments							
1.	0.02	0.24^*					0.07
	(0.28)	(3.58)					
2.			-0.03	-0.01			0.00
			(-1.04)	(-0.50)			
3.					0.11^*	0.16^*	0.03
					(2.66)	(3.99)	
4.	-0.00	0.23^*	-0.01	0.01	0.11^*	0.13^*	0.32
	(-0.01)	(3.48)	(-0.25)	(0.51)	(2.81)	(3.38)	

Notes to Table V: This table reports the results of a panel regression of industry real investment growth on changes in the price consumption ratio of the industry portfolio, normalized by the average price consumption ratio, $\left(P_t^i/C_t\right)/\overline{PC}$, changes in relative share $\overline{s}^i/s_t^i$ and changes in conditional betas β_t, and their lags. In Panel A, industry investments are defined as the industry total Capital Expenditures (Capex) over total Property, Plant and Equipment (PPE). Panel B and C industry investments are defined a Weighted Average or Equally Weighted Average of individual firms Capex over PPE. The industry conditional beta at time t, β_t is computed from a rolling regression using the 24 months prior to t. Industry dummies are included in the regression. t-statistics, computed using robust standard errors clustered by year, are reported in parenthesis. * and ** denotes significance at the 5% and 10% respectively.

TABLE VI
Changes in betas and investment growth with year dummies

|	$\Delta\left(\frac{P_t^i/C_t}{PC}\right)$	$\Delta\left(\frac{P_{t-1}^i/C_{t-1}}{PC}\right)$	$\Delta\left(\frac{\overline{s}^i}{s_t^i}\right)$	$\Delta\left(\frac{\overline{s}^i}{s_{t-1}^i}\right)$	$\Delta\beta_t^i$	$\Delta\beta_{t-1}^i$	Adj. R^2
Panel A: Total Investment							
1.	0.08	0.13					0.32
	(1.27)	(1.48)					
2.			-0.06^*	-0.04^*			0.32
			(-3.28)	(-1.82)			
3.					0.09^*	0.06	0.32
					(2.47)	(1.57)	
4.	-0.00	0.11	-0.06^*	-0.02	0.09^*	0.06	0.33
	(-0.01)	(1.13)	(-3.01)	(-0.97)	(2.57)	(1.43)	
Panel B: Value-weighted Investments							
1.	0.04	0.07					0.29
	(0.59)	(0.71)					
2.			-0.02	-0.01			0.29
			(-1.03)	(-0.23)			
3.					0.14^*	0.07	0.30
					(2.71)	(1.55)	
4.	-0.00	0.08	-0.02	0.01	0.14^*	0.07	0.30
	(-0.01)	(0.86)	(-0.92)	(0.19)	(2.71)	(1.50)	
Panel C: Equal-weighted Investments							
1.	-0.01	0.14					0.27
	(-0.08)	(1.74)					
2.			-0.03	-0.03			0.27
			(-1.15)	(-1.59)			
3.					0.14^*	0.10^*	0.29
					(3.37)	(2.43)	
4.	-0.07	0.13	-0.04	-0.02	0.14^*	0.10^*	0.29
	(-0.84)	(1.48)	(-1.29)	(-0.73)	(3.40)	(2.45)	

Notes to Table VI: This table reports the results of a panel regression of industry real investment growth on changes in the price consumption ratio of the industry portfolio, normalized by the average price consumption ratio, $\left(P_t^i/C_t\right)/\overline{PC}$, changes in relative share $\overline{s}^i/s_t^i$ and changes in conditional betas β_t, and their lags. In Panel A, industry investments are defined as the industry total Capital Expenditures (Capex) over total Property, Plant and Equipment (PPE). Panel B and C industry investments are defined a Weighted Average or Equally Weighted Average of individual firms Capex over PPE. The industry conditional beta at time t, β_t is computed from a rolling regression using the 24 months prior to t. Industry dummies and year dummies are included in the regression. t-statistics, computed using robust standard errors clustered by year, are reported in parenthesis. * and ** denotes significance at the 5% and 10% respectively.

