Leverage

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Abstract

A frictionless general equilibrium model featuring heterogeneous time-varying risk tolerance explains the business cycle dynamics of intermediary leverage, aggregate credit, and many asset markets' facts. In booms, when risk tolerance is high, households borrow more and aggregate credit increases funded by higher intermediary debt. In recessions, credit contracts and intermediaries delever. Yet, their debt-to-equity ratios increase as equity drops when risk aversion increases. Because households borrow more or less as their risk tolerance increases or decreases, the intermediary's balance sheet forecasts stock returns both in the time series and the cross section. Moreover, credit expansions correlate with negatively skewed stock returns, low credit spreads, and predict lower future returns.

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1 Introduction

As shown by the macrofinance literature, financial intermediaries' balance sheets expand and contract over the business cycle: In good times, intermediaries' debt increases together with their credit to the economy, while in bad times, intermediaries delever and lending dries up (see e.g. Adrian and Shin (2014)). While intermediaries' debt is procyclical, their debt-to-equity is countercyclical, as equity values also drop in bad times (He and Krishanmurthy (2013)). A large literature emerged in the wake of the 2008 financial crisis emphasizing the impact of intermediaries' borrowing and lending activities, and various frictions and behavioral biases, on asset prices. This view places the excessive growth in intermediaries' leverage and their subsequent deleveraging at the heart of the 2008 financial crisis. Consider for instance Panels A, C, and D of Figure 1. These panels show that the 2008 stock market crash was accompanied by large increases in the VIX and trading volume, significant mutual fund outflows, a widening of the credit spreads and an increase in intermediaries' debt-to-equity ratio. To corroborate the importance of intermediaries' on asset prices, the recent empirical literature has also documented that increasing intermediary balance sheets forecast asset returns in the time-series and in the cross-section, negatively correlate with credit spreads, and increase the likelihood of market crashes.

In this paper we propose a rational, frictionless model with heterogeneous agents that can address these empirical regularities. The key novel modeling ingredient is time-varying heterogeneity in households' risk attitudes over the business cycle. This heterogeneous variation in risk attitudes introduces incentives for risk sharing and trading, which households in our model implement through financial intermediaries. As a result, the balance sheet of these intermediaries reflects the changing risk sharing needs of the household sector. Time-series variation in risk aversion heterogeneity also implies time-varying discount rates, which affect asset prices. As a result, intermediaries' balance sheets comove with asset prices over the business cycle, although there is no causal link between the two. We show how this mechanism can reconcile a variety of facts in the intermediary asset pricing literature.

There are multiple reasons why risk aversion of some households may increase more than others in downturns, such as demographics, wealth heterogeneity, changes in background risk (e.g. Gollier and Pratt (1996)) or consumption commitments (e.g. Chetty and Szdeil (2007)). There is evidence of countercyclical variation in risk aversion. Bricker et al. (2015, p. 342-3) use the Survey of Consumer Finances to show that households' willingness to take financial risk decreased during the recession of 2008-09, especially for families that moved down in the wealth distribution by more than 10 percentage points. Similarly, Guiso et al. (2018) provide empirical evidence of heterogeneous variation in risk preferences amongst Italian households during the 2008 crisis. Cohn et al. (2015) provide experimental evidence of countercyclical risk aversion in financial professionals.

We model time-varying risk aversion by assuming households preferences feature external habits but with different degrees of intensity. The model aggregates to a version of the well-known external habit persistence model of Campbell and Cochrane (1999), which we calibrate to match standard asset pricing moments. We use this calibration to assess the impact of aggregate shocks on households' borrowing and lending, and thus their impact on intermediaries' balance sheets, and their co-movement with asset prices, credit spreads and other variables of interest.

The mechanism advanced in this paper can shed light on the joint behavior of asset prices and financial intermediaries' balance sheets beyond episodes such as the global financial crisis. Consider, for example, Panels B, D, and F of Figure 1. These panels plot the same quantities as in the left-hand-side panels but now for the first quarter of 2020, which was dominated by the COVID-19 pandemic. The patterns are indistinguishable from those observed in 2008 but clearly intermediary leverage is not the proximate cause of the events in 2020. These dynamics are consistent instead with our model, in which leverage is an endogenous variable driven by heterogeneous time varying risk aversion. Moreover, the mutual funds outflows that took place in both crisis (Panels E and F) highlight the active role of households' rebalancing during crises, in general.

Ours is an endowment economy. Households' endowments feature both aggregate and idiosyncratic risk, which induces additional motives for risk sharing and trading. We follow others and assume that the households access some financial services through a set of competitive financial intermediaries, (e.g. He and Krishnamurthy (2013)) which offer two benefits to households. First, intermediaries eliminate idiosyncratic risks by pooling endowments and marketing shares on the aggregate output. Second, they borrow and lend from and to households via a money market account. Households go on to trade dynamically the shares on aggregate output and the money market account to achieve their desired allocation across time and states. Intermediaries borrowing and lending is thus endogenous and depend on households' trading needs.

We fully characterize the competitive equilibrium and derive novel implications for household and intermediary leverage and their relation with asset prices. Consider households first. In our model, income is exogenous but both household debt and net worth are endogenous. As aggregate income grows households become more risk tolerant and some of them become more tolerant than others. The more risk tolerant households are willing to take on additional risk through debt and increase their exposure to aggregate market conditions. This second effect gives an additional "kick" to the amount of debt borrowing households take and thus the debt taken by borrowing households grows faster than income. To accommodate this increase in borrowing needs, the intermediaries' balance sheet needs to expand.

Households differ along two dimensions in our model: risk preferences and initial wealth. We characterize borrowing patterns in the cross section and investigate the model's ability to match some stylized facts. The model is flexible enough to match two important characteristics of the data, shown in Figure 2. First, the lower the net worth quartile to which the household belongs, the higher the ratio of debt to net worth. Second, during the 2008 crisis, the debt-to-asset ratio increased, and especially for households in the lower quartiles. While our model matches both patterns, it misses on the levels and is only able to explain half of the leverage of low wealth households in the data. The model thus illustrates the limits associated with fluctuations in the households' attitudes towards risk as an explanatory variable for household leverage and its behavior over the cycle: Other factors need to be brought to bear to make up for the difference. Similarly, while heterogeneity in risk preferences and endowments yields wealth inequality, our calibration is not able to match the extreme wealth inequality in the data. However, an extension of the model featuring temporary idiosyncratic shocks to risk aversion is able to match the dispersion in idiosyncratic and systematic consumption growth as in the data.

The model features a financial intermediary, whose balance sheet inherits the properties of the balance sheet of the household sector. The financial intermediaries issues the safe short term deposits that risk averse households demand to smooth consumption. But the financial intermediary faces a constraint: It is only able to supply these securities if it is able to originate the assets to back them up, that is, only if there are households that are willing to borrow. This is the role of risk tolerant households: They are willing to lever up and thus provide the assets that back the financial intermediaries' liabilities. In benign economic conditions, risk tolerant households become even more tolerant and thus willing to lever up more, producing the pro-cyclicality of the intermediaries' balance sheet. As the economy experiences negative shocks, premia and volatility increase, and risk tolerant households delever, shrinking the intermediaries' balance sheet.

Our framework can address several empirical regularities that have been the focus of the macro-finance literature. Consider three such regularities. First, some, such as Adrian and Shin (2014) for instance, have argued that the financial intermediary leverage is driven by value-at-risk (VaR)-like constraints: There is a negative relation between changes in VaR and changes in leverage. To illustrate, the top two panels of Figure 3 reproduce Figure 3 in Adrian and Shin (2014). They plot the change in assets against changes in book (left panel) and enterprise leverage (right panel). Given that asset growth is procyclical, the plots show that market leverage is strongly countercyclical whereas book leverage is procyclical. The bottom panel of Figure 3 shows that indeed an increase in Value-of-Risk of financial institutions – that is, the volatility of their risky assets – correlates with a decline in their (book) leverage as intermediaries' deleverage. Our model shows that this pattern is due to time-varying discount rate effects, which increase asset volatility (and hence VaR) and decrease debt in bad times, as discussed above.

Second, Adrian, Etula and Muir (AEM, 2014) and He, Kelly and Manela (HKM, 2017) argue that measures of financial intermediary leverage are priced factors in tests of the cross section of returns. This evidence is reported in Panel A of Table 1 which uses the same data in AEM and HKM to price the Fama French 25 value and size sorted portfolios. The first column reports the CAPM regression, in which the aggregate market portfolio is the main risk factor. The failings of the CAPM are well known: The R^2 is a puny 6.5%, the alpha is strongly positive, and the average market return is negative. The second column shows that HKM market leverage is able to explain a large fraction of the variation of the portfolios. The market return becomes positive (but not statistically significant), the alpha is zero, and the market price of risk is negative, and significant. Similarly, column III shows the same results for AEM book leverage, and obtains similar results, but now with a positive market price of risk. Column IV and V show that the interaction of market return and the *level* of lagged leverage also lead to a substantially improved fit, especially for market leverage. Finally, Panels B and C of Table 1 are the time series counterparts of Panel A, in which we regress future excess returns on book and market leverage, respectively. Book leverage has only mild predictive power of future returns, and with a negative sign. In contrast, market leverage displays stronger predictability of future returns, with a positive sign: High aggregate market leverage predicts higher future returns.

This evidence is seen as supporting He and Krishnamurthy's (2013) view that financial institutions are the "marginal agent" in asset markets. We instead make a simple point: The size of the balance sheet of the financial intermediary, and any measure related to it, is limited by the willingness by part of agents in the economy to borrow and lend. This willingness fluctuates with the agents' attitudes towards risk and thus so will the financial intermediaries' balance sheet. While measuring the risk attitude of households is challenging, the intermediary's balance sheet may simply act as a proxy of this aggregate risk aversion of the economy, which is easy to observe. Hence the empirical results obtain in the intermediary asset pricing literature. In addition, we clarify a debate regarding the sign of the market price of leverage risk. This price can be positive or negative depending on whether debt is normalized by some measure of income, which is slow moving, or the intermediaries equity, which incorporates the discount shocks that are key in our analysis. Our model shows that "market leverage" is better able to predict future returns than "book leverage," a robust finding in the data. Moreover, our model reproduces the different signs of the factors suggested by AEM and HKM.

Finally, our model speaks to the dynamics credit cycles and the variation in credit spreads. Recent research (Baron and Xiong (2017), and Muir (2019)) show that credit expansions are correlated with low credit spreads, lower future stock returns, and negatively skewed future returns, in which a market crash is more likely than a market boom. Our model predicts these empirical regularities. The balance sheet of intermediaries expands in good times as households' lower risk aversion increases their borrowing needs. Lower risk aversion, however, also implies low credit spreads and low future expected equity returns, thereby establishing the negative relation between credit/output, low credit spreads, and future returns. Moreover, the high credit/output ratio in good times is also associated with negatively skewed returns, in that there is a higher probability of a large decrease in equity prices in the future than a symmetric increase. Intuitively, in good times aggregate risk aversion is at its lowest level. Negative shocks to output decrease equity prices directly, because of lower output, and indirectly, because of an increase in risk aversion. This double kick to prices makes it more likely to have a large drop in good times than in bad times, generating the asymmetry observed in the data. Simulations of our model thus reproduce salient empirical regularities without frictions or any causality link.

Related literature. This paper is obviously connected to the literature on optimal risk sharing, starting with Borch (1962). Much of this literature is concerned with assessing to what extent consumers are effectively insured against idiosyncratic shocks to income and wealth (See for instance Dynarski and Sheffrin (1987), Cochrane (1991), Mace (1991) and Townsend (1994).) Our paper is closely related to Dumas (1989), Wang (1996), Bolton and Harris (2013), Longstaff and Wang (2012), and Bhamra and Uppal (2014). These papers consider two groups of agents with constant risk aversion, and trading and asset prices are generated by aggregate shocks through the variation in the wealth distribution. While similar in spirit, our model generates several novel results that do not follow from this previous work, such as procyclical debt to income ratios, countercyclical debt to wealth ratios, higher leverage amongst poorer households, consistency with asset pricing facts, and so on. Our model is more closely related to Chan and Kogan (2002), who also consider a continuum of households with habit preferences and heterogeneous risk aversion. In their setting, however, households' risk aversions are constant, while in our setting they are time varying in response to business cycle variation, a crucial ingredient in our model. Moreover, Chan and Kogan (2002) do not investigate the leverage dynamics implied by their model, which is our focus. Finally, our model has implications for the dynamics of the supply of safe assets and their relation to aggregate variables and thus connects as well to the recent literature on safe assets such as Barro and Mollerus (2014) and Caballero and Fahri (2014).

Our model is related to Campbell, Grossman and Wang (1993), which explores the implications for trading volume and asset prices in a model where the motivation for trade is driven by shocks to agents' risk tolerance. More recently Alvarez and Atkenson (2017) consider a model where agents' risk tolerance is subject to uninsurable idiosyncratic shocks. In our paper instead variation in risk tolerance is driven by exposure to a business cycle factor, and the source of heterogeneity, in addition to initial endowment, is the degree of exposure to that factor. Neither Campbell, Grossman and Wang (1993) nor Alvarez and Atkenson (2017) analyze the dynamics of leverage and the distribution of leverage in the population.

Finally, our model is also related to the literature on the leverage cycle. Geanakoplos (2009) and Gorton and Metrick (2012) argue that procyclical leverage is the mirror image of increased collateral requirement during downturns (increased "haircuts"). Geanakoplos terms such dynamics "the leverage cycle," arguing that leverage is high when volatility is low and prices are high. He and Krishnamurthy (2013) provide a theoretical model of procyclical market leverage also based on the constraints of financial institutions. All these theories however fail to simultaneously reconcile the pro- and countercylicality of book and market leverage, respectively, and the other features of the data we highlighted above. To be clear, constraints do matter and are likely to explain some of what happens during crises, but it is also equally obvious that agents are much more risk averse in crises than in periods of benign economic conditions and that these changes in the attitudes towards risk transmit to the financial sector. It is difficult to judge the quantitative importance of the financial intermediation channel without simultaneously accounting for the importance of these discount rate shocks.

2 The model

Preferences and endowments. We posit a continuous time single good exchange economy populated by a continuum of households indexed by i. These households have preferences for period $t \in [0, \infty)$ over consumption C_{it} given by

$$u\left(C_{it}; \psi_{it}, Y_t, t\right) = e^{-\rho t} \log\left(C_{it} - \psi_{it} Y_t\right). \tag{1}$$

Utility is then derived from the distance between individual consumption and aggregate output Y_t, scaled by the process ψ_{it}. Y_t follows

$$\frac{dY_t}{Y_t} = \mu_Y \; dt \; + \; \sigma_Y(I_t) \; dZ_t. \tag{2}$$

where Z_t is a Brownian motion, μ_Y is constant, and the volatility $\sigma_Y(I_t)$, which refer to as *economic uncertainty*, depends on a state variable I_t that summarizes the state of the economy. I_t is a *recession indicator* and follows the mean reverting process:

$$dI_t = k \; (\overline{I} - I_t) \; dt - v \; I_t \; \left[\frac{dY_t}{Y_t} - \mu_Y dt\right]. \tag{3}$$

That is, I_t increases after bad aggregate shocks, $\frac{dY_t}{Y_t} < \mu_Y dt$, and it hovers around its central tendency $\overline{I}$. The parameter k is the speed of mean reversion and it measures the average length of booms and recessions.

