Self-Image Bias and Talent Loss

On-Line Appendix

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This on-line appendix contains additional analysis and the proofs of our propositions. In particular

- A1.: Extensions
  - A1.1.: Endogenous Entry
  - A1.2.: Seniors and Juniors
  - A1.3.: Co-authorship
  - A1.4.: General Distance Functions from Referees
- A2.: Additional Analysis and Results
- A3.: Proofs

A1. Extensions

A1.1. Endogenous Entry

In this section we extend the model to consider the optimal choice of young researchers on whether to undertake a research career (Section A1.1.1.) and the optimal choice of hiring institutions on whether to hire young researchers (Section A1.1.4.).

A1.1.1. Endogenous Choice of Young Researchers

Consider a potential researcher choosing between an academic career and an outside option. The prospective researcher knows her type θ, and is aware of both the likelihood of producing quality research, and the evaluation criteria used by the referees. Attempting to pursue research entails a cost C, which is identical across agents. If the potential researcher is hired (accepted), he or she receives a payoff of P; finally, the outside option is normalized to 0. Thus, the total payoff is $P - C$ if the researcher is hired, and $-C$ otherwise. What types of agents decide to pay the cost C and thus take their chance with the academic career?

Assume that the entry decision, research activity, and hiring decision all occur at time t. Then, given the time-t distribution $\lambda_t = (\lambda_t^\theta)_{\theta\in\Theta}$ of referees' types, a prospective researcher of type θ pursues an academic career—"applies"—if and only if

$$\gamma^\theta \lambda_t^\theta (P - C) + (1 - \gamma^\theta \lambda_t^\theta)(-C) > 0. \tag{A.18}$$

Consequently, the accepted mass of researchers is as follows: for $g = f, m$,

$$a_t^{\theta,g} = \begin{cases} \gamma^\theta \cdot \lambda_{t-1}^\theta \cdot p^{\theta,g} & \text{if } \gamma^\theta \lambda_{t-1}^\theta \geq \frac{C}{P} \\ 0 & \text{otherwise} \end{cases} \tag{A.19}$$

$$\lambda_t^{\theta,g} = \lambda_{t-1}^{\theta,g} (1 - a_t) + a_t^{\theta,g} \tag{A.20}$$

Expression (A.19) shows that if the mass of type-θ reviewers drops below $\frac{C}{\gamma^\theta P}$ at time $t-1$, both M and F young type-θ researchers will not apply at date t. From Eq. (A.20), this implies that the total mass of such types will decrease, at least weakly, because some type-θ established researchers will have to retire in order to make room for researchers of other types who are accepted. In fact, the mass of such types will decrease strictly, except in case no young researcher wants to apply.

While the dynamics with endogenous entry is considerably more complicated than in the benchmark case, we prove the following Proposition:

Proposition A.1 *Assume that at time 0, all referees are from the M-group with $\lambda_0 = p^m$.*

(a.1) If $\rho < \bar{\rho}(\phi, N)$ and $\frac{C}{P} \leq (1-\phi)^N \gamma_0 \sqrt{\rho}$, then the steady state is as in Proposition 3(a).

(a.2) If $\rho < \bar{\rho}(\phi, N)$ and $(1-\phi)^N \gamma_0 \sqrt{\rho} < \frac{C}{P} \leq \phi^N \gamma_0 \sqrt{\rho}$, then only type θ^m survives in the limit, i.e. $\bar{\lambda}^{\theta^m} = 1$. The limiting mass of M researchers is strictly larger than in (a.1):

$$\bar{\Lambda}^m = \lim_{t\to\infty} \sum_\theta \lambda_t^{m,\theta} = \frac{\phi^N}{\phi^N + (1-\phi)^N} > \frac{1 + \left(\frac{\phi}{1-\phi}\right)^{2N}}{1 + \left(\frac{\phi}{1-\phi}\right)^{2N} + 2\left(\frac{\phi}{1-\phi}\right)^N}. \tag{A.21}$$

(b) If $\rho > \bar{\rho}(\phi, N)$ and $[\phi(1-\phi)]^{N/2} \geq \frac{C}{\gamma_0 \rho P}$, then the steady state is as in Proposition 3(b).

In each of the above cases, if $\bar{\lambda}^\theta = 0$*, then there is* $t^\theta \geq 0$ *such that* $\lambda_t^\theta = 0$ *for all* $t \geq t^\theta$*.*

Part (a.1) and (b) of this proposition shows that if the cost C is low enough, then the the steady state is the same as in the basic model in Section 2. for the same two conditions about ρ, respectively. This is intuitive. The only difference is that all types other than surviving ones drop out in finite time, rather than only in the limit.

The interesting new part is (a.2). In this case, the only type that survives in the long-run is θ^m, the most prevalent type in the M-population. In particular, θ^f now disappears. Thus, the characteristics that are mildly more frequent in the F-population, but also common in the M-population, eventually disappear. In this case, endogenous entry greatly exacerbates the loss of talent compared to the base case. Indeed, the total mass of M researchers, $\bar{\Lambda}^m$, is now even larger than in its counterpart without endogenous entry, whose expression is in Eq. (10) in Proposition 3. Thus, if the conditions in part (a.2) are satisfied, the distribution of established researchers will be even more skewed towards the M group.

Parts (a.1)–(b) do not exhaust all possible cases; for instance, they do not analyze the possibility that the first condition in part (b) holds, but the second does not—that is, θ^* is not willing to apply. The following section illustrates a stark instance of one such possibility. The proof of the above Proposition in the Appendix provides a general characterization that can be used to further explore different parametric choices.

A1.1.2. Example of Group Imbalance due to Endogenous Entry

We first illustrate how endogenous entry can exacerbate group imbalance, provided the cost of entry is not too small. Consider the parameterization in Section 3. In our basic model, M-researchers represent 91% of the overall population in the limit. If we add endogenous entry, Proposition A.1 shows that the steady state either remains the same, if the cost C is sufficiently low, as in case (a.1), or it becomes even more skewed towards the M group, as in case (a.2). In the latter case, the limiting fraction of M-researchers is $\bar{\Lambda}^m = \phi^N/(\phi^N + (1-\phi)^N) = 95\%$.

We now illustrate how endogenous choice may prevent convergence to group balance even when group balance would in fact attain in the basic model. We use the same parameterization as in Section 3., except that the number of characteristics is $N = 8$ instead of $N = 10$. With these parameter values, Proposition 3 part (b) implies that the system will converge to an equal mass of M and F researchers, because

$\rho = 5 > 3.61 = \bar{\rho}(\phi, N)$. The solid and dashed lines in Figure A.1 confirm this.

However, assume now that entry is endogenous; the payoff if a researcher is hired is $P = 1,000$, and the cost of entry is $C = 4$ (i.e., 0.4% of the payoff of becoming a researcher over the outside option). Note that these parameters apply equally to M and F researchers. The key point is that now the efficient type θ^* (M or F) does not want to apply at date 0:

$$\lambda_0^{\theta^*} = p^{\theta^*,m} = \phi^{N/2}(1-\phi)^{N/2} = 0.3574\% < 0.4\% = \frac{C}{\gamma^{\theta^*}P}.$$

Moreover, type θ^f (M or F) does not want to apply either:

$$\lambda_0^{\theta^f} = p^{\theta^f,m} = (1-\phi)^N = 0.1081\% < 0.8944\% = \frac{C}{\gamma^{\theta^f}P}.$$

On the other hand, type θ^m (M or F) does:

$$\lambda_0^{\theta^m} = p^{\theta^m,m} = \phi^N = 1.18\% > 0.8944\% = \frac{C}{\gamma^{\theta^m}P}.$$

Therefore, while other types are also willing to apply, type θ^m will prevail, which will lead to a severe imbalance between M and F researchers in the limit, as shown in Figure A.1. Indeed, in this case the talent loss is rather severe, as the only surviving type $\theta^m = (1, ..., 1, 0, ...0)$ has none of the research characteristics that are (mildly) more common in the F-population. Figure A.2 shows that both F and M researchers are of type θ^m in the long run.

To sum up, even if the basic environment is meritocratic, in the sense that differences in talents γ^θ across types are sufficient to lead to group balance, endogenous entry introduces a bias in favor of M-researchers which leads to an imbalance steady state. In this case, policies aimed at lowering the cost C can lead to group balance in the long run.

A1.1.3. Characterization of the Applicant Pool

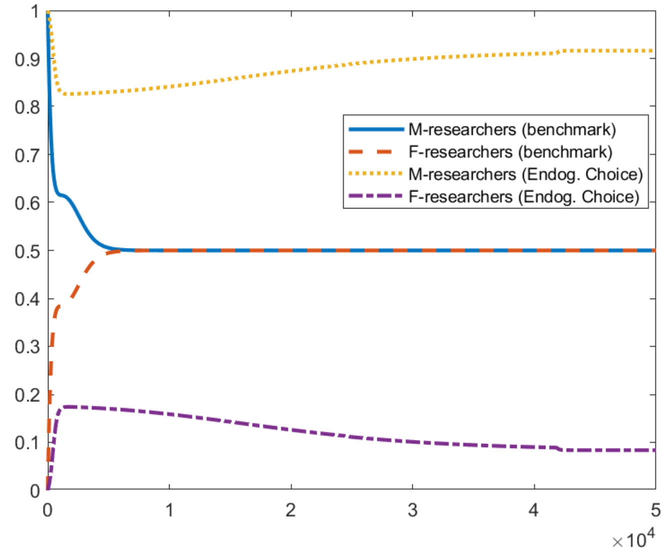
Due to variation in the distribution of characteristics, Proposition A.1 also has implications for the mass of young M and F researchers who decide to apply for an academic job:

Proposition A.2 *For every t, let*

$$A_t^m = \sum_{\theta:\lambda_t^\theta \geq \frac{C}{\gamma^\theta P}} p^{\theta,m} \quad \text{and} \quad A_t^f = \sum_{\theta:\lambda_t^\theta \geq \frac{C}{\gamma^\theta P}} p^{\theta.,f}$$

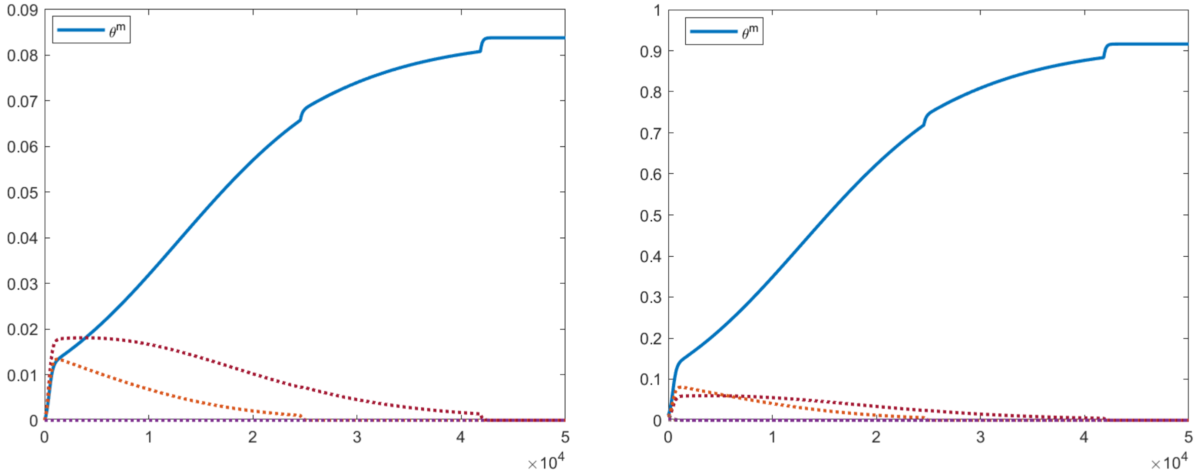
Then $A_t^m \geq A_t^f$. Moreover, if $\lambda_0^{\theta^m} > \frac{C}{\gamma_0\sqrt{\rho}P} > \lambda_0^{\theta^f}$, then $A_t^f \to 1 - \bar{\Lambda}^m$, where $\bar{\Lambda}^m$ is as in part (a.2) of Proposition A.1.

Figure A.1: Fraction of M and F Researchers with Endogenous Entry



Fraction of M and F researchers when $\lambda_0 = p^m$. Parameters: $\phi = 0.5742$ ($d = 0.3$), $\gamma_0 = 0.2$, $\rho = 5$, $N = 8$, $P = 1000$, and $C = 4$.

Figure A.2: Types of Established F and M Researchers with Endogenous Entry

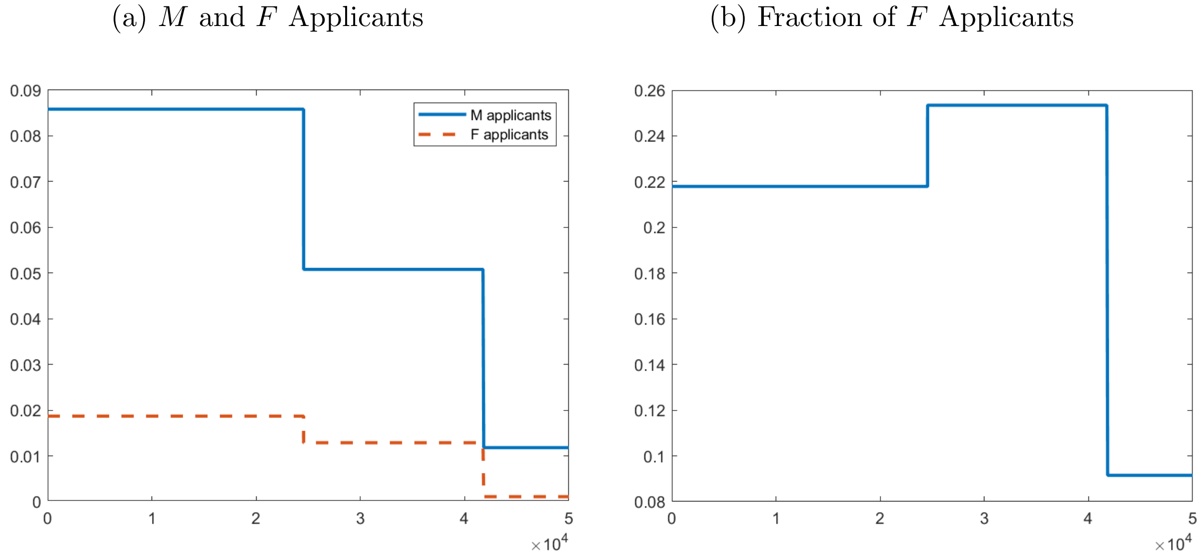

(a) F researchers

(b) M researchers

Types of established F (left) and M (right) researchers with endogenous entry. $\theta^m = (1, ..., 1, 0, ..., 0)$ dominates; all other types eventually vanish. Parameters: $\phi = 0.0.5742$ ($d = 0.3$), $\gamma_0 = 0.2$, $\rho = 5$, $N = 8$, $P = 1000$, and $C = 4$.

The intuition stems from the fact that when the majority of referees is from the M-group, it is more likely for an M-researchers to be accepted than for a F-researcher, on average.

Figure A.3: Endogenous entry: applicants


(a) M and F Applicants

(b) Fraction of F Applicants

Total mass of M and F applicants (left) and fraction of F applicants (right). Parameters: $\phi = 0.5742$ ($d = 0.3$), $\gamma_0 = 0.2$, $\rho = 5$, $N = 8$, $P = 1000$, and $C = 4$.

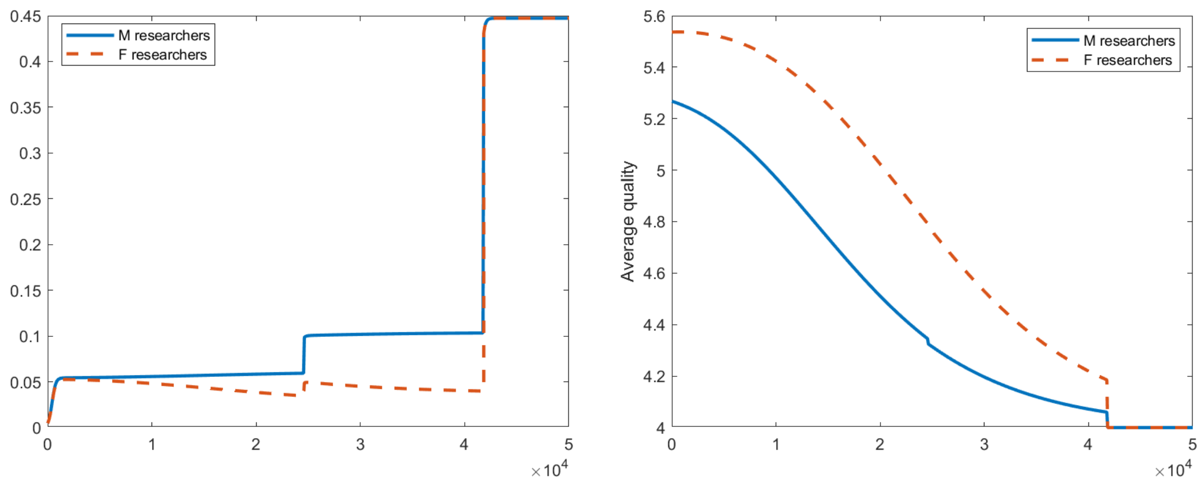
Thus, mass of applicants from the M-group is higher than from the F-group.

Figures A.3a and A.3b show the total masses of M and F applicants and, respectively, the percentage of F applicants over the total application pool. The parameter values are the same as for Figure A.1. Consistently with Corollary A.2, the mass of M applicants is always greater than that of F applicants; furthermore, the latter declines over time. The discrete jumps in these masses occur whenever, for some type θ, the population fraction λ_t^θ falls below the cutoff $C/(\gamma^\theta P)$. In the limit, the fraction of F applicants equals the fraction of F researchers of the only surviving type θ^m over the total:

$$\lim_{t\to\infty} \frac{A_t^f}{A_t^m + A_t^f} = \frac{p^{\theta^m,f}}{p^{\theta^m,f} + p^{\theta^m,m}} = \frac{(1-\phi)^N}{\phi^N + (1-\phi)^N} = \frac{0.4258^8}{.4258^8 + .5742^8} = 0.0838$$

Finally, the left panel of Figure A.4 shows the total acceptance rates of M and F applicants. In the initial period, the acceptance rates of M and F applicants are similar. They though diverge in the intermediate period, in which M applicants are accepted more often than the (fewer) F applicants, and then they finally converge, when only type θ^m survives. Interestingly, the right panel shows that the average quality of F researchers is uniformly higher until the time of convergence. This implies that in the initial period our model predicts similar acceptance rates of M and F researchers, even if the latter have higher objective

Figure A.4: Endogenous entry: Acceptance Rates



(a) Acceptance rate of applicants

(b) Average quality

Acceptance rate of M and F applicants (left) and average quality of accepted ones (right). Parameters: $\phi = 0.5742$ ($d = 0.3$), $\gamma_0 = 0.2$, $\rho = 5$, $N = 8$, $P = 1000$, and $C = 4$.

quality. This result is reminiscent of Card et al. (2020), who show that unconditionally, acceptance rates of men- and women-authored papers are similar, but that the average quality of accepted women-authored papers, proxied by their future citations, is higher.