TABLE VII

Simulations - Changes in betas and investment growth

Panel A: Panel regression without year dummies - Table V

	Mean	Median	5%	95%
A.1		θ_{CF}^i-Cash-Flow		
$\Delta\left(\frac{P_t^i/C_t}{\overline{PC}}\right)$	0.510	0.596	0.052	0.777
$\Delta\left(\frac{\overline{s}^i}{s_t^i}\right)$	−0.011	−0.00	−0.051	0.000
$\Delta\beta_t^i$	−0.094	−0.059	−0.519	0.204
A.2		θ_{CF}^i-Returns		
$\Delta\left(\frac{P_t^i/C_t}{\overline{PC}}\right)$	0.211	0.194	0.026	0.461
$\Delta\left(\frac{\overline{s}^i}{s_t^i}\right)$	−0.001	−0.00	−0.012	0.000
$\Delta\beta_t^i$	0.022	0.023	−0.029	0.059

Panel B: Panel regression with year dummies - Table VI

|	Mean	Median	5%	95%
B.1		θ_{CF}^i-Cash-Flow		
$\Delta\left(\frac{P_t^i/C_t}{\overline{PC}}\right)$	0.352	0.443	−0.057	0.804
$\Delta\left(\frac{\overline{s}^i}{s_t^i}\right)$	−0.029	−0.00	−0.142	0.000
$\Delta\beta_t^i$	0.540	0.547	0.358	0.709
B.2		θ_{CF}^i-Returns		
$\Delta\left(\frac{P_t^i/C_t}{\overline{PC}}\right)$	0.078	0.041	0.001	0.207
$\Delta\left(\frac{\overline{s}^i}{s_t^i}\right)$	−0.002	−0.00	−0.014	−0.000
$\Delta\beta_t^i$	0.110	0.119	0.026	0.154

Notes to Table VII: This table reports the results of the multivariate panel regression of industry real investment growth on changes in the price consumption ratio of the industry portfolio, normalized by the average price consumption ratio, $\left(P_t^i/C_t\right)/\overline{PC}$, changes in relative share $\overline{s}^i/s_t^i$ and changes in conditional betas β_t in simulated data. To handle the dimensionality problem with year dummies, we divide the 10,000 simulation years in 20 series of 500 years each for our two sets of estimates of the cash-flow parameter θ_{CF}^i. Panel A: Panel regression without time dummies as in Table V. Panel B: Panel regression with time dummies as in Table VI. For each panel regression we report the mean, median, 5 % and 95% estimates of the corresponding coefficient across the 20 simulations.