Finally, ψ_{it} in (1) is a function on I_t, $\psi_{it} \equiv \psi_i(I_t)$. To obtain closed form solutions for prices and quantities we assume a specific functional form for $\psi_i(\cdot)$:

$$\psi_i(I_t) = \gamma_i \left(1 - I_t^{-1}\right), \tag{4}$$

where γ_i are positive constants normalized so that $\int \gamma_i di = 1$ and we assume throughout that $I_t > 1$ so that $\psi_i(I_t) > 0$. We achieve this by assuming that $\sigma_Y(I_t) \to 0$ as $I_t \to 1$. (The simple notation $\int di$ indicates the integration over agents' density $f(\gamma_i, \omega_i)$.)

Intuitively, ψ_{it} regulates the local curvature of the utility function, with higher ψ_{it} implying a higher curvature. Indeed, for given consumption C_{it} and output Y_t, the local risk aversion (LRA) is

$$\text{LRA}_{it} \equiv -\frac{u_{CC} \; C_{it}}{u_C} = 1 + \frac{\psi_{it}}{C_{it}/Y_t - \psi_{it}}. \tag{5}$$

As we will show below (see equation (9)), $C_{it}/Y_t > \psi_{it}$ and thus households have a *LRA* strictly greater than that of log utility. Attitudes towards risk are thus determined by both ψ_{it} and the consumption share C_{it}/Y_t. For a given consumption share, a higher ψ_{it} implies a higher LRA. From (4), ψ_{it} is monotonically increasing in γ_i and the recession indicator I_t. Therefore, our preference specification implies that households with higher γ_i have higher LRA and, in addition, all households' LRAs increase in recessions, when I_t is higher, albeit heterogeneously depending on γ_i. Our model thus allows us to introduce heterogeneous variation in households' risk preferences during the business cycle in a simple way.

To conclude the model, each agent is born at time 0 endowed with a tree that produces Y_{it}. We do not need to make assumptions on Y_{it} except that its aggregate $Y_t = \int Y_{i,t} di$

follows the dynamics in (2). The time-0 values of households' stochastic endowments are heterogeneous and denoted by ω_i. We normalize prices so that $\int \omega_i di = 1$.

Financial intermediary and financial markets. As we will show, households want to shed off idiosyncratic risk but they cannot do so on their own: They need a financial intermediary to pool all idiosyncratic risks and create shares of the aggregate output, $Y_t = \int Y_{i,t}di$. Specifically, at date $t = 0$ households form a competitive financial intermediary, in which the paid in capital consists of their individual trees. In exchange, households receive a portfolio of shares on the aggregate output Y_0, short term loans, L_0, and deposits D_0. After that households trade shares on the aggregate output and borrow and lend from the intermediary. Let r_t be the borrowing and lending rate. The initial capital of the financial intermediary is then the household sector paid in capital, $P_0 = \int_i P_{i,0}\, di$.

Figure 4 shows the balance of sheets of the two sectors in this economy, which consists of the household and the financial sector, at time t. Start with line (1) in Figure 4. This is the initial position of the household sector, namely its tree, which is valued at $P_{i,t}$. As shown in line (2), household i has a liability towards the financial intermediary, valued at $P_{i,t}$; in exchange the household receives loans L_{it}, deposits, $D_{i,t}$ and shares of the aggregate output issued by the financial intermediary, as shown in lines (3), (4), and (5) of Figure 4, respectively. The financial intermediary then operates like an "ETF" bundled with a money market account from which the households can borrow and lend freely.

Let $N_{i,t}$ then be the number of shares of the aggregate stock held by household i at time t. We adopt the convention that if a household i borrows from the financial intermediary, then $L_{i,t} > 0$ and $D_{i,t} = 0$ and $L_{i,t} = 0$ and $D_{i,t} > 0$ if it lends. As we will show below, households are either risk tolerant (RT) and they lever up, $L_{i,t} > 0$, or risk averse (RA) in which case they save, $D_{i,t} > 0$. The net worth $W_{i,t}$ of each household i at time $t \geq 0$ is then

$$W_{i,t} = \begin{cases} N_{i,t}P_t - L_{i,t} & \text{for the Risk Tolerant (RT) household} \\ N_{i,t}P_t + D_{i,t} & \text{for the Risk Averse (RA) household} \end{cases} \tag{6}$$

Given ω_i is the value of household's initial endowment, it has to be that $\omega_i = W_{i,0}$.

Discussion. As shown in equation (5), the key ingredient in our model is cross sectional heterogeneity in risk attitudes and the dependence of that heterogeneity on aggregate economic conditions. There is substantial evidence of cross sectional dispersion in attitudes towards risk in the population (see, for instance, Barsky, Juster, Kimball and Shapiro (1997), Guiso and Paiella (2008) and Chiappori and Paillea (2011)). As for the time series variation, Guiso, Sapienza and Zingales (2018), using a large sample of clients of an Italian bank, find that measures of risk aversion increased after the 2008 financial crisis, and such increases are heterogeneous in the population. They find that the increase in risk aversion is more pronounced for those experiencing large losses in wealth, though the increase in risk aversion occurs even for those agents who did not experience any loss. In our model all variables are perfectly correlated and thus we cannot produce this "pure" discount effect. Using the Survey of Consumer Finances Bricker et al. (2015) show that households willingness to take financial risk decreased during the recession of 2008-09, and that this effect that was particularly pronounced for families that moved down in the wealth distribution by more than 10 percentage points. Cohn, Engelman, Fehr and Marechal (2015) provide experimental evidence of countercyclical risk aversion in financial professionals.

In addition, for a given ψ_{it}, households who are richer consume more of aggregate output and thus from (5) result in a lower curvature. That is, our preference specification imply non-homotheticity at the individual level. There is strong evidence in favor of this property in the data; roughly, richer households are less risk averse. Households with higher endowment thus increase the share of wealth invested in the risky asset, an empirical regularity found in surveys of household finances even when restricted to those who participate in the stock market (Wachter and Yogo, 2010). More generally, the portfolio allocation predictions of our model are consistent with the empirical evidence of Calvet and Sodini (2014).

In sum, there are two sources of differentiation across households, wealth and attitudes towards risk. To illustrate the role of each, we also investigate the two polar cases of homogeneous preferences ($\gamma_i = 1$ for all i) or homogeneous endowments ($\omega_i = 1$ for all i).

Finally, we need an assumption to guarantee that households' marginal utility is positive in all possible states. The following is a sufficient condition and assumed throughout,

$$\frac{\omega_i}{\gamma_i} > \left(1 - \overline{I}^{-1}\right) \qquad \text{for all} \quad i. \tag{A1}$$

Below we show that the cross-section of equilibrium risk aversion solely depends on the ratio ω_i/γ_i. Thus (A1) restricts preferences to a minimum curvature of the utility function.

In our model, the role of the intermediary is to complete financial markets even in the absence of Arrow-Debreu securities, pooling idiosyncratic risks and selling claims on the aggregate endowment.Notice that its balance sheet is perfectly hedged, due to the law of large numbers. More broadly, we have ruled the possibility of default entirely: Neither borrowing households, nor the financial intermediary can default on promises made.

Finally, our model can also help shed light on the patterns of international capital flows. Indeed, if foreign households are more risk averse (see e.g. Gourinchas, Rey, and Govillot (2017), Maggiori (2017)), US households can lever up in order to supply the safe assets needed by foreigners to achieve their consumption smoothing needs. Moreover, shocks that increase the aversion to risk by the rest of the world translate into higher US household indebtness, as in Bernanke (2005).

3 Equilibrium

The portfolio problem. Given prices $\{P_t, r_t\}$ households choose consumption C_{it}, the amount of shares of the intermediary stock N_{it}, and the amount of deposits D_{it} or loans L_{it} to maximize their expected utilities

$$\max_{\{C_{it}, N_{it}, D_{it}, L_{it},\}} E_0 \left[\int_0^\infty e^{-\rho t} \log \left(C_{it} - \psi_{it} Y_t \right) dt \right]$$

subject to the dynamic budget constraint

$$dW_{it} = N_{it}(dP_t + Y_t dt) + (D_{it} - L_{it}) r_t dt - C_{it} dt \qquad \text{with} \qquad W_{i,0} = \omega_i.$$

The optimal allocation only depends on the net position $(D_{it} - L_{it})$. We break the indeterminacy by assuming that only either D_{it} or L_{it} can be positive at any given time t.

Definition of a competitive equilibrium. A competitive equilibrium is a series of stochastic processes for prices $\{P_t, r_t\}$ and allocations $\{C_{it}, N_{it}, D_{it}, L_{it}\}_{i \in \mathcal{I}}$ such that households maximize their intertemporal utility and markets clear $\int C_{it} di = Y_t$, $\int N_{it} di = 1$, and the intermediary balance sheet clears $\int D_{it} di = \int L_{it} di$. The economy starts at time 0 in its stochastic steady state $I_0 = \overline{I}$. Without loss of generality, we normalize the initial output $Y_0 = \rho$ for notational convenience.

For later reference, it is useful to illustrate some steps of the derivation of the competitive equilibrium. Details are contained in the Internet Appendix. Because markets are dynamically complete, each agent solves the static problem

$$\max_{\{C_{it}\}} E_0 \left[\int_0^\infty e^{-\rho t} \log \left(C_{it} - \psi_{it} Y_t \right) dt \right] \text{ subject to } E_0 \left[\int_0^\infty M_t \; C_{it} dt \right] \leq w_i \; M_0, \qquad (7)$$

where M_t is the state price density. The first order condition of the corresponding Lagrangean implies that

$$u_C(C_{it}; \psi_{it}, Y_t, t) = \frac{e^{-\rho t}}{C_{it} - \psi_{it} Y_t} = \frac{1}{\phi_i} M_t \qquad \text{for all } i, \qquad (8)$$

where ϕ_i is the inverse of the Lagrange multiplier of the static budget constraint in (7), normalized such that $\int \phi_i di = 1$. It is easy to show that[1]

$$M_t = e^{-\rho t}\, Y_t^{-1}\, I_t \qquad \text{and} \qquad C_{it} = \left[\psi_{it} + \phi_i I_t^{-1}\right] Y_t. \tag{9}$$

Consumption for all households is increasing in aggregate output but it increases more for those households for whom ψ_{it} is larger as their marginal utility increases more with increases in aggregate output.[2]

Proposition 1 *(Efficient allocation). Let the economy be at its stochastic steady state at time 0, $I_0 = \overline{I}$, and normalize $Y_0 = \rho$. Then (a) the (inverse of) Lagrange multipliers are*

$$\phi_i = \gamma_i + (\omega_i - \gamma_i)\overline{I} \tag{10}$$

(b) The optimal consumption path for household i is given by

$$C_{it} = s_i\left(I_t\right) Y_t \qquad \textit{with} \qquad s_i\left(I_t\right) = \gamma_i + \left(w_i - \gamma_i\right) \frac{\overline{I}}{I_t} \in (0,1) \tag{11}$$

The inverse of the Lagrange multipliers ϕ_i in (10) are increasing in the initial aggregate endowment ω_i and decreasing in γ_i (as $\overline{I} > 1$). Higher initial endowment loosens the financial constraint and thus reduces the Lagrange multiplier. Similarly, higher γ_i increases the marginal utility of consumption and thus the desire to increase consumption, making the financial constraint tighter. The inverse of the Lagrange multipliers ϕ_i can also be interpreted as Pareto weights in a planner problem, and we refer to them as such at times.

Equation (11) shows the equilibrium sharing rule. Households with high endowment w_i or low γ_i enjoy a high consumption share $s_i\left(I_t\right) = C_{it}/Y_t$ during good times, that is, when the recession indicator I_t is low, and vice versa. This is intuitive given the discussion of local risk aversion in (5). Indeed, substituting now in LRA the equilibrium consumption we find

$$\text{LRA}_{it} = -\frac{C_{it} u_{cc}(C_{it}, \psi_{it}, Y_t, t)}{u_c(C_{it}, \psi_{it}, Y_t, t)} = \frac{I_t + (\omega_i/\gamma_i - 1)\overline{I}}{1 + (\omega_i/\gamma_i - 1)\overline{I}} \tag{12}$$

Given that $I_t > 1$, in equilibrium, each agent i's LRA_{it} is decreasing in ω_i/γ_i: Households with low initial endowment ω_i relative to γ_i are more risk averse in equilibrium than those

[1] It is enough to solve for C_{it} in (8), integrate across households, and use the resource constraint $\int C_{it} di = Y_t$ to yield M_t. Plugging this expression in (8) yields C_{it}. Note that because optimal consumption only depends on aggregate shocks, households do not want to hold any idiosyncratic risk in their portfolios.

[2] Marginal utilities remain positive, $\left(C_{it} - \psi_{it} Y_t\right)^{-1} = \frac{I_t}{\phi_i Y_t} > 0$, and thus households' utilities are well defined. The marginal utility is lower the higher the aggregate output and the lower the recession indicator.

with high endowment relative to γ_i. Efficiency calls for households with $\omega_i > \gamma_i$ to consume a bigger share of aggregate output in good times in exchange for a lower share in bad times thus insuring households with $\omega_i < \gamma_i$. This effect is standard in the risk sharing literature that features households with CRRA preferences (see Longstaff and Wang (2014) and Veronesi (2018)). In our model though there are two additional effects relative to that literature. First, in our framework the amount of risk sharing also depends on the recession index because it generates systematic heterogeneous variation in household' risk aversion over the business cycle. For instance, households with $\omega_i > \gamma_i$ are relatively more risk tolerant in the peak of the cycle than in the trough which expands risk sharing possibilities. Second, and unlike in the CRRA case, our preferences are non-homothetic and thus initial endowment affects households' risk aversion: Even if all agents had identical preferences, $\gamma_i = 1$ for all i, agents with higher ω_i still consume more in good times and less in bad times.

