Internet Appendix for

Option-Implied Spreads and Option Risk Premia

Christopher L. Culp
Johns Hopkins Institute for Applied Economics
and Swiss Finance Institute

Mihir Gandhi
University of Chicago

Yoshio Nozawa
University of Toronto

Pietro Veronesi
University of Chicago, NBER, and CEPR

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The appendix contains technical details and additional results omitted from the text. The contents are as follows:

Appendix A. Proofs

Proof of Proposition 1. (a) Taking the first partial derivative of NIS with respect to LGD we obtain

$$\frac{\partial NIS}{\partial LGD} = \frac{-E_f^*[1_{x_T<1}]}{\left(1 - LGD \times E_f^*[1_{x_T<1}]\right)} \frac{1}{\log\left(1 - E_f^*[1_{x_T<1}]\right)} > 0 \ .$$

(b) Write $NIS = \frac{\log(1-LGD\times P)}{\log(1-P)}$. Then some algebra shows

$$\frac{\partial NIS}{\partial P} = NIS \times (f(1) - f(LGD))$$

where

$$f(x) = \frac{x}{(1-xP)\log(1-xP)}$$

We now show that $f'(x) < 0$, thus proving the claim, as $LGD < 1$. Indeed, taking the first derivative and after some algebra we obtain

$$f'(x) = \frac{\log(1-xP) + xP}{[(1-xP)\log(1-xP)]^2}$$

which is negative, as $\log(1 - xP) < xP$ for every $xP > 0$. Q.E.D.

Appendix B. Data and Methodologies

Filters. The filters are similar to those in Culp, Nozawa, and Veronesi (2018). In short,

- We drop all but one of any duplicate quotes.
- We drop quotes with bid prices of zero, time-to-maturity less than seven days, and implied volatility less than 5% or greater than 100%.
- We drop quotes with negative time value ("unable to compute implied volatility" filter in CNV).
- We drop quotes with implied volatility more than one standard deviation from the average among options in the same moneyness bin ("IV" filter in CNV).
- We drop quotes with put-call parity implied interest rate more than one standard deviation from the average among options in the same moneyness bin ("put-call parity" filter in CNV).

We do not drop quotes with negative implied interest rates from put-call parity or drop quotes with zero open interest, as do CNV. The former would bind too often in the post-crisis, low interest rate environment. The latter would be too restrictive: we require options in the full moneyness/maturity plane and not only deep OTM options, as in CNV. For more details about the filters, see Constantinides, Jackwerth, and Savov (2013) and Appendix A of Culp, Nozawa, and Veronesi (2018).

Moneyness/Maturity Portfolios. We construct a panel of put option and implied bond portfolios with target moneyness $K/S = 0.90$, 0.925, 0.95, 0.975, 1.00, 1.025, 1.05, 1.075, 1.10 and target maturity $T = 30$, 60, 91, 122, 152, 182, 273, 365 days.

For the regressions, we select the option contract with moneyness/maturity closest to these targets on each date. If there is no contract closer than 5% from the target moneyness and 91 days from the target maturity, we set the portfolio return on that date to missing. Panels A to D of Table D1 tabulate portfolio moneyness/maturity relative to these targets. While the differences between portfolio and target in terms of moneyness are trivial, the differences in terms of maturity are less so. Panel E tabulates the number of observations by portfolio. Missing portfolio returns concentrate among long-maturity, OTM options.

For the plots, we compute a weighted average of option contracts via a Gaussian kernel smoother of the form

$$\omega\left(M,T\right) \sim \mathcal{N}\left(\begin{bmatrix} M \\ T \end{bmatrix}, \begin{bmatrix} \left(0.05/2\right)^2 & 0 \\ 0 & \left(91/2\right)^2 \end{bmatrix}\right)$$

where M is the target moneyness and T is the target maturity in days. We impose the same moneyness/maturity bounds as above. We interpolate any missing spreads via piecewise constant interpolation.

Estimating the Risk-Neutral Default Probability. We follow Aït-Sahalia and Duarte (2003) to estimate the probability of default. The probability is the first derivative of the cross-section of put options

$$\text{Prob}\left[S_T < K\right] = e^{r(T-t)} \frac{dP\left(K\right)}{dK}$$

To estimate the derivative, we implement their two-step procedure.

1. We impose shape restrictions from no-arbitrage on the put option surface. On each day and for every maturity, we solve the following constrained least squares problem. We estimate put option prices $m(K)$ that minimize the squared pricing errors

$$\min_{m(\cdot)} \sum_{i=1}^{N} \left(P\left(K_i\right) - m\left(K_i\right)\right)^2$$

subject to the slope and convexity constraints

$$\begin{aligned} \frac{m\left(K_{i+1}\right) - m\left(K_i\right)}{K_{i+1} - K_i} &> 0 \text{ for } i = 1, \ldots, N-1 \\ \frac{m\left(K_{i+1}\right) - m\left(K_i\right)}{K_{i+1} - K_i} - \frac{m\left(K_i\right) - m\left(K_{i-1}\right)}{K_i - K_{i-1}} &> 0 \text{ for } i = 2, \ldots, N-1 \end{aligned}$$

These constraints ensure the function $m(K)$ satisfies standard no-arbitrage bounds on the cross-section of option prices.

2. We fit locally linear regressions on $m(K)$ with a Gaussian kernel. Aït-Sahalia and Duarte find that the locally linear regression is optimal to other polynomial functions in a small sample of S&P 500 index options. On each date and for every contract with strike price K_j, we solve

$$\min_{m(\cdot)} \sum_{i=1}^{N} \left(m(K_i) - \beta_{0,j} - \beta_{1,j} K_i \right)^2 \theta \left(\frac{K_i - K_j}{h} \right)$$

where $\theta(x) = \exp(-x^2/2)/\sqrt{2\pi}$ is the normal density and h is a bandwidth parameter. As Aït-Sahalia and Duarte find that pricing errors are minimized with $h \approx 70$ in 1999 when the range of available strikes is approximately 700, we set h to 10% of the range of available strikes on each date.