A1.1.4. Endogenous Selection by Hiring Institutions

The previous section demonstrates that endogenizing the choice of entry into academia may shrink the supply of talent. We now show that the a similar mechanism operates on the demand side: when hiring decisions are based on the expectation of academic success, the anticipation of self-image bias in the refereeing process (Section 2.2.) induces institutions to hire only those types θ that can produce research that is more likely to be "accepted" by the established refereeing population.

Consider the following alternative interpretation of our model. When a hiring institution evaluates a candidate, it takes into account whether or not the candidate will produce quality work that the profession recognizes, or—in the language of Section 2.2.—"accepts." A candidate who is accepted by the profession yields a payoff P to the institution; this reflects e.g. visibility, grant money, or increased ability to attract top students. Hiring a candidate involves a cost C, which may be monetary but may also reflect mentoring resources and/or opportunity cost. This cost is borne by the institution whether or not the candidate is eventually accepted, and it is the same for M and F researchers. If the candidate is eventually not accepted or if the institution does not hire any candidate, the institution's payoff is zero. As above, a candidate of type θ produces quality work with probability γ^{θ}. To analyze demand effects, we reinterpret the key assumption of Section 2.2. as follows: the hiring institution anticipates that referees are subject to self-image bias, so that a type-θ researcher will be accepted by the profession with probability $\gamma^{\theta}\,\lambda_t^{\theta}$ at the end of time t.

Under these conditions, the institution hires an agent of type θ if and only if

$$\gamma^{\theta}\lambda_t^{\theta}\,(P-C)+\left(1-\lambda_t^{\theta}\gamma^{\theta}\right)(-C)>0 \tag{A.22}$$

This is the same condition as in Equation (A.18) in the previous section. Thus, the mass of established researchers λ_t^{θ} follows the system dynamics described by Equations (A.19) - (A.20). Proposition A.1 then applies and group imbalance and loss of talent obtains.

Moreover, under the conditions of case (a.2) of Proposition A.1, the system converges, in finite time, to a steady state in which only type θ^m survives. That is, if institutions *only* take acceptance by the profession into account at the hiring stage, type θ^f eventually disappears, even when such type would survive without endogenous selection. Again, this implies talent loss: research characteristics that are (mildly) more common in the F-population disappear.

We can also re-interpret the example in subsection A1.1.2. as a consequence of the hiring practices of hiring institutions. In the absence of endogenous selection, the parametric choices in that example lead to group balance, with both types θ^m and θ^f being represented in the limit. However, if institutions wish to hire only young researchers who are sufficiently likely to be accepted by the *current* population of referees, then group imbalance emerges, as in Figure A.1. Again, in this example type θ^f then disappears completely, as in Figure A.2.

This mechanism may explain the patterns in Figure 1. From the top panel, the female representation of undergraduate students with economics major in the top-20 schools has been rising over the past 25 years, reaching almost 40% by the late 2010s. This shows interests in economics among female undergraduates. Yet, in the same period, the percentage of female PhD students has been flat at around 30%, and that of assistant professors has been flat at around 22%. The bottom panel shows a striking difference between schools with and without PhD programs: In the latter group, the share of assistant professors is over 40%, while in the former is below 30%, with the top 10 schools at 20%.[1] These differences do not apply to the female share of teaching faculty, which are around 37% across all schools. This is consistent with our model: when a school has research as the guiding principle in hiring, it tends to skew towards the characteristics of established researchers, i.e. θ^m in our model.

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[1] We use the "top-X schools" terminology as in Chevalier (2020). School names are not reported.

A1.2. Seniors and Juniors

We now extend the basic model (without endogenous entry) in a different direction, namely, to the case in which there are different levels of seniority in the population of established researchers, with the seniors judging the research of the juniors, before accepting them onto their group. For instance, junior assistant professors may judge candidates from the rookie market and senior professors judge both assistant professors and rookies.

To avoid introducing new symbols, we add a subscript "1" to denote the mass of junior established researchers, and a subscript "2" for the senior established researchers. The difference from the previous case is mainly the mass of candidates of each type θ at each time t. For simplicity, we assume that, at time 0 and thereafter, the mass of seniors is fixed at σ and the mass of juniors is $1-\sigma$, so that the overall population of established researchers has mass 1, as in previous sections. That is, for all t, we must have

$$\sum_\theta \lambda_{1,t}^\theta = 1-\sigma, \quad \sum_\theta \lambda_{2,t}^\theta = \sigma.$$

The flows are similar to before: young researchers are evaluated by all, and juniors are evaluated by seniors only. For each group $g \in \{f, m\}$ and type $\theta \in \Theta$, the flows of juniors $a_{1,t}^{\theta,g}$ and seniors $a_{2,t}^{\theta,g}$ evolve according to

$$a_{1,t}^{\theta,g} = \gamma^\theta \cdot p^{\theta,g} \cdot (\lambda_{1,t-1}^\theta + \lambda_{2,t-1}^\theta) \tag{A.23}$$

$$a_{2,t}^{\theta,g} = \gamma^\theta \cdot \lambda_{1,t-1}^{\theta,m} \cdot \lambda_{2,t-1}^\theta. \tag{A.24}$$

Again, we assume that current seniors are randomly replaced by newly promoted juniors, and current juniors are randomly replaced by newly accepted young researchers. However, we now must take into account the fact that juniors promoted to seniors leave the junior pool. We thus obtain the dynamics

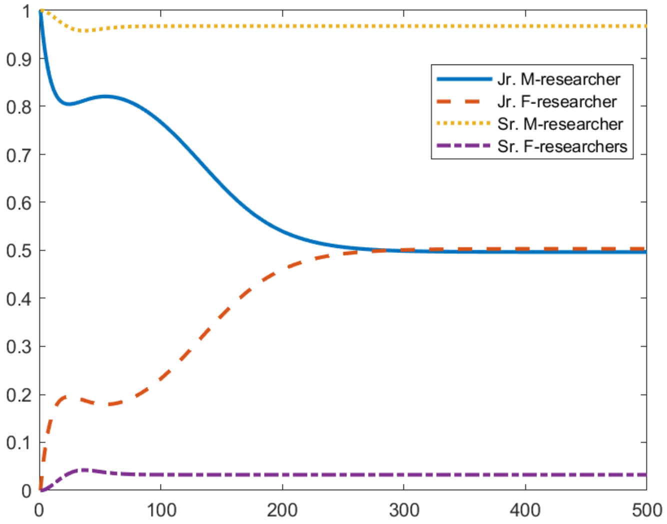
$$\lambda_{1,t}^{\theta,g} = \lambda_{1,t-1}^{\theta,m}\left(1 - \frac{1}{1-\sigma}(a_{1,t} - a_{2,t})\right) + a_{1,t}^{\theta,g} - a_{2,t}^{\theta,g} \tag{A.25}$$

$$\lambda_{2,t}^{\theta,g} = \lambda_{2,t-1}^{\theta,g}\left(1 - \frac{1}{\sigma}a_{2,t}\right) + a_{2,t}^{\theta,g} \tag{A.26}$$

for $g \in \{f, m\}$, where $a_{j,t} = \sum_\theta (a_{j,t}^{\theta,f} + a_{j,t}^{\theta,m})$ for $j = 1, 2$.

The dynamics are far more complex than in the base case, and we rely on numerical simulations.

Figure A.5: Leaky pipeline



Fraction of senior and junior M and F researchers, relative to σ (seniors) and $1-\sigma$ (juniors), when $\lambda_0 = p^m$. Parameters: $\phi = 0.7$, $\gamma_0 = 0.2$, $\rho = 4$, $N = 4$ and $\sigma = 0.5$.

A1.2.1. Leaky Pipeline

Here we focus on the most interesting case, namely, the fact that this extension can also account for the "leaky pipeline" pattern highlighted in the CSWEP report (Chevalier, 2020). Figure A.5 provides a stark illustration: under the given parametric assumptions, group balance attains among juniors, but not among seniors. A rough intuition is that the self-image bias may not be strong enough to result in a prevalence of θ^m types among juniors, given the constant influx of new researchers with a more balanced distribution of types. However, it may be strong enough if the candidates' types are themselves more biased towards the M researchers' distribution—as is the case for junior up for promotion to the senior rank.

A1.3. Co-authorship

This section briefly explores the implications of our model's dynamics for inferences about the relative (objective) quality of coauthors in a joint project.

We show that, consistently with the findings in Sarsons et al. (2021), if research co-authored by a young M-researcher and a young F-researcher is accepted, then the expected quality of the M-researcher is higher. For simplicity, we consider an economy that has reached its steady state, and such that only types θ^m and θ^f are represented in the population of established scholars. Hence, a joint research project is accepted if and only if its vector of characteristics is θ^m or θ^f.

Proposition A.3 *Let the economy be at its steady state with only types θ^f and θ^m surviving. For each researcher of type θ, define $L(\theta) = \sum_{n=1}^{N} \theta_n$ its objective quality. Let a research that is coauthored by type θ^a and θ^b be of type $\theta = \theta^a \vee \theta^b$, where $\vee$ denotes the component-wise maximum. Let researcher $a \in M$ and $b \in F$. Then, conditional on acceptance of the joint work, i.e. $\theta^a \vee \theta^b \in \{\theta^m, \theta^f\}$, we have*

$$\mathrm{E}[L(\theta^a)|\theta^a \vee \theta^b \in \{\theta^m, \theta^f\}] > \mathrm{E}[L(\theta^b)|\theta^a \vee \theta^b \in \{\theta^m, \theta^f\}]$$

The intuition of the result is that referees are more frequently of type θ^m, and, in addition, θ^m is more frequent in the M population than in the F population. It follows that conditional on the joint work being accepted, it is then more likely it is due for the M characteristics than the F characteristics.

A1.4. Similarity in Research Characteristics

In this section we extend the model to investigate the case in which referees accept researchers who have characteristics close but not necessarily identical to their own. In particular, we assume that referee r of type θ^r accepts the research of young researcher θ if

$$D(\theta^r, \theta) = \sum_n (\theta_n^r - \theta_n)^2 \leq \eta \tag{A.27}$$

where η is a non-negative integer. That is, referee θ^r treats candidate θ as "close enough" if it differs from his or her own type in no more than η characteristics.

Our models so far correspond to $\eta = 0$. If instead $\eta > 0$, the dynamics for λ_t^θ are still as in Eq. (6), but the mass $a_t^{\theta,g}$ of accepted researchers of type θ in group $g \in \{f, m\}$ is given by

$$a_t^{\theta,g} \;\; = \;\; \gamma^\theta \sum_{\theta^r : D(\theta^r, \theta) \leq \eta} \lambda_{t-1}^{\theta^r} p^{\theta,g} \tag{A.28}$$

Unfortunately, obtaining general analytical results in this case seems difficult. Therefore, we consider illustrative special cases.

A1.4.1. Connected Set of Types

The set Θ of types we have considered so far enjoys a special structure that is relevant to the relaxed definition of "acceptance" in Eq. (A.27). For every $\eta \geq 1$, and every pair $\theta, \theta' \in \Theta$, there is a finite ordered list $\theta_1, \ldots, \theta_K \in \Theta$ such that $\theta_1 = \theta$, $\theta_K = \theta'$, and $D(\theta_k, \theta_{k+1}) \leq \eta$ for all $k = 1, \ldots, K-1$. In this sense, we say that $\Theta = \{0,1\}^N$ is *η-connected* for every $\eta \geq 1$. Of course, being 1-connected implies being η-connected for $\eta > 1$; we shall see in the next subsection that a subset of $\{0,1\}^N$ may be η-connected for some $\eta > 1$, but for any smaller integer η' (including $\eta' = 1$).

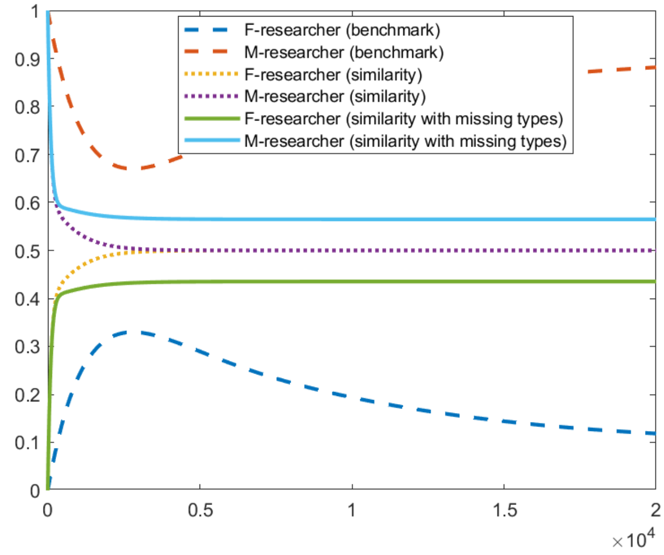
With $\Theta = \{0,1\}^N$, and for the parameter values used in the examples of Sections 2. and 3., the relaxed acceptance criterion in Eq. (A.27) leads to convergence. For instance, Figure A.6 illustrates the parameterization used in Section 3.. The dashed lines represent the benchmark case $\eta = 0$, where there is no convergence. The dotted lines reflect the assumption that referees accept young researchers that are closely similar to them: specifically, taking $\eta = 1$. Notably, group balance obtains. (The solid lines are discussed in the next section.) Moreover, we have not been able to find parameterizations for which convergence did *not* occur. We conjecture that this is a general property of the special structure of the type space $\Theta = \{0,1\}^N$. Intuitively, a referee of type θ accepts a positive mass of young researchers of similar, but not identical type θ'; these become referees in the following period, and accept a positive mass of young researchers of type θ'' that type-θ referees would reject; and so on. A contagion argument suggests that, in the limit, the impact of self-image bias should vanish, so that group balance should emerge.

A1.4.2. Disconnected Set of Types

A subset of $\{0,1\}^N$ may well be η-disconnected for some η. For a trivial example, $\{\theta^m, \theta^f\}$ is $(N-1)$-disconnected, because each of the N coordinates of θ^f is different from the corresponding coordinate of θ^f. A fortiori, it is η-disconnected for every $\eta \leq N-1$.

Intuition suggests that the contagion argument given above breaks down with a disconnected set of types. We now verify this intuition. The solid lines in Figure A.6 represent the same parameterization as in the previous subsection, with $\eta = 1$, but applied to a state space Θ obtained by randomly removing 20% of the elements of $\{0,1\}^N$ and suitably renormalizing probabilities. As expected, the system does not attain group balance in the limit.

Figure A.6: Fraction of M and F Researchers under the Research Similarity Assumption



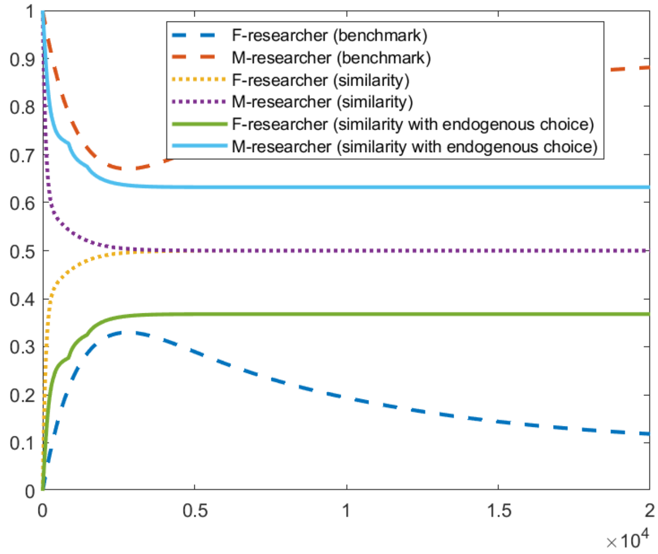
Fraction of M and F researchers when $\lambda_0 = p^m$. Parameters: $\phi = 0.5742$, which implied $d = 0.3$, $\gamma_0 = 0.2$, $\rho = 5$, $N = 10$, and, under research similarity, $\eta = 1$.

A1.4.3. Endogenous Entry

Finally, return to the case in which $\Theta = \{0,1\}^N$ (a connected set of types) but consider endogenous entry, as in Section A1.1.. In this case, even if the connected set of types would lead to convergence (see subsection A1.4.1.), the endogenous entry prevents such convergence, as shown in Section A1.1.2.. This is shown in Figure A.7. Again, the dashed lines and the dotted lines show the total fraction of M- and F-researchers in the benchmark case ($\eta = 0$) and, respectively, the research similarity case ($\eta = 1$). The solid lines now show the the fraction of M- and F-researchers under research similarity ($\eta = 1$) but with endogenous entry. The intuition is the same as the one given in Section A1.1..

In sum, this section suggests that the main results of the paper are robust to a weaker assumption about the referees' selection mechanism.

Figure A.7: Fraction of M and F Researchers under Research Similarity and Endogenous Entry



Fraction of M and F researchers when $\lambda_0 = p^m$. Parameters: $\phi = 0.5742$, which implied $d = 0.3$, $\gamma_0 = 0.2$, $\rho = 5$, $N = 10$, and, under research similarity, $\eta = 1$. Endogenous choice assume $P = 1000$ and $C = 6$.

A2. Additional Analysis and Results

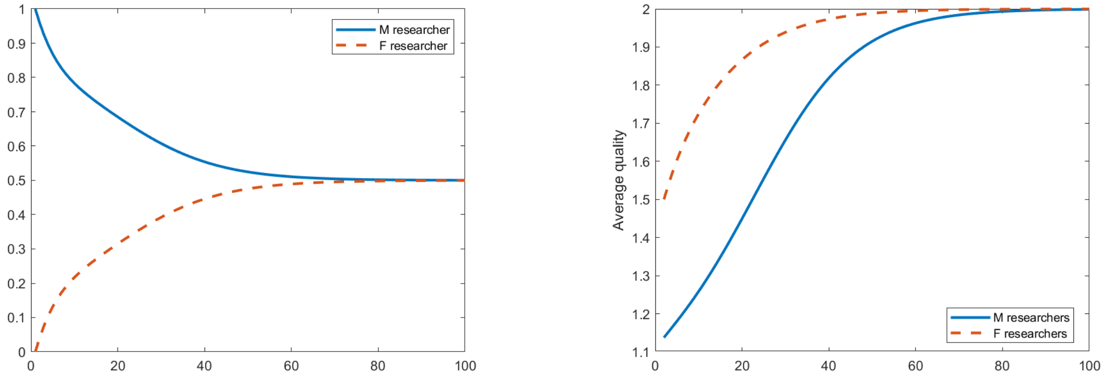
A2.1. Convergence to Efficiency

In Section 2. we considered a simple numerical example with only two characteristics ($N = 2$), which led to types $\Theta = \{(0,0),(0,1),(1,0),(1,1)\}$. In this section we continue that example and demonstrate how group balance may arise even with an unbalanced initial population. We continue to assume that the initial population is M-dominated: $\lambda_0 = p^m$, and that $N = 2$ and $\phi = 0.8$. However, we now take $\gamma_0 = 0.1$ and $\rho = 9$. Compared with our previous parameterization, research characteristics now have a greater impact on the likelihood of producing quality research. For instance, type θ^* is 3 times as likely to produce quality research as types θ^f and θ^m, who are themselves 3 times as likely to do so as type $(0,0)$. Thus, the system is now more "meritocratic." Now

$$\rho = 9 > 4.25 = \frac{1}{4}\left(\frac{0.2}{0.8} + \frac{0.8}{0.2}\right)^2 = \bar{\rho}(0.8, 2),$$

so Proposition 3 part (b) implies that type θ^* will dominate in the limit. Figures A.8 and A.9 illustrate the dynamics. Now the percentage of F-researchers indeed converges to 50%.