3.1 Asset Prices

Proposition 2 *(Competitive equilibrium). The equilibrium stock price and interest rate are*

$$
\begin{aligned}
P_t &= \left(\frac{\rho + k\overline{I}I_t^{-1}}{\rho\left(\rho+k\right)}\right) Y_t && (13)\\
r_t &= \rho + \mu_Y - (1+v)\sigma_Y^2(I_t) + k\left(1 - \overline{I}I_t^{-1}\right) && (14)
\end{aligned}
$$

(13) is also the stock price in Menzly, Santos and Veronesi (MSV, 2004), once we define $S_t = I_t^{-1}$. Indeed, the state price density M_t in (9) is similar to the one in that paper:

Proposition 3 *(Stochastic discount factor). Given the risk-free rate r_t in (14), the stochastic discount factor follows*

$$
\frac{dM_t}{M_t} = -r_t dt - \sigma_{M,t} dZ_t \qquad \textit{with} \qquad \sigma_{M,t} = (1+v)\sigma_Y(I_t), \tag{15}
$$

These results allow for a calibration of the economy that yields reasonable asset pricing quantities. Intuitions for the asset pricing implications are well understood and we refer the reader to MSV. Briefly, a negative aggregate shock $dZ_t < 0$ decreases the price directly through its impact on Y_t and it also increases households risk aversion through I_t which produces an additional drop in prices, thus the higher volatility of returns when compared with that of traditional CRRA models:

$$
\sigma_P(I_t) = \sigma_Y(I_t)\left(1 + \frac{vk\overline{I}}{\rho I_t + k\overline{I}}\right). \tag{16}
$$

In addition, returns are predictable both because the market price of risk is time varying (see (15)) and there is variation in aggregate consumption volatility ($\sigma_Y(I_t)$). This generates the predictability of stock returns. Indeed,

$$E_t\left[dR_P - r_t dt\right] = \sigma_M(I_t)\sigma_P(I_t)dt \qquad \text{where} \qquad dR_P = (dP_t + Y_t dt)/dt \tag{17}$$

All these effects combine to generate a higher equity premium. Formulas (13) and (14) of P_t and r_t in Proposition 2 also imply the following result:

Corollary 1 *Asset prices are independent of the endowment distribution across households as well as the distribution of preferences. In particular the model has identical asset pricing implications even if all households are identical, i.e.* $\gamma_i = 1$ *and* $\omega_i = 1$ *for all* i.

In our framework standard Gorman aggregation results hold and thus there is a representative household that one can use for pricing purposes. Corollary 1 simply emphasizes that the preferences of this representative household are independent of the distribution of endowments and preferences of the underlying households. Thus P_t in equation (13) and r_t in (14) are independent of the distribution of either current consumption or wealth in the population. This property distinguishes our model from the existing literature, such as Longstaff and Wang (2012) or Chan and Kogan (2002). In these papers, the variation in risk premia is driven by endogenous changes in the cross-sectional distribution of wealth. Roughly more risk-tolerant households hold a higher proportion of their wealth in stocks. A drop in stock prices reduces the fraction of aggregate wealth controlled by such households and hence their contribution to the aggregate risk aversion. The conditional properties of returns thus rely on strong fluctuations in the cross sectional distribution of wealth. Instead, in our model households' risk aversions change, which in turn induces additional variation in premia and puts less pressure on the changes in the distribution of wealth to produce quantitatively plausible conditional properties for returns. Indeed, Corollary 1 shows that the asset pricing implications are identical even when households are homogeneous and thus there is no variation in cross-sectional distribution of wealth. Our model then features a clean separation between its asset pricing implications and its implications for trading, leverage and risk sharing. In particular, the corollary clarifies that equilibrium prices and quantities do not need to be causally related to each other, but rather comove with each other because of fundamental state variables, such as I_t in our model.

4 Leverage

4.1 Households' Investments and Saving Decisions

Proposition 4 *(Optimal Allocations). In equilibrium:*

a) *Households with $\gamma_i > \omega_i$ are risk averse (RA) and save in risk-free deposits:*

$$D_{it} = v\left(\gamma_i - \omega_i\right) H\left(I_t\right) Y_t > 0 \tag{18}$$

b) *Household with $\omega_i > \gamma_i$ are risk tolerant (RT) and borrow from the intermediary:*

$$L_{it} = v\left(\omega_i - \gamma_i\right) H\left(I_t\right) Y_t > 0 \tag{19}$$

c) *All households buy N_{it} shares in the intermediary stock:*

$$N_{it} = \gamma_i + (\rho + k)(1 + v)\left(\omega_i - \gamma_i\right) H\left(I_t\right) \tag{20}$$

where

$$H\left(I_t\right) = \frac{\overline{I}}{\rho I_t + k(1+v)\overline{I}} > 0 \tag{21}$$

Implementation of the efficient allocation described in Proposition 1 requires that households with $\omega_i < \gamma_i$ save and households with $\omega_i > \gamma_i$ borrow. Thus, the terminology introduced in (6): Households for whom $\omega_i < \gamma_i$ are the RA households and households with $\omega_i > \gamma_i$ are the RT households. Notice that both D_{it} and L_{it} vary with I_t. This follows from the dependence of attitudes towards risk on economic conditions. We explore this dependence in the next section.

The next corollary shows that RT households borrow to achieve a position in stocks that is higher than 100% of their wealth.

Corollary 2 *(Household positions in stocks).*

a) *The investment in stock of household i in proportion to wealth is*

$$\frac{N_{it} P_t}{W_{it}} = \frac{1 + v\left(1 - \frac{\rho I_t}{\rho I_t + \overline{I}[k + (\rho + k)(\omega_i/\gamma_i - 1)]}\right)}{1 + v\left(1 - \frac{\rho I_t}{\rho I_t + \overline{I}k}\right)} > 1 \quad \textit{if and only if} \quad \omega_i > \gamma_i. \tag{22}$$

b) *The risky share* $\frac{N_{it}P_t}{W_{it}}$ *decreases with* I_t *if and only if* $\omega_i > \gamma_i$.

c) *The risky share* $\frac{N_{it}P_t}{W_{it}}$ *increases in* ω_i *and decreases in* γ_i.

Expression (22) shows that RT households invest comparatively more in stocks. Moreover, (b) shows that these agents increase their position in stocks, as a percentage of wealth, during good times (I_t small). In particular, because households' preferences are non-homothetic in wealth, for given preference parameter γ_i there is a positive relation between wealth and the share of the portfolio held in risky assets, a result consistent with the empirical findings of Wachter and Yogo (2010, section 2.2).[3] Nonhomotheticity obtains in a variety of settings, but (22) has specific implications that have been tested by Calvet and Sodini (2014). Indeed we show in the Internet Appendix that (22) can be written as

$$\frac{N_{it}P_t}{W_{it}} = \frac{\mathsf{SR}\left(I_t\right)}{\sigma_P\left(I_t\right)}\left(1 - \frac{\theta_i Y_t}{W_{it}}\right), \tag{23}$$

where $\mathsf{SR}\left(I_t\right) = \left(1+v\right)\sigma_P\left(I_t\right)$ is the Sharpe ratio of the risky asset and θ_i is a household specific constant. Equation (23) is a version of equation (2) in Calvet and Sodini (2014, page 876).[4] These authors test a variety of implications of (23) in a large panel of Swedish twins (which serves to control for differences in risk preferences) and find strong support for them.

4.1.1 Leverage and consumption

Our framework has tight implications for the relation between leverage and current and future consumption at the individual household level.

Corollary 3 *Household i's consumption growth satisfies*

$$\frac{dC_{it}}{C_{it}} = \mu_{C,it}dt + \sigma_{C,it}dZ_t \tag{24}$$

where

$$\mu_{C,it} = \mu_Y + \frac{(\omega_i/\gamma_i - 1)\bar{I}}{I_t + (\omega_i/\gamma_i - 1)\bar{I}}F(I_t) \tag{25}$$

$$\sigma_{C,it} = \left(1 + \frac{v(\omega_i/\gamma_i - 1)\bar{I}}{I_t + (\omega_i/\gamma_i - 1)\bar{I}}\right)\sigma_Y(I_t) \tag{26}$$

[3] Table 4 of Wachter and Yogo (2010) shows that higher wealth correlates with higher risky share after controlling for households characteristics, which in our model are captured by γ_i.

[4] Equation (2) in Calvet and Sodini (2014) is $\phi_{it} = \frac{\mathsf{SR}}{\gamma\sigma_P}\left(1 - \theta_i X_{it}/W_{it}\right)$, where X_{it} is a subsistence or habit level in consumption. This equation obtains in a variety of habit setups (see Section II of the Internet Appendix of Calvet and Sodini (2014)). In expression (1) of our model, aggregate output, Y_t, takes the place of "habit" in traditional models.

with

$$F(I_t) = k(1 - \overline{I} I_t^{-1}) + (1 + v)v \ \sigma_Y^2(I_t) \tag{27}$$

If $\sigma_Y(I_t)$ is increasing in I_t with $\sigma_Y(1) = 0$, then there exists a unique solution I^* to $F(I^*) = 0$ such that for all i and j with $\omega_i/\gamma_i > 1 > \omega_j/\gamma_j$ we have

$$E\left[\frac{dC_{it}}{C_{it}}\right] < \mu_Y < E\left[\frac{dC_{jt}}{C_{jt}}\right] \qquad \textit{for} \qquad I_t > I^* \tag{28}$$

$$E\left[\frac{dC_{it}}{C_{it}}\right] > \mu_Y > E\left[\frac{dC_{jt}}{C_{jt}}\right] \qquad \textit{for} \qquad I_t < I^* \tag{29}$$

Cross-sectionally then, RT households, those with $\gamma_i < \omega_i$, have lower expected growth rate of consumption than RA households when I_t is low, and viceversa. These are also times when such households are heavily in debt. Thus heavily leveraged households enjoy both a high consumption boom in good times and lower *future* expected consumption growth.[5]

Corollary 4 *Highly leveraged households enjoy high consumption shares in good times but have lower expected consumption going forward.*

To reiterate, leverage and consumption patterns are not casually related. They are both driven by changes in the attitudes towards risk: After a sequence of good economic shocks aggregate risk aversion declines. Thus, RT households borrow more and experience a consumption "boom". The increase in consumption is due to the higher investment in stocks that have higher payoffs in good times. Good times mean lower individual (and aggregate) risk aversion and thus these same households take on more leverage. Hence, our model predicts a positive comovement of leverage and consumption at the household level. Finally, mean reversion in I_t also implies that RT households are also those that suffer a bigger drop in consumption growth once I_t increases.

This implication of our model speaks to some of the recent debates regarding the low consumption growth experienced by levered households following the Great Recession. Some have argued that the observed drop in consumption growth was purely due to a wealth effect, as levered households tend to live in counties that experienced big drops in housing values, whereas others have emphasized the critical role of debt in explaining this drop (see for instance Mian and Sufi (2015) for a summary of these differing views.) Clearly these effects are important but our contribution is to show that because leverage is an endogenous

[5] Parker and Vissing-Jørgensen (2009) use the Consumer Expenditure (CEX) Survey to show that the consumption growth of high-consumption households is significantly more exposed to aggregate fluctuations than that of the typical household.

variable, high leverage followed by low consumption is precisely the prediction of our model even without a causal effect of leverage on consumption.

4.2 Financial intermediary leverage

The total amount of debt of the intermediary is

$$\mathcal{D}_t = \int_{i:\gamma_i<\omega_i} D_{it} di. \tag{30}$$

We define two measures of intermediary leverage: The first measure, $\mathcal{D}_t/Y_t$ normalizes the amount of debt on the liability side of the intermediary's balance sheet by the aggregate output at time t, the income flowing from the assets held. The second, $\mathcal{D}_t/P_t$, instead normalizes by the equity of the intermediary, which recall is the paid in capital of the individual households, $P_t = \int_i P_{i,t} di$. We are interested in the cyclical properties of these measures and whether they can serve as factors priced in the cross section.

4.2.1 Time series properties of financial intermediary leverage

Proposition 5 *(Financial intermediary leverage)*

a) *The amount of debt issued by the financial intermediary, $\mathcal{D}_t$, is given by*

$$\mathcal{D}_t = vK_1 H(I_t) Y_t \qquad \textit{where} \qquad K_1 \equiv \int_{i:\omega_i>\gamma_i} (\omega_i - \gamma_i)\, di > 0 \tag{31}$$

and $H(I_t)$ is given in expression (21).

b) *Debt-to-output ratio and debt-to-equity ratio are given by, respectively:*

$$\mathcal{D}_t/Y_t = vK_1 H(I_t) \qquad \textit{and} \qquad \mathcal{D}_t/P_t = \frac{v\rho(\rho+k) K_1 H(I_t)}{\rho + k\bar{I} I_t^{-1}}. \tag{32}$$

c) *Intermediary's debt-to-output ratio, $\mathcal{D}_t/Y_t$, is procyclical. The intermediary debt-to-equity ratio, $\mathcal{D}_t/P_t$, it is countercyclical provided $I_t < I^{**}$, where I^{**} is given in equation* (IA.17) *in the Internet Appendix.*

The implications for the leverage of the financial intermediary follow immediately from the results on household leverage. After all, the financial intermediary's leverage is directly linked to the short debt issued to saving households who use it to hedge their exposure against aggregate shocks. In turn the amount of short debt issued by the financial intermediary is backed by the loans granted to borrowing households.

Financial intermediary debt, $\mathcal{D}_t$, increases as the economy improves (as I_t decreases) on account in turn of the increase of the risk bearing capacity of RT households who take on leverage. This allows the financial intermediary to grant more loans to those household which in turn allows it to issue more short term debt to the RA households. Conversely, the risk bearing capacity of borrowing households diminishes as the economy deteriorates and with it the supply of safe assets, precisely when it is most needed. Our model thus provides a theory of the supply of safe assets that is determined by the risk bearing capacity of borrowing households.[6]

Whether we normalize financial intermediary debt, $\mathcal{D}_t$, by aggregate output or by equity matters for whether aggregate leverage is pro- or countercyclical. Proposition 5 establishes that as the economy improves debt grows more than aggregate output, Y_t, and thus the procyclicality of $\mathcal{D}_t/Y_t$. The countercyclicality of intermediary's debt-to-equity ratio $\mathcal{D}_t/P_t$ instead has to do with the discount effects that characterize our model: A large realization of Y_t increases aggregate wealth on account of both the direct effect but also the additional increase in valuations as the households' risk aversion drops (see (13)).

The time-series properties of intermediary leverage described in Proposition 5 are consistent with the findings of the macrofinance literature and Adrian and Shin (2014) in particular. Figure 3 reproduces Figure 3 in Adrian and Shin (2014, p. 379): book leverage decreases when firms' asset value decrease, while market leverage increases when the intermediaries' valuations decrease. While in this literature this dynamics is interpreted as the active deleveraging of intermediaries, our results show that similar results obtain in general equilibrium due to the demand and supply of credit from households.

Corollary 5 *On average, the intermediary debt-to-output ratio, $\mathcal{D}_t/Y_t$, is high when aggregate volatility $\sigma_P(I_t)$ is low and the price of risky assets P_t is high.*

In our model, both the volatility of returns and the intermediary's debt-to-output ratio depend on I_t, which induces the comovement of both variables. This is consistent with the

[6] This is an issue studied by other papers. See for instance Barro and Mollerus (2014), who propose a model based on Epstein-Zin preferences to offer predictions about the ratio of safe assets to output in the economy. Gorton, Lewellen and Metrick (2012) and Krishnamurthy and Vissing-Jorgensen (2012) provide empirical evidence regarding the demand for safe assets. In all these papers the presence of "outside debt" in the form of government debt plays a critical role in driving the variation of the supply of safe assets by the private sector, a mechanism that is absent in this paper.

evidence in the bottom panel of Figure 3: VaR measures correlate negatively with leverage, as VaR is in turn positively related to measures of asset return volatility.