Using the locally linear regressions, the risk-neutral default probability is $\text{Prob}[S_T < K_j] = e^{r(T-t)} \widehat{\beta}_{1,j}$, and so the normalized implied spread is

$$NIS(K_j, T) = \frac{IS(K_j, T)}{-\log(1 - \text{Prob}[S_T < K_j]) / (T - t)} = \frac{IS(K_j, T)}{-\log\left(1 - e^{r(T-t)} \widehat{\beta}_{1,j}\right) / (T - t)}$$

For computational reasons, we instead estimate the risk-neutral default probability in simulations via the naive estimator as follows:

$$IDF(K_j, T) = -\frac{1}{T - t} \log \left(1 - \frac{1}{Z(T)} \frac{dP(K_j, T)}{dK} \right)$$

where

$$\frac{dP(K_j, T)}{dK} = \begin{cases} \dfrac{P(K_2, T) - P(K_1, T)}{K_2 - K_1} & i = 1 \\ \dfrac{P(K_{i+1}, T) - P(K_{i-1}, T)}{K_{j+1} - K_{j-1}} & 2 \leqslant i \leqslant N - 1 \\ \dfrac{P(K_N, T) - P(K_{N-1}, T)}{K_N - K_{N-1}} & i = N \end{cases}$$

if $dP/dK > 0$ and is missing otherwise (see discussion in Section 2.2). In particular, Figure 10, Figure 11, and Table 9 in the text and Figure D1 and Figure D3 in the appendix use the naive estimator.

Appendix C. Stochastic Volatility, Stochastic Jump Intensity Model

This appendix describes estimation of the stochastic volatility, stochastic jump intensity (SVSIJ) model.

Dynamics. The stock, volatility, and intensity dynamics under the physical measure P are

$$d\log S_t = \left[\mu_S - \delta - \frac{1}{2}v_t - \lambda_t E(J_S - 1)\right]dt + \sqrt{v_t}dW_{S,t} + J_S dQ_t^{\lambda_t} \tag{C1}$$

$$dv_t = k_v\left(\theta_v - v_t\right)dt + \sigma_v\sqrt{v_t}dW_{v,t} \tag{C2}$$

$$d\lambda_t = k_\lambda\left(\theta_\lambda - \lambda_t\right)dt + \sigma_\lambda\sqrt{\lambda_t}dW_{\lambda,t} \tag{C3}$$

For the stock dynamics in (C1), μ_S is the expected stock return, and δ is the dividend yield. For the volatility dynamics in (C2), κ_v is the speed of mean reversion, θ_v is the long-run mean, and σ_v is the volatility of volatility. For the intensity dynamics in (C3), κ_λ is the speed of mean reversion, θ_λ is the long-run mean, and σ_λ is the volatility of intensity. Finally, J_S and $dQ_t^{\lambda_t}$ govern the jump dynamics: J_S is the stochastic jump size with probability distribution $J_S \sim \mathcal{N}\left(\mu_J^P, \left(\sigma_j^P\right)^2\right)$, $E\left(J_S - 1\right) = \exp\left(\mu_j^P + \frac{1}{2}\left(\sigma_j^P\right)^2\right) - 1$, and $dQ_t^{\lambda_t}$, the increment of a Poisson process with intensity λ_t, determines jump arrivals.

The analogous dynamics under the risk-neutral measure Q are

$$d\log S_t = \left[r_f - \delta - \frac{1}{2}v_t - \lambda_t E(J_S^Q - 1)\right]dt + \sqrt{v_t}dW_{S,t}^Q + J_S^Q dQ_t^{\lambda_t} \tag{C4}$$

$$dv_t = k_v^Q\left(\theta_v^Q - v_t\right)dt + \sigma_v\sqrt{v_t}dW_{v,t}^Q \tag{C5}$$

$$d\lambda_t = k_\lambda^Q\left(\theta_\lambda^Q - \lambda_t\right)dt + \sigma_\lambda\sqrt{\lambda_t}dW_{\lambda,t}^Q \tag{C6}$$

For the volatility dynamics in (C5), $\kappa_v^Q = \kappa_v + \xi_v\sigma_v$ is the speed of mean reversion, $\theta_v^Q = \frac{\theta_v\kappa_v}{\kappa_v^Q}$ is the long-run mean, and ξ_v is the market price of volatility risk. For the intensity dynamics in (C6), $\kappa_\lambda^Q = \kappa_\lambda + \xi_\lambda\sigma_\lambda$ is the speed of mean reversion, $\theta_\lambda^Q = \frac{\theta_\lambda\kappa_\lambda}{\kappa_\lambda^Q}$ is the long-run mean, and ξ_λ is the market price of intensity risk. The remainder of the parameters under Q are analogous to those under P.

$\rho = E\left[W_{S,t}W_{v,t}\right]$ is the correlation between return and volatility shocks. We assume the correlation between return and intensity shocks and that between volatility and intensity shocks is zero: $E\left[W_{S,t}W_{\lambda,t}\right] = E\left[W_{v,t}W_{\lambda,t}\right] = 0$.

The assumption that $E\left[W_{v,t}W_{\lambda,t}\right] = 0$ is not ideal. On the one hand, Gormsen and Jensen (2020) document a negative correlation between volatility and higher-moment risk under the physical measure, which suggests the correlation between volatility and intensity shocks ought to be negative. On the other hand, Santa-Clara and Yan (2010) estimate a 16.8% correlation in a short sample from 1996 to 2002, which suggests zero may not be a bad approximation. We make this assumption largely for tractability: it is incredibly difficult to estimate correlations among latent processes, even between returns and volatility (see Chernov, Gallant, Ghysels, and Tauchen 2003).

We do not estimate μ_S, δ, or r and instead set these parameters to $\mu_S = 5.83\%$, $\delta = 2.00\%$, and $r = 3.70\%$ in Monte Carlo simulations. The equity premium and risk-free rate are from Chambers, Foy, Liebner, and Lu (2014). The dividend yield is our own estimate.