Figure A.8: Fraction of M and F Researchers and Acceptance Rates with More Meritocracy

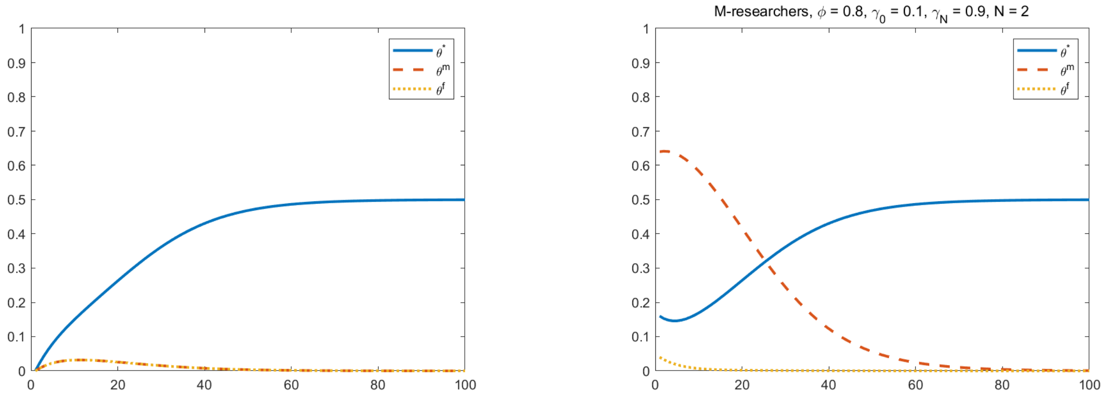


(a) M and F researchers

(b) Average Quality

Fraction of M and F researchers (panel a) and average acceptance rates of M and F researchers, i.e. $\sum_\theta L^\theta w_t^{\theta,g}$ where $L^\theta = \sum_{n=1}^N \theta_n$ and $w_t^{\theta,g} = a_t^{\theta,g} / \sum_{\theta'} a_t^{\theta',g}$, $g = f, m$ (Panel b). Initially $\lambda_0 = p^m$. Parameters: $\phi = 0.8$, $\gamma_0 = 0.1$, $\rho = 9$, $N = 2$.

Figure A.9: Types of Established Female and Male Researchers with More Meritocracy



(a) F researchers

(b) M researchers

Types of established F (left) and M (right) researchers. We show types $\theta^* = (1, 1)$, $\theta^m = (1, 0)$, and $\theta^f = (0, 1)$. Initially $\lambda_0 = p_m$. Parameters: $\phi = 0.8$, $\gamma_0 = 0.1$, $\rho = 9$, $N = 2$.

Moreover, the system weeds out those researchers that do not possess both characteristics. Panel (b) of Figure A.8 shows that accepted F researchers are of higher quality than accepted M researchers, as in Proposition 6, until convergence to quality $L = 2$.

A2.2. Balanced Steady State

In Section 2. we considered a simple numerical example with only two characteristics ($N = 2$), which led to types $\Theta = \{(0,0),(0,1),(1,0),(1,1)\}$. In that section, we showed that when $\rho < \overline{\rho}(\phi, N)$ and the initial population of referees is only from the M-group, $\lambda_0^{\theta,m} = p^{\theta,m}$, then the dynamics never converges. Here we now consider a different initial condition.

Indeed, the dynamics of the mass of each type depends upon their frequencies in the population of young researchers, p_m and p_f, as well as the initial conditions λ_0. In particular, suppose that the initial mass of referees is composed of M- and F-researchers in equal proportions: $\lambda_0 = \frac{1}{2}p_m + \frac{1}{2}p_f$. One implication is that then the two M-prevalent and F-prevalent types $\theta^m = (1,0)$ and $\theta^f = (0,1)$ both represent 34% of the initial mass of referees, whereas the other two types $(0,0)$ and $(1,1)$ each represent 16% of the initial population. While we can no longer invoke the results in Sections 2.2.-2.4.3., we can plot the dynamics of the fractions of established M- and F-researchers, as well as those of established M-and F-researcher types. (Theorem A.1 in the Appendix characterizes the limiting behavior of the system for arbitrary initial conditions and type distributions.)

Figures A.10 and A.11 display the results. The figures are self explanatory: an equal proportion of M- and F-researchers is maintained throughout. However, importantly, type θ^f (resp. θ^m) will eventually become prevalent among F-researchers (resp. M-researchers), which means that established F- (resp. M-) economists are oversampled from those who possess characteristic 2 (resp. 1). Furthermore, the efficient type θ^* will disappear in the limit.

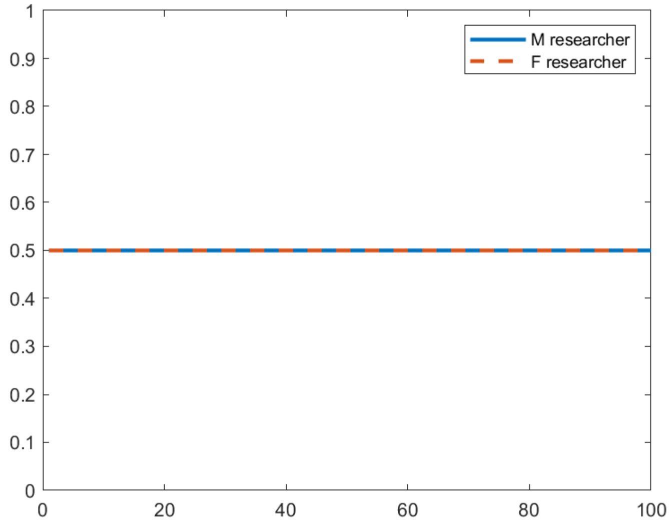
A2.3. Dynamics under Mentoring

In this section we provide additional intuition on the dynamics of the system under mentoring, and further illustration. The mass of young researchers from group $g \in \{f, m\}$ of type θ accepted at time t is then

$$a_t^{\theta,g} = \gamma^\theta \, \lambda_{t-1}^\theta \left[\left(p^{\theta,g} \sum_{\theta^a : \widetilde{C}(\theta,\theta^a) \geq \gamma^{\theta^a} \lambda_{t-1}^{\theta^a} - \gamma^\theta \lambda_{t-1}^\theta} \lambda_{t-1}^{\theta^a} \right) + \left(\lambda_{t-1}^\theta \sum_{\theta' : \theta' \neq \theta, \widetilde{C}(\theta',\theta) < \gamma^\theta \lambda_{t-1}^\theta - \gamma^{\theta'} \lambda_{t-1}^{\theta'}} p^{\theta',g} \right) \right] \tag{A.29}$$

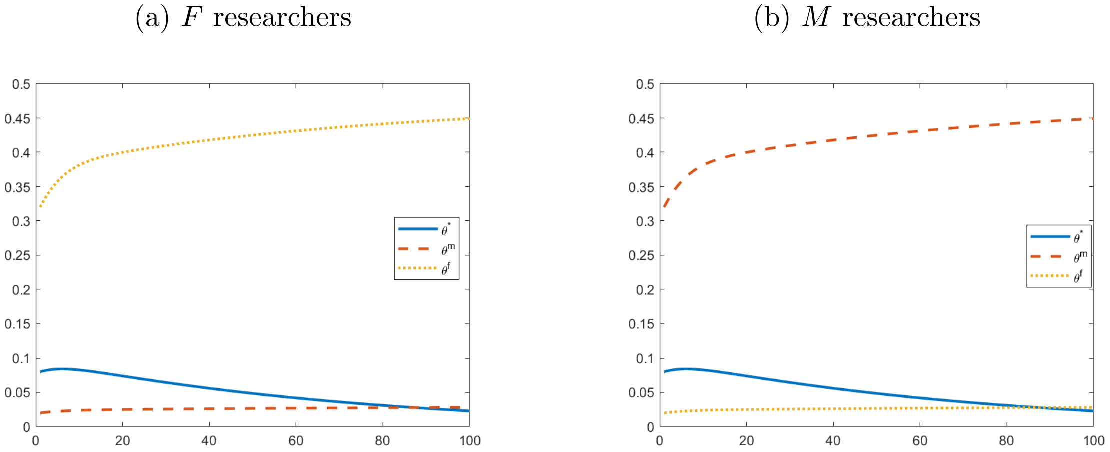
The first term in brackets captures all of the young researchers of type θ from group g who are matched with mentors of types θ^a (with probability $\lambda_{t-1}^{\theta^a}$) and choose not to be advised as the cost is too large; these young researchers thus remain of type θ. The inequality is

Figure A.10: Fraction of M and F researchers with Start from Equal Proportions



Fraction of M and F researchers when $\lambda_0 = \frac{1}{2}p_m + \frac{1}{2}p_f$. Parameters: $\phi = 0.8$, $\gamma_0 = 0.2$, $\rho = 4$, $N = 2$.

Figure A.11: Types of Established F and M Researchers with Start from Equal Proportions


(a) F researchers

(b) M researchers

Types of established F (left) and M (right) researchers. We show types $\theta^* = (1, 1, ..., 1)$, $\theta^m = (1, ..., 1, 0, ..., 0)$, and $\theta^f = (0, ..., 0, 1, ..., 1)$. Initially $\lambda_0 = \frac{1}{2}p_m + \frac{1}{2}p_f$. Parameters: $\phi = 0.8$, $\gamma_0 = 0.2$, $\rho = 4$, $N = 2$.

weak to reflect the fact that type $\theta^a = \theta$ will also not want to pay the cost to "acquire" his or her own current type. The second term in the bracket captures young g-researchers of type $\theta' \neq \theta$ who are matched with a mentor of type θ (whose mass is λ^{θ}_{t-1}) and decide to be advised by them. The remaining dynamics for $\lambda^{\theta,m}_t$ and $\lambda^{\theta,f}_t$ are the same as in the main model. Note that if $\widetilde{C}(\theta,\theta^a) \to \infty$ for all types (e.g. $P \to U$) then the first term in the bracket converges to $p^{\theta,m}$ and the second to 0, returning to the original dynamics.

To gauge the type of dynamics that emerges from Eq. (A.29), note that initially all λ^{θ}_{t-1} are likely small, and thus for a given cost function, both conditions $\widetilde{C}(\theta,\theta^a) \geq \gamma^{\theta^a}\lambda^{\theta^a}_{t-1} - \gamma^{\theta}\lambda^{\theta}_{t-1}$ and $\widetilde{C}(\theta',\theta) < \gamma^{\theta}\lambda^{\theta}_{t-1} - \gamma^{\theta'}\lambda^{\theta'}_{t-1}$ are likely to hold. That is, in this case, the system runs as in the benchmark case in Section 2.2.. However, as we know from our preceding analysis, λ^{θ}_{t} converges to zero for all $\theta \notin \{\theta^*, \theta^m, \theta^f\}$. Specifically, consider the case in which eventually only θ^m and θ^f survive, so $\lambda^{\theta^m}_{t}$ and $\lambda^{\theta^f}_{t}$ increase, and the former does so at a faster rate. Intuitively, suppose t is large enough so the mass of established researchers satisfies $\lambda^{\theta^m}_{t-1} + \lambda^{\theta^f}_{t-1} \approx 1$. By symmetry, recall also that $\gamma^{\theta^m} = \gamma^{\theta^f} = \overline{\gamma}$ and the distance between θ^f and θ^m is just $\widetilde{C}(\theta^m,\theta^f) = \widetilde{C}(\theta^f,\theta^m) = \widetilde{C}$. Consistently with the assumption of a large M-group mass of referees initially, let $\left(\lambda^{\theta^m}_{t-1} - \lambda^{\theta^f}_{t-1}\right) > 0$ with $0 < \widetilde{C} < \overline{\gamma}\left(\lambda^{\theta^m}_{t-1} - \lambda^{\theta^f}_{t-1}\right)$. The dynamics then specializes to

$$a^{\theta^m,g}_{t} = \overline{\gamma}\ \lambda^{\theta^m}_{t-1}\left[p^{\theta^m,g} + \lambda^{\theta^m}_{t-1}\left(p^{\theta^f,g} + \sum_{\theta':\theta'\neq\theta^m,\theta^f,\widetilde{C}(\theta',\theta^m)<\overline{\gamma}\lambda^{\theta^m}_{t-1}} p^{\theta',g}\right)\right] \tag{A.30}$$

$$a^{\theta^m,f}_{t} = \overline{\gamma}\ \lambda^{\theta^m}_{t-1}\left[p^{\theta^m,f} + \lambda^{\theta^m}_{t-1}\left(p^{\theta^f,f} + \sum_{\theta':\theta'\neq\theta^m,\theta^f,\widetilde{C}(\theta',\theta^m)<\overline{\gamma}\lambda^{\theta^m}_{t-1}} p^{\theta',f}\right)\right] \tag{A.31}$$

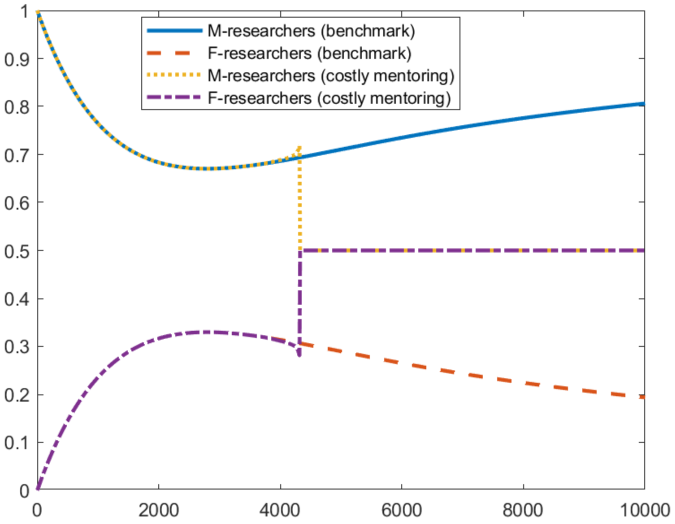
$$a^{\theta^f,g}_{t} = \overline{\gamma}\ \left(\lambda^{\theta^f}_{t-1}\right)^2\left(\sum_{\theta':\theta'\neq\theta^m,\theta^f,\widetilde{C}(\theta',\theta^f)<\overline{\gamma}\lambda^{\theta^f}_{t-1}} p^{\theta',g}\right) \tag{A.32}$$

$$a^{\theta^f,f}_{t} = \overline{\gamma}\ \left(\lambda^{\theta^f}_{t-1}\right)^2\left(\sum_{\theta':\theta'\neq\theta^m,\theta^f,\widetilde{C}(\theta',\theta^f)<\overline{\gamma}\lambda^{\theta^f}_{t-1}} p^{\theta',f}\right) \tag{A.33}$$

for $g \in \{f,m\}$. Comparing these expressions with the benchmark case, we see that each $a^{\theta^m g}_{t}$ is weakly larger than then in the benchmark case, and hence $\lambda^{\theta^m}_{t}$ increases further over time. If $\lambda^{\theta^m}_{t}$ becomes sufficiently large, for many young researchers of type θ' the condition $\widetilde{C}(\theta',\theta^m) < \overline{\gamma}\lambda^{\theta^m}_{t-1}$ will hold, but the condition $\widetilde{C}(\theta',\theta^f) < \overline{\gamma}\lambda^{\theta^f}_{t-1}$ will not hold (the details depend on the cost structure). Indeed, if $\max_{\theta'}\left\{\widetilde{C}(\theta',\theta^m)\right\} = \overline{C} < \overline{\gamma}$ and $\lambda^{\theta^m}_{t-1} > \overline{C}/\overline{\gamma}$, then *all* young researchers will be willing to pay a cost to become type θ^m and none will be willing to pay to became any other type. The system then quickly converges to $\lambda^{\theta^m}_{t} = 1$.

Figure A.12 illustrates the dynamics resulting from Eq. (A.29), under the same parameters as in Section 3. and a cost function $C(\theta,\theta') = \beta\sum_{n=1}^{N}(\theta_n - \theta'_n)^2$, with $\beta = 0.025$. Initially, the dynamics are as in the base case, as all θ^{θ}_{t} are small and thus no young researcher wants

Figure A.12: Fraction of F and M Researchers with Costly Mentoring (low costs)



Fraction of M and F researchers when $\lambda_0 = p^m$. Parameters: $\phi = 0.5742$ ($d = 0.3$), $\gamma_0 = 0.2$, $\rho = 5$, $N = 10$, cost function $C(\theta, \theta') = 0.0250 \sum_{n=1}^{N} (\theta_n - \theta'_n)^2$.

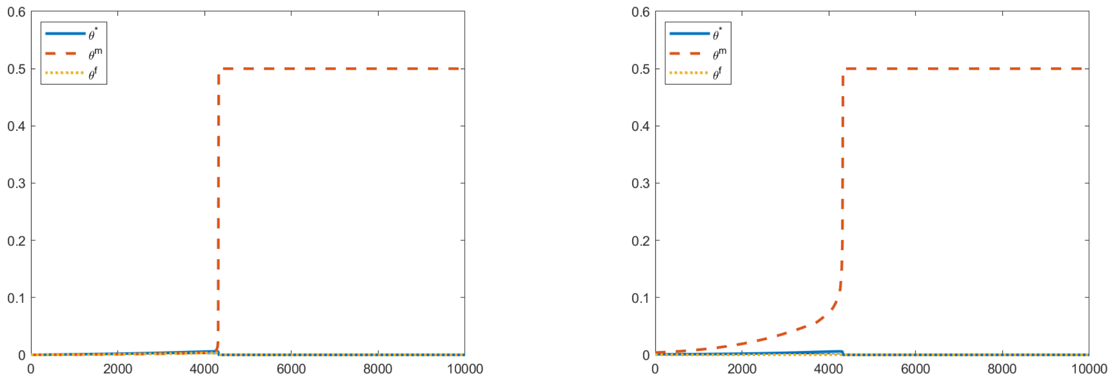
to pay the cost of mentoring. In this dynamics, as we know, $\theta_t^{\theta m}$ and $\theta_t^{\theta f}$ increase, with the former increasing faster, as shown in the in the right panel of Figure A.13. At some point, the mass of $\lambda_t^{\theta m}$ is sufficiently large to induce all young researchers, M and F, decide to pay the cost and the system (nearly) jumps. The reason is that *all* young researchers now expect that their advisor will likely be of type θ^m, which is also the type of established researchers who will evaluate their research. They are thus happy to pay the cost and become like their advisors. Moreover, we reach group balance, as all young M- and F-researchers decide to become θ^m, and there are equal masses of them. However, the downside is that group balance is achieved at the expense of weeding out valuable research characteristics that are more prevalent among young F-researchers—there is, again, loss of talent.