4.2.2 Intermediary asset pricing and leverage risk price

The intermediary asset pricing literature finds that proxies for the financial sector's leverage factor are useful predictors of returns in the cross section. Our model also sheds light on these findings. Our's is a one-factor model and thus the conditional CAPM holds. Tests of the conditional CAPM require proxies for conditioning information, any variable that forecasts future excess returns. This is exactly what the leverage ratio of the financial intermediaries, whether measured agains income or equity, does. Formally, let $\mathcal{L}_t$ denote either of the two measures of financial intermediary leverage that we have considered. Then if $\mathcal{L}_t$ is monotonic in I_t there exists a function $q(\cdot)$ such that[7] $I_t = q(\mathcal{L}_t)$. The state price density is then

$$M_t = e^{-\rho t} Y_t^{-1} I_t = e^{-\rho t} Y_t^{-1} q(\mathcal{L}_t).$$

The volatility of the SDF is thus $\sigma_{M,t} = \sigma_{Y,t} - \frac{q'(\mathcal{L}_t)}{q(\mathcal{L}_t)} \sigma_{\mathcal{L},t}$ where $\sigma_{\mathcal{L},t}$ is the volatility of leverage. The risk premium for *any* asset with return dR_{it} can then be written as

$$E_t[dR_{it} - r_t dt] = Cov_t\left(\frac{dY_t}{Y_t}, dR_{it}\right) + \lambda_t^{\mathcal{L}} Cov_t\left(d\mathcal{L}_t, dR_{it}\right) \qquad \text{with} \quad \lambda_t^{\mathcal{L}} = -\frac{q'(\mathcal{L}_t)}{q(\mathcal{L}_t)} \tag{33}$$

The first term of (33) corresponds to the usual log-utility, consumption-CAPM term, while the second term corresponds to the additional risk premium due to shocks to $\mathcal{L}_t$. $\lambda_t^{\mathcal{L}}$ is the market price of leverage risk.[8] We then obtain

Corollary 6 *(Price of leverage risk)*

a) *The price of leverage risk is positive,* $\lambda_t^{\mathcal{D}/Y} > 0$*, when leverage is defined as* $\mathcal{L}_t = \mathcal{D}_t/Y_t$.

b) *The price of leverage risk is negative,* $\lambda_t^{\mathcal{D}/P} < 0$*, when leverage is defined as* $\mathcal{L}_t = \mathcal{D}_t/P_t$.

As shown in Proposition 5, $\mathcal{D}_t/Y_t$ is procyclical. That is, leverage is high when on average, marginal utilities are low. Thus the market price of risk associated with this measure of leverage is positive. Instead $\mathcal{D}_t/P_t$ is countercyclical and thus it is high when on average marginal utilities are high and thus the market price of risk is in this case negative.

[7] For this heuristic argument, we restrict $I_t < I^{**}$ so that $\mathcal{D}_t/P_t$ is monotonic. See Proposition 5.

[8] This decomposition is for illustrative purposes only. All shocks are perfectly correlated in our model and so there is only one priced of risk factor.

We link these results to the empirical evidence in AEM and HKM (see Table 1) by equating $\mathcal{D}_t/Y_t$ to the intermediaries' "book leverage" and $\mathcal{D}_t/P_t$ to their "market leverage". Indeed, like our measure of debt-to-output $\mathcal{D}_t/Y_t$, book leverage does not depend on market prices and thus it is procyclical. In contrast, market leverage depends on asset prices, and thus the discount effect that renders it countercyclical.

To further elaborate, in our one-factor model the conditional CAPM holds

$$E_t[dR_{it} - r_t dt] = \beta_{it} \, E_t[dR_{Pt} - r_t dt]$$

where the stock $\beta_{it} = cov(dR_P, dR_{it})/\sigma^2_{Pt}$ depends on the state variable I_t, or, $\mathcal{L}_t$. Assuming for simplicity an approximate linear relation, $\beta_{it} \approx \beta_{i0} + \beta_{i1} \, \mathcal{L}_t$, by substituting and taking unconditional expectations on both sides, we obtain:

$$E[dR_{it} - r_t dt] \approx \beta_{i0} E[dR_{Pt} - r_t dt] + \beta_{i1} E\left[\mathcal{L}_t \times (dR_{Pt} - r_t dt)\right] \tag{34}$$

(34) explains the increase in explanatory power of the factor $f_t = (r_t - r_f) \times \mathcal{L}_{t-1}$ in columns IV and V in Panel A of Table 1. We test for this relation also in our simulations below.

Finally, notice that good time, when I_t is low, are also periods when expected excess returns are low and so is typically aggregate uncertainty $\sigma_Y(I_t)$.[9] Because $\mathcal{D}_t/Y_t$ is procyclical and $\mathcal{D}_t/P_t$ is countercyclical the following corollary obtains immediately:

Corollary 7 *(Predictability of Future Excess Returns) Let the risk risk premium $E[dR_{it} - r_t dt] = \sigma_M(I_t)\sigma_P(I_t)$ be countercyclical. Then a high intermediary debt-to-output ratio $\mathcal{D}_t/Y_t$ predicts lower future excess returns, while a high intermediary debt-to-equity ratio $\mathcal{D}_t/P_t$ predicts high future excess returns. The regression coefficient is negative for intermediary debt-to-output ratio and positive for intermediary debt-to-equity ratio.*

These theoretical results are consistent with Panel B and C of Table 1 that show that high book leverage weakly predicts future lower excess returns, while high market leverage predicts higher future returns. Our simulations below also show that interestingly our model is also consistent with market leverage being better in predicting returns than book leverage, as the price at denominator induces far more variation of market leverage than book leverage.

5 Idiosyncratic Preference Shocks

The model in Section 2 features preference shocks to subsistence consumption level, ψ_{it}, that increase risk aversion in bad times, as shown by much of the evidence. Evidence also shows

[9] Note that we have not made any assumptions yet on $\sigma_Y(I_t)$, except that it vanishes for $I_t \to 1$.

that risk aversion moves for idiosyncratic reasons as well. For instance, households who are hit by negative health shocks tend to become more risk averse (Decker and Schmits, 2016)) and background risk also tends to increase risk aversion (Cohn et al (2018), Guiso et al (2015)). We now extend the model to incorporate idiosyncratic preference shocks.

We now assume that ψ_{it} in households' preferences (1) is given by

$$\psi_{it} = \gamma_i \epsilon_{it} \left(1 - I_t^{-1}\right) \qquad \text{with} \quad d\epsilon_{it} = \kappa \left(1 - \epsilon_{it}\right) dt + \sigma_\epsilon \epsilon_{it} dZ_{it} \tag{35}$$

ϵ_{it} is a persistent, mean-reverting stochastic variable and dZ_{it} are idiosyncratic Brownian shocks independent of each other and of the aggregate shock dZ_t. The time series and cross-sectional average of ϵ_{it} are thus given by $E[\epsilon_{it}] = E^{CS}[\epsilon_{it}] = 1$. To help with our aggregation results, we also assume that there is a continuum of households for each type (γ^i, ω^i). Finally, we assume markets are complete and hence households can hedge these idiosyncratic shocks through financial contracts offered by the intermediary. For instance, health shocks, which indeed impact risk aversion, can be hedge through health insurance policies. Some types of background risk shocks can be hedge through unemployment insurance (Cochrane (1995) for instance discusses the contingent claim approach to health insurance. See also Koijen, Van Neuwerburgh, and Yogo (2016).)

We briefly discuss our main results here and leave the details to the internet appendix. First, the state price density and thus the asset pricing results are unchanged, because idiosyncratic preference shocks wash out in the aggregate. Second, the intermediary balance sheet and its dynamics are also unchanged, as positive and negative idiosyncratic shocks that affect households' demand and supply of safe assets cancel each other. Idiosyncratic preference shocks instead affect households' consumption, asset allocation, and wealth:

Proposition 6 *With idiosyncratic shocks to preferences:*

(a) The consumption of household i is

$$\frac{C_{it}^{idio}}{Y_t} = \frac{C_{it}}{Y_t} + \gamma_i \left(\epsilon_{it} - 1\right) \left(1 - \frac{1}{I_t}\right) \tag{36}$$

(b) The net deposit of household i is

$$\left(D_{it}^{idio} - L_{it}^{idio}\right) = \left(D_{it} - L_{it}\right) + \gamma_i \left(\epsilon_{it} - 1\right) vQH\left(I_t\right) Y_t \tag{37}$$

where recall $H(I_t)$ is in (21) and $Q > 0$ is a constant reported explicitly in (IA.20) in the on-line appendix.

(c) The investment in risky assets of household i is

$$N_{it}^{idio} = N_{it} + \gamma_i \left(1 - \epsilon_{it}\right) H_1(I_t) \tag{38}$$

where $H(I_t)$ is in (21) and $H_1(I_t)$ is in equation (IA.18) on the on-line appendix, it is positive for $I_t < I^*$, and globally increasing.

(d) The wealth of household i is

$$W_{it}^{idio} = W_{it} + \gamma_i \left\{ \frac{(\rho + \kappa) + \left[\bar{I}k - (k + \rho + \kappa)\right] I_t^{-1}}{(\rho + \kappa)(\rho + k + \kappa)} (\epsilon_{it} - 1) \right\} Y_t \tag{39}$$

This proposition shows that consumption, net deposits, stock holdings, and wealth are all given by two terms. The first terms are the same as in the corresponding quantities in the first part of the paper. The second terms depend on the difference $(\epsilon_{it} - 1)$. For instance, equation (36) shows that households with $\epsilon_{it} > 1$ consume more than households with $\epsilon_{it} < 1$. Intuitively, the former experience an increase in subsistence consumption, ψ_{it}, and thus need to consume more to maximize utility. Such agents have the funding to support additional consumption thanks to the inflow from their hedges against ϵ_{it} shocks. Intuitively, if a household is subject to a negative health shock, the households subsistence consumption increases (literally) and health insurance pays for the additional consumption. Thus, total measured consumption, including health services, increases.

Similarly, expression (37) shows that net deposits increase when agents have $\epsilon_{it} > 1$. Again, intuitively, higher risk aversion induces households to deposit the additional funds from the hedges in deposits, rather than in risky stocks. In fact, the investment in stocks declines when $\epsilon_{it} > 1$. Finally, other things constant, the financial wealth of the agent even increases when it is hit by such preference shocks through the insurance payouts.

6 Quantitative implications

6.1 Parameter specification

In order to assess the model quantitatively we need to decide on two sets of parameters, those that pertain to the aggregate time series properties of the model and those that relate to the cross sectional dispersion in households' attitudes towards risk and wealth. For the time series parameters we follow MSV closely as our model aggregates to a representative consumer household which is identical to the one there. The only difference is that in the present model the aggregate endowment process is heteroskedastic. Our theoretical results do not depend on the functional form of $\sigma_Y(I_t)$ but obviously to simulate the model we need to specify one. We assume that

$$\sigma_Y(I_t) = \sigma^{max}\left(1 - I_t^{-1}\right) \in [0, \sigma^{max}]. \tag{40}$$

Consistent with the empirical evidence (Jurado, Ludvigson, and Ng (2015)), (40) implies that output volatility increases when the recession index increases,[10] and moreover that the condition $\sigma_Y(I_t) \to 0$ as $I_t \to 1$ is met. The dynamics of the recession indicator are then

$$dI_t = k(\overline{I} - I_t)dt - (I_t - 1)\overline{v}dZ_t,$$

with $\overline{v} = v\sigma^{max}$ which is similar to the one in MSV.

σ^{max} is chosen to match the average consumption volatility $E[\sigma_Y(S_t)] = std[\Delta \log(C_t^{data})]$, where the expectation can be computed from the stationary density of I_t.[11] The rest of the parameters are similar but not identical to MSV and are reported in Panel A of Table A.1 in Appendix A1. Panels B and C of that table shows that, similarly to MSV, the model is able to match the main properties of stock returns, both conditionally and unconditionally. Figure A.1 in the Appendix reproduces Figure 1 in MSV.

To assess the quantitative performance of the model we need to specify the distribution of initial endowments w_i and of the preference parameter γ_i. We proceed by fitting the distribution to match some basic stylized facts regarding the time series properties of individual consumption. But, as Attanasio and Pistaferri (2016, p. 3) note "household surveys on household expenditure are rare, small, and lack a consistent longitudinal component." Here we follow others and use the Consumer Expenditure Survey to draw inferences about some basic properties of individual consumption process (see e.g. Kocherlakota and Pistaferri, 2009, and Constantinides and Ghosh, 2017). In what follows, we sketch the procedure and leave most of the details for the appendix

We assume first that the risk aversion parameters γ_i are uniformly distributed $\gamma_i \sim U[1-\gamma, 1+\gamma]$, so as $\int \gamma_i di = 1$. As for the distribution of endowments, recall first that ω_i must meet Assumption A1. To achieve this, we assume that the Pareto weights ϕ_i are distributed independently of preferences γ_i and obtain the endowments by inverting (10):

$$\omega_i = \gamma_i\left(1 - \overline{I}^{-1}\right) + \phi_i\overline{I}^{-1}. \tag{41}$$

[10] The alternative of assuming e.g. $\sigma_Y(I_t)$ as linear in I_t would result in $\sigma_Y(I_t)$ potentially diverging to infinity as I_t increases. We also assume that $\sigma_Y(I)$ is multiplied by a "killing function" $k(I^{-1})$ such that $k(x) \to 0$ when $x \to 0$ to ensure that integrability conditions are satisfied (see Ceriditto and Gabaix (2008)). We do not make such function explicit for notational convenience.

[11] See the Appendix in MSV. In addition, note that in MSV, $\alpha = \overline{v}/\sigma$ and therefore we compute $\overline{v} = \alpha\sigma$. Finally, MSV has I_t bounded below by a parameter $\lambda > 1$ while in our model I_t is bounded below by 1.

To ensure a skewed distribution of wealth, we assume $\phi_i = \exp\left(\sigma_\phi \varepsilon_i - \frac{1}{2}\sigma_\phi^2\right)$ with $\varepsilon_i \sim N(0,1)$. Thus, $\phi_i > 0$ and $\int_i \phi_i di = E^{CS}[\phi_i] = 1$. These parametric assumptions imply that the correlation between ω_i/γ_i – which affects the local risk aversion LRA in (12) – and the endowment ω_i can be positive or negative depending on the distributional assumptions. In our parametric choice, $Corr\left(\omega_i/\gamma_i, \omega_i\right) = -0.0269$, which implies an essentially null correlation between endowments and local risk aversion. Finally, to match both idiosyncratic and systematic consumption quantities, we select the volatility of idiosyncratic risk aversion ϵ_{it}, σ_ϵ, in equation (35) and set the mean reversion parameters sufficiently large to ensure an stationary distribution for ϵ_{it}, namely, $\kappa = \frac{1}{2}\sigma_\epsilon^2 + 0.01$. We complete the procedure by choosing γ, σ_ϕ, and σ_ϵ with an eye on relevant moments of individual households' consumption growth, such as average household consumption growth (arithmetic or log), its mean and median total and systematic volatility, and the cross-sectional dispersion of both.