We impose several practical, economically-motivated constraints on these parameters to reduce the computational burden and obtain sensible estimates. First, we assume the correlation between return and volatility shocks satisfies $-0.90 < \rho < -0.76$. Second, we assume the mean jump size is negative: $-0.31 < \mu_j^P < 0$ and $\mu_j^Q < 0$. Third, we assume the jump risk premium is positive: $\mu_j^P - \mu_j^Q > 0$. Fourth, we assume the jump size standard deviation is 8.94%: $\sigma_j^P = \mu_j^Q = 8.94\%$. Fifth, we assume the volatility and intensity dynamics satisfy the Feller condition: $2\kappa_v\theta_v < \sigma_v^2$ and $2\kappa_\lambda\theta_\lambda < \sigma_\lambda^2$, respectively. The Feller condition guarantees that volatility and intensity are strictly positive. Since $2\kappa_v\theta_v = 2\kappa_v^Q\theta_v^Q$ and likewise for intensity, this assumption applies to both the physical and risk-neutral dynamics. Finally, we assume the steady state volatility and steady state intensity satisfy $0.10 < \sqrt{\theta_v} < 0.25$ and $0.15 < \theta_\lambda < 1.50$, respectively. That is, we assume that average volatility is economically reasonable and that jumps occur about once per year on average.

The least innocuous of these constraints is that on the jump size standard deviation. From a numerical standpoint, our methodology struggles to disentangle the mean and standard deviation of the jump size distribution. We thus set the standard deviation to that under SVJ from Broadie, Chernov, and Johannes (2009) and Chambers, Foy, Liebner, and Lu (2014). While these constraints need not bind at the optimum, they do so in practice. More important, the constraints help the optimizer converge in a reasonable time frame.

As is usual, we refer to the difference between P and Q parameters as risk premia. The diffusive volatility premium is $\kappa_v - \kappa_v^Q$, the diffusive intensity premium is $\kappa_\lambda - \kappa_\lambda^Q$, and the *mean* jump risk premium is $\mu_j^P - \mu_j^Q$. Given the assumption that $\sigma_j^P = \sigma_j^Q$, the *volatility* jump risk premium is zero. As is such, we refer to the mean jump risk premium as simply the jump risk premium.

In sum, we estimate five volatility parameters $\Theta_v = [\kappa_v, \theta_v, \sigma_v, \rho, \xi_v]$, four intensity parameters $\Theta_\lambda = [\kappa_\lambda, \theta_\lambda, \sigma_\lambda, \xi_\lambda]$, two jump parameters $\Theta_j = \left[\mu_J^P, \mu_J^Q\right]$, and eleven economic parameters in total $\Theta = [\Theta_v, \Theta_\lambda, \Theta_j]$. The remainder of the economic parameters, $\left[\kappa_v^Q, \theta_v^Q, \kappa_\lambda^Q, \theta_\lambda^Q, \sigma_j^P, \sigma_j^Q\right]$, follow from the restrictions above. We discuss four additional econometric parameters below.

Option Pricing. The representation above belongs to the class of affine jump-diffusion models of Duffie, Pan, and Singleton (DPS henceforth). That is, the state vector $X_t = [\log S_t, v_t, \lambda_t]^T$ dynamics under the risk-neutral measure

$$dX_t = \mu_X dt + \sigma_X dW_t^Q + dZ_t$$

are affine in the state vector with drift $\mu_X = K_0 + K_1 X_t$, instantaneous covariance $\sigma_X \sigma_X^T = H_0 + H_1 X_t$, jump process dZ_t, and jump intensity $\lambda_X = L_0 + L_1 X_t$. We also assume the interest rate is constant and all regularity conditions in DPS hold. Because our model is in fact an affine jump-diffusion, we simply follow the steps in DPS to compute option prices. We first derive the conditional characteristic function and then use standard Fast Fourier transform methods of Carr and Madan (1999) to price call options, as in DPS Section 3.1

and Section 2.2, respectively. We find equivalent put option prices via put-call parity.[1]

Maximum Likelihood Estimation. We estimate parameters via maximum likelihood subject to a return predictability constraint. In what follows, we refer to maximum likelihood as the estimation and the constraint as the calibration.

Our estimation methodology builds on the approximate maximum likelihood estimator in Kitagawa (1987). Kitagawa develops an iterated, numerical integration approach to filter non-Gaussian, latent variable models. As in Kitagawa, we also discretize the state-space to approximate the likelihood function, and so our methodology is approximate, not exact, maximum likelihood. We use this approach to estimate parameters and filter latent volatility/intensity of the two-factor SVSIJ model.

We emphasize our estimation methodology is far from original. The closest precursors to our methodology are Gala and Veronesi (2002), Bates (2006), and Bégin and Boudreault (2020). Gala and Veronesi estimate parameters and filter latent volatility of a SVJ model via Kitagawa. We instead estimate a SVSIJ model. Bates estimates a SVJ model via approximate maximum likelihood but in the space of characteristic functions. Our approximate maximum likelihood works in the usual space of probability densities. Bégin and Boudreault estimate a SVSIJ model via a high-dimensional version of Kitagawa but use only the time-series of index returns in the estimation. We also use option prices in the estimation.

Of course, our approach is one of many valid methodologies – which include simulated method of moments, simulated maximum likelihood, generalized method of moments, Markov chain Monte Carlo, among others – to estimate and filter affine jump-diffusion models. The major advantage of our methodology is that we are able to jointly estimate parameters and filter latent states. The major disadvantage is the computational burden of a two-factor model. As is such, most papers that build on Kitagawa are one-factor models (see, for example, Fridman and Harris 1998).

As it is necessary to estimate physical and risk-neutral parameters, we use both the time-series index returns and the cross-section of option prices in the estimation. The option panel consists of six series: put option prices with target moneyness K/F = 0.95, 1.00, 1.05 and target maturity 91 and 273 days. To construct this option panel, we first interpolate the implied volatility surface at these targets on each date and then convert implied volatilities to option prices via the Black-Scholes-Merton formula. Although the regressions in Section 4 use spot moneyness K/S and the estimation uses forward moneyness K/F, the definition of moneyness does not materially affect any of our results.