A2.3.1. Seniors and Juniors: Other Patterns

We now consider other cases, for illustration. All the simulations in this section assume equal fractions of juniors and seniors ($\sigma = 0.5$).

First, the presence of a second screening—and hence a second opportunity for self-image bias to exert its influence—can exacerbate group imbalance in the senior rank, at least in the short run. Figure A.14 demonstrates this. Model parameters are as in Figure 3, so in a single-

Figure A.13: Types of Established F and M Researchers with Costly Mentoring (low cost) .


(a) F researchers

(b) M researchers

Types of established F (left) and M (right) researchers with costly mentoring. We show the masses of types $\theta^* = (1, 1, ..., 1)$, $\theta^m = (1, ..., 1, 0, ..., 0)$, and $\theta^f = (0, ..., 0, 1, ..., 1)$. Initial reviewers: $\lambda_0 = p^m$. Parameters: $\phi = 0.0.5742$ $(d = 0.3)$, $\gamma_0 = 0.2$, $\rho = 5$, $N = 10$; cost function: $C(\theta, \theta') = 0.0250 \sum_{n=1}^{N} (\theta_n - \theta'_n)^2$.

cohort environment significant group imbalance emerges. The same is true with two ranks; however, in the short run, the imbalance is more pronounced in the senior rank. The reason is that, in order to be promoted to the senior rank, a researcher must match with a referee of the same type *twice*. Initially, both junior and senior referees have the same type distribution, which by assumption coincides with that of M researchers. Hence, whatever effect is present at the junior rank is compounded at the senior rank.[2] The difference between the two ranks vanishes in the long run because, as type θ^m becomes prevalent among established juniors and seniors, promotion eventually is driven solely by objective research quality—matching with a senior reviewer of the junior candidate's own type is virtually guaranteed.

A more pronounced group imbalance can also arise, in the short / medium run, for parameter values for which convergence is eventually attained. This is demonstrated in Figure A.15, where we take $\phi = 0.6$ rather than $\phi = 0.8$. Again, the need to match with a like type twice, coupled with the assumption that the initial population consists entirely of M-researchers, leads to a lower representation of F researchers at the senior rank. However, over time, type θ^* prevails among juniors and seniors, so matching with like types is virtually guaranteed; and since convergence is attained amongst juniors, it must obtain among seniors as well.

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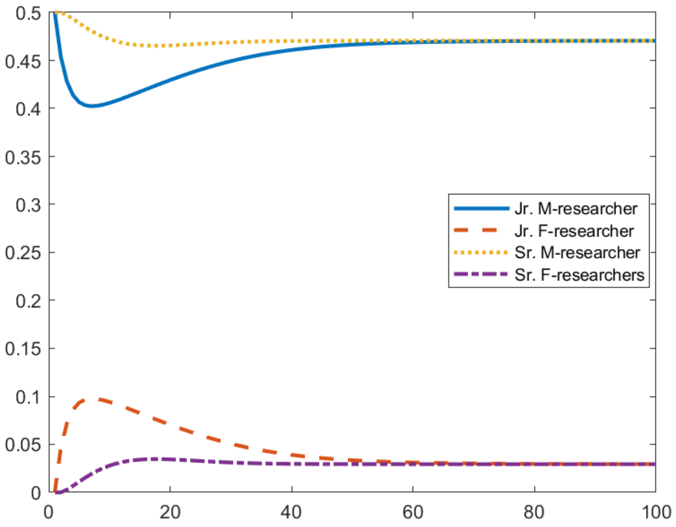
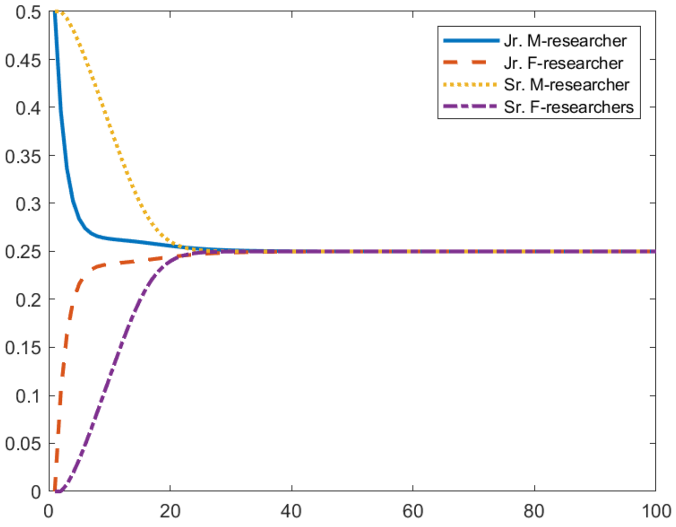
[2] In fact, the bias becomes stronger over time at the senior rank. The reason is that the initial population of junior candidates up for promotion is characterized by types distributed as among male researchers, whereas the initial population of young researchers applying for a junior position is balanced.

Figure A.14: More extreme imbalance for senior rank



Fraction of senior and junior M and F researchers when $\lambda_0 = p_m$. Parameters: $\phi = 0.8$, $\gamma_0 = 0.2$, $\rho = 4$, $N = 2$.

Figure A.15: Convergence, but greater short-run imbalance among seniors



Fraction of senior and junior M and F researchers when $\lambda_0 = p^m$. Parameters: $\phi = 0.6$, $\gamma_0 = 0.2$, $\rho = 4$, $N = 2$.

A3. Proofs

We first characterize key features of the population dynamics for an arbitrary, finite set Θ of types, with initial distribution $\lambda_0 \in \Delta(\Theta)$, such that $\lambda_0 = \lambda_0^m + \lambda_0^f$ for $\lambda_0^m, \lambda_0^f \in \mathbb{R}_+^\Theta$, and per-period inflows $q^g = (q^{\theta,g})_{\theta\in\Theta} \in \mathbb{R}_+^\Theta \setminus \{0\}$, for $g \in \{f, m\}$. It is also convenient to define $q = q^m + q^f$. Then, for $g \in \{f, m\}$, the dynamics are given by

$$\lambda_t^{\theta,g} = \lambda_{t-1}^{\theta,g}\left(1 - \sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'}\right) + \lambda_{t-1}^{\theta} q^{\theta,g} \tag{A.34}$$

$$\lambda_t^{\theta} = \lambda_t^{\theta,m} + \lambda_t^{\theta,f}. \tag{A.35}$$

The body of the paper focuses on the special case $q^{\theta,m} = \gamma^\theta p^{\theta,m}$, $q^{\theta,f} = \gamma^\theta p^{\theta,f}$.

Theorem A.1 *Assume that $q^\theta \le 1$ for all $\theta \in \Theta$. Then, for all $t \ge 0$, $\lambda_t \in \Delta(\Theta)$, and $\lambda_t^m, \lambda_t^f \in \mathbb{R}_+^\Theta$. Moreover:*

1. *if $\lambda_0^\theta = 0$, then $\lambda_t^\theta = 0$ for all $t \ge 0$;*
2. *if $\lambda_0^\theta > 0$, then $\lambda_t^\theta > 0$ for all $t \ge 0$;*
3. *for $\theta, \tilde\theta \in \Theta$ with $\lambda_0^\theta \cdot \lambda_0^{\tilde\theta} > 0$:*
    - *(a) $\frac{\lambda_t^\theta}{\lambda_{t-1}^\theta} - \frac{\lambda_t^{\tilde\theta}}{\lambda_{t-1}^{\tilde\theta}} = q^\theta - q^{\tilde\theta}$ for all $t \ge 1$, and*
    - *(b) $q^\theta > q^{\tilde\theta}$ implies $\frac{\lambda_t^\theta}{\lambda_t^{\tilde\theta}} \to \infty$, and $q^\theta = q^{\tilde\theta}$ implies $\frac{\lambda_t^\theta}{\lambda_t^{\tilde\theta}} = \frac{\bar\lambda_o^\theta}{\bar\lambda_o^{\tilde\theta}}$ for all $t \ge 0$;*
4. *define the set*

$$\Theta^{\max} = \{\theta \in \Theta \;:\; \lambda_0^\theta > 0,\ \theta \in \arg\max_{\theta'\in\Theta} q^{\theta'}\} \tag{A.36}$$

and let $\bar\lambda \in \Delta(\Theta)$ be such that

$$\bar\lambda^{\tilde\theta} = \begin{cases} \frac{\lambda_0^{\tilde\theta}}{\sum_{\theta\in\Theta^{\max}} \lambda_0^\theta} & \tilde\theta \in \Theta^{\max} \\ 0 & \tilde\theta \notin \Theta^{\max} \end{cases} : \tag{A.37}$$

then $\lim_{t\to\infty} \lambda_t = \bar\lambda$;

5. *define*

$$\bar\lambda^{\tilde\theta,f} = \begin{cases} \frac{\lambda_0^{\tilde\theta} q^{\tilde\theta,f}}{\sum_{\theta\in\Theta^{\max}} \lambda_0^\theta q^\theta} & \tilde\theta \in \Theta^{\max} \\ 0 & \tilde\theta \notin \Theta^{\max} \end{cases} \quad \text{and} \quad \bar\lambda^{\tilde\theta,m} = \begin{cases} \frac{\lambda_0^{\tilde\theta} q^{\tilde\theta,m}}{\sum_{\theta\in\Theta^{\max}} \lambda_0^\theta q^\theta} & \tilde\theta \in \Theta^{\max} \\ 0 & \tilde\theta \notin \Theta^{\max} \end{cases} : \tag{A.38}$$

then $\lim_{t\to\infty} \lambda_t^f = \bar\lambda^f$ and $\lim_{t\to\infty} \lambda_t^m = \bar\lambda^m$.

Proof: Eqs. (A.34) and (A.35) imply that

$$\lambda_t^\theta = \left(1 - \sum_{\theta' \in \Theta} \lambda_{t-1}^{\theta'} q^{\theta'}\right) \lambda_{t-1}^\theta + \lambda_{t-1}^\theta q^\theta. \tag{A.39}$$

By assumption $\lambda_0 \in \Delta(\Theta)$. Inductively, suppose $\lambda_{t-1} \in \Delta(\Theta)$ and $\lambda_{t-1}^m, \lambda_{t-1}^f \in \mathbb{R}_+^\Theta$. Summing over Θ on both sides of Eq. (A.39) yields $\sum_\theta \lambda_t^\theta = (1 - \sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'})(\sum_\theta \lambda_{t-1}^\theta) + \sum_\theta \lambda_{t-1}^\theta q^\theta = (1 - \sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'}) + \sum_\theta \lambda_{t-1}^\theta q^\theta = 1$. Furthermore, since $\lambda_{t-1} \in \Delta(\Theta)$, $\sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'} \in [\min_{\theta'} q^{\theta'}, \max_{\theta'} q^{\theta'}] \subseteq [0,1]$; moreover, $q^\theta \geq 0$ and $\lambda_{t-1}^\theta \geq 0$, so Eq. (A.39) implies that $\lambda_t^\theta \geq 0$ as well. By the same argument, $q^\theta \geq 0$ and $\lambda_{t-1}^{\theta,g} \geq 0$ for $g \in \{f, m\}$ imply $\lambda_t^{\theta,g} \geq 0$ for $g \in \{f, m\}$ as well by Eq. (A.34). Thus, $\lambda_t \in \Delta(\Theta)$, and $\lambda_t^g \in \mathbb{R}_+^\Theta$ for each g.

Claim 1 is immediate. For Claim 2, again we argue by induction. For $t = 0$, the claim is trivially true. Inductively, assume $\lambda_{t-1}^\theta > 0$. By Eq. (A.39), since as was just shown $1 - \sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'} \geq 0$, and the inductive hypothesis implies that $\lambda_{t-1}^\theta > 0$, if $q^\theta > 0$ then $\lambda_t^\theta \geq \lambda_{t-1}^\theta q^\theta > 0$. Suppose instead $q^\theta = 0$. If $\sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'} = 1$, then, since $q^{\theta'} \leq 1$ for all θ' by assumption, and $\lambda_{t-1} \in \Delta(\Theta)$, it must be that $\lambda_{t-1}^{\theta'} > 0$ implies $q^{\theta'} = 1$: but then $\lambda_{t-1}^\theta = 0$, which contradicts the inductive hypothesis. Thus, $0 \leq \sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'} < 1$, so Eq. (A.39) implies that $\lambda_t^\theta = \left(1 - \sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'}\right) \lambda_{t-1}^\theta > 0$.

For Claim 3, divide both sides of Eq. (A.39) for type θ by λ_{t-1}^θ, which is assumed to be positive; this yields

$$\frac{\lambda_t^\theta}{\lambda_{t-1}^\theta} = 1 + q^\theta - \sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'}. \tag{A.40}$$

A similar equation holds for $\tilde\theta$. This immediately yields 3(a). To derive 3(b), since $\lambda_t^{\theta'} = \lambda_0^{\theta'} \cdot \prod_{s=1}^t \frac{\lambda_s^{\theta'}}{\lambda_{s-1}^{\theta'}}$ for $\theta' = \theta, \tilde\theta$,

$$\frac{\lambda_t^\theta}{\lambda_t^{\tilde\theta'}} = \frac{\lambda_0^\theta}{\lambda_0^{\tilde\theta}} \cdot \frac{\prod_{s=1}^t \frac{\lambda_s^\theta}{\lambda_{s-1}^\theta}}{\prod_{s=1}^t \frac{\lambda_s^{\tilde\theta}}{\lambda_{s-1}^{\tilde\theta}}} = \frac{\lambda_0^\theta}{\lambda_0^{\tilde\theta}} \cdot \prod_{s=1}^t \frac{\frac{\lambda_s^\theta}{\lambda_{s-1}^\theta}}{\frac{\lambda_s^{\tilde\theta}}{\lambda_{s-1}^{\tilde\theta}}} = \frac{\lambda_0^\theta}{\lambda_0^{\tilde\theta}} \cdot \prod_{s=1}^t \frac{\frac{\lambda_s^{\tilde\theta}}{\lambda_{s-1}^{\tilde\theta}} + q^\theta - q^{\tilde\theta}}{\frac{\lambda_s^{\tilde\theta}}{\lambda_{s-1}^{\tilde\theta}}} = \frac{\lambda_0^\theta}{\lambda_0^{\tilde\theta}} \cdot \prod_{s=1}^t \left(1 + \frac{q^\theta - q^{\tilde\theta}}{\frac{\lambda_s^{\tilde\theta}}{\lambda_{s-1}^{\tilde\theta}}}\right).$$

If $q^\theta = q^{\tilde\theta}$, then every term in parentheses equals 1, and the claim follows. If instead $q^\theta > q^{\tilde\theta}$, recall that, by Eq. (A.40), for all $s \geq 1$, since $\lambda_{s-1} \in \Delta(\Theta)$ and $q \in [0,1]^{|\Theta|}$, $\frac{\lambda_s^{\tilde\theta}}{\lambda_{s-1}^{\tilde\theta}} \leq 1 + q^{\tilde\theta}$. Therefore, each term in parentheses is not smaller than $1 + \frac{q^\theta - q^{\tilde\theta}}{1 + q^{\tilde\theta}} > 1$. It follows that

$$\frac{\lambda_t^\theta}{\lambda_t^{\tilde\theta'}} = \frac{\lambda_0^\theta}{\lambda_0^{\tilde\theta}} \cdot \prod_{s=1}^t \left(1 + \frac{q^\theta - q^{\tilde\theta}}{\frac{\lambda_s^{\tilde\theta}}{\lambda_{s-1}^{\tilde\theta}}}\right) \geq \frac{\lambda_0^\theta}{\lambda_0^{\tilde\theta}} \cdot \left(1 + \frac{q^\theta - q^{\tilde\theta}}{1 + q^{\tilde\theta}}\right)^t \to \infty.$$

For Claim 4, consider first $\tilde{\theta} \notin \Theta^{\max}$, and fix $\theta \in \Theta^{\max}$ arbitrarily. Then $\frac{\lambda_t^\theta}{\lambda_t^{\tilde{\theta}}} \to \infty$ by Claim 3(b). Suppose that there is a subsequence $(\lambda_{t(\ell)})_{\ell \geq 0}$ such that $\lambda_{t(\ell)}^{\tilde{\theta}} \geq \epsilon$ for some $\epsilon > 0$ and all $\ell \geq 0$. Since $\frac{\lambda_{t(\ell)}^\theta}{\lambda_{t(\ell)}^{\tilde{\theta}}} \to \infty$ as well, there is ℓ large enough such that $\frac{\lambda_{t(\ell)}^\theta}{\lambda_{t(\ell)}^{\tilde{\theta}}} > \frac{1}{\epsilon}$: but then $\Lambda_{t(\ell)}^\theta > 1$ for such ℓ: contradiction. Thus, for every $\epsilon > 0$, eventually $\lambda_t^{\tilde{\theta}} < \epsilon$: that is, $\lambda_t^{\tilde{\theta}} \to 0$.

Next, consider $\tilde{\theta} \in \Theta^{\max}$. By Claim 2, $\lambda_t^{\tilde{\theta}} > 0$ and $\sum_{\theta \in \Theta^{\max}} \lambda_t^\theta > 0$, and

$$\frac{\lambda_t^{\tilde{\theta}}}{\sum_{\theta \in \Theta^{\max}} \lambda_t^\theta} = \frac{1}{\sum_{\theta \in \Theta^{\max}} \frac{\lambda_t^\theta}{\lambda_t^{\tilde{\theta}}}} = \frac{1}{\sum_{\theta \in \Theta^{\max}} \frac{\lambda_0^\theta}{\lambda_0^{\tilde{\theta}}}} = \frac{\lambda_0^{\tilde{\theta}}}{\sum_{\theta \in \Theta^{\max}} \lambda_0^\theta} = \bar{\lambda}^{\tilde{\theta}},$$

where the third inequality follows from Claim 3(b). Therefore,

$$\lambda_t^{\tilde{\theta}} = \frac{\lambda_t^{\tilde{\theta}}}{\sum_{\theta \in \Theta^{\max}} \lambda_t^\theta} \cdot \left(\sum_{\theta \in \Theta^{\max}} \lambda_t^\theta \right) = \bar{\lambda}^{\tilde{\theta}} \cdot \left(1 - \sum_{\theta \notin \Theta^{\max}} \lambda_t^\theta \right) \to \bar{\lambda}^{\tilde{\theta}},$$

because, as was just shown above, $\lambda_t^\theta \to 0$ for $\theta \notin \Theta^{\max}$.