Panels A of Table 2 reports the relevant consumption moments in the data. The average quarterly (arithmetic) growth rate is about 5.3%, which is due to the large cross-sectional heterogeneity in quarterly growth rates. Indeed, the median is slightly negative and the cross-sectional standard deviation is 36%, in line with estimates by e.g. Constantinides and Ghosh (2017). The log-growth indeed shows a slightly negative mean, which is close to the median, highlighting the positive skewness of the consumption data. The total quarterly volatility is also large, at 34.2%, and it displays a strong positive skewness, as its median is much lower at 26.8%, and its dispersion (standard deviation) is at 35.7%. Clearly, much of this quarterly consumption volatility is due to idiosyncratic shocks and residual seasonalities. Quarterly systematic volatility is of course lower than the total volatility: the average is about 7%, and the median is just 5.5%. The dispersion is still large, but reasonable, at 7.3%.

Panel B of Table 2 contains the same moments as Panel A but from the simulated model. Our parameterization generates a distribution of idiosyncratic and systematic consumption growth quite close to the one estimated in the data. The cross-sectional dispersion of consumption growth is nearly identical to the one in the data, and so is the mean idiosyncratic volatility. The mean systematic volatility is slightly larger than in the data, but the median is slightly smaller. Our model however generates a larger idiosyncratic and systematic dispersion of consumption volatility compared to the data. But overall, the chosen parameterization matches well the consumption data. Next section shows that it also brings about reasonable patterns of households' debt/wealth across the wealth distribution.

6.2 The cross-section of household leverage

Figure 5 plots L_{it}/W_{it} of the borrowing RT households by wealth percentile in simulations, which aims to replicate Figure 2. As for Figure 2, we sort households by their wealth in crisis time.[12] First, in general, households with lower net worth (W_{it}) in crisis time take on more debt as a fraction of assets. Intuitively, for given endowment ω, the poor households in a crisis are exactly those that took on higher leverage than others.

The second important pattern in Figure 2 is that debt-to-wealth ratios L_{it}/W_{it} increase markedly during crises, that is, those rare times in which S_t is on the left-hand-side of its distribution (see Panel A of Figure A.1). This is an important channel in our model: While households who borrow deleverage when I_t increases, and hence reduce their amount of debt, the debt-to-asset ratio actually increases. The reason is that the value of assets declines by even more. That is, households engage in active debt repayment but household leverage when debt is normalized by wealth, L_{it}/W_{it}, increases nonetheless. This pattern is particularly evident during the financial crisis of 2008.[13]

Notice though that we miss on the magnitudes. As shown in Figure 2 the leverage of low net worth households at the trough of the crisis, in 2009, is above 130%, whereas it is less than half of that in our simulations. The disparity in magnitudes is even more pronounced in good times. Other ingredients need to be brought in to explain why low net worth households levered as much as they did during the years leading up to the financial crisis of 2008.

In sum, our model captures an important fact in the cross section, that the less wealthy lever more, unlike in most models with heterogeneous agents, such as Dumas (1989) and Longstaff and Wang (2012). There, less risk averse households lever up, invest in risky stocks, and become richer as a result. These models thus imply counterfactually that leverage is more pronounced amongst richer agents and are unable to explain the patterns in Figure 2. In contrast, in our model the two different sources of heterogeneity, combined with the implicit assumption that households with low endowment have lower habit loading γ_i, imply that poor households lever up more. There is obviously an important difference model and data: in our model agents poorer households lever to purchase equity issued by the financial intermediary, while in the data they lever to purchase durable goods and housing.

[12]Sorting period by period also produces a decreasing pattern of debt/wealth ratios, although less prominent. See Figure IA.3 in the Internet Appendix. This emphasizes the impact of wealth declines in determining the leverage ratio, as in Figure 2.

[13]The Internet Appendix documents that a similar plot obtains in the case of Spain which also has a comprehensive household survey (the "Encuesta Financiera de las Familias" or EFF). We thank Olympia Bover of the Bank of Spain for pointing out this to us.

6.3 Aggregate leverage, panic deleveraging and stock prices

Our model has implications for the dynamics of the aggregate household leverage, which is also the leverage of the financial intermediary in our model. Panel A of Figure 6 shows the aggregate debt-to-output ratio as a function of $S_t = I_t^{-1}$ for our choice of parameter values. The aggregate debt-to-income is about 125% in boom (S_t high), which is close to the maximum of 135% observed at the peak of the housing cycle. Moreover, from expression (32) the behavior of $\mathcal{D}_t/Y_t$ with respect to S_t depends on the shape of the function $H(I_t)$. Recall that this function is decreasing and convex in I_t, and thus increasing and concave in S_t. In particular, our parametric choices imply that aggregate deleveraging accelerates as bad times morph into severe distress as households' risk aversions skyrocket.[14] Instead for high values of S_t the function is relatively flat and variation of S_t in that domain do not result in big swings in either aggregate aggregate debt-to-output ratio or stock holdings of leveraged agents (Panel B). Because the state variable does not visit that range of values very often (see the stationary density of S_t in Panel A of Figure A.1) it follows then that the extreme periods of deleveraging do not happen often.

Figure 7 further emphasizes the point. It shows the time series behavior of several quantities of interest over a 100 years of artificial quarterly data. Panel A shows the realization of the surplus consumption ratio $S_t = I_t^{-1}$ and the corresponding economic uncertainty $\sigma_Y(I_t)$. Economic uncertainty increases in bad times but not unreasonably so as the conditional volatility is only slightly above 6% when the economy is in deep distress.

Panels B, and D of Figure 7 illustrate the behavior in simulations of the same variables shown in the top panels and bottom panels of Figure 1 for the 2008 and 2020 crisis. Specifically, the solid line in Panel B shows the variation in the price-dividend ratio due to variation in the surplus consumption ratio, with a visible drop from the mid 30s to around 15 early on in the sample and again in the middle of the sample. At these times of deep distress, volatility of stock returns (dashed line) increases substantially, to almost 60%. This is the standard behavior of asset prices in external habit economies in the presence of negative economic shocks, as shown in the top panels of Figure 1 for the 2008 and 2020 crisis.

The solid line in Panel D shows the behavior of the aggregate stock holdings of the RT households, who are the ones who are leveraged. They sell during bad times to RA households, as shown by the drop in holdings around market crisis. Trading volume (dashed line), computed as the average absolute change in holdings multiplied by the stock price, increases

[14] It is important to emphasize that these results do not depend on the specific assumptions made on the functional form for $\sigma_Y(I_t)$ as the function $H(I_t)$ does not depend on it.

substantially exactly at such times. This figure replicates the behavior of households during financial crisis, as shown in the bottom panels of Figure 1. As shown in that figure, households actively redeem shares from mutual funds, i.e. liquidate risky holdings, during bad times, as our model predicts. That is, households do trade, both in the model and in the data, and especially so during financial crisis.

Panel C in Figure 7 reports the intermediary's market leverage $\mathcal{D}_t/P_t$ together with "book leverage" $\mathcal{D}_t/Y_t$. As it is apparent, they mostly move opposite to each other. The variation in both quantities is rather limited most of the time, except during extreme bad events. It is thus in these occasions, as the surplus consumption ratio drops and economic uncertainty increases, that levered households decrease their indebtness and liquidate their positions in risky assets (Panel D). Comparing Panels C and D with Panel B, we see that during such times prices drop substantially and leveraged households delever as well by selling stocks. A common interpretation of the comovemet of these time series is that the "price pressure" generated by the stock selling shown in Panel F is causing the price decline in Panel B. This interpretation is incorrect. As shown in Corollary 1 the asset pricing implication of our model are identical to those that obtain in a representative household framework and the same sequence of aggregate shocks would have led to the same path for asset prices.

Our model does not produce large swings in household leverage, except in situations of deep distress but the speed of adjustment is much faster, as should be expected from a frictionless model. For example,aggregate household debt-to-income peaked at about 135% in early 2007 and dropped to about 100% in the years following the crisis (figure not shown). Our model delivers slightly higher magnitudes. In simulations, debt-to-income is close to 125% and drops to about 112% when the surplus consumption ratio suffers a strong drop, but it does it so much faster than in the data.

In our framework the balance sheet of the financial intermediary responds passively to the households' portfolio decisions. As shown in Panel C, $\mathcal{D}_t/Y_t$ drops when stock prices drop but this deleveraging is simply a reflection of the fact that as the economy deteriorates RT households become more risk averse and decrease the amount they borrow. Because the asset side of the intermediary's balance sheet contracts so does the liability side and thus the reduction in the amount of deposits that the intermediary can issue to RA households. The dynamics of the balance sheet of the intermediary are delinked, for example, from any form VaR constraints such as in Adrian and Shin (2014), as illustrated in Figure 3 in the introduction. Our point is not that frictions do not matter but rather that there are more fundamental forces at work driving the low frequency dynamics of balance sheets and that frictions are likely to be an amplification factor rather than the primal cause of fluctuations.

Figure 8 reproduces Figure 3 in the introduction. Panels A and B shows the relation between asset growth (on the y-axis) and leverage growth (on x-axis), where leverage is "book leverage" (Panel A) and market leverage (Panel B) in simulations. The intermediary's assets, $\mathcal{A}_t$, are equal to the sum of the values of the individual trees plus the total value of the loans L_{it} granted, that is $\mathcal{A}_t \equiv \int P_{it} di + \int_{i:\omega_i>\gamma_i} L_{it} di$. There is negative relation in Panel A and positive relation in Panel B, exactly as in Panels A and B of Figure 3, respectively. Panels C and D plot the relation between the change in the intermediary value-at-risk and leverage growth in the two cases, respectively. We approximate the intermediary value-at-risk as $VaR_t = 2.326 \times \sigma_{\mathcal{A},t}$, that is, assuming assets are (approximately) normally distributed. The volatility of assets is given by $\sigma_{\mathcal{A},t} = \frac{P_t}{\mathcal{A}_t} \times \sigma_{P,t}$ as the volatility of loans L_{it} is zero.

As can be seen from Panel C, book leverage growth is negatively related on average with change in value at risk, consistently with the evidence put forth by Adrian and Shin (2014) (see Panel C of Figure 3). But the relation is not causal: Both leverage ratios and asset volatility are driven by I_t. As I_t increases, intermediaries delever and asset volatility increases. Panel D reports the same simulation results but with market leverage on the x-axis, in which case a clear positive relation appears. Unfortunately, we do not have an empirical counterpart to which we can compare this plot. Still, the message of Panels C and D is that passive deleveraging and market price variation can as well generate the type of empirical predictions that are usually argued as evidence of active balance sheet management by financial intermediaries.

6.4 Credit Cycles

Our model matches several stylized facts about credit cycles. For instance, Muir (2019) and Baron and Xiong (2017) show that credit expansions, first, forecast low returns and second that the probability of market crashes increases after such expansions. Figure 9 and Table 3 illustrate the credit cycle implications of our model. Start with Figure 9, which reports the model's implications for the credit cycle in the same simulation as in Figure 7. Panels A and B plot the total intermediary risk-free lending and market leverage, respectively. Intermediary lending is high in good times, but declines quickly in bad times, as evidenced from the downward spikes. High intermediary lending also correlates with a high probability of a crash and a lower probability of boom in the subsequent year. Panel C plots the probability of a 20% stock-market crash in the next year minus the probability of 20% stock-market rise. This difference is high and positive in good times and low and negative in bad times. That is, the return distribution is strongly negatively skewed in booms, and positively skewed in busts. Intuitively, in good times a negative shock to output Y_t results in a drop in prices for two reasons: A direct effect of a negative cash-flow shock and an additional negative "kick" associated with the increase in the discount rate. In bad times, however, risk aversion is already high and it is unlikely it will increase further. In fact, it is more likely that it will decline in the future. While stock return volatility is high in bad times, volatility is itself symmetric, implying similar probabilities of an increase and decrease in stock prices. Our model thus reproduces the empirical finding that high credit/gdp correlates with future stock market crashes, as shown in e.g. Baron and Xiong (2017) and Muir (2019).

Table 3 provides further simulation evidence on 20,000 year of quarterly artificial data. Panel A shows that a higher aggregate credit-to-output ratio predicts lower future excess returns while Panel B shows that credit expansions, defined as a 3-year increase in the total amount of credit/output ratio, also predict lower future returns. In our model, aggregate lending to the economy increases in good times, when risk aversion is low, and therefore it predicts lower future excess returns. These results are consistent with the empirical evidence, such as Muir (2019), with the caveat that the intermediary in our model offer risk-free loans to investors who need to increase their exposure to systematic risk.

Panels C and D provide evidence that after a credit expansion, defined as either an increase in credit-to-output or a three-year credit expansion, stock returns are more negatively skewed in good times than in bad times. That is, in good times market crashes are more likely than market booms, compared to bad times. Bad times however are characterized by higher volatility. To disentangle the two effects — volatility versus negative skewness — we follow Baron and Xiong (2017, Table III). Specifically, we define a market crash as a 20% drop in stock prices in a given quarter. We then run a predictive regression of a dummy variable that takes the value of 1 if a quarterly crash occurs over the subsequent one, two, and three year period. We do the same for a market boom, which is defined symmetrically as a quarterly market increase of 20% (other thresholds yield similar results). We then test whether the regression coefficient of the difference (Crash minus Boom) is positive. A positive coefficient implies that it is more likely to see a market crash than a market boom in good times compared to bad times. The results are reported in Panels C and D. A high credit/output ratio (Panel C) or a three-year credit expansion (Panel D) makes it more likely to see a large quarterly decline than a large quarterly increase, as show in Baron and Xiong (2017) and Muir (2019). One important difference from Baron and Xiong (2017), however, is that their regression coefficient on "crash" is positive, while ours is negative, although it is "less negative" than for the boom regression. As mentioned, as in Campbell and Cochrane (1999) and MSV (2004), in our model stock return volatility is one-to-one related to expected excess return, while this relation is not so strong in the data.

Our model also has implications for credit spreads (see Panel D of Figure 9). To compute credit spreads, we return to the household balance sheet in Figure 4 and assume that households, as owners of their endowment tree P_{it}, issue risky debt $\mathcal{B}_{it}$ and equity $\mathcal{E}_{it}$ to the intermediary sector. We may think of $\mathcal{B}_{it}$ as "corporate" debt backed by the value of the tree P_{it}. Because there are no costs of default, this assumption does not change any of our results but allows us to study the dynamics of credit spreads. For simplicity, we assume that individual trees have no idiosyncratic risk. Because capital structure is immaterial, we assume that at every t some firms issue debt with maturity T and principal K and we plot its credit spread. We set "corporate leverage" at a constant 20%, as in the data (Graham et al.(2015)), and thus assume $K = 20\% P_t$. Maturity is $T = 5$. The value of debt is $\mathcal{B}_{it} = E_t\left[\frac{M_T}{M_t}\min\left(P_{iT}, K\right)\right]$ and the credit spread is computed against the yield of risk-free zero-coupon bond with five-year to maturity. The appendix contains additional details and shows that the probability of default of a bond with 20% leverage is always lower than 0.16%, corresponding to a AAA credit rating.

Panel D shows that credit spreads are strongly countercyclical, being very low in good times and very high in bad times, even controlling for default probability. The main driver of credit spreads is investors' required risk premium, which varies with the business cycle. In our model, however, the growth rate of output $E[dY/Y] = \mu_Y dt$ is constant and thus, unlike in the data, credit spreads in our model do not predict future growth. An extension of the model with time varying μ_Y may bring about such additional result but we are not exploring this extension here.