To evaluate the likelihood, the high-level steps are as follows:

1. Following Kitagawa, we discretize the volatility state space $(0, \infty)$ into a grid $\Gamma_v = [v_1, v_2, \cdots, v_{n_v}]$, the intensity state space $(0, \infty)$ into a grid $\Gamma_\lambda = [\lambda_1, \lambda_2, \cdots, \lambda_{n_\lambda}]$, and the jump state space $(-1, \infty)$ into a grid $\Gamma_v = [J_1, J_2, \cdots, J_{n_J}]$. We choose n_v such that $\Pr[v_t > v_{n_v}]$ is negligible and likewise for n_λ and n_J. We also discretize the stock, volatility, and intensity dynamics in the usual way with orthogonalized Gaussian

[1] In practice, we use the characteristic function option pricing toolbox in Matlab for option pricing.

shocks. These shocks preserve the correlation structure among the Brownian motions.

2. Following the option pricing literature, the likelihood function is not based on pricing errors but rather price-to-strike ratio errors. We assume the econometrician observes the option price-to-strike ratio with observational or specification error

$$\frac{\widehat{P}_{t+\Delta}(M,H)}{K_{t+\Delta}(M,H)} = \frac{P_{t+\Delta}(M,H)}{K_{t+\Delta}(M,H)} + \varepsilon_{t+\Delta}(M,H)$$

where $\widehat{P}_{t+\Delta}(M,H)$ is the observed price of a put option with moneyness $M = K/F$ and time-to-maturity H, $P_{t+\Delta}$ is the corresponding theoretical option price, $K_{t+\Delta}$ is the corresponding strike price, and $\varepsilon_{t+\Delta}$ is the corresponding error with probability distribution $\varepsilon_{t+\Delta} \mid \varepsilon_t \sim \mathcal{N}(\rho_H \varepsilon_t, \sigma_H^2)$. We estimate ρ_H and σ_H separately by maturity (four additional parameters, not tabulated). In other words, we assume the errors are independent and heteroskedastic across maturities but autocorrelated among strike prices within the same maturity.

3. The remainder of the steps are a straightforward implementation of the approximate maximum likelihood estimator in Kitagawa. At each month, we evaluate the likelihood contribution by integrating out latent variables. As in Kitagawa, we approximate the integral via a probability weighted sum on the grid. We initialize the state density at the stationary distribution. At each month, we update the state density via Bayes rule. The likelihood function is simply the product of the likelihood contributions at each month.

Calibration. With the estimated parameters and state variable dynamics, we compute model-implied put option prices on a dense moneyness/maturity grid. With prices from the model and risk-free rates from the data, we are able produce the model-implied analog to our data. That is, we compute put option prices, implied bond prices, implied spreads, and the corresponding realized returns for the moneyness/maturity targets above. To ensure an apples to apples comparison between moments, we set model-implied spreads and returns to missing if missing in the data.

Our calibration focuses on implied spreads. We omit normalized implied spreads because of attenuation bias: the implied default intensity estimator is noisy, and so implied spreads better predict implied bond returns (see discussion in Section 4.2).

Table D6 reports the target predictability moments. The key empirical results guide our choice of moments. First, implied spreads predict implied bond returns at both short and long horizons. Second, implied spreads predict put option returns at neither short nor long horizons. Third, implied bond return predictability is strongest for OTM options. To keep the number of moments feasible, we select some but not all moments that capture these results. In particular, while we target only one-month moments, we continue to compare our calibration to the non-target long-horizon moments. In practice, we impose these targets on the optimization via a nonlinear constraint.[2]

[2] First, our approach is somewhat reminiscent of that in David and Veronesi (2013). David and Veronesi

The devil is of course in the details. First, we use the model to compute prices outside the range of options ($0.90 \leqslant K/F \leqslant 1.10$, $T = 91$ and 273 days) we use to estimate the model. For instance, an option with moneyness $K/F = 0.90$ today (inside the range) would have $K/F < 0.90$ tomorrow (outside the range) if the underlying stock price rises. Second, we do not compute exact option prices. Instead, we compute option prices on a dense moneyness/maturity grid and interpolate to match the moneyness/maturity targets. While not ideal, these simplifications are necessary for computational tractability. That is, as the optimizer evaluates the likelihood over the parameter space, it must also simultaneously reevaluate the constraint, and so speed is of the essence.

There are a number of caveats in order. First, it is unlikely our optimization converges to a global optimum given these simplifications. Second, our parameters are not necessarily unique: there are certainly other parameters that also match the target predictability moments. Finally, our use of option prices in the estimation may seem somewhat circular, as we use option-implied information to explain option return predictability. However, we think this is not the case. Our goal is not to address option mispricing (as in Broadie, Chernov, and Johannes (2009)) but rather estimate parameters consistent with the target moments. In doing so, we take option prices as given.

Ultimately, the calibration is simply a means to end. The calibration ensures the parameters indeed match the return predictability (or lack thereof) in the data. Without the calibration, the SVSIJ model would not necessarily resolve the counterfactual predictions of the SVJ model.

Parameters and Fit. Panel A of Table D7 reports properties of the parameter estimates in Table 8. The average diffusive volatility is 13.86%. The average total volatility would be higher due to jumps in returns. Loosely speaking, the average number of jumps per year is 0.34. The diffusive volatility premium and diffusive intensity premium are both positive but small. The jump risk premium is 2.6%: the mean jump size under the physical measure is -4.0% and under the risk-neutral measure -6.6%. Although we do not undertake an exhaustive comparison here, our parameters are roughly in line with those in the literature.

Panel B of Table D7 evaluates the in-sample option price fit via two metrics. The first is the implied volatility absolute error:

$$IV\left(M,H\right) \text{ Absolute Error} = \left|\widehat{IV}_t\left(M,H\right) - IV_t\left(M,H\right)\right|$$

The second is the implied volatility absolute error relative to the implied volatility in the data:

$$IV\left(M,H\right) \text{ Absolute Error Ratio} = \left|\frac{\widehat{IV}_t\left(M,H\right) - IV_t\left(M,H\right)}{IV\left(M,H\right)}\right|$$

combine maximum likelihood with the method of moments to estimate a regime-switching model via the generalized method of moments (GMM). Unfortunately, we are unable to use GMM in this context because neither the score of the likelihood function nor the target return predictability moments are available in closed form. Second, we are unaware of any paper in the option pricing literature that explicitly targets return predictability moments. Of course, there are many such papers outside option pricing (see, for example, Lustig, Roussanov, and Verdelhan 2014).