Finally, consider Claim 5. Fix $g \in \{f, m\}$. First, since $0 \leq \lambda_t^{\theta,g} \leq \lambda_t^\theta$ for all $t \geq 0$, if $\theta \notin \Theta^{\max}$ then by Claim 4 $\lambda_t^\theta \to \bar{\lambda}^\theta = 0$, and so $\lambda_t^{\theta,g} \to 0 = \bar{\lambda}^{\theta,g}$ as well. Thus, focus on the case $\theta \in \Theta^{\max}$, so that by Claim 4 $\bar{\lambda}^\theta > 0$.

If $\sum_{\theta'} \bar{\lambda}^{\theta'} q^{\theta'} = 1$, then Eq. (A.34) and the fact that $\sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'} \in [0,1]$ and $0 \leq \lambda_{t-1}^{\theta,g} \leq \lambda_{t-1}^\theta \leq 1$ for all θ imply that

$$\lambda_t^{\theta,g} = \left(1 - \sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'} \right) \lambda_{t-1}^{\theta,g} + \lambda_{t-1}^\theta q^{\theta,g} \in \left[\lambda_{t-1}^\theta q^{\theta,g}, 1 - \sum_{\theta'} \lambda_{t-1}^{\theta'} q^{\theta'} + \lambda_{t-1}^\theta q^{\theta,g} \right]$$

and both endpoints of the interval in the r.h.s. converge to $\bar{\lambda}^\theta q^{\theta,g}$ by Claim 4 if $\sum_{\theta'} \bar{\lambda}^{\theta'} q^{\theta'} = 1$. Furthermore, the same assumption implies that $\bar{\lambda}^\theta q^{\theta,g} = \bar{\lambda}^{\theta,g}$, so $\lambda_t^{\theta,g} \to \bar{\lambda}^{\theta,g}$.

Now consider the case $0 < \sum_{\theta'} \bar{\lambda}^{\theta'} q^{\theta'} < 1$. (The set $\Theta^{\max}$ is non-empty, and since $q \in \mathbb{R}_+^\Theta \setminus \{0\}$, there is $\theta^+ \in \Theta^{\max}$ with $q^{\theta^+} > 0$; by Claim 4, $\bar{\lambda}^{\theta'} > 0$ for $\theta' \in \Theta^{\max}$, so in particular $\bar{\lambda}^{\theta^+} > 0$; but then $\sum_{\theta'} \bar{\lambda}^{\theta'} q^{\theta'} \geq \bar{\lambda}^{\theta^+} q^{\theta^+} > 0$.) It is convenient to let $q_t = \sum_{\theta'} \lambda_t^{\theta'} q^{\theta'}$ and $\bar{q} = \sum_{\theta'} \bar{\lambda}^{\theta'} q^{\theta'} = \lim_{t \to \infty} q_t$, where the second equality follows from Claim 4. Thus, Eq. (A.34) can be written as

$$\lambda_t^{\theta,g} = (1 - q_{t-1}) \lambda_{t-1}^{\theta,g} + \lambda_{t-1}^\theta q^{\theta,g}. \tag{A.41}$$

In addition, $\bar{q} \in (0,1)$.

We claim that, for all $T \geq 0$ and $t > T$,

$$\lambda_t^{\theta,g} = \lambda_T^{\theta,g} \prod_{s=T}^{t-1} (1 - q_s) + q^{\theta,g} \sum_{s=T}^{t-1} \lambda_s^\theta \prod_{r=s+1}^{t-1} (1 - q_r). \tag{A.42}$$

For $t = T+1$, this follows from Eq. (A.41). Inductively, assume it holds for $t-1 > T$. Then, by Eq. (A.41) and the inductive hypothesis,

$$\lambda_t^{\theta,g} = (1-q_{t-1})\left[\lambda_T^{\theta,g}\prod_{s=T}^{t-2}(1-q_s) + q^{\theta,g}\sum_{s=T}^{t-2}\lambda_s^\theta \prod_{r=s+1}^{t-2}(1-q_r)\right] + \lambda_{t-1}^\theta q^{\theta,g} =$$
$$= \lambda_T^{\theta,g}\prod_{s=T}^{t-1}(1-q_s) + q^{\theta,g}\sum_{s=T}^{t-1}\lambda_s^\theta \prod_{r=s+1}^{t-1}(1-q_r),$$

as claimed.

Fix $\epsilon > 0$ such that $\bar\lambda^\theta - \epsilon > 0$, $\bar q - \epsilon > 0$, $1-\bar q+\epsilon < 1$, and $1-\bar q-\epsilon > 0$. This is possible because $\bar\lambda^\theta > 0$ and $\bar q \in (0,1)$, hence $1-\bar q \in (0,1)$.

Since $\lambda_t^\theta \to \bar\lambda^\theta$ and $q_t \to \bar q$, there is $T \geq 0$ such that, for all $t > T$, $\lambda_t^\theta < \bar\lambda^\theta + \epsilon$ and $q_t > \bar q - \epsilon$. Hence, for such $t > T$, Eq. (A.42) implies that

$$\begin{aligned}\lambda_t^{\theta,g} \leq & \lambda_T^{\theta,g}\prod_{s=T}^{t-1}(1-\bar q+\epsilon) + q^{\theta,g}\sum_{s=T}^{t-1}(\bar\lambda^\theta+\epsilon)\prod_{r=s+1}^{t-1}(1-\bar q+\epsilon) =\\ =&\lambda_T^{\theta,g}(1-\bar q+\epsilon)^{t-T} + q^{\theta,g}(\bar\lambda^\theta+\epsilon)\sum_{s=T}^{t-1}(1-\bar q+\epsilon)^{t-1-s} =\\ =&\lambda_T^{\theta,g}(1-\bar q+\epsilon)^{t-T} + q^{\theta,g}(\bar\lambda^\theta+\epsilon)\sum_{s=0}^{t-1-T}(1-\bar q+\epsilon)^{s} =\\ =&\lambda_T^{\theta,g}(1-\bar q+\epsilon)^{t-T} + q^{\theta,g}(\bar\lambda^\theta+\epsilon)\frac{1-(1-\bar q+\epsilon)^{t-T}}{\bar q-\epsilon} \to \frac{q^{\theta,g}(\bar\lambda^\theta+\epsilon)}{\bar q-\epsilon}.\end{aligned}$$

This implies that $\limsup_t \lambda_t^{\theta,g} \leq \frac{q^{\theta,g}(\bar\lambda^\theta+\epsilon)}{\bar q-\epsilon}$. Since this must hold for all $\epsilon > 0$, it must be that $\limsup_t \lambda_t^{\theta,g} \leq \frac{q^{\theta,g}\bar\lambda^\theta}{\bar q} = \bar\lambda^{\theta,g}$.

Similarly, $\lambda_t^\theta \to \bar\lambda^\theta$ and $q_t \to \bar q$ imply that there is $T \geq 0$ such that, for all $t > T$, $\lambda_t^\theta > \bar\lambda^\theta - \epsilon > 0$ and $q_t < \bar q + \epsilon < 1$. Then

$$\begin{aligned}\lambda_t^{\theta,g} \geq & \lambda_T^{\theta,g}\prod_{s=T}^{t-1}(1-\bar q-\epsilon) + q^{\theta,g}\sum_{s=T}^{t-1}(\bar\lambda^\theta-\epsilon)\prod_{r=s+1}^{t-1}(1-\bar q-\epsilon) =\\ =&\lambda_T^{\theta,g}(1-\bar q-\epsilon)^{t-T} + q^{\theta,g}(\bar\lambda^\theta-\epsilon)\frac{1-(1-\bar q-\epsilon)^{t-T}}{\bar q+\epsilon} \to \frac{q^{\theta,g}(\bar\lambda^\theta-\epsilon)}{\bar q+\epsilon},\end{aligned}$$

so $\liminf_t \lambda_t^{\theta,g} \geq \frac{q^{\theta,g}(\bar\lambda^\theta-\epsilon)}{\bar q+\epsilon}$. Again, since this must hold for all $\epsilon > 0$, $\liminf_t \lambda_T^{\theta,g} \geq \frac{q^{\theta,g}\bar\lambda^\theta}{\bar q} = \bar\lambda^{\theta,g}$. Hence, $\lambda_t^{\theta,g} \to \bar\lambda^{\theta,g}$. *Q.E.D.*

Next, we establish certain basic properties of the symmetric model considered in the paper. Claims 1 and 3 characterize the set $\Theta^{\max}$ for this specification. Claim 2 ensures that the parameterization satisfies the conditions in Theorem A.1.

Lemma A.1 *Assume that, for every $\theta \in \Theta$, γ^θ, $p^{\theta,m}$ and $p^{\theta,f}$ are as defined in Section 2.. Then, for every $\phi \in (\frac{1}{2}, 1)$, N even, $\gamma_0 \in (0,1)$, and $\rho \in (1, \frac{1}{\gamma_0})$:*

1. *the set of maximizers of $\gamma^\theta \cdot (p^{\theta,m} + p^{\theta,f})$ is $\{\theta^m, \theta^f\}$ if $\rho < \bar{\rho}(\phi, N)$ and $\{\theta^*\}$ if $\rho > \bar{\rho}(\phi, N)$.*
2. *$0 < \gamma^\theta \cdot [p^{\theta,m} + p^{\theta,f}] \leq 1$.*
3. *there is $\bar{N} > 0$ such that, for all even $N \geq \bar{N}$, the maximizers of $\gamma^\theta \cdot (p^{\theta,m} + p^{\theta,f})$ are θ^m and θ^f.*

Recall that $\bar{\rho}(\cdot)$ is defined in Eq. (9).

Proof: Write

$$\begin{aligned} p^{\theta,m} = {} & \phi^{\sum_{n=1}^{N/2}\theta_n}(1-\phi)^{N/2-\sum_{n=1}^{N/2}\theta_n} \cdot (1-\phi)^{\sum_{n=N/2+1}^{N}\theta_n}\phi^{N/2-\sum_{n=N/2+1}^{N}\theta_n} = \\ = {} & \phi^{N/2+\sum_{n=1}^{N/2}\theta_n-\sum_{n=N/2+1}^{N}\theta_n}(1-\phi)^{N/2+\sum_{n=N/2+1}^{N}\theta_n-\sum_{n=1}^{N/2}\theta_n} = \\ = {} & \phi^{N/2}(1-\phi)^{N/2}\left(\frac{\phi}{1-\phi}\right)^{\sum_{n=1}^{N/2}\theta_n-\sum_{n=N/2+1}^{N}\theta_n}. \end{aligned}$$

Similarly

$$p^{\theta,f} = \phi^{N/2}(1-\phi)^{N/2}\left(\frac{\phi}{1-\phi}\right)^{\sum_{n=N/2+1}^{N}\theta_n-\sum_{n=1}^{N/2}\theta_n}.$$

Then $F(\theta) \equiv \gamma^\theta(p^{\theta,m} + p^{\theta,f})$ equals

$$\gamma_0\, \rho^{\sum_n \theta_n/N} \cdot \phi^{N/2}(1-\phi)^{N/2}\left[\left(\frac{\phi}{1-\phi}\right)^{\sum_{n=1}^{N/2}\theta_n-\sum_{n=N/2+1}^{N}\theta_n} + \left(\frac{\phi}{1-\phi}\right)^{-\sum_{n=1}^{N/2}\theta_n+\sum_{n=N/2+1}^{N}\theta_n}\right].$$

Since Θ is finite, there exists at least one maximizer θ of $F(\cdot)$. We claim that, if θ satisfies $\theta_n = \theta_m = 0$ for some $n \in \{1, \ldots, N/2\}$ and $m \in \{N/2+1, \ldots, N\}$, then it is not a maximizer. To see this, define θ' by $\theta'_\ell = \theta_\ell$ for $\ell \in \{1, \ldots, N\} \setminus \{n, m\}$ and $\theta'_n = \theta'_m = 1$. Then $\sum_n \theta'_n > \sum_n \theta_n$, so for $\rho > 1$, $\gamma^{\theta'} > \gamma^\theta$. On the other hand, the term in square brackets is the same for θ and θ' (and it is strictly positive). Hence, θ is not a maximizer of $F(\cdot)$. It follows that the only candidate maximizers of $F(\cdot)$ have either $\theta_n = 1$ for all $n = 1, \ldots, N/2$, or $\theta_n = 1$ for all $n = N/2, \ldots, N$, or both.

If $\theta_n = 1$ for $n = 1, \ldots, N/2$, then $F(\theta) = F(\theta')$, where $\theta'_n = 1$ for $n = N/2+1, \ldots, N$ and $\theta'_n = \theta_{n+N/2}$ for $n = 1, \ldots, N/2$. Hence, it is enough to consider θ such that $\theta_n = 1$ for $n = N/2+1, \ldots, N$. Let Θ^f be the collection of such types, and notice that it contains both

θ^f (for which $\theta^f_n = 0$ for $n = 1, \ldots, N/2$) and $\theta^* = (1, \ldots, 1)$. We show that the maximizer of $F(\cdot)$ on Θ^f is either θ^f or θ^*.

For each $\theta \in \Theta^f$, factoring out all terms not involving $\sum_{n=1}^{N/2} \theta_n$, $F(\theta)$ is proportional to

$$\rho^{\sum_{n=1}^{N/2} \theta_n / N} \cdot \left[\left(\frac{\phi}{1-\phi} \right)^{\sum_{n=1}^{N/2} \theta_n} + \left(\frac{1-\phi}{\phi} \right)^{\sum_{n=1}^{N/2} \theta_n} \right].$$

Hence, $F(\theta)$ is proportional to $\tilde{F}(\sum_{n=1}^{N/2} \theta_n)$, where $\tilde{F} : [0, \frac{1}{2}] \to \mathbb{R}_+$ is defined by

$$\tilde{F}(x) = \rho^x \left[\left(\frac{\phi}{1-\phi} \right)^x + \left(\frac{1-\phi}{\phi} \right)^x \right].$$

The functions $x \mapsto \rho^{\frac{x}{N}} \Phi^x = \left(\rho^{\frac{1}{N}} \right)^x \Phi^x = \left(\rho^{\frac{1}{N}} \cdot \Phi \right)^x$, for $\Phi = \frac{\phi}{1-\phi} \neq 1$ and $\Phi = \frac{1-\phi}{\phi} \neq 1$ respectively, are non-constant and exponential, hence strictly convex on $[0, \frac{1}{2}]$. Hence, $\tilde{F}(\cdot)$ is also strictly convex on $[0, \frac{1}{2}]$, so its maximum is either at 0 or at $\frac{1}{2}$. Correspondingly, $F(\cdot)$ attains a maximum either at θ^f or at θ^* on the set Θ^f.

To conclude the proof of Claim 1, we calculate the values attained by $F(\cdot)$ at these two extremes:

$$\begin{aligned} F(\theta^f) &= \gamma_0 \sqrt{\rho} \cdot [(1-\phi)^N + \phi^N] \\ F(\theta^*) &= \gamma_0 \rho \cdot 2\phi^{N/2}(1-\phi)^{N/2}. \end{aligned}$$

Dividing $F(\theta^*)$ and $F(\theta^f)$ by $\gamma_0 \sqrt{\rho} \phi^{N/2}(1-\phi)^{N/2}$ and comparing the resulting quantities, we conclude that θ^* is (uniquely) optimal iff

$$2\sqrt{\rho} > \left[\left(\frac{\phi}{1-\phi} \right)^{-\frac{N}{2}} + \left(\frac{1-\phi}{\phi} \right)^{-\frac{N}{2}} \right]$$

or equivalently

$$\rho > \frac{1}{4} \left(\left(\frac{1-\phi}{\phi} \right)^{\frac{N}{2}} + \left(\frac{\phi}{1-\phi} \right)^{\frac{N}{2}} \right)^2 = \bar{\rho}(\phi, N), \tag{A.43}$$

which is Claim 1.

For Claim 2, we show that $(1-\phi)^N + \phi^N \leq 1$ and $\phi^{N/2}(1-\phi)^{N/2} \leq \frac{1}{2}$; this is sufficient, because $\gamma_0 \in (0, 1)$ and $\rho \in (1, \frac{1}{\gamma_0})$ by assumption, so also $\gamma_0 \sqrt{\rho} \leq \gamma_0 \rho < 1$.

The function $N \mapsto (1-\phi)^N + \phi^N$ is strictly decreasing in N, so it is enough to prove the claim for $N = 2$. In this case, $(1-\phi)^2 + \phi^2 = 1 - 2\phi + \phi^2 + \phi^2 = 1 + 2\phi(\phi - 1) < 1$, because $\phi < 1$. Similarly, $N \mapsto [\phi(1-\phi)]^{N/2}$ is decreasing in N, and for $N = 2$ it reduces to $\phi(1-\phi) = \phi - \phi^2$; this is concave and maximized at $\phi = \frac{1}{2}$, where it takes the value $\frac{1}{4} < \frac{1}{2}$.

Finally, for Claim 3, as $N \to \infty$, the first term in the rhs of Eq. (A.43) converges to zero, but the second diverges to infinity. Thus, for N large, only θ^m and θ^f maximize $F(\cdot)$. *Q.E.D.*

We now turn to the proofs of the main Propositions and Corollaries in the text.

Proof of Proposition 3 and Corollary 1: convergence of $(\lambda_t)_{t\geq 0}$, $(\lambda_t^m)_{t\geq 0}$ and $(\lambda_t^f)_{t\geq 0}$ follows from Theorem A.1 and Claim 2 of Lemma A.1. Parts (a) and (b) follow from Claim 1 in Lemma A.1 and Claim 4 in Theorem A.1. Corollary 1 follows from Claim 3 in Lemma A.1. *Q.E.D.*

Proposition 2 follows from Proposition 3.

Proof of Proposition 5: Fix $\theta \in \Theta$, and define θ^{sym} by $\theta_n^{\mathrm{sym}} = \theta_{N+1-n}$ for all $n = 1, \ldots, N$. (Notice that, for some θ, it may be the case that $\theta^{\mathrm{sym}} = \theta$.) We first claim that

$$a_t^{\theta,m} + a_t^{\theta^{\mathrm{sym}},m} \geq a_t^{\theta,f} + a_t^{\theta^{\mathrm{sym}},f}. \tag{A.44}$$

Notice that, if $\theta^{\mathrm{sym}} = \theta$, the above inequality just says that $a_t^{\theta,m} \geq a_t^{\theta,f}$.

Let $m_0 = \sum_{n=1}^{N/2} \theta$ and $m_1 = \sum_{n=N/2+1}^{N} \theta_n$. By definition, $p^{\theta,m} = \phi^{m_0}(1-\phi)^{N/2-m_0}\phi^{N/2-m_1}(1-\phi)^{m_1} = \phi^{(m_0-m_1)+N/2}(1-\phi)^{N/2-(m_0-m_1)} = [\phi(1-\phi)]^{N/2}\left(\frac{\phi}{1-\phi}\right)^{m_0-m_1}$, and similarly $p^{\theta^{sym},m} = [\phi(1-\phi)]^{N/2}\left(\frac{1-\phi}{\phi}\right)^{m_0-m_1}$. Moreover, since p_f is defined with the roles of ϕ and $1-\phi$ reversed, $p^{\theta,f} = p^{\theta^{\mathrm{sym}},m}$ and $p^{\theta,m} = p^{\theta^{\mathrm{sym}},f}$, so $p^{\theta,m} + p^{\theta,f} = p^{\theta^{\mathrm{sym}},m} + p^{\theta^{\mathrm{sym}},f}$. Finally, by construction $\gamma^\theta = \gamma^{\theta^{sym}}$.