6.5 Intermediary Asset Pricing

We showed in subsection 4.2.2 that financial intermediary leverage should be expected to be a predictor of the cross section of stock returns and that the sign of the market price of risk depends on the specific definition of leverage used. To check that this result obtains in simulations we perform standard Fama-MacBeth cross-sectional regressions and use both measures of leverage as risk factors.

In our one-factor model, the conditional CAPM holds. As discussed in Section 4.2.2 the unconditional CAPM may fail if betas are time varying. To study this channel, we introduce a continuum of trees as follows:

$$Y_{it} = Y_t\left(1 + \alpha_{it}\overline{I}I_t^{-1}\right) \tag{42}$$

where α_{it} are stochastic processes with a cross-sectional average of zero $E^{CS}[\alpha_{it}] = 0$, and with $\alpha_{it} > -\overline{I}^{-1}$. We assume that

$$d\alpha_{it} = -h\ \alpha_{it}\ dt + (\alpha_{it} + \overline{I}^{-1})\ \sigma_\alpha\ dZ_{i,t}^{\alpha} \tag{43}$$

where $dZ_{i,t}^{\alpha}$ are Brownian motions that are independent of each other and all the others in the model. The price of stocks is

$$P_{it} = P_t + Y_t \frac{\alpha_{it}\overline{I}I_t^{-1}}{\rho + h}$$

In essence, stocks i with higher α_{it} are more sensitive to the "boom" indicator $S_t = I_t^{-1}$, and thus have a higher cash-flow "beta". This "beta" is persistent but time varying, which induces a violation of the unconditional CAPM.

Panel A of Table 4 shows the results of Fama-MacBeth cross-sectional regressions with simulated data and should be compared with Panel A of Table 1. The first column of Panel A shows a strong quarterly coefficient of 2.96 and with cross-sectional $R^2 = 73\%$. The time-varying nature of betas make the unconditional CAPM fail, although the one-factor nature of the model makes the CAPM betas still line up positively with expected returns. Column II shows that the estimated market price of risk of market leverage is negative, while column III shows that the estimated market price of risk of book leverage is positive, consistently with Panel A of Table 1, and with the results in Corollary 6. The magnitudes are smaller though – about half of the data for market leverage and one-tenth for book leverage – and the R^2's increased only mildly.

Columns IV and V, however, show simulation results that are more in line with the data in Panel A of Table 1. These two columns report the cross-sectional regressions with the interaction term $f_t = \mathcal{L}_{t-1}R_t$ as additional factor. In this case, the R^2 increases substantially to 81% in the case of market leverage, which is consistent with the large increase in R^2 in Table 1. The interaction factor with "book leverage" in Column V also increases the R^2 but not as much as market leverage, again consistently with the result in data. The regression coefficients are also closer to the empirical ones. In general, while all coefficients on the leverage factors are smaller than their counterpart in Table 1, they are not statistically significantly different, except for the factor coefficient in Column III. Finally, we note that the coefficient on the market return is large in all specifications. The reason, as it is well known, is that betas are positively correlated with average excess returns, but have a smaller dispersion compared to their average excess returns; thus, the market price of risk has to be larger "to compensate" and better fit the dispersion of average returns (see e.g. Santos and Veronesi (2010) for a discussion).

An additional prediction of our model is at the time-series level (see Corollary 7), which is consistent with Panels B and C of Table 1. Panels B and C of Table 4 provides evidence from artificial data. In this case, book leverage (Panel B) is always significant, but we note both a lower R^2 and $t-$statistics compared to market leverage (Panel C). That is, our model is consistent with the empirical finding that market leverage should be a better predictor of future stock returns. Indeed, while book leverage and market leverage are clearly related to each other, they are not perfectly correlated. In our simulated data, market leverage and book leverage have a correlation of -47% in levels, and -48% in first differences. In the data, they have a correlation of -39% in levels and -31% in first difference. The imperfect correlation in simulation is due to the non-linearities implicit in the model.

In sum, measures of financial intermediary leverage show up as risk factors in tests of the cross section of stock returns. This has been interpreted as evidence that financial intermediaries act as marginal investors in many markets. Our contribution is to show that this is not necessarily the case. In our model fluctuations in the intermediary's balance sheet are driven in turn by fluctuations in the households' attitudes towards risk. Thus it might be the case that the predictive success of measures of financial intermediaries' leverage is simply due to the fact that it proxies for these changes in the attitudes towards risk and is unrelated to leverage ratio constraints.

7 Conclusions

We propose a general equilibrium exchange economy populated with heterogeneous households. Households differ in risk preferences, which are differentially impacted by aggregate economic conditions. During bad times some agents become more risk averse than others which induces motives for risk sharing and trading. A financial intermediary issues deposits and grant loans to allow households to achieve their optimal allocation through a dynamic trading strategy that combines aggregate stock market positions and either borrowing from or lending to the intermediary. The model aggregates to a representative household that features also time changing attitudes towards risk. Our framework is thus able to generate the strong discount effects that have been shown to be key in addressing well known asset pricing regularities in the data.

We show that the discount rate component induces a negative correlation between the two measures of intermediary leverage: book leverage and market leverage. The intermediaries' balance sheets reflect the economy's aggregate risk aversion and they expand and contract as households' demand for loans and deposits change over the business cycle. Because the intermediaries' balance sheet reflects the state of the economy and are easier to measure than households risk preferences, intermediaries' leverage ratios can serve as proxies for the potentially poorly measured marginal rates of substitution of the representative household. We are able to qualitatively replicate standard tests in the financial intermediation and asset pricing literature which have been put forth as evidence of the existence of the asset pricing role of frictions and capital constraints. We argue that these tests offer no such proof as our results obtain in a frictionless complete markets framework.

Our model is simple, however, in that it only has one state variable, all quantities move in lock-step and thus there is an unrealistic perfect (positive or negative) correlation between leverage, prices, volatility, expected return, consumption, and so on. It is this assumption which allows for closed form solutions in quantities and prices and thus obtain a better understanding of the various economic forces at work. Future research should focus on generalizing our simple setting to obtain more realistic dynamics. In particular, it would be useful to explicitly model housing as a second risky asset that also provides housing services to households. In our calibration, poor agents borrow more to buy the risky asset, which would match well the data if the class of risky assets were to also comprise housing. A second useful extension is to solve for the incomplete market version of the model, as in Alvarez and Atkenson (2017). These authors show in a three-period model but with more general recursive utilities and incomplete markets that preference shocks impact asset prices and trading. The extension to a dynamic economy such our ours may bring about additional dynamics and possibly allow the model to better match the distribution of consumption growth and its total volatility.

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Figure 1: The 2008 and the 2020 crisis

Panel A and B plot the S&P 500 index (solid line) and return volatility (VIX, dashed line) during the 2008 financial crisis and the 2020 coronavirus crisis, respectively. Panel C and D plot intermediary market leverage (solid line) and credit spreads (dashed line) during the same two crisis periods. Panel E and F plot the trading volume (solid line) and the cumulative net mutual fund flow (dashed line) during the same periods. Data source: CRSP, CBOE, He, Kelly, Manela (2018, updated series)

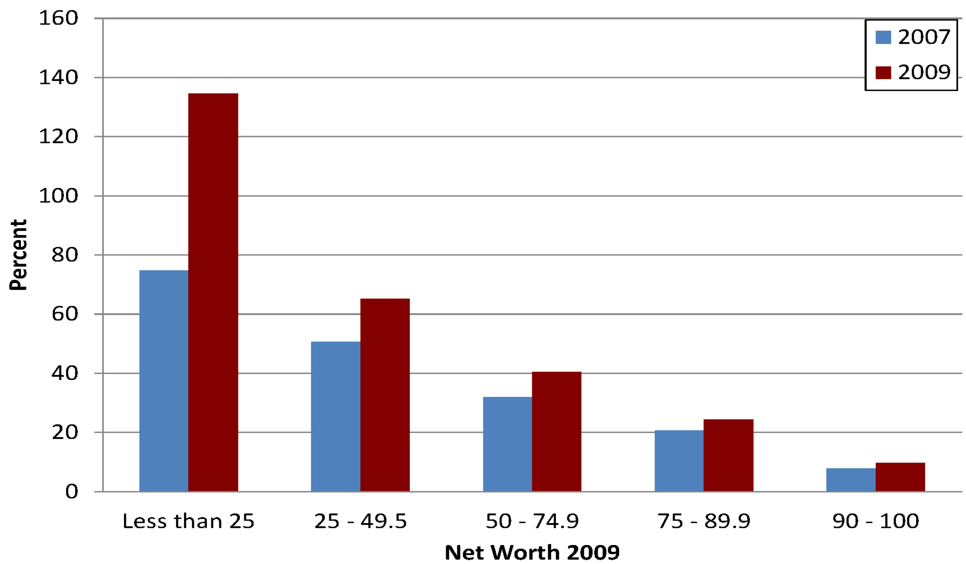

Figure 2: The cross section of household leverage

This figure plots the distribution of debt-to-asset ratios from the Survey of Consumer Finances in 2007 and in 2009, which was conducted on the same sample of households. The sample is restricted to households who own stocks.

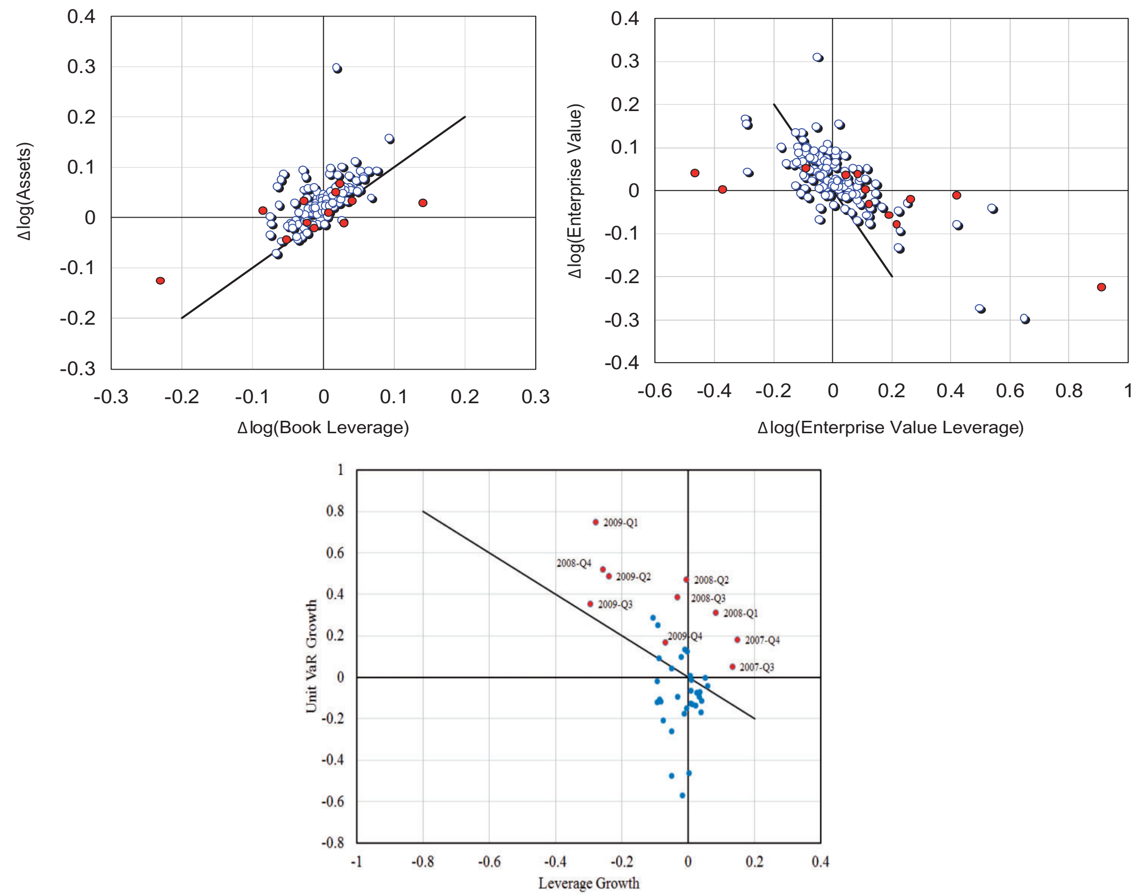

Figure 3: Leverage and Risk (Adrian and Shin, 2014)

The top two panels reproduce Figure 3 in Adrian Shin (2014). The left panel shows the scatter chart of the asset-weighted growth in book leverage and total assets for the eight largest U.S. broker dealers and banks. The right panel is the scatter for the asset-weighted growth in enterprise value leverage and enterprise value. The dark dots correspond to the period 2007−2009. The bottom panel reproduces Figure 6 in Adrian and Shin (2014) and it plots the annual growth rate in unit VaR against the annual growth rate in leverage.

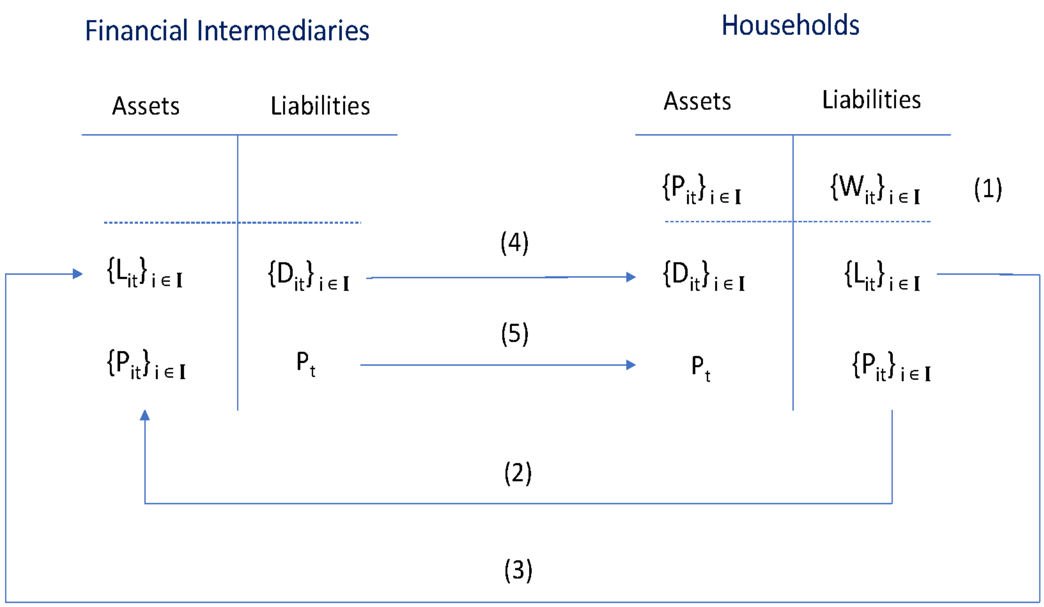

Figure 4: Balance sheets of households and the financial intermediary

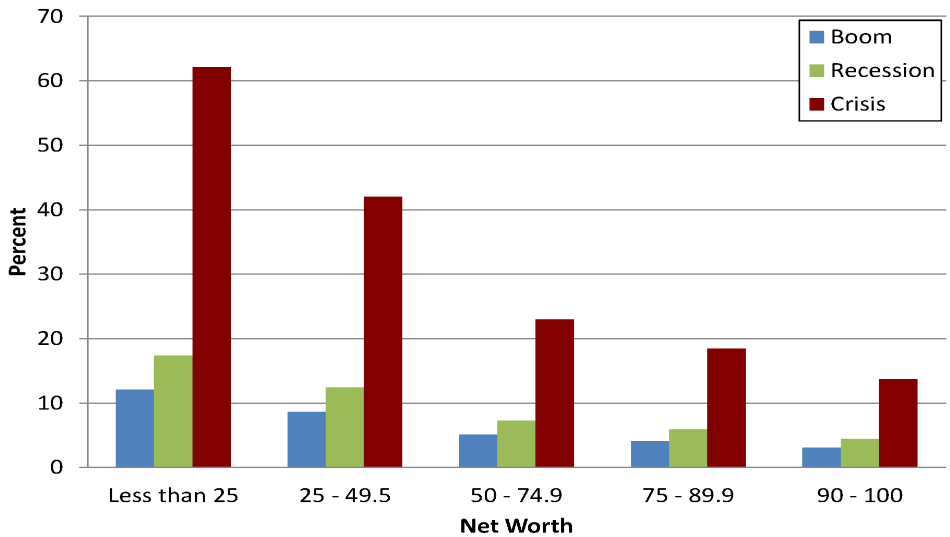

Figure 5: Debt-to-assets ratios across the wealth distribution in the model . This figure plots the distribution of debt-to-asset ratios of households who take on debt in simulations during three types of periods: Booms (I_t low), recessions (I_t high) and crisis (I_t very high). Households are sorted by the net worth in crisis time.