Because we not only target fit via the estimation but also predictability moments via the calibration, the mean pricing errors are somewhat larger than that in the literature (see, for example, Bégin, Dorion, and Gauthier 2020). However, the median pricing errors are notably smaller than the means, which suggests outliers, in terms of fit, drive the large means. Figure D2 plots the corresponding model-implied and actual put option prices. The pricing errors are largest among short-maturity, ITM options.

Figure D3 plots model-implied and actual option-implied spreads. Both implied spreads and normalized implied spreads fit the data reasonably well, though the fit worsens somewhat after the Financial Crisis.

Panel A of Figure D4 plots the time-series of volatility and intensity in the estimation. The volatility varies from 1.6% to 57.7%. The jump intensity varies from 0.2 to 4.8. While there is a clear association between volatility and intensity, the correlation between the two is only 38% (not tabulated). For example, in bad times, volatility may be high, but intensity may be low (e.g. October 2002 and March 2020), or both volatility and intensity may be high (e.g. the Financial Crisis). In other words, the mapping between volatility and intensity is not deterministic, as in Bates (2000), Pan (2002), and Eraker (2004), among others. Santa-Clara and Yan (2010) document a similar pattern between 1996 and 2002. The volatility and intensity dynamics reinforce the need for a two-factor model.

Panel B of Figure D4 plots option-implied spreads in the data. Both implied spreads and normalized implied spreads are strongly correlated with the state variables. For implied spreads, the correlations with volatility and intensity are about the same: 67% and 66%, respectively (not tabulated). For normalized implied spreads, the correlation with intensity is a bit larger than that with volatility: 61% and 67%, respectively (not tabulated). The positive correlations are intuitive because option-implied spreads are countercyclical (see discussion in Section 3).

Panel C of Figure D4 plots the convexity of normalized implied spreads in the data. The correlations with volatility and intensity are negative: -41% and -42%, respectively (not tabulated). The negative correlations are intuitive because tail risk is countercyclical (see discussion in Section 3).

Figure D5 plots expected put option returns in the model and average put option returns in the data. Although average put option returns are difficult to reconcile with standard models (see, for example, Broadie, Chernov, and Johannes 2009 and Chambers, Foy, Liebner, and Lu 2014, among others), the SVSIJ model quantitatively matches average put option returns.

Simulated Data. We simulate 100,000 artificial datasets of length $T = 371$ months from the SVSIJ model. The steps are as follows:

1. We first simulate the dynamics of volatility and intensity under the physical measure. To initialize each path, we set the state variables to their steady state means and impose a burn-in period of 25 years.

2. We then map the state variables to option-implied spreads and one-month expected

returns from Monte Carlo simulations (see Figure 11) via interpolation.

3. We finally compute one-month realized returns as the sum of expected returns and a Gaussian shock. The shock variance is the corresponding error variance in the data (from a predictive regression of excess returns on spreads) appropriately scaled to match the variance of expected returns in the simulation:

$$\sigma_{\varepsilon,sim}^2 = \sigma_{\varepsilon,data}^2 \times \frac{\sigma^2\left(E_{t,sim}\right)}{\sigma^2\left(E_{t,data}\right)}$$

   In other words, expected returns vary less in the simulation than in the data, and so the same must hold for shocks. We assume shocks to put options and implied bonds obey the correlation structure in the data. Because the scaling depends on the predictor (e.g. $\sigma_{\varepsilon,data}^2$), we compute realized returns separately for regressions on implied spreads and regressions on normalized implied spreads.

With this, we have artificial data of (normalized) implied spreads and realized returns with which to run regressions. To be clear, the artificial data are not independent tests of the model. Their performance stems directly from the conditional expected return dynamics in the Monte Carlo simulations and the shock structure.

But the Monte Carlo simulations alone can only *qualitatively* validate the model. The goal of artificial data is to translate the Monte Carlo simulations to observable moments in the data to *quantitatively* validate the model. In each artificial dataset, we run the same predictive regressions in our empirical work. We are thus able to directly compare moments in artificial data with that in actual data.

There are two caveats. First, unlike the actual data, the artificial data do not have missing observations. Since missing observations are more likely in bad times, the moments in artificial and actual data may not be fully comparable. Second, given the shock structure, the R^2s in artificial data do not validate the model. By construction, the population R^2 in artificial data closely approximates that in actual data.

Appendix D. Additional Tables and Figures

This appendix contains additional tables and figures omitted from the text.

- Table D1 reports summary statistics for the put option and implied bond portfolios.
- Table D2 decomposes the predictive content of normalized implied spreads into that from implied spreads (the numerator) and that from implied default frequencies (the denominator). For ease of interpretation, we scale the coefficients such that the NIS coefficient equals the IS coefficient minus the IDF coefficient, and the NIS R^2 equals the IS R^2 plus the IDF R^2. Table D3 and Table D4 report long-horizon predictive regressions for implied bond excess returns in subsamples.

- Table D5 and Figure D1 show that the implications of the SVJ model are robust to alternative parameter estimates and alternative risk adjustments.

- The next set of tables and figures concern the SVSIJ model. Table D6 reports target predictability moments in the calibration. Table D7 reports properties and pricing errors in the estimation. Figure D2 (Figure D3) plots fitted and actual put option prices (option-implied spreads). Figure D4 plots risk-neutral volatility/intensity in the estimation and option-implied quantities in the data. Figure D5 plots expected put option returns in the model and average put option returns in the data. Table D8 reports estimates from predictive regressions in simulations.

Table D1: Monthly Returns: Summary Statistics

This table reports average portfolio moneyness (Panel A), the standard deviation of portfolio moneyness (Panel B), average portfolio time-to-maturity (Panel C), the standard deviation of portfolio time-to-maturity (Panel D), and the number of portfolio observations (Panel E) for the put option and implied bond portfolios. OTM is moneyness K/S $\leqslant$ 0.95, ATM is 0.975 $\leqslant$ K/S $\leqslant$ 1.025, and ITM is K/S $\geqslant$ 1.05. The sample is monthly from January 1990 to December 2020.