Suppose that $m_0 \geq m_1$. Since $\phi > \frac{1}{2}$, $p^{\theta,m} \geq p^{\theta^{\mathrm{sym}},m}$. At time 0 we thus have $\lambda_0^\theta = p^{\theta,m} \geq p^{\theta^{\mathrm{sym}},m} = \lambda_0^{\theta^{\mathrm{sym}}} > 0$. Then, since $q^\theta = \gamma^\theta(p^{\theta,m} + p^{\theta,f}) + \gamma^{\theta^{sym}}(p^{\theta^{sym},m} + p^{\theta^{sym},f}) = q^{\theta^{sym}}$, by part 3(a) of Theorem A.1, for every $t > 0$, $\frac{\lambda_t^\theta}{\lambda_{t-1}^\theta} = \frac{\lambda_t^{\theta^{\mathrm{sym}}}}{\lambda_{t-1}^{\theta^{\mathrm{sym}}}}$, and hence $\frac{\lambda_t^\theta}{\lambda_t^{\theta^{\mathrm{sym}}}} = \frac{\lambda_{t-1}^\theta}{\lambda_{t-1}^{\theta^{\mathrm{sym}}}} = \frac{\lambda_0^\theta}{\lambda_0^{\theta^{\mathrm{sym}}}} \geq 1$. Thus, $\lambda_t^\theta \geq \lambda_t^{\theta^{\mathrm{sym}}}$ for all $t > 0$ as well. Finally, letting $\bar{\gamma} \equiv \gamma^{\theta^{\mathrm{sym}}} = \gamma^\theta$, for every $t \geq 1$,

$$a_t^\theta = a_t^{\theta,m} + a_t^{\theta,f} = \bar{\gamma}\lambda_{t-1}^\theta(p^{\theta,m} + p^{\theta,f}) \geq \bar{\gamma}\lambda_{t-1}^{\theta^{\mathrm{sym}}}(p^{\theta^{\mathrm{sym}},m} + p^{\theta^{\mathrm{sym}},f}) = a_t^{\theta^{\mathrm{sym}},m} + a_t^{\theta^{\mathrm{sym}},f} = a_t^{\theta^{\mathrm{sym}}}.$$

All the inequalities in the above paragraph are strict if $m_0 > m_1$; they are reversed if $m_0 \leq m_1$; and hold as equalities if $m_0 = m_1$.

Now, regardless of the values of m_0 and m_1,

$$\begin{aligned}
& a_t^{\theta,m} + a_t^{\theta^{\text{sym}},m} \geq a_t^{\theta,f} + a_t^{\theta^{\text{sym}},f} \\
\Leftrightarrow \quad & \bar{\gamma}(\lambda_{t-1}^{\theta} p^{\theta,m} + \lambda_{t-1}^{\theta^{sym}} p^{\theta^{sym},m}) \geq \bar{\gamma}(\lambda_{t-1}^{\theta} p^{\theta,f} + \lambda_{t-1}^{\theta^{sym}} p^{\theta^{\text{sym}},f}) \\
\Leftrightarrow \quad & \lambda_{t-1}^{\theta}[p^{\theta,m} - p^{\theta,f}] \geq \lambda_{t-1}^{\theta^{\text{sym}}}[p^{\theta^{\text{sym}},f} - p^{\theta^{\text{sym}},m}] \\
\Leftrightarrow \quad & [\lambda_{t-1}^{\theta} - \lambda_{t-1}^{\theta^{\text{sym}}}] \cdot [p^{\theta,m} - p^{\theta,f}] \geq 0,
\end{aligned}$$

where the last step follows from $p^{\theta,m} = p^{\theta^{\text{sym}},f}$ and $p^{\theta,f} = p^{\theta^{\text{sym}},m}$.

If $m_0 = m_1$, then both terms in square brackets equal zero, so equality obtains; in particular, this is true if $\theta = \theta^{\text{sym}}$. If $m_0 > m_1$, then both terms are positive, if $m_0 < m_1$, then both terms are negative. Thus, in any event, the last inequality, and hence Eq. (A.44), holds; furthermore, if $\theta = \theta^{\text{sym}}$, then $a_t^{\theta,m} = a_t^{\theta,f}$.

Now fix $L \in \{0, \ldots, N\}$. Then

$$\begin{aligned}
\sum_{\theta:\sum_n \theta_n = L} a_t^{\theta,m} &= \sum_{\theta:\sum_n \theta_n = L, \theta = \theta^{\text{sym}}} a_t^{\theta,m} + \sum_{\theta:\sum_n \theta_n = L, \theta \neq \theta^{\text{sym}}} a_t^{\theta,m} = \\
&= \sum_{\theta:\sum_n \theta_n = L, \theta = \theta^{\text{sym}}} a_t^{\theta,m} + \frac{1}{2} \sum_{\theta:\sum_n \theta_n = L, \theta \neq \theta^{\text{sym}}} [a_t^{\theta,m} + a_t^{\theta^{\text{sym}},m}] \geq \\
&\geq \sum_{\theta:\sum_n \theta_n = L, \theta = \theta^{\text{sym}}} a_t^{\theta,f} + \frac{1}{2} \sum_{\theta:\sum_n \theta_n = L, \theta \neq \theta^{\text{sym}}} [a_t^{\theta,f} + a_t^{\theta^{\text{sym}},f}] = \\
&= \sum_{\theta:\sum_n \theta_n = L} a_t^{\theta,f}.
\end{aligned}$$

The second equality follows from the observation that, restricting attention to types θ with $\sum_n \theta_n = L$, also $\sum_n \theta_n^{\text{sym}} = L$, so that adding $a_t^{\theta,m} + a_t^{\theta^{\text{sym}},m}$ over all θ with $\theta \neq \theta^{\text{sym}}$ counts each type twice. The inequality follows from Eq. (A.44), which in particular implies that $a_t^{\theta,m} = a_t^{\theta,f}$ if $\theta = \theta^{\text{sym}}$. This inequality is strict if the second summation is non-empty, i.e., if there is θ with $\sum_n \theta_n = L$ and $\theta_n \neq \theta_{N+1-n}$ for some n, because the latter condition implies $\theta \neq \theta^{\text{sym}}$. Finally, the last equality follows by repeating the first two steps backwards, for F-group researchers. *Q.E.D*

Proof of Proposition 6: We begin with a preliminary result.

Lemma A.2 *For all parameter values and initial conditions, and for all $\theta \in \Theta$ and $t \geq 1$,*

$$\frac{\lambda_t^{\theta}}{\lambda_{t-1}^{\theta}} = (1 - a_t) + \gamma^{\theta}(p^{\theta,m} + p^{\theta,f});$$

and for $t \geq 2$,

$$\frac{a_t^{\theta}}{a_{t-1}^{\theta}} = \frac{a_t^{\theta,m}}{a_{t-1}^{\theta,m}} = \frac{a_t^{\theta,f}}{a_{t-1}^{\theta,f}} = \frac{\lambda_{t-1}^{\theta}}{\lambda_{t-2}^{\theta}}.$$

Proof: From Eq. (6), $\lambda_t^\theta = \lambda_t^{\theta,m} + \lambda_t^{\theta,f} = (\lambda_{t-1}^{\theta,m} + \lambda_{t-1}^{\theta,f})(1-a_t) + \gamma^\theta(p^{\theta,m} + p^{\theta,f})$, which yields the first equation because $\lambda_\tau^\theta > 0$ for all θ and τ.

From Eq. (5), for $t \geq 2$,

$$\frac{a_t^{\theta,g}}{a_{t-1}^{\theta,g}} = \frac{\lambda_{t-1}^\theta \gamma^\theta p^{\theta,g}}{\lambda_{t-2}^\theta \gamma^\theta p^{\theta,g}} = \frac{\lambda_{t-1}^\theta}{\lambda_{t-2}^\theta};$$

similarly,

$$\frac{a_t^\theta}{a_{t-1}^\theta} = \frac{\lambda_{t-1}^\theta \gamma^\theta (p^{\theta,m} + p^{\theta,f})}{\lambda_{t-2}^\theta \gamma^\theta (p^{\theta,m} + p^{\theta,f})} = \frac{\lambda_{t-1}^\theta}{\lambda_{t-2}^\theta}.$$

Q.E.D.

We now prove Proposition 6. For $N = 2$ we only have 4 types, $\Theta = \{(0,0),(1,0),(0,1),(1,1)\}$. Let $a^{L,g} = \sum_{sum_{n=1}^2 \theta_n = L} a^{\theta,g}$ and $a_t^g = \sum_{\ell=0}^2 a_t^{\ell,g}$. From Proposition 5, for all t, $a_t^{1,m} > a_t^{1,f}$, $a_t^{2,m} = a_t^{2,f}$, and $a_t^{0,m} = a_t^{0,f}$. Therefore, $a_t^m > a_t^f$, which implies that the weight on $L = 1$ for accepted M researchers is

$$\frac{a_t^{1,m}}{a_t^m} = 1 - \frac{a_t^{2,m} + a_t^{0,m}}{a_t^m} = 1 - \frac{a_t^{2,f} + a_t^{0,f}}{a_t^m} > 1 - \frac{a_t^{2,f} + a_t^{0,f}}{a_t^f} = \frac{a_t^{1,f}}{a_t^f}.$$

Similarly, $a_t^m > a_t^f$ and $a_t^{0,m} = a_t^{0,f}$, $a_t^{2,m} = a_t^{2,f}$ imply

$$\frac{a_t^{0,m}}{a_t^m} < \frac{a_t^{0,f}}{a_t^f}, \qquad \frac{a_t^{2,m}}{a_t^m} < \frac{a_t^{2,f}}{a_t^f}.$$

Moreover, we claim that, $a_t^{2,g} > a_t^{0,g}$. For $t = 0$, $a_0^{2,g} = a_0^{(1,1),g} = p^{(1,1),m}\gamma^{(1,1)}p^{(1,1),g} > p^{(0,0),m}\gamma^{(0,0)}p^{(0,0),g} = a_0^{(0,0),g} = a_0^{0,g}$, because $p^{(0,0),g} = p^{(1,1),g}$ but $\gamma^{(1,1)} > \gamma^{(0,0)}$. Inductively, from Lemma A.2,

$$\begin{aligned}
a_t^{2,g} = a_t^{(1,1),g} &= a_{t-1}^{(1,1),g} \cdot \frac{a_t^{(1,1),g}}{a_{t-1}^{(1,1),g}} = a_{t-1}^{(1,1,g)} \left(1 - a_{t-1} + \gamma^{(1,1)}(p^{(1,1),m} + p^{(1,1),f}\right) > \\
&> a_{t-1}^{(1,1,g)} \left(1 - a_{t-1} + \gamma^{(0,0)}(p^{(0,0),m} + p^{(0,0),f}\right) > a_{t-1}^{(0,0,g)} \left(1 - a_{t-1} + \gamma^{(0,0)}(p^{(0,0),m} + p^{(0,0),f}\right) = \\
&= a_{t-1}^{(0,0,g)} \frac{a_t^{(0,0),g}}{a_{t-1}^{(0,0,g)}} = a_t^{(0,0),g} = a_t^{0,g}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
0 &< \frac{a_t^{0,f}}{a_t^{1,f} + a_t^{2,f} + a_t^{0,f}} - \frac{a_t^{0,m}}{a_t^{1,m} + a_t^{2,m} + a_t^{0,m}} = \frac{a_t^{0,f}}{a_t^{1,f} + a_t^{2,f} + a_t^{0,f}} - \frac{a_t^{0,f}}{a_t^{1,m} + a_t^{2,m} + a_t^{0,m}} < \\
&< \left(\frac{a_t^{2,f}}{a_t^{0,f}}\right) \cdot \left(\frac{a_t^{0,f}}{a_t^{1,f} + a_t^{2,f} + a_t^{0,f}} - \frac{a_t^{0,f}}{a_t^{1,m} + a_t^{2,m} + a_t^{0,m}}\right) = \frac{a_t^{2,f}}{a_t^{1,f} + a_t^{2,f} + a_t^{0,f}} - \frac{a_t^{2,f}}{a_t^{1,m} + a_t^{2,m} + a_t^{0,m}} = \\
&= \frac{a_t^{2,f}}{a_t^{1,f} + a_t^{2,f} + a_t^{0,f}} - \frac{a_t^{2,m}}{a_t^{1,m} + a_t^{2,m} + a_t^{0,m}};
\end{aligned}$$

the first inequality follows from $a_t^{1,f} < a_t^{1,m}$ and $a_t^{0,f} = a_t^{0,m}$ and $a_t^{2,f} = a_t^{2,m}$, the next equality from $a_t^{0,m} = a_t^{0,f}$, the second inequality from $a_t^{2,f} > a_t^{0,f} > 0$ and the fact that the difference of fractions is positive, and the last equality from $a_t^{2,m} = a_t^{2,f}$.

The result then follows from a symmetry argument.

$$\begin{aligned} E[L|F] &= \frac{0 \times a_t^{0,f} + a_t^{1,f} + 2a_t^{2,f}}{a_t^f} \\ E[L|M] &= \frac{0 \times a_t^{0,m} + a_t^{1,m} + 2a_t^{2,m}}{a_t^m} \end{aligned}$$

which, since $a_t^{1,g} = 1 - a_t^{0,g} - a_t^{2,g}$, implies

$$E[L|F] = -1\frac{a_t^{0,f}}{a_t^f} + 1 + \frac{a_t^{2,f}}{a_t^f}$$

$$E[L|M] = -1\frac{a_t^{0,m}}{a_t^m} + 1 + \frac{a_t^{2,m}}{a_t^m}$$

It follows that

$$E[L|F] - E[L|M] = -\left(\frac{a_t^{0,f}}{a_t^f} - \frac{a_t^{0,m}}{a_t^m}\right) + \left(\frac{a_t^{2,f}}{a_t^f} - \frac{a_t^{2,m}}{a_t^m}\right) > 0$$

Q.E.D

Proof of Proposition 7. Letting $L = \sum_{n=1}^N \theta_n$,

$$\begin{aligned} [\gamma^\theta]^N =& \alpha^N \frac{1}{2^N} \sum_{\vartheta \in \Theta} \beta^{\sum_{n:\theta_n=1} \max(1-\vartheta_n,\theta_n) + \sum_{n:\theta_n=0} \max(1-\vartheta_n,\theta_n)} \\ &= \frac{1}{2^N}\alpha^N \sum_{\vartheta\in\Theta} \beta^{\sum_{n=1}^N \theta_n + \sum_{n:\theta_n=0}(1-\vartheta_n)} = \frac{1}{2^N}\alpha^N\beta^L \sum_{\vartheta\in\Theta}\beta^{\sum_{n:\theta_n=0}(1-\vartheta_n)} \\ &= \frac{1}{2^N}\alpha^N\beta^L \sum_{\vartheta\in\Theta}\beta^{\sum_{n:\theta_n=0} 1 - \sum_{n:\theta_n=0}\vartheta_n} = \frac{1}{2^N}\alpha^N\beta^L\beta^{N-L}\sum_{\vartheta\in\Theta}\beta^{-\sum_{n:\theta_n=0}\vartheta_n} \\ &= \frac{1}{2^N}\alpha^N\beta^N \sum_{\vartheta\in\Theta}\beta^{-\sum_{n:\theta_n=0}\vartheta_n} = \frac{1}{2^N}\alpha^N\beta^N 2^L \sum_{\ell=0}^{N-L}\binom{N-L}{\ell}\beta^{-\ell} = \\ &= \alpha^N\beta^N 2^{L-N}\sum_{\ell=0}^{N-L}\binom{N-L}{\ell}\beta^{-\ell} = \alpha^N\beta^N\sum_{\ell=0}^{N-L}\binom{L}{\ell}\left(\frac{1}{2}\right)^\ell\left(\frac{1}{2}\right)^{(N-L)-\ell}\left(\frac{1}{\beta}\right)^\ell \\ &= \alpha^N\beta^N\sum_{\ell=0}^{N-L}\binom{N-L}{\ell}\left(\frac{1}{2}\right)^\ell\left(\frac{1}{2}\right)^{(N-L)-\ell}\left(e^{\log\frac{1}{\beta}}\right)^\ell = \alpha^N\beta^N\left(\frac{1}{2}+\frac{1}{2}e^{\log\frac{1}{\beta}}\right)^{N-L} \\ &= \left[\alpha\beta\left(\frac{1}{2}+\frac{1}{2\beta}\right)\right]^N\left(\frac{1}{\frac{1}{2}+\frac{1}{2\beta}}\right)^L = \left[\alpha\beta\frac{\beta+1}{2\beta}\right]^N\left(\frac{2\beta}{\beta+1}\right)^L = \left[\alpha\frac{\beta+1}{2}\right]^N\left[\frac{2\beta}{\beta+1}\right]^L. \end{aligned}$$

The last equality in the penultimate row follows from the formula for the moment-generating function of a binomial r.v. Now we can match coefficients: by definition $\gamma^{\theta^r} = \gamma_0 \rho^{L/N}$, so $[\gamma^\theta]^N = \gamma_0^N \rho^L$ and we can take $\gamma_0 = \alpha \frac{\beta+1}{2}$ and $\rho = \frac{2\beta}{\beta+1}$.

As noted in the text, to ensure that $\rho > 1$, we need $\frac{2\beta}{\beta+1} > 1$, i.e. $\beta > 1$.

Q.E.D.

Detailed dynamics of the mass of M and F accepted agents. It is useful to rewrite Equation (6) for each group $g = m, f$ as follows:

$$\lambda_t^{\theta,m} - \lambda_{t-1}^{\theta,m} = -\lambda_{t-1}^{\theta,m}\, a_t + \lambda_{t-1}^{\theta,m}\gamma^\theta p^{\theta,m} + \lambda_{t-1}^{\theta,f}\gamma^\theta p^{\theta,m} \tag{A.45}$$

$$\lambda_t^{\theta,f} - \lambda_{t-1}^{\theta,f} = -\lambda_{t-1}^{\theta,f}\, a_t + \lambda_{t-1}^{\theta,f}\gamma^\theta p^{\theta,f} + \lambda_{t-1}^{\theta,m}\gamma^\theta p^{\theta,f} \tag{A.46}$$

Consider the dynamics of F-researchers in (A.46), for instance. The change in the mass of F-researchers of type θ decreases due to replacement at the rate a_t, and it then increases due to the young F-researchers who produce quality research and are matched with referees from the F group who share their type and hence view them positively ($\lambda_{t-1}^{\theta,f}\gamma^\theta p^{\theta,f}$), plus the young F-researchers who produce quality research and are matched with M-referees of their own type ($\lambda_{t-1}^{\theta,m}\gamma^\theta p^{\theta,f}$). The asymmetry between the two dynamics (A.45) and (A.46) is apparent in the last two terms of each. If θ is a type that is more prevalent among M-researchers—for instance, $\theta = \theta^m$—then $p^{\theta,f}$ will be small while $p^{\theta,m}$ will be large. If the current mass of M-researchers of type θ is large, then $\lambda_{t-1}^{\theta}\gamma^\theta p^{\theta,m}$ will act to further increase the mass of M-researchers, while the respective term $\lambda_{t-1}^{\theta}\gamma^\theta p^{\theta,f}$ in the F-group dynamics will lead to a smaller increase in the mass of type-θ F-researchers. In particular, if we start from a situation in which *all* referees of type θ are in M-group, then, while they will accept some F-researchers of type θ, they will accept a much larger mass of M-researchers.