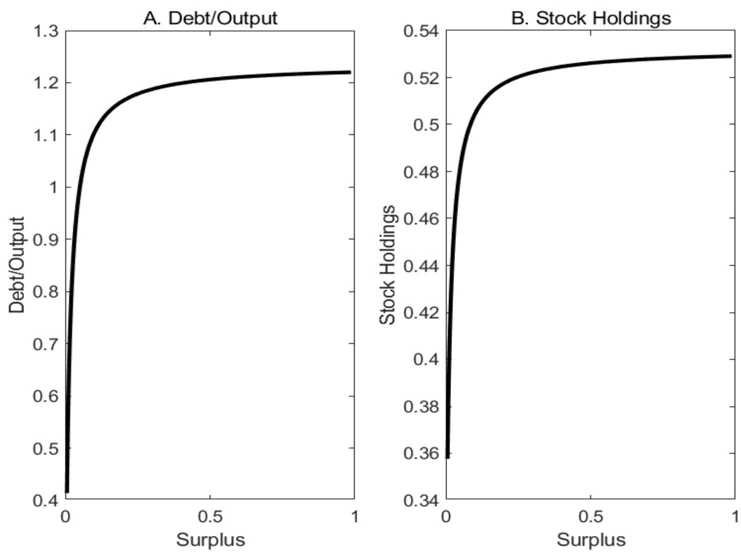

Figure 6: Aggregate leverage and stock holdings of levered households Panel A plots $\mathcal{D}_t/Y_t$, the aggregate debt to output ratio in the economy (see expression (32)) as a function of the surplus consumption ratio S_t. Panel B reports the aggregate holdings of stocks for the RT households.

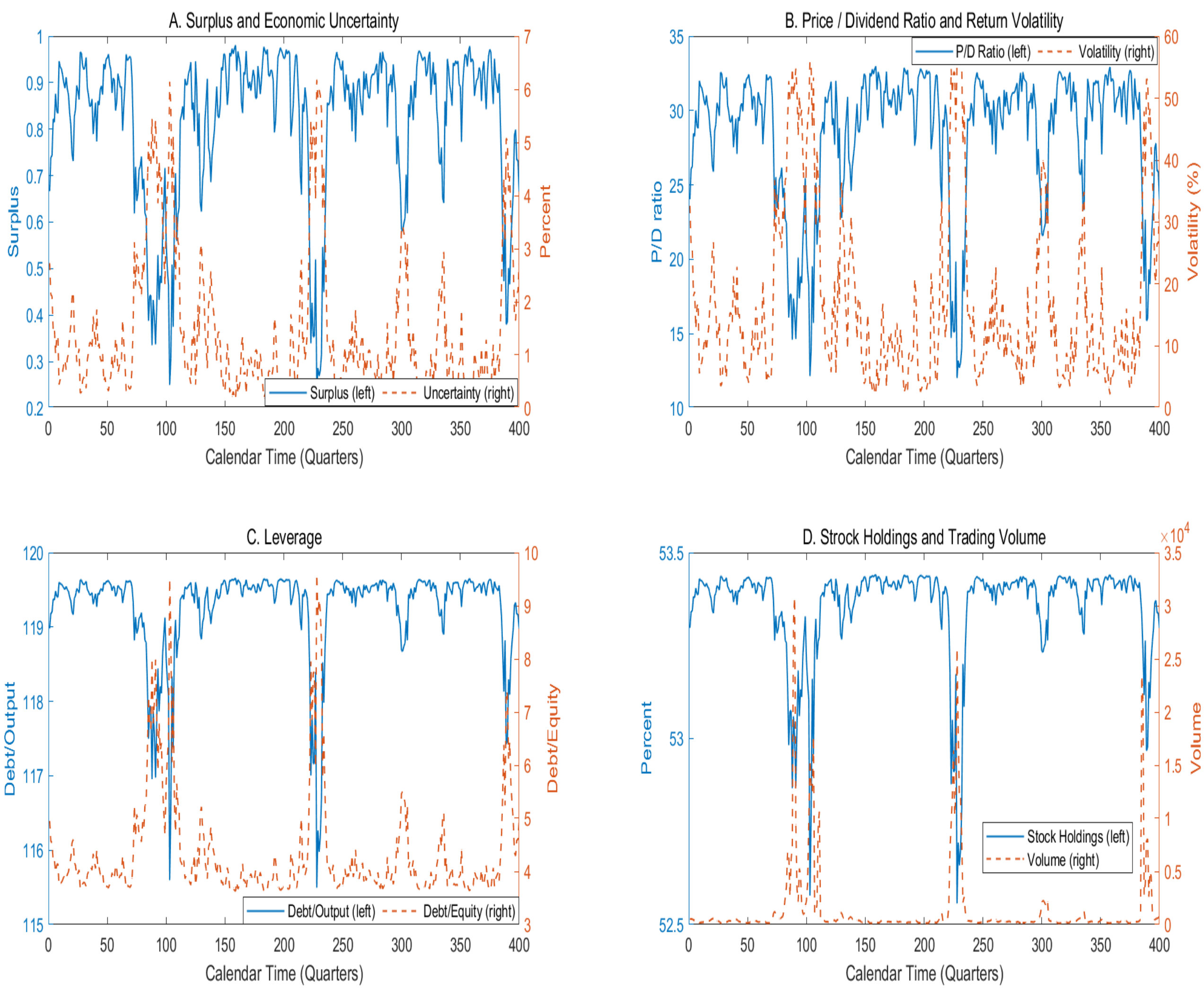

Figure 7: "Fire Sales" in a Simulation Run . This figure plots the time series of several quantities in 100 years of quarterly artificial data. Panel A reports the "surplus consumption ratio" $S_t = I_t^{-1}$ and output volatility $\sigma_Y(I_t)$. Panel B reports the price-dividend ratio (solid line) and the stock return volatility (dashed line). Panel C reports the aggregate debt-to-wealth ratio of levered agents (dashed line) and the aggregate debt-to-output ratio (solid line). Panel D reports the aggregate position in risky stock of levered households (solid line) and the trading volume (dashed line).

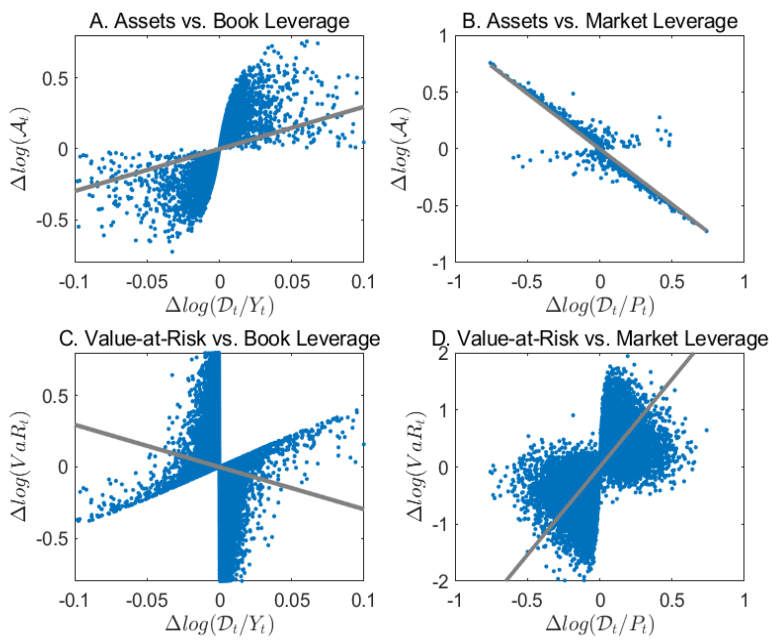

Figure 8: Simulations: Panel A and B plot log changes in total assets, defined as $\mathcal{A}_t \equiv \int P_{it}di + \int_{i:\omega_i>\gamma_i} L_{it}di$, against two measures of financial intermediary leverage, our proxy for book leverage $\mathcal{D}_t/Y_t$, and market leverage, $\mathcal{D}_t/P_t$. Panels C and D show the VaR_t, defined as $VaR_t = 2.325 \times \sigma_{\mathcal{A},t}$ against the corresponding measures of leverage.

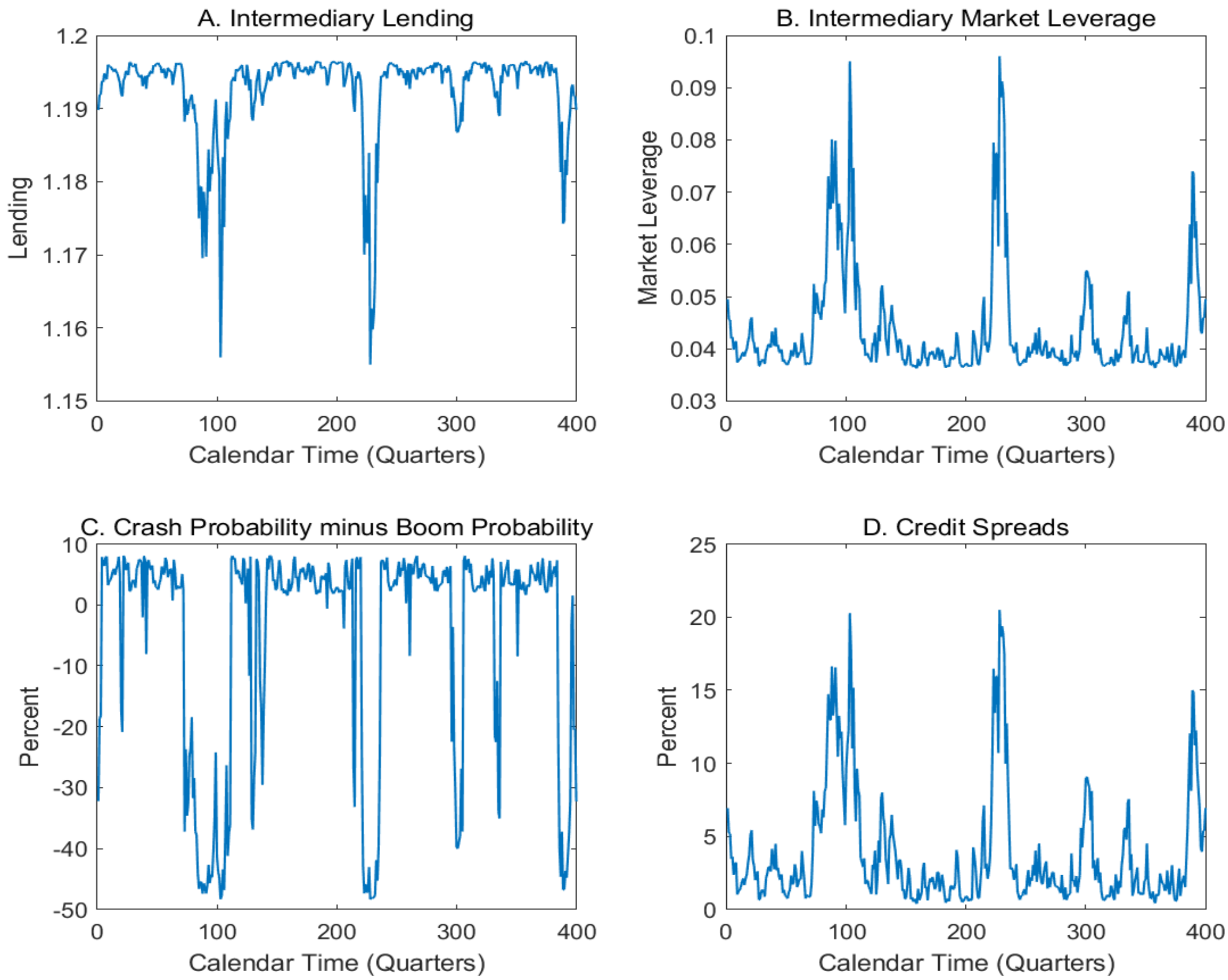

Figure 9: Credit Cycles in a Simulation Run . This figure plots the time series of credit quantities in 100 years of quarterly artificial data. Panel A reports intermediary lending to investors. Panel B reports the intermediary market leverage. Panel C reports the probability of market crash, defined as a decline in equity values by more than 20% over the subsequent year, minus the probability of a market boom, defined symmetrically. This is a measure of negative skewness. Panel D reports the credit spreads for a firm with 20% leverage.

Table 1: The Market Price of Leverage Risk. Panel A reports Fama-MacBeth regressions where the set of test portfolios are the standard 25 Fama-French portfolios sorted on size and book to market. Column I reports the standard CAPM regression. Column II adds to the market, the market leverage, defined as Debt/Equity = 1/(capital ratio) − 1, which is a transformation of the capital ratio= Equity/(Debt + Equity) introduced in He, Kelly and Manela (2017). Column III reports the same regression where instead of using market leverage we use book leverage, defined as in Adrien, Etula, and Muir (2014). Columns IV and V report the results of rather adding the interaction term $xr_t^{Mkt} \times Leverage_{t-1}$ for the two definitions of leverage. Panel B reports time-series predictability regressions of future excess returns on book leverage, while Panel C reports time-series predictability regression on market leverage. The sample period is 1970-2012. t-statistics are in parenthesis.