	Target Maturity (Days)							
Target K/S	30	60	91	122	152	182	273	365
Panel A: Average Portfolio Moneyness								
0.900	0.9071	0.9058	0.9029	0.9020	0.9031	0.9018	0.9028	0.9028
0.925	0.9295	0.9270	0.9284	0.9268	0.9271	0.9290	0.9309	0.9320
0.950	0.9530	0.9506	0.9513	0.9505	0.9501	0.9512	0.9528	0.9525
0.975	0.9786	0.9756	0.9757	0.9748	0.9742	0.9746	0.9761	0.9762
1.000	1.0028	1.0000	1.0005	1.0012	1.0018	1.0009	0.9988	0.9982
1.025	1.0264	1.0252	1.0256	1.0254	1.0251	1.0250	1.0261	1.0256
1.050	1.0476	1.0481	1.0492	1.0502	1.0502	1.0504	1.0518	1.0511
1.075	1.0675	1.0674	1.0694	1.0723	1.0742	1.0745	1.0756	1.0759
1.100	1.0885	1.0880	1.0900	1.0921	1.0936	1.0950	1.0973	1.0986
Panel B: Standard Deviation of Portfolio Moneyness								
0.900	0.0166	0.0154	0.0172	0.0150	0.0173	0.0195	0.0228	0.0234
0.925	0.0121	0.0093	0.0107	0.0099	0.0096	0.0106	0.0117	0.0136
0.950	0.0120	0.0081	0.0090	0.0087	0.0096	0.0097	0.0133	0.0130
0.975	0.0113	0.0061	0.0069	0.0070	0.0084	0.0078	0.0104	0.0109
1.000	0.0111	0.0070	0.0071	0.0093	0.0113	0.0114	0.0136	0.0138
1.025	0.0094	0.0066	0.0067	0.0069	0.0073	0.0084	0.0105	0.0109
1.050	0.0113	0.0084	0.0079	0.0069	0.0078	0.0092	0.0115	0.0125
1.075	0.0158	0.0141	0.0129	0.0104	0.0086	0.0092	0.0090	0.0099
1.100	0.0195	0.0184	0.0173	0.0156	0.0135	0.0134	0.0113	0.0112
Panel C: Average Portfolio Time-to-Maturity								
0.900	46.86	53.82	84.01	113.22	144.16	173.81	267.71	347.82
0.925	46.27	53.50	83.91	113.26	144.67	173.61	268.25	346.92
0.950	45.51	53.11	83.64	113.99	144.86	174.38	267.77	346.24
0.975	45.10	53.23	83.93	113.61	145.12	174.39	267.02	346.96
1.000	44.66	53.25	84.00	114.18	145.60	173.81	266.62	347.13
1.025	44.22	53.07	83.86	114.22	145.09	173.81	266.88	347.16
1.050	44.11	53.07	83.83	114.06	145.51	174.35	266.70	347.08
1.075	44.90	53.50	83.86	114.02	145.49	174.29	267.10	346.92
1.100	48.43	56.67	85.40	115.20	145.70	173.79	267.54	347.17
Panel D: Standard Deviation of Portfolio Time-to-Maturity								
0.900	9.36	7.30	10.32	19.92	23.21	24.78	30.25	34.87
0.925	9.30	7.09	10.43	20.09	22.77	24.10	30.24	33.74
0.950	8.17	5.53	9.45	19.74	22.06	24.13	28.29	31.47
0.975	9.12	6.20	9.16	19.68	22.25	24.20	27.57	31.92
1.000	9.53	6.22	9.38	19.62	21.84	23.81	27.06	32.37
1.025	9.04	5.37	9.47	19.51	21.95	24.32	28.33	32.46
1.050	9.11	5.37	8.79	19.38	21.66	23.54	26.48	32.26
1.075	10.21	6.78	9.21	19.36	21.65	23.59	26.59	32.38
1.100	15.39	14.38	15.04	20.73	21.69	23.69	26.82	32.32
Panel E: Number of Portfolio Observations								
0.900	371	371	371	371	370	366	347	335
0.925	371	371	371	371	370	366	351	342
0.950	371	371	371	371	370	370	366	357
0.975	371	371	371	371	370	370	366	360
1.000	371	371	371	371	371	371	366	362
1.025	371	371	371	371	371	371	365	359
1.050	371	371	371	371	371	371	366	365
1.075	371	371	371	371	371	371	366	365
1.100	361	364	368	369	369	369	365	365

Table D2: One-Month Return Predictability Decomposition

This table reports estimates from pooled regressions of the form

$$\text{Excess Return}_{i,t+1} = a + b \times \log\left(\text{Implied Quantity}_{i,t}\right) + \varepsilon_{i,t+1}$$

The excess return is either the put option return in excess of the risk-free rate or the implied bond return in excess of the risk-free rate. The implied quantity is either the put's implied spread, implied default frequency, or normalized implied spread. The coefficients on the implied spread and implied default frequency are scaled such that

$$\widetilde{b}_{IS} = \frac{b_{IS}\sigma_{IS}^2}{\sigma_{NIS}^2} \qquad \widetilde{b}_{IDF} = \frac{b_{IDF}\sigma_{IDF}^2}{\sigma_{NIS}^2} \qquad b_{NIS} = \widetilde{b}_{IS} - \widetilde{b}_{IDF}$$

and so

$$\widetilde{R}_{IS}^2 = \left(\widetilde{b}_{IS}^2 - \widetilde{b}_{IS}\widetilde{b}_{IDF}\right)\frac{\sigma_{NIS}^2}{\sigma_R^2} \qquad \widetilde{R}_{IDF}^2 = -\left(\widetilde{b}_{IDF}^2 - \widetilde{b}_{IS}\widetilde{b}_{IDF}\right)\frac{\sigma_{NIS}^2}{\sigma_R^2} \qquad R_{NIS}^2 = \widetilde{R}_{IS}^2 + \widetilde{R}_{IDF}^2$$

OTM is moneyness K/S $\leqslant 0.95$, ATM is $0.975 \leqslant$ K/S $\leqslant 1.025$, and ITM is K/S $\geqslant 1.05$. Standard errors are clustered by month. The sample is monthly from January 1990 to December 2020.