This force is at play regardless of the parameter values, and for all types. However, its implications for the limiting group (im)balance in the population depend upon whether or not we are in a "meritocratic" scenario. If research characteristics have a limited effect on the probability of quality research, as in Part (a) of Proposition 3, then θ^m and θ^f are the only types that survive in the limit. These are also the types for which the difference in proportions among young M- and F-researchers is greatest. Thus, in the scenario of Part (a), the force thus described has the greatest effect, which is further reinforced if initially *all* referees are in M-group. The result is that, in the limit, despite the fact that the mass of young M- and F-researchers appearing at each time t is the same, the referees' self-image bias leads to a limiting population in which the majority of scholars are in M group.

By way of contrast, in the meritocratic scenario of Part (b) in Proposition 3, the type that prevails in the limit is the efficient one, namely θ^*. In our symmetric model, the *same* fraction of young M- and F-researchers are of type θ^*. Therefore, the effect described above becomes more and more muted over time. Consequently, in the limit, the mass of M- and F-scholars is the same.

The following Proposition formalizes the above discussion. We denote by $\Lambda_t^m \equiv \sum_\theta \lambda_t^{\theta,m}$ and $\Lambda_t^f \equiv \sum_\theta \lambda_t^{\theta,f}$ the total mass of M- and F-scholars at date t; $\bar{\Lambda}^m$ and $\bar{\Lambda}^f$ are the corresponding limiting quantities.

Proposition A.4 *Assume that all referees are initially from the M-group, i.e., $\lambda_0 = p^m$.*

(a) If $\rho < \bar{\rho}(\phi, N)$, then the limiting masses are

$$\text{(M-researchers of type θ^m): } \bar{\lambda}^{\theta^m,m} = \frac{(\phi^N)^2}{(\phi^N + (1-\phi)^N)^2}; \tag{A.47}$$

$$\text{(F-researchers of type θ^m): } \bar{\lambda}^{\theta^m,f} = \frac{\phi^N\,(1-\phi)^N}{(\phi^N + (1-\phi)^N)^2}; \tag{A.48}$$

$$\text{(M-researchers of type θ^f): } \bar{\lambda}^{\theta^f,m} = \frac{((1-\phi)^N)^2}{(\phi^N + (1-\phi)^N)^2}; \tag{A.49}$$

$$\text{(F-researchers of type θ^f): } \bar{\lambda}^{\theta^f,f} = \frac{(1-\phi)^N\,\phi^N}{(\phi^N + (1-\phi)^N)^2}; \tag{A.50}$$

with

$$\bar{\lambda}^{\theta^m,m} > \bar{\lambda}^{\theta^m,f} = \bar{\lambda}^{\theta^f,f} > \bar{\lambda}^{\theta^f,m} \tag{A.51}$$

In addition, the total mass of M and F researchers are

$$\bar{\Lambda}^m = 1 - \bar{\Lambda}^f = \frac{1 + \left(\frac{\phi}{1-\phi}\right)^{2N}}{1 + \left(\frac{\phi}{1-\phi}\right)^{2N} + 2\left(\frac{\phi}{1-\phi}\right)^{N}} > 0.5. \tag{A.52}$$

(b) If $\rho > \bar{\rho}(\phi, N)$, then $\bar{\lambda}^{\theta^*,m} = \bar{\lambda}^{\theta^*,f} = \bar{\Lambda}^m = \bar{\Lambda}^f = \frac{1}{2}$.

Proof of Proposition 4, A.4 and Corollary 2. For Part (a), since $\gamma^{\theta^m} = \gamma^{\theta^f} = \gamma_0\,(\rho)^{N/2}$ and, by Proposition 3, $\Theta^{\max} = \{\theta^m, \theta^f\}$, $\bar{\lambda}^{\tilde{\theta},m} = \frac{\lambda_0^{\tilde{\theta}} p^{\tilde{\theta},m}}{\lambda_0^{\theta^m} p^{\theta^m,m} + \lambda_0^{\theta^f} p^{\theta^f,m}}$ for $\tilde{\theta} \in \Theta^{\max}$, and $\bar{\lambda}^{\tilde{\theta},m} = 0$ otherwise; a similar expression holds for $\bar{\lambda}^{\tilde{\theta},f}$. Equations (A.47) through (A.50) then follow from the specification of p^m and p^f. Eq. (11) follows from $\bar{\Lambda}^g = \bar{\lambda}^{\theta^m,g} + \bar{\lambda}^{\theta^f,g}$.

Part (b) follows from the fact that, by Proposition 3 part (b), $\Theta^{\max} = \{\theta^*\}$ in this scenario. Corollary 2 follows from Lemma A.1 Claim (3).

Proposition 4 consists of (b) and the last claim in (a) of Proposition A.4. *Q.E.D.*

Proof of Proposition A.1: let $\Theta_{-1} = \Theta$ and $t(-1) = 0$. Also let $\lambda^m_{0,0} = \lambda^m_{1,0} = \lambda^m_0$, $\lambda^f_{0,0} = \lambda^f_{1,0} = \lambda^f_0$, and $\lambda_{0,0} = \lambda_{1,0} = \lambda^m_{1,0} + \lambda^f_{1,0}$. Finally, let $\Theta_0 = \left\{\theta \in \Theta : \lambda^\theta_{1,0} \geq \frac{C}{\gamma^\theta P}\right\}$.

For $j \geq 0$, say that *Conditions $C(j)$ hold* if there is a set $\Theta_j \subseteq \Theta_{j-1}$, a period $t(j) > t(j-1)$, and for $\tau = 0, \ldots, t(j) - t(j-1)$, vectors $\lambda^m_{\tau,j}, \lambda^f_{\tau,j}, \lambda_{\tau,j} \in \mathbb{R}^\Theta_+$ such that

(i) for $0 \leq \tau \leq t(j) - t(j-1)$, $\lambda^m_{\tau,j} = \lambda^m_{t(j-1)+\tau}$, $\lambda^f_{\tau,j} = \lambda^f_{t(j-1)+\tau}$, and $\lambda_{\tau,j} = \lambda^m_{\tau,j} + \lambda^f_{\tau,j}$;

(ii) for $0 \leq \tau < t(j) - t(j-1)$, $\lambda^\theta_{\tau,j} \geq \frac{C}{\gamma^\theta P}$ for all $\theta \in \Theta_j$;

(iii) $\lambda^\theta_{\tau,j} < \frac{C}{\gamma^\theta (P-U)}$ for $0 \leq \tau \leq t(j) - t(j-1)$ and all $\theta \in \Theta \backslash \Theta_j$, and $\lambda^{\theta_0}_{t(j)-t(j-1),j} < \frac{C}{\gamma^{\theta_0}(P-U)}$ for some $\theta_0 \in \Theta_j$.

We claim that, for every $k \geq 0$, if either $k = 0$ or $k > 0$ and Conditions $C(k-1)$ hold, then either Conditions $C(k)$ hold as well, with $\Theta_k \subsetneq \Theta_{k-1}$ in case $k > 0$, or else there exist vectors $\lambda^m_{\tau,k}, \lambda^f_{\tau,k}, \lambda_{\tau,k} \in \mathbb{R}^\Theta_+$ for all $\tau \geq 1$ such that (i) holds for $j = k$, and $\lambda^\theta_{\tau,j} \geq \frac{C}{\gamma^\theta P}$ for all $\theta \in \Theta_k$. In the latter case, if the sequences of such vectors converge, then $\lim_{\tau\to\infty} \lambda^m_{\tau,k} = \lim_{t\to\infty} \lambda^m_t$ and similarly for $\lambda^f_{\tau,k}$ and $\lambda_{\tau,k}$.

Let $\lambda^{\theta,g}_{0,k} = \lambda^{\theta,g}_{t(k-1)}$ for $g = f, m$; also let $\lambda_{0,k} = \lambda^m_{0,k} + \lambda^f_{0,k}$. Let $\Theta_k = \left\{\theta \in \Theta : \lambda^\theta_{0,k} \geq \frac{C}{\gamma^\theta P}\right\}$. If $k = 0$, then $\Theta_0 \subseteq \Theta = \Theta_{-1}$. Otherwise, $C(k-1)$ must hold, so $\lambda_{0,k} = \lambda_{t(k-1)} = \lambda_{t(k-1)-t(k-2),k-1}$. By (iii), if $\theta \notin \Theta_{k-1}$ then $\lambda^\theta_{0,k} = \lambda^\theta_{t(k-1)-t(k-2),k-1} < \frac{C}{\gamma^\theta P}$, so $\theta \notin \Theta_k$ as well; firthermore, there exists $\theta_0 \in \Theta_{k-1}$ such that $\lambda^{\theta_0}_{0,k} = \lambda^{\theta_0}_{t(k-1)-t(k-2),k-1} < \frac{C}{\gamma^\theta P}$. Therefore, if $k > 0$, then $\Theta_k \subsetneq \Theta_{k-1}$.

Define $q^g_k \in \mathbb{R}^\Theta_+ \setminus \{0\}$ for $g = f, m$ by $q^{\theta,g}_k = \gamma^\theta p^{\theta,g}$ if $\theta \in \Theta_k$, and $q^{\theta,g}_k = 0$ otherwise. Then $q^{\theta,m}_k + q^{\theta,f}_k \leq 1$ for all θ. Consider the sequences $(\lambda^{\theta,g}_{\tau,k})_{\tau\geq 0}$ for $g = f, m$ and $(\lambda^\theta_{\tau,k})_{\tau\geq 0}$ defined by Eqs. (A.34)–(A.35) for the vectors q^f_k, q^m_k .

Suppose first that there are $\bar\tau > 0$ and $\theta_0 \in \Theta_k$ such that $\lambda^{\theta_0}_{\bar\tau,k} < \frac{C}{\gamma^{\theta_0}(P-U)}$. Let $t(k) = t(k-1) + \bar\tau$. Then, for each group $g = f, m$, the dynamics in Eqs. (A.34)–(A.35) induced by the vectors q^f_k, q^m_k for the subsequence $(\lambda^g_{\tau,k})_{\tau=0,\ldots,\bar\tau}$ coincide with those in Eq. (A.20) for the subsequences $(\lambda^g_t)_{t=t(k-1),\ldots,t(k)}$; thus, (i) holds for $j = k$. Furthermore, (ii) and the second part of (iii) hold for $j = k$ by the definition of $\bar\tau$. For the first part of (iii) with $j = k$, recall that by definition $q^{\theta,m}_k + q^{\theta,f}_k = 0$ for $\theta \in \Theta \setminus \Theta_k$; hence, for all $\theta' \in \Theta$ and all $\theta \in \Theta \setminus \Theta_k$, $q^{\theta,m}_k + q^{\theta,f}_k \leq q^{\theta'}_{m,k} + q^{\theta'}_{f,k}$. By part 3(a) in Theorem A.1, it must be the case that

$\lambda^{\theta}_{\tau+1,k}/\lambda^{\theta}_{\tau,k} \leq 1$: otherwise, $\sum_{\theta' \in \Theta} \lambda^{\theta'}_{\tau+1,k} > \sum_{\theta' \in \Theta} \lambda^{\theta'}_{\tau,k} = 1$, which contradicts the fact that $\lambda_{\tau+1,k} \in \Delta(\Theta)$ per Theorem A.1. Since by definition $\lambda^{\theta}_{0,k} < \frac{C}{\gamma^{\theta} P}$ for $\theta \notin \Theta_k$, it follows that also $\lambda^{\theta}_{\tau,k} < \frac{C}{\gamma^{\theta} P}$ for $\tau = 0, \ldots, \bar{\tau}$ and for any such θ. Thus, in this case Conditions $C(k)$ hold.

If instead $\lambda^{\theta}_{\bar{\tau},k} \geq \frac{C}{\gamma^{\theta}(P-U)}$ for all $\theta \in \Theta_k$, then for each group $g = f, m$, the dynamics in Eqs. (A.34)–(A.35) induced by the vectors $q_{m,k}, q_{f,k}$ for the subsequence $(\lambda^{g}_{\tau,k})_{\tau \geq 0}$ coincide with those in Eq. (A.20) for the subsequence $(\lambda^{g}_{t})_{t \geq t(k-1)}$. Again, in this case (i) holds for $j = k$. This completes the proof of the claim.

Since the set Θ is finite, there exists $K \geq 0$ such that the induction stops—that is, $\lambda^{\theta}_{\bar{\tau},K} \geq \frac{C}{\gamma^{\theta}(P-U)}$ for all $\theta \in \Theta_K$. Let $\Theta^{\max}_{k} = \arg\max\{q^{\theta,m}_{k} + q^{\theta,f}_{k} : \theta \in \Theta\}$. Since $\Theta_0 \supsetneq \Theta_1 \supsetneq \ldots \supsetneq \Theta_K$, by the definition of the vectors q^{g}_{k} for $g = f, m$, also $\Theta^{\max}_{0} \supseteq \Theta^{\max}_{1} \supseteq \ldots \supseteq \Theta^{\max}_{K}$. Moreover, for every $k = 0, \ldots, K-1$, and every $\theta \in \Theta^{\max}_{k}$, $\lambda^{\theta}_{\tau+1,k}/\lambda^{\theta}_{\tau,k} \geq 1$ for $0 \leq \tau < t(k) - t(k)$; otherwise, by part 3(a) in Theorem A.1, $\sum_{\theta \in \Theta} \lambda^{\theta}_{\tau+1,k} < \sum_{\theta \in \Theta} \lambda^{\theta}_{\tau,k} = 1$, which contradicts the fact that $\lambda_{\tau+1} \in \Delta(\Theta)$ per Theorem A.1.

Now assume that $\Theta^{\max}_{0} \subseteq \Theta_0$. Then, for every $\theta \in \Theta^{\max}_{0}$,

$$\frac{C}{\gamma^{\theta} P} \leq \lambda^{\theta}_{0,0} \leq \lambda^{\theta}_{t(1)-t(0),0} = \lambda^{\theta}_{0,1} \leq \lambda^{\theta}_{t(2)-t(1),1} \cdots \leq \lambda^{\theta}_{0,K},$$

so $\theta \in \Theta_k$ for all $k = 0, \ldots, K$, and thus $\Theta^{\max}_{0} = \Theta^{\max}_{1} = \ldots = \Theta^{\max}_{K} \equiv \Theta^{\max}$. In addition, again by part 3(a) of Theorem A.1, if $\theta, \theta' \in \Theta^{\max}$, then $\frac{\lambda^{\theta}_{\tau+1,k}}{\lambda^{\theta}_{\tau,k}} = \frac{\lambda^{\theta'}_{\tau+1,k}}{\lambda^{\theta'}_{\tau,k}}$ for all $k = 0, \ldots, K-1$ and $\tau = 0, \ldots, t(k) - t(k-1)$, and for $k = K$ and all $\tau \geq 0$. Rearranging terms, $\frac{\lambda^{\theta}_{\tau+1,k}}{\lambda^{\theta'}_{\tau+1,k}} = \frac{\lambda^{\theta}_{\tau,k}}{\lambda^{\theta'}_{\tau,k}}$ for such k and τ. Therefore, (i) in Conditions $C(0)$...$C(K)$ imply that

$$\frac{\lambda^{\theta}_{0,K}}{\lambda^{\theta'}_{0,K}} = \frac{\lambda^{\theta}_{t(K-1)}}{\lambda^{\theta'}_{t(K-1)}} = \frac{\lambda^{\theta}_{t(K-1)-t(K-2),K-1}}{\lambda^{\theta'}_{t(K-1)-t(K-2),K-1}} = \frac{\lambda^{\theta}_{0,K-1}}{\lambda^{\theta'}_{0,K-1}} = \ldots = \frac{\lambda^{\theta}_{t(0)-t(-1),0}}{\lambda^{\theta'}_{t(0)-t(-1),0}} = \frac{\lambda^{\theta}_{0,0}}{\lambda^{\theta'}_{0,0}} = \frac{\lambda^{\theta}_{0}}{\lambda^{\theta'}_{0}}.$$

Therefore, for $\theta \in \Theta^{\max} = \Theta^{\max}_{K}$, from Theorem A.1 part (4),

$$\bar{\lambda}^{\theta} = \bar{\lambda}^{\theta}_{K} = \frac{\lambda^{\theta}_{0,K}}{\sum_{\theta' \in \Theta^{\max}} \lambda^{\theta'}_{0,K}} = \frac{1}{\sum_{\theta' \in \Theta^{\max}} \frac{\lambda^{\theta'}_{0,K}}{\lambda^{\theta}_{0,K}}} = \frac{1}{\sum_{\theta' \in \Theta^{\max}} \frac{\lambda^{\theta'}_{0}}{\lambda^{\theta}_{0}}} = \frac{\lambda^{\theta}_{0}}{\sum_{\theta' \in \Theta^{\max}} \lambda^{\theta'}_{0}}. \tag{A.53}$$

Similarly, for $\theta \in \Theta^{\max}$, part (5) in the same Theorem implies that

$$\bar{\lambda}^{\theta,m} = \bar{\lambda}^{\theta,m}_{K} = \frac{\lambda^{\theta}_{0,K} q^{\theta,m}_{K}}{\sum_{\theta' \in \Theta^{\max}} \lambda^{\theta'}_{0,K} q^{\theta'}_{K}} = \frac{q^{\theta,m}_{K}}{\sum_{\theta' \in \Theta^{\max}} \frac{\lambda^{\theta'}_{0,K}}{\lambda^{\theta}_{0,K}} q^{\theta'}_{K}} = \frac{q^{\theta,m}_{K}}{\sum_{\theta' \in \Theta^{\max}} \frac{\lambda^{\theta'}_{0}}{\lambda^{\theta}_{0}} q^{\theta'}_{K}} = \frac{\lambda^{\theta}_{0} q^{\theta,m}_{K}}{\sum_{\theta' \in \Theta^{\max}} \lambda^{\theta'}_{0} q^{\theta'}_{K}}, \tag{A.54}$$

and analogously for $\bar{\lambda}^{\theta,f}$.