Panel A. Cross-sectional Regressions

	I	II	III	IV	V
α	3.19	0.76	1.07	0.87	3.58
	(3.05)	(0.62)	(0.97)	(0.74)	(3.65)
Market Return	-0.85	0.97	0.82	0.89	-1.28
	(-0.72)	(0.69)	(0.61)	(0.65)	(-1.10)
Market Leverage		-0.22			
		(-2.13)			
Book Leverage			0.63		
			(3.07)		
(Lagged Market Leverage)×(Market Return)				8.09	
				(2.63)	
(Lagged Book Leverage)×(Market Return)					-6.37
					(-2.33)
R^2 (%)	6.21	49.57	53.22	45.33	11.06

Panel B. Time-Series Predictability with Book Leverage

	1 year	2 year	3 year	4 year	5 year
Coef (×100)	-1.78	-1.79	-2.17	-3.13	-9.89
	(-0.83)	(-0.72)	(-0.89)	(-1.03)	(-3.29)
R^2	0.01	0.01	0.01	0.01	0.07

Panel C. Time-Series Predictability with Market Leverage

	1 year	2 year	3 year	4 year	5 year
Coef (×100)	3.66	6.21	8.56	10.03	13.06
	(1.57)	(1.50)	(2.18)	(2.51)	(3.84)
R^2	0.04	0.07	0.10	0.12	0.19

Table 2: Cross-Sectional Parameters and Household Consumption Moments. Panel A reports the distribution of household consumption growth and their quarterly volatility and systematic volatility estimated from the Survey of Consumption Expenditures. The estimation methodology is discussed in the Appendix. The data are from Kocherlakota and Pistaferri (2009) and span the period 1980 to 2005. Panel B reports the same quantities in artificial data. We simulate the model with 250,000 households for a period of 1,000 years. As in the data, estimates are performed on quarterly data. The parameters of the model are as in Table A.1, except for the preference and Pareto weight parameters, γ_i and ϕ_i, and the idiosyncratic shock process. We assume $\gamma_i \sim U[0,2]$ and $\phi_i \sim$ Log Normal$(-\frac{1}{2}\sigma_\phi^2, \sigma_\phi^2)$ with $\sigma_\phi = 0.5$. The idiosyncratic shock process uses $\sigma_\varepsilon = 3$ and $\kappa = 4.5$.

Panel A. Households Quarterly Consumption Moments. Data

	Growth Rate (%)				Volatility (%)		
	Mean	Median	Std. Dev.		Mean	Median	Std. Dev.
Arithmetic	5.33	-0.40	36.52	Total	34.17	26.83	35.74
Logarithmic	-0.39	-0.41	33.37	Systematic	7.03	5.51	7.35

Panel B. Households Quarterly Consumption Moments. Model

	Growth Rate (%)				Volatility (%)		
	Mean	Median	Std. Dev.		Mean	Median	Std. Dev.
Arithmetic	5.34	0.79	37.13	Total	31.02	14.20	65.42
Logarithmic	0.53	0.50	24.38	Systematic	9.43	4.32	19.88

Table 3: Credit Cycles in Simulations. Panel A Panels B and C report time series regressions of market returns on book and market leverage, respectively lagged one to five years. $t-$statistics are in parenthesis.

Panel A. Predictability of Excess Returns with Aggregate Credit / Output

	1 year	2 year	3 year	4 year	5 year
β	-1.98	-3.49	-4.79	-5.86	-6.67
$t(\beta)$	(-4.72)	(-4.50)	(-4.75)	(-5.21)	(-5.57)
$R^2(\%)$	4.57	8.54	12.35	15.91	18.63

Panel B. Predictability of Excess Returns with Credit Expansion

	1 year	2 year	3 year	4 year	5 year
β	-0.57	-1.07	-1.62	-2.14	-2.54
$t(\beta)$	(-3.09)	(-3.10)	(-3.50)	(-4.05)	(-4.49)
$R^2(\%)$	0.56	1.18	2.07	3.10	3.96

Panel C. Predictability of Crash with Credit/Output

		1 year			2 year			3 year	
	Crash	Boom	Diff.	Crash	Boom	Diff.	Crash	Boom	Diff.
β	-1.60	-3.78	2.45	-2.33	-4.50	3.13	-2.64	-4.51	3.53
$t(\beta)$	(-4.00)	(-4.57)	(4.78)	(-3.71)	(-4.28)	(4.54)	(-4.19)	(-4.34)	(4.45)
$R^2(\%)$	1.51	5.73	1.95	2.03	5.38	2.30	2.05	4.48	2.40

Panel D. Predictability of Crash with Credit Expansion

		1 year			2 year			3 year	
	Crash	Boom	Diff.	Crash	Boom	Diff.	Crash	Boom	Diff.
β	-0.53	-1.16	0.64	-0.81	-1.50	0.86	-1.01	-1.51	1.00
$t(\beta)$	(-3.29)	(-3.62)	(2.76)	(-3.25)	(-3.71)	(2.80)	(-3.78)	(-3.67)	(3.00)
$R^2(\%)$	0.25	0.79	0.19	0.36	0.88	0.25	0.44	0.73	0.28

Table 4: The Market Price of Leverage Risk in Simulations. Panel A reports Fama-MacBeth regressions in a sample of simulated data from our model. The set of test portfolios are stocks that pay dividends $Y_{it} = Y_t(1+\alpha_{it}\overline{I}I_t^{-1})$ where α_{it} follow mean-zero idiosyncratic persistent processes bounded below by $1/\overline{I}$ (see (43)). Panels B and C report time series regressions of market excess returns on debt/output ratio and to debt/price ratio, respectively lagged one to five years. $t-$statistics are in parenthesis.

Panel A. Cross-sectional Regressions

	I	II	III	IV	V
α	−1.35	−1.48	−1.59	−2.59	−1.92
	(−14.27)	(−14.37)	(−15.09)	(−18.83)	(−16.32)
Market Return	2.96	3.09	3.20	4.19	3.53
	(29.50)	(28.59)	(28.95)	(29.70)	(28.88)
Market Leverage		−0.11			
		(−27.87)			
Book Leverage			0.05		
			(12.35)		
(Lagged Market Leverage)×(Market Return)				4.25	
				(20.89)	
(Lagged Book Leverage)×(Market Return)					−1.65
					(−11.42)
R^2 (%)	73.06	73.78	74.53	81.73	77.03

Panel B. Predictability with Book Leverage

	1 year	2 year	3 year	4 year	5 year
β	−1.98	−3.49	−4.79	−5.86	−6.67
$t(\beta)$	(−4.72)	(−4.50)	(−4.75)	(−5.21)	(−5.57)
$R^2(\%)$	4.57	8.54	12.35	15.91	18.63

Panel C. Predictability with Market Leverage

	1 year	2 year	3 year	4 year	5 year
β	6.02	10.04	12.92	14.87	16.36
$t(\beta)$	(25.87)	(27.94)	(32.54)	(38.04)	(43.68)
$R^2(\%)$	14.99	24.93	31.81	36.33	39.74

A1. Appendix: Quantitative implications of the model

Panel A of Table A.1 reports the preference parameters, that are similar to those of Menzly, Santos, and Veronesi (2004). Figure A.1 reports the conditional moments implied by the model as a function of the surplus-consumption ratio S_t. As in MSV Figure 1, Panel A reports the stationary distribution of the surplus-consumption ratio S_t and shows that most of the probability mass is around $\overline{S} = 0.667$, although S_t drops considerably below occasionally. The price-dividend ratio is increasing in S_t (panel B), while volatility, risk premium and interest rates decline with S_t (panel C) for the area with positive mass. Note that our choice of parameters is such to give near zero mass to the area in which $\sigma_P(I_t)$ and expected return $E_t[dR_t - r_t dt]$ are increasing in $S_t = I_t^{-1}$.[15] Finally, the Sharpe ratio is also strongly time varying, and it is higher in bad times (low S_t) and lower in good times (high S_t). These first four panels are very similar to the corresponding panels of Figure 1 in MSV.

In addition to the quantities discussed in MSV, Panel E and F of Figure A.1 plot the "corporate" bond credit spreads and default probabilities, respectively, for two levels of corporate leverage, 20% and 40%, and for maturity $T = 5$ years. In particular, corporate bond prices are given by $\mathcal{B}_{it} = E_t\left[\frac{M_{t+T}}{M_t} min\left(K, P_{i,t+T}\right)\right]$, where $K/P_{it} = 20\%$ (solid line), or 40% (dashed line). We assume no idiosyncratic risk, for simplicity, and therefore $P_{it} = P_t$. In other words, credit spreads are for a representative firm in the economy. The credit spreads in Panel E are computed as

$$cs_t(S) = -log(K/\mathcal{B}_{it})/T - r_{i,t,T}$$

where $r_{i,t,T} = -log(\mathcal{B}_t^{RF})/T$ is the yield of the risk-free zero coupon bond $\mathcal{B}_t^{RF} = E_t\left[M_{t+T}/M_t\right]$. The probability of default in Panel F is $Pr(P_{i,t+T} < K) = E_t\left[1_{P_{i,t+T}<K}\right]$. All these quantities are computed by Monte Carlo simulations.

Panel F shows that for 20% (40%) leverage, the default probability is below 0.16% (1.5%), which correspond to AAA (BBB) credit ratings.[16] Panel E shows that the credit spreads increase in bad times, even if the probability of default does not, reaching 10% and even 20%. Given the low probability of default, the size of the credit spreads is mostly due to the large risk premium that occur during crisis times.

Given the parameters in Panel A of Table A.1, we simulate 10,000 years of quarterly data and report the aggregate moments in Panel B. As in MSV, Table 1, the model fits well the asset pricing data, though both the volatilities of stock returns and of the risk free rate are higher than their empirical counterparts.[17] Still, the model yields a respectable Sharpe

[15] Figures 4 and 5 of Campbell and Cochrane (1999) also display expected excess return and return volatility that are monotonically decreasing in the surplus consumption ratio S_t.

[16] The default probability bends backwards for S_t low as the volatility of stocks also declines, as shown in Panel C.

[17] The volatility of the risk free rate can be substantially reduced by making the natural assumption that expected dividend growth μ_Y decreases in bad times, i.e. when the recession indicator I_t is high. Indeed, in the extreme, by assuming $\mu_Y(I_t) = \overline{\mu}_Y + (1-v)\sigma_Y(I_t)^2 - k(1 - \overline{I}I_t^{-1})$, which is decreasing in I_t, we would obtain a constant interest rates $r = \rho + \overline{\mu}_Y$. No other result in the paper depend on $\mu_Y(I_t)$ and thus all the other results would remain unaltered by the change.

Table A.1: Parameters and Moments. Panel A reports the parameters for our calibration of the time series properties of the model. σ^{max} which is chosen to match the average volatility of consumption, which is the new parameter relative to MSV. Panel B reports a set of moments for the aggregate stock market and interest rates, as well as consumption growth, and compares with the same moments in artificial data obtained from a 10,000-year Monte Carlo simulation of the model. Panel C similarly reports the R^2 of predictability regressions in the model and in the data, using the price-dividend ratio as predictor.

Panel A. Parameter Estimates

ρ	k	$\overline{I}$	$\overline{v}$	μ	σ^{max}
0.0416	0.1567	1.5	1.1353	0.0218	0.0819

Panel B. Moments (1952 – 2014)

	$E[R]$	$Std(R)$	$E[r_f]$	$Std(r_f)$	$E[P/D]$	$Std[P/D]$	SR	$E[\sigma_t]$	Std(σ_t)
Data	7.13%	16.55%	1.00%	1.00%	38	15	43%	1.41%	0.52%
Model	6.00%	21.24%	2.01%	2.76 %	28.60	4.65	28.22%	1.42%	1.34%

Panel C. P/D Predictability R^2

	1 year	2 year	3 year	4 year	5 year
Data	5.12%	8.25%	9.22%	9.59%	12.45%
Model	15.12%	25.12%	31.76%	35.83%	38.77%

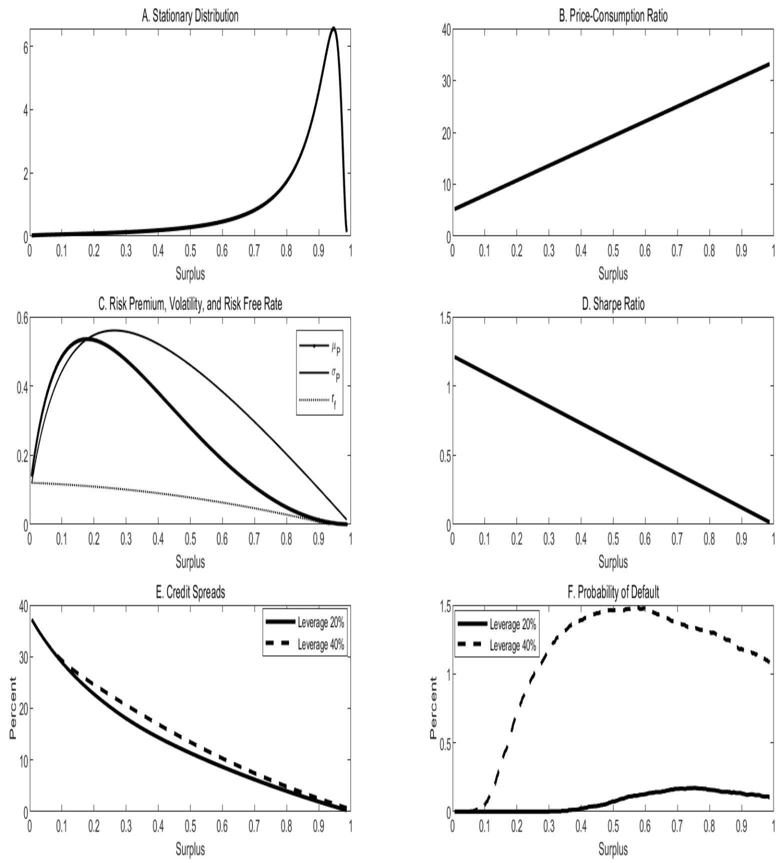

Figure A.1: Conditional Moments. Panel A shows the stationary probability density function of the surplus consumption ratio S_t. Panel B shows the P/D ratio as a function of S_t. Panel C plot the expected excess return $E_t[dR_P - r_t dt]$, the return volatility $\sigma_P(S_t)$ and the interest rate $r(S_t)$ as functions of S_t. Panel D shows the Sharpe ratio $E_t[dR_P - r_t dt] / \sigma_P(S_t)$ against S_t. Panels E and F show the credit spreads and the default probability, respectively, of "corporate" zero-coupon bonds with maturity $T = 5$ and leverage $K/P_t(S)$ equal 20% (solid line) or 40% (dashed line). Credit spreads are computed against the a five-year risk-free bond.

ratio of 32.64%. Finally, the simulated model generates an average consumption volatility of 1.43% with a standard deviation of 1.18%. This latter variation is a bit higher than the variation of consumption volatility in the data (0.52%), where the latter is computed fitting a GARCH(1,1) model to quarterly consumption data, and then taking the standard deviation of the annualized GARCH volatility. Our calibrated number is however lower than the standard deviation of dividend growth' volatility, which is instead around 1.50%.

The calibrated model also generates a strong predictability of stock returns (Panel C), with R^2 ranging between 14.18% at one year to 35.92% at 5 year. This predictability is stronger than the one generated in MSV and also the one in the data. This is due to the combined effect of time varying economic uncertainty (*i.e.* the quantity of risk) and time varying risk aversion (*i.e.* the market price of risk), which move in the same direction.