		(1)	(2)	(3)	(4)	(5)	(6)
		Implied Volatility		Implied Spread		Normalized Implied Spread	
		Put	Implied Bond	Put	Implied Bond	Put	Implied Bond
OTM	b	18.72	0.88	-1.52	0.51	20.24	0.36
$T \leqslant 122$	t	(1.86)	(3.00)	(-0.24)	(3.33)	(3.65)	(2.37)
	R2	0.01	0.04	-0.00	0.02	0.01	0.02
	N	4452	4452	4452	4452	4452	4452
ATM	b	-1.51	0.88	-6.47	0.45	4.96	0.43
$T \leqslant 122$	t	(-0.20)	(2.75)	(-1.60)	(3.21)	(0.70)	(1.65)
	R2	-0.00	0.01	-0.00	0.01	0.00	0.01
	N	4452	4452	4452	4452	4452	4452
ITM	b	-3.37	0.65	-4.19	0.44	0.81	0.21
$T \leqslant 122$	t	(-0.88)	(2.08)	(-0.93)	(1.41)	(0.18)	(0.53)
	R2	-0.00	0.00	-0.00	0.00	0.00	0.00
	N	4430	4430	4430	4430	4430	4430
OTM	b	-1.54	0.94	-4.45	0.30	2.91	0.64
$T \geqslant 152$	t	(-0.21)	(2.41)	(-0.88)	(1.42)	(0.65)	(2.87)
	R2	-0.00	0.03	-0.00	0.01	0.00	0.02
	N	4310	4310	4310	4310	4310	4310
ATM	b	-2.92	0.73	-5.40	0.11	2.49	0.62
$T \geqslant 152$	t	(-0.56)	(1.78)	(-1.65)	(0.60)	(0.60)	(2.05)
	R2	-0.00	0.01	-0.00	0.00	0.00	0.01
	N	4402	4402	4402	4402	4402	4402
ITM	b	-4.87	0.73	-7.42	0.28	2.55	0.45
$T \geqslant 152$	t	(-1.42)	(1.81)	(-2.18)	(0.91)	(0.69)	(1.09)
	R2	-0.00	0.00	-0.00	0.00	0.00	0.00
	N	4414	4414	4414	4414	4414	4414

Table D3: Long-Horizon Implied Bond Return Predictability: January 1990 to June 2005

This table reports estimates from predictive regressions of the form

$$\sum_{i=1}^{H} \log\left(1 + R^{IB}_{t+i}(M,H)\right) - r^f_{t-1+i} = a + \mathbf{b}'\mathbf{X}_t(M,H) + \varepsilon_{t+i}$$

r^f_{t-1+i} is the continuously compounded risk-free rate at $t-1+i$. $R^{IB}_{t+i}(M,H)$ is the monthly return from $t+i-1$ to $t+i$ of an implied bond with moneyness $M = K/S$ and time-to-maturity H. $X_t(M,H)$ is either the implied volatility IV, implied spread IS, or normalized implied spread NIS of a put option with moneyness M and time-to-maturity H. Test statistics are calculated with the Hansen and Hodrick standard error correction for overlapping data. The sample is monthly from January 1990 to June 2005. The corresponding full sample regressions are in Table 5.

Panel A: Univariate Regressions

		Implied Volatility				Implied Spread				Normalized Implied Spread			
		Horizon (Months)				Horizon (Months)				Horizon (Months)			
K/S		3	6	9	12	3	6	9	12	3	6	9	12
0.900	b	0.13	0.21	0.28	0.31	0.24	0.52	0.69	0.87	0.26	0.34	0.38	0.44
	t	(4.28)	(4.29)	(2.85)	(2.63)	(5.82)	(5.28)	(3.23)	(2.79)	(3.98)	(5.15)	(6.53)	(5.25)
	R2	0.18	0.16	0.14	0.12	0.23	0.21	0.15	0.15	0.13	0.15	0.13	0.13
	N	180	162	136	108	180	162	136	108	180	162	136	108
0.950	b	0.14	0.21	0.23	0.22	0.17	0.37	0.33	0.54	0.37	0.35	0.35	0.34
	t	(3.37)	(2.74)	(1.54)	(1.08)	(2.49)	(2.26)	(0.86)	(1.06)	(3.92)	(3.47)	(2.62)	(1.98)
	R2	0.10	0.08	0.04	0.03	0.08	0.08	0.02	0.03	0.09	0.08	0.06	0.04
	N	180	174	165	157	180	174	165	157	180	174	165	157
0.975	b	0.14	0.18	0.17	0.15	0.19	0.35	0.51	0.46	0.44	0.36	0.35	0.31
	t	(2.52)	(1.93)	(0.95)	(0.63)	(2.44)	(1.89)	(1.20)	(0.81)	(3.61)	(2.75)	(2.19)	(1.33)
	R2	0.06	0.05	0.02	0.01	0.07	0.06	0.04	0.02	0.07	0.05	0.04	0.02
	N	180	174	168	162	180	174	168	162	180	174	168	162
1.000	b	0.15	0.16	0.12	0.10	0.17	0.29	0.44	0.51	0.47	0.42	0.43	0.33
	t	(2.22)	(1.26)	(0.56)	(0.36)	(1.88)	(1.37)	(0.91)	(0.85)	(2.82)	(2.38)	(2.25)	(1.15)
	R2	0.04	0.02	0.00	-0.00	0.04	0.03	0.02	0.02	0.05	0.04	0.04	0.02
	N	180	180	168	162	180	180	168	162	180	180	168	162