Statements (a.1)–(b) now follow. Recall that $\lambda_0 = p^m$. In (a.1), by assumption $\Theta^{\max} = \Theta^{\max}_{0} = \{\theta^m, \theta^f\} \subseteq \Theta_0$. Substituting $\lambda^{\theta^m}_{0} = \phi^N$ and $\lambda^{\theta^f}_{0} = (1-\phi)^N$ in Eq. (A.53) yields

$\bar{\lambda}^{\theta^m} = \frac{\phi^N}{\phi^N+(1-\phi)^N}$. Similarly, substituting for q_K^g, $g = f, m$, and $q_K = q_K^f + q_K^m$ in Eq. (A.54) yields the same expression for $\bar{\lambda}^{\theta^m,m}$ as in Proposition 3, because $\theta \in \Theta^{\max}$ implies that $q_K^{\theta,g} = \gamma^\theta p^{\theta,g}$; ditto for $\bar{\lambda}^{\theta^m,f}$, $\bar{\lambda}^{\theta^f,m}$ and $\bar{\lambda}^{\theta^f,f}$, and hence for $\bar{\Lambda}^m$.

For (a.2), $\Theta^{\max} = \Theta_0^{\max} = \{\theta^m\}$. This immediately implies that $\bar{\lambda}^{\theta^m} = \bar{\lambda}_K^{\theta^m} = 1$. Furthermore, from Eq. (A.54), $\bar{\Lambda}^m = \bar{\lambda}^{m,\theta^m} = \bar{\lambda}_K^{m,\theta^m} = \frac{\gamma^{\theta^m} p^{\theta^m,m}}{\gamma^{\theta^m}(p^{\theta^m,m}+p^{\theta^m,f})} = \frac{p^{\theta^m,m}}{p^{\theta^m,m}+p^{\theta^m,f}} = \frac{\phi^N}{\phi^N+(1-\phi)^N}$, as asserted. Finally, we compare this quantity with its counterpart in Eq. (11):

$$\frac{1+\left(\frac{\phi}{1-\phi}\right)^{2N}}{1+\left(\frac{\phi}{1-\phi}\right)^{2N}+2\left(\frac{\phi}{1-\phi}\right)^{N}} = \frac{(1-\phi)^{2N}+\phi^{2N}}{\left[(1-\phi)^N+\phi^N\right]^2} <$$

$$< \frac{(1-\phi)^N\phi^N+\phi^{2N}}{\left[(1-\phi)^N+\phi^N\right]^2} = \frac{(1-\phi)^N+\phi^N}{(1-\phi)^N+\phi^N}\cdot\frac{\phi^N}{(1-\phi)^N+\phi^N} = \frac{\phi^N}{(1-\phi)^N+\phi^N} = \bar{\Lambda}^m,$$

where the inequality follows from the assumption that $\phi > 0.5$.

The analysis of (b) is analogous to that of (a.2), with θ^* in lieu of θ^m; in this case, $p^{\theta^*,m} = p^{\theta^*,f} = \phi^{N/2}(1-\phi)^{N/2}$, so $\bar{\Lambda}^m = \bar{\lambda}^{\theta^*,m} = \frac{1}{2}$.

The statements about t^θ for $\theta \notin \Theta^{\max}$ follow from the construction of $t(0), \ldots, t(K)$. *Q.E.D.*

Proof of Proposition A.2. For part 1, the key step is analogous to the proof of Proposition 5, modified to allow for endogenous entry. Let $m_0 = \sum_{n=1}^{N/2}\theta$ and $m_1 = \sum_{n=N/2+1}^{N}\theta_n$. By assumption, $m_0 > m_1$. By definition, $p^{\theta,m} = \phi^{m_0}(1-\phi)^{N/2-m_0}\phi^{N/2-m_1}(1-\phi)^{m_1} = \phi^{(m_0-m_1)+N/2}(1-\phi)^{N/2-(m_0-m_1)} = [\phi(1-\phi)]^{N/2}\left(\frac{\phi}{1-\phi}\right)^{m_0-m_1}$, and similarly $p^{\theta^{sym},m} = [\phi(1-\phi)]^{N/2}\left(\frac{1-\phi}{\phi}\right)^{m_0-m_1}$; since $\phi > \frac{1}{2}$, $p^{\theta,m} > p^{\theta^{\mathrm{sym}},m}$. At time 0 we thus have $\lambda_0^\theta = p^{\theta,m} > p^{\theta^{\mathrm{sym}},m} = \lambda_0^{\theta^{\mathrm{sym}}}$. Moreover, since p_f is defined with the roles of ϕ and $1-\phi$ reversed, $p^{\theta,f} = p^{\theta^{\mathrm{sym}},m} < p^{\theta,m} = p^{\theta^{\mathrm{sym}},f}$.

Since $\gamma^{\theta^{\mathrm{sym}}} = \gamma^\theta$, it follows that at time 0, if $\lambda_0^{\theta^{\mathrm{sym}}} > \frac{C}{\gamma^{\theta^{\mathrm{sym}}}P}$, then also $\lambda_0^\theta > \frac{C}{\gamma^\theta P}$. In addition, $p_m^\theta + p_f^\theta = p_m^{\theta^{\mathrm{sym}}} + p_f^{\theta^{\mathrm{sym}}}$. Thus, in the notation of Proposition A.1, for $t < \min(t^\theta, t^{\theta^{\mathrm{sym}}})$, both θ and θ^{sym} apply, and applying part 3(a) of Theorem A.1 to the relevant subsequence of $(\lambda_t)_{t\geq 0}$ as in the proof of Proposition A.1, $\frac{\lambda_t^\theta}{\lambda_{t-1}^\theta} = \frac{\lambda_t^{\theta^{\mathrm{sym}}}}{\lambda_{t-1}^{\theta^{\mathrm{sym}}}}$, and hence $\frac{\lambda_t^\theta}{\lambda_t^{\theta^{\mathrm{sym}}}} = \frac{\lambda_{t-1}^\theta}{\lambda_{t-1}^{\theta^{\mathrm{sym}}}} = \frac{\lambda_0^\theta}{\lambda_0^{\theta^{\mathrm{sym}}}} > 1$. Thus, $\lambda_t^\theta > \lambda_t^{\theta^{\mathrm{sym}}}$, so again, if $\lambda_t^{\theta^{\mathrm{sym}}} > \frac{C}{\gamma^{\theta^{\mathrm{sym}}}P}$, then also $\lambda_t^\theta > \frac{C}{\gamma^\theta P}$, i.e., $t^\theta \geq t^{\theta^{\mathrm{sym}}}$. In particular, if the inequality is strict and $t^{\theta^{\mathrm{sym}}} < t < t^\theta$, then researchers of type θ will apply at time t, but those of type θ^{sym} will not.

For part 2, We have

$$\begin{aligned}
A_t^m - A_t^f = & \sum_{\theta:\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P} p^{\theta,m} - \sum_{\theta:\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P} p^{\theta,f} = \\
& = \sum_\theta p^{\theta,m} 1_{\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P} - \sum_\theta p^{\theta,f} 1_{\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P} = \\
& = \sum_\theta p^{\theta,m} 1_{\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P} - \sum_\theta p^{\theta^{\mathrm{sym}},f} 1_{\lambda_t^{\theta^{\mathrm{sym}}} \geq \frac{C}{\gamma^{\theta^{\mathrm{sym}}}}P} = \\
& = \sum_\theta p^{\theta,m} \left(1_{\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P} - 1_{\lambda_t^{\theta^{\mathrm{sym}}} \geq \frac{C}{\gamma^{\theta^{\mathrm{sym}}}}P}\right) = \\
& = \sum_{\theta:\sum_{n=1}^{N/2}\theta_n > \sum_{n=N/2+1}^{N}\theta_n} p^{\theta,m} \left(1_{\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P} - 1_{\lambda_t^{\theta^{\mathrm{sym}}} \geq \frac{C}{\gamma^{\theta^{\mathrm{sym}}}}P}\right) + \\
& + \sum_{\theta:\sum_{n=1}^{N/2}\theta_n = \sum_{n=N/2+1}^{N}\theta_n} p^{\theta,m} \left(1_{\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P} - 1_{\lambda_t^{\theta^{\mathrm{sym}}} \geq \frac{C}{\gamma^{\theta^{\mathrm{sym}}}}P}\right) + \\
& + \sum_{\theta:\sum_{n=1}^{N/2}\theta_n < \sum_{n=N/2+1}^{N}\theta_n} p^{\theta,m} \left(1_{\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P} - 1_{\lambda_t^{\theta^{\mathrm{sym}}} \geq \frac{C}{\gamma^{\theta^{\mathrm{sym}}}}P}\right) = \\
& = \sum_{\theta:\sum_{n=1}^{N/2}\theta_n > \sum_{n=N/2+1}^{N}\theta_n} p^{\theta,m} \left(1_{\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P} - 1_{\lambda_t^{\theta^{\mathrm{sym}}} \geq \frac{C}{\gamma^{\theta^{\mathrm{sym}}}}P}\right) + \\
& + \sum_{\theta:\sum_{n=1}^{N/2}\theta_n > \sum_{n=N/2+1}^{N}\theta_n} p^{\theta^{\mathrm{sym}},m} \left(1_{\lambda_t^{\theta^{\mathrm{sym}}} \geq \frac{C}{\gamma^{\theta^{\mathrm{sym}}}}P} - 1_{\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P}\right) = \\
& = \sum_{\theta:\sum_{n=1}^{N/2}\theta_n > \sum_{n=N/2+1}^{N}\theta_n} (p^{\theta - p_m^{\theta^{\mathrm{sym}}},m}) \left(1_{\lambda_t^\theta \geq \frac{C}{\gamma^\theta}P} - 1_{\lambda_t^{\theta^{\mathrm{sym}}} \geq \frac{C}{\gamma^{\theta^{\mathrm{sym}}}}P}\right) \geq 0.
\end{aligned}$$

The third equality follows from the fact that $\theta \mapsto (1-\theta_n)_{n=1}^N$ is a bijection. The fourth follows from the fact that $p^{\theta^{\mathrm{sym}},f} = p^{\theta,f}$. To obtain the fifth, we break up the sum into types θ with more (resp. as many, resp. fewer) characteristics between 1 and $N/2$ than between $N/2+1$ and N. For the sixth, observe that if a type θ has the same number of features between 1 and $N/2$ and between $N/2+1$ and N, then $p^{\theta,m} = p^{\theta^{\mathrm{sym}},m}$ and so $\lambda_0^\theta = \lambda_0^{\theta^{\mathrm{sym}}}$; arguing as in Proposition A.2, $\lambda_t^\theta = \lambda_t^{\theta^{\mathrm{sym}}}$ for all $t \geq 0$ (note that as soon as one type stops applying, so does the other); but then, since also $\gamma^\theta = \gamma^{\theta^{\mathrm{sym}}}$, the term in parentheses for such types is identically zero. In addition, we express the sum over θ's for which $\sum_{n=1}^{N/2}\theta_n < \sum_{n=N/2+1}^{N}\theta_n$ iterating over types θ for which $\sum_{n=1}^{N/2}\theta_n > \sum_{n=N/2+1}^{N}\theta_n$, but adding up terms corresponding to the associated symmetric types θ^{sym}. The seventh equality is immediate. Finally, the inequality follows because, for θ such that $\sum_{n=1}^{N/2}\theta_n > \sum_{n=N/2+1}^{N}\theta_n$, the term in parentheses is non-negative by Proposition A.2, and in addition $p^{\theta > p_m^{\theta^{\mathrm{sym}}},m}$. *Q.E.D.*

Proof of Proposition A.3 Let θ^a and θ^b be the types of the two young researchers. We assume that the type of the joint project is the elementwise maximum of θ^a and θ^b: that is, the project displays characteristics i if and only if at least one of the researchers displays it.

For $g = m, f$, let $\Theta^g = \{(\theta, \theta') : \theta \vee \theta' = \theta^g\}$, where $\vee$ denotes the component-wise maximum. Note that, if $(\theta, \theta') \in \Theta^m$, then $\theta_i = \theta'_i = 0$ for $i = N/2 + 1, \ldots, N$; similarly, if $(\theta, \theta') \in \Theta^f$, then $\theta_i = \theta'_i = 0$ for $i = 1, \ldots, N/2$. Moreover, $(\theta, \theta') \in \Theta^g$ iff $(\theta', \theta) \in \Theta^g$ for $g = m, f$. Finally, $(\theta, \theta') \in \Theta^m$ if and only if $(\bar{\theta}, \bar{\theta}') \in \Theta^f$, where $\bar{\theta}, \bar{\theta}'$ are defined by $\bar{\theta}_{i+N/2} = \theta_i$, $\bar{\theta}'_{i+N/2} = \theta'_i$ and $\bar{\theta}_i = \bar{\theta}'_i = 0$ for $i = 1, \ldots, N/2$; furthermore, these types satisfy

$$p^{\theta,m} = p^{\bar{\theta},f} \qquad \text{and} \quad p^{\theta',f} = p^{\bar{\theta}',m}. \tag{A.55}$$

Then, invoking the above properties, the probability that the joint project is accepted—that is, the probability that $\theta^a \vee \theta^b \in \{\theta^m, \theta^f\}$—is

$$\begin{aligned}
&\gamma^{\theta^m}\bar{\lambda}^{\theta^m} \sum_{(\theta,\theta')\in\Theta^m} p^{\theta,m}\cdot p^{\theta',f} + \gamma^{\theta^f}\bar{\lambda}^{\theta^f} \sum_{(\theta,\theta')\in\Theta^f} p^{\theta,m}\cdot p^{\theta',f} \\
=&\gamma^{\theta^m}\bar{\lambda}^{\theta^m} \sum_{(\theta,\theta')\in\Theta^m} p^{\theta,m}\cdot p^{\theta',f} + \gamma^{\theta^f}\bar{\lambda}^{\theta^f} \sum_{(\theta,\theta')\in\Theta^m} p^{\bar{\theta},m}\cdot p^{\bar{\theta}',f} \\
=&\gamma^{\theta^m}\bar{\lambda}^{\theta^m} \sum_{(\theta,\theta')\in\Theta^m} p^{\theta,m}\cdot p^{\theta',f} + \gamma^{\theta^f}\bar{\lambda}^{\theta^f} \sum_{(\theta',\theta)\in\Theta^m} p^{\bar{\theta}',m}\cdot p^{\bar{\theta},f} \\
=&\gamma^{\theta^m}\bar{\lambda}^{\theta^m} \sum_{(\theta,\theta')\in\Theta^m} p^{\theta,m}\cdot p^{\theta',f} + \gamma^{\theta^f}\bar{\lambda}^{\theta^f} \sum_{(\theta',\theta)\in\Theta^m} p^{\theta',f}\cdot p^{\theta,m} \\
=&\gamma^{\theta^m}\bar{\lambda}^{\theta^m} \sum_{(\theta,\theta')\in\Theta^m} p^{\theta,m}\cdot p^{\theta',f} + \gamma^{\theta^f}\bar{\lambda}^{\theta^f} \sum_{(\theta,\theta')\in\Theta^m} p^{\theta,f}\cdot p^{\theta',m} \\
=&(\gamma^{\theta^m}\bar{\lambda}^{\theta^m} + \gamma^{\theta^f}\bar{\lambda}^{\theta^f}) \sum_{(\theta,\theta')\in\Theta^m} p^{\theta,m}\cdot p^{\theta',f} \\
=&\gamma_0\rho^{N/2} \sum_{(\theta,\theta')\in\Theta^m} p^{\theta,m}p^{\theta',f} \equiv \gamma_0\rho^{N/2}\Pi,
\end{aligned}$$

where the last equality follows from the definition of γ^θ and the fact that θ^m, θ^f are the only surviving types.

Now let $L(\theta) = \sum_i \theta_i$. We claim that the expectation of $L(\theta^a) - L(\theta^b)$ conditional on $\theta^a \vee \theta^b \in \{\theta^m, \theta^f\}$ is strictly positive—that is, the expected quality of a, the young M coauthor, is strictly higher than the expected quality of that of the young F coauthor b.

First,

$$\begin{aligned}
\Delta \equiv & \sum_{(\theta,\theta')\in\Theta^m} p^{\theta,m}\cdot p^{\theta',f}[L(\theta)-L(\theta')] \\
= & \sum_{(\theta,\theta')\in\Theta^m:L(\theta)>L(\theta')} p^{\theta,m}\cdot p^{\theta',f}[L(\theta)-L(\theta')] + \sum_{(\theta,\theta')\in\Theta^m:L(\theta)<L(\theta')} p^{\theta,m}\cdot p^{\theta',f}[L(\theta)-L(\theta')] \\
= & \sum_{(\theta,\theta')\in\Theta^m:L(\theta)>L(\theta')} [p^{\theta,m}\cdot p^{\theta',f}-p^{\theta',m}\cdot p^{\theta,f}][L(\theta)-L(\theta')] > 0.
\end{aligned}$$

The last equality follows because $(\theta,\theta')\in\Theta^m$ if and only if $(\theta',\theta)\in\Theta^m$, and of course $L(\theta)>L(\theta')$ iff $L(\theta')<L(\theta)$. The inequality follows because, if $L(\theta)>L(\theta')$, then by assumption $p^{\theta,m}>p^{\theta',m}$ and $p^{\theta',f}>p^{\theta,f}$.

Repeating the calculations for Θ^f and again appealing to the properties of pairs $(\theta,\theta')\in\Theta^m$ and the corresponding types $(\bar\theta,\bar\theta')\in\Theta^f$,

$$\begin{aligned}
& \sum_{(\theta,\theta')\in\Theta^f} p^{\theta,m}\cdot p^{\theta',f}[L(\theta)-L(\theta')] = \sum_{(\theta,\theta')\in\Theta^f:L(\theta)>L(\theta')} [p^{\theta,m}\cdot p^{\theta',f}-p^{\theta',m}\cdot p^{\theta,f}][L(\theta)-L(\theta')] \\
= & \sum_{(\theta,\theta')\in\Theta^m:L(\theta)>L(\theta')} [p^{\bar\theta,m}\cdot p^{\bar\theta',f}-p^{\bar\theta',m}\cdot p^{\bar\theta,f}][L(\bar\theta)-L(\bar\theta')] \\
= & \sum_{(\theta,\theta')\in\Theta^m:L(\theta)>L(\theta')} [p^{\theta,f}\cdot p^{\theta',m}-p^{\theta',f}\cdot p^{\theta,m}][L(\theta)-L(\theta')] = \\
= & -\sum_{(\theta,\theta')\in\Theta^m} p^{\theta,m}\cdot p^{\theta',f}[L(\theta)-L(\theta')] = -\Delta.
\end{aligned}$$

Finally, the expected difference in the number of characteristics of θ^a and θ^b is

$$\mathrm{E}[L(\theta^a)-L(\theta^b)|\theta^a\vee\theta^b\in\{\theta^m,\theta^f\}] = \frac{\gamma^{\theta^m}\bar\lambda^{\theta^m}\Delta-\gamma^{\theta^f}\bar\lambda^{\theta^f}\Delta}{\gamma_0\rho^{N/2}\Pi} = \frac{\rho^{N/2}\Delta}{\Pi}(\bar\lambda^{\theta^m}-\bar\lambda^{\theta^f}) > 0,$$

as asserted.

Q.E.D