Figure 1: Model-Implied Betas
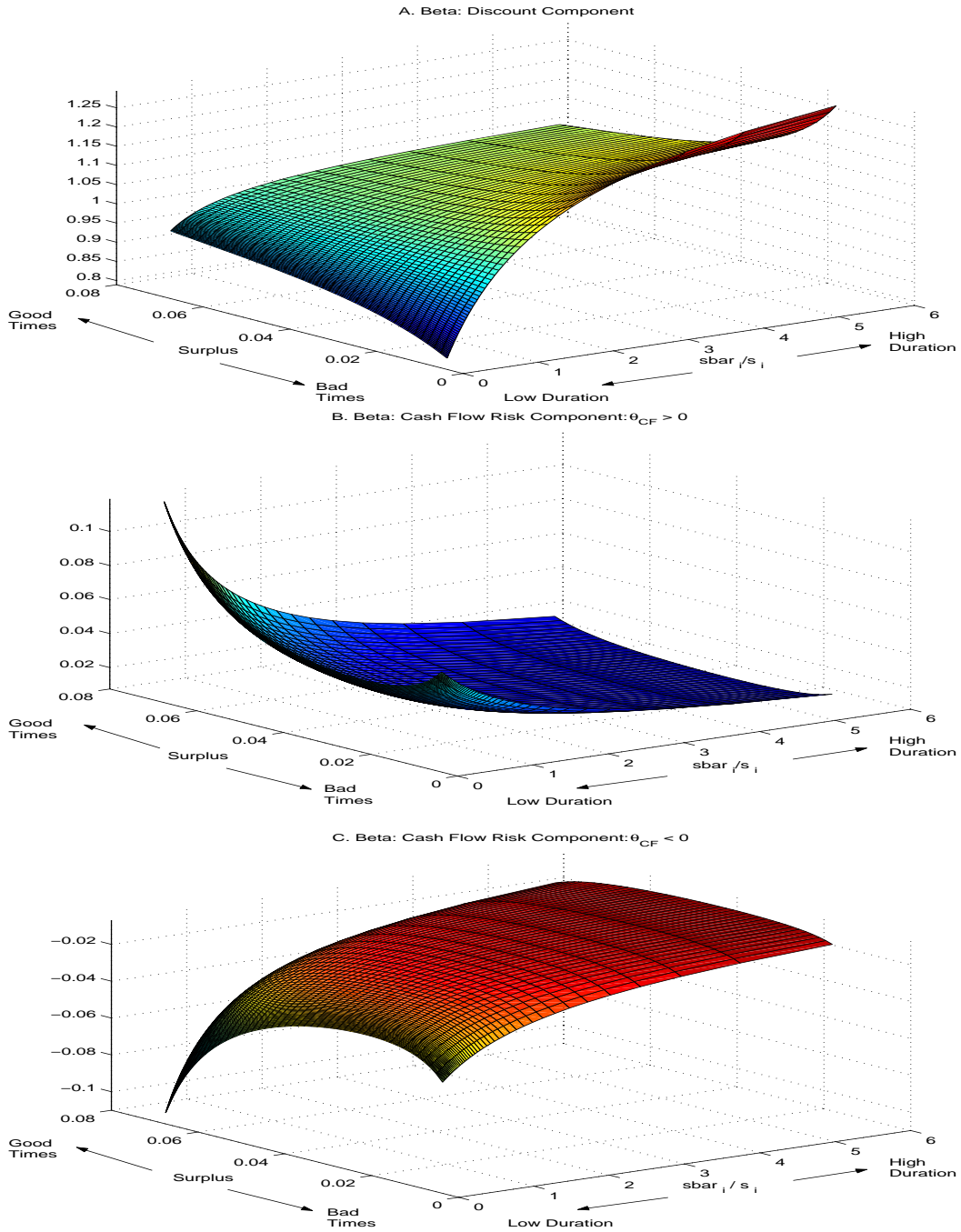
Panel A: Discount Beta $\beta_{DIS}(S_t, \overline{s}^i/s_t^i)$; Panel B: Cash-Flow Beta $\beta_{CF}(S_t, \overline{s}^i/s_t^i)$ with positive unconditional cash flow risk index $\theta_{CF}^i > 0$; Panel C: Cash-Flow Beta $\beta_{CF}(S_t, \overline{s}^i/s_t^i)$ with negative unconditional cash flow risk index $\theta_{CF}^i < 0$.

Figure 2: Model-Implied Betas

95% simulation bands: θ^i_{CF} from fundamentals

95% simulation bands: θ^i_{CF} from returns
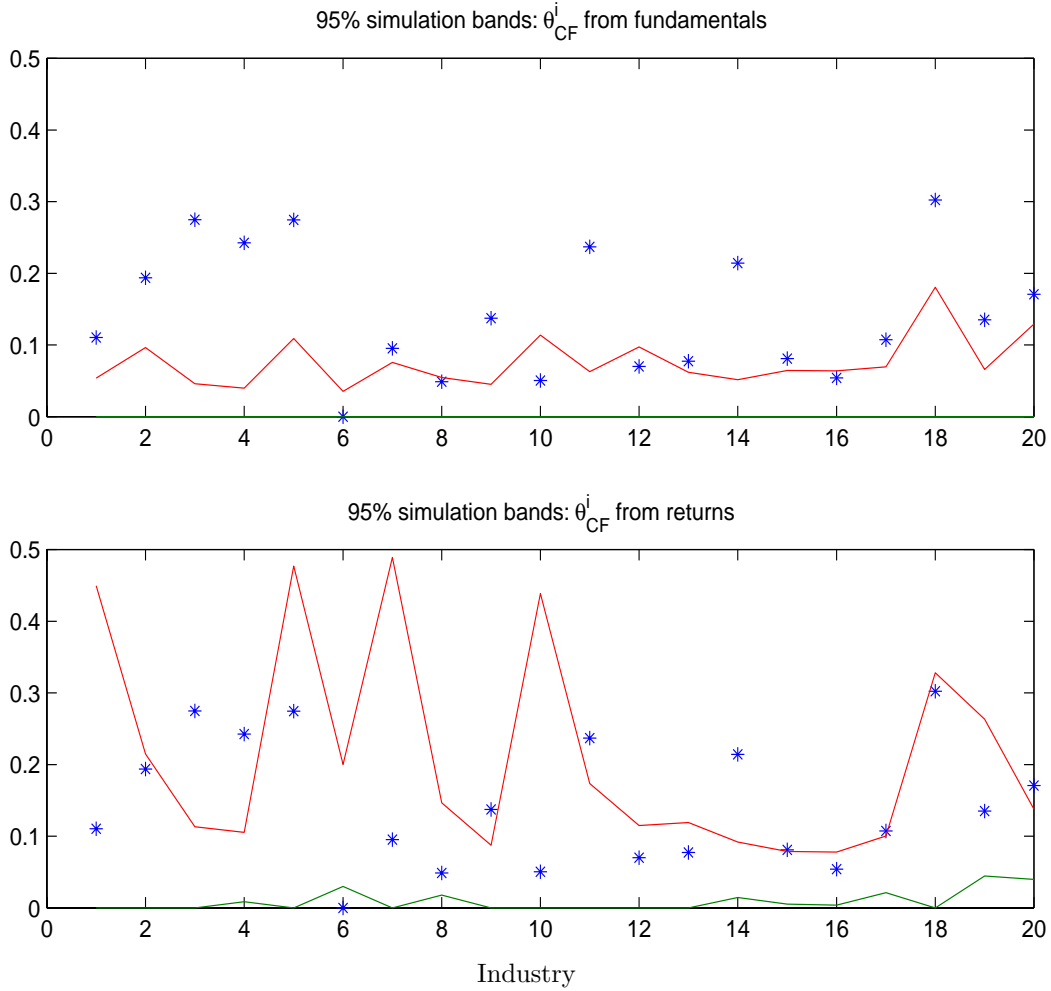
Industry

Panel A: Empirical estimates of the time-series variation of industry betas (stars), computed as in Fama and French (1997):

$$\sigma(\beta_t^{\text{true}}) = \sqrt{\sigma^2\left(\widehat{\beta}_t^{\text{rolling-regress.}}\right) - \sigma^2\left(\varepsilon_t\right)},$$

where $\sigma^2\left(\widehat{\beta}_t^{\text{rolling-regress.}}\right)$ is the time series variance of betas estimated using a 20 quarter rolling regression, and $\sigma^2\left(\varepsilon_t\right)$ is the average variance of the residuals of the rolling regressions. The solid lines provide the 95 % confidence interval for the same statistic computed on 1000, 54-year samples of artificial data (the lower bound coincides with the zero axis). The parameter choices correspond to the case where θ^i_{CF} are computed using fundamental variables. Panel B: Same as panel A, but with parameter choices corresponding to the case where θ^i_{CF} are estimated by GMM using stock returns.