Panel B: Multivariate Regressions

		IS and IV					NIS and IV					IS and NIS			
		Horizon (Months)					Horizon (Months)					Horizon (Months)			
K/S		3	6	9	12		3	6	9	12		3	6	9	12
0.900	IS	0.21	0.51	0.45	0.86	NIS	0.06	0.15	0.19	0.27	IS	0.22	0.43	0.45	0.62
	t	(3.55)	(3.03)	(1.67)	(1.83)	t	(0.92)	(0.81)	(0.96)	(0.76)	t	(4.66)	(2.43)	(1.36)	(1.30)
	IV	0.02	0.00	0.11	0.01	IV	0.11	0.14	0.17	0.16	NIS	0.04	0.10	0.20	0.20
	t	(0.55)	(0.02)	(0.97)	(0.04)	t	(3.23)	(1.20)	(0.79)	(0.48)	t	(0.62)	(0.77)	(1.97)	(1.45)
	R2	0.23	0.21	0.15	0.15	R2	0.18	0.16	0.14	0.13	R2	0.23	0.21	0.16	0.16
	N	180	162	136	108	N	180	162	136	108	N	180	162	136	108
0.950	IS	-0.01	0.15	-0.47	0.54	NIS	0.15	0.16	0.34	0.46	IS	0.08	0.21	-0.06	0.18
	t	(-0.06)	(0.28)	(-0.77)	(0.57)	t	(0.69)	(0.52)	(0.84)	(0.99)	t	(0.76)	(0.77)	(-0.11)	(0.22)
	IV	0.15	0.13	0.44	-0.00	IV	0.10	0.13	0.01	-0.11	NIS	0.24	0.20	0.37	0.27
	t	(1.83)	(0.47)	(2.23)	(-0.00)	t	(1.10)	(0.63)	(0.03)	(-0.23)	t	(1.30)	(0.89)	(1.82)	(0.90)
	R2	0.09	0.08	0.05	0.03	R2	0.09	0.08	0.05	0.04	R2	0.09	0.09	0.05	0.04
	N	180	174	165	157	N	180	174	165	157	N	180	174	165	157
0.975	IS	0.20	0.39	0.82	0.70	NIS	0.39	0.28	0.57	0.57	IS	0.10	0.22	0.27	0.16
	t	(1.22)	(0.81)	(1.26)	(0.67)	t	(1.44)	(0.70)	(1.29)	(1.11)	t	(0.84)	(0.72)	(0.48)	(0.19)
	IV	-0.01	-0.02	-0.17	-0.12	IV	0.02	0.05	-0.19	-0.23	NIS	0.26	0.18	0.23	0.24
	t	(-0.08)	(-0.10)	(-0.63)	(-0.27)	t	(0.17)	(0.20)	(-0.44)	(-0.42)	t	(1.36)	(0.70)	(1.17)	(0.70)
	R2	0.07	0.05	0.04	0.01	R2	0.07	0.05	0.04	0.03	R2	0.08	0.06	0.04	0.02
	N	180	174	168	162	N	180	174	168	162	N	180	174	168	162
1.000	IS	0.08	0.35	0.87	1.18	NIS	0.41	0.63	0.98	0.87	IS	0.06	0.13	0.09	0.29
	t	(0.37)	(1.06)	(1.30)	(1.16)	t	(1.18)	(1.50)	(1.98)	(1.48)	t	(0.45)	(0.49)	(0.14)	(0.36)
	IV	0.08	-0.05	-0.26	-0.37	IV	0.02	-0.13	-0.44	-0.45	NIS	0.35	0.31	0.38	0.20
	t	(0.54)	(-0.27)	(-0.93)	(-0.74)	t	(0.17)	(-0.49)	(-0.96)	(-0.75)	t	(1.26)	(1.38)	(1.47)	(0.55)
	R2	0.04	0.03	0.03	0.03	R2	0.05	0.04	0.07	0.04	R2	0.05	0.04	0.04	0.02
	N	180	180	168	162	N	180	180	168	162	N	180	180	168	162

Table D4: Long-Horizon Implied Bond Return Predictability: July 2005 to December 2020

This table reports estimates from predictive regressions of the form

$$\sum_{i=1}^{H} \log\left(1 + R^{IB}_{t+i}(M,H)\right) - r^f_{t-1+i} = a + \mathbf{b}'\mathbf{X}_t(M,H) + \varepsilon_{t+i}$$

r^f_{t-1+i} is the continuously compounded risk-free rate at $t-1+i$. $R^{IB}_{t+i}(M,H)$ is the monthly return from $t+i-1$ to $t+i$ of an implied bond with moneyness $M = K/S$ and time-to-maturity H. $X_t(M,H)$ is either the implied volatility IV, implied spread IS, or normalized implied spread NIS of a put option with moneyness M and time-to-maturity H. Test statistics are calculated with the Hansen and Hodrick standard error correction for overlapping data. The sample is monthly from July 2005 to December 2020. The corresponding full sample regressions are in Table 5.

Panel A: Univariate Regressions

		Implied Volatility				Implied Spread				Normalized Implied Spread			
		Horizon (Months)				Horizon (Months)				Horizon (Months)			
K/S		3	6	9	12	3	6	9	12	3	6	9	12
0.900	b	0.14	0.30	0.42	0.46	0.17	0.53	0.90	1.14	0.36	0.60	0.65	0.60
	t	(2.49)	(4.17)	(3.63)	(3.72)	(1.52)	(3.08)	(4.05)	(6.93)	(3.61)	(4.65)	(3.04)	(2.66)
	R2	0.14	0.20	0.19	0.16	0.10	0.19	0.22	0.22	0.12	0.16	0.13	0.09
	N	186	186	186	186	186	186	186	186	186	186	186	186
0.950	b	0.14	0.31	0.42	0.55	0.16	0.51	0.82	1.21	0.40	0.65	0.71	0.86
	t	(1.71)	(2.61)	(2.82)	(3.42)	(1.43)	(3.12)	(4.22)	(6.46)	(2.68)	(4.17)	(3.13)	(3.32)
	R2	0.08	0.12	0.13	0.15	0.07	0.13	0.15	0.19	0.08	0.12	0.11	0.13
	N	186	186	186	186	186	186	186	186	186	186	186	186
0.975	b	0.15	0.31	0.43	0.53	0.18	0.50	0.85	1.24	0.40	0.57	0.70	0.84
	t	(1.50)	(2.45)	(2.64)	(3.09)	(1.69)	(2.79)	(3.73)	(6.12)	(2.48)	(2.95)	(2.52)	(3.25)
	R2	0.07	0.11	0.12	0.12	0.08	0.12	0.15	0.18	0.07	0.08	0.09	0.11
	N	186	186	186	186	186	186	186	186	186	186	186	186
1.000	b	0.13	0.29	0.43	0.53	0.14	0.47	0.86	1.19	0.26	0.57	0.70	0.85
	t	(1.11)	(1.79)	(2.34)	(2.83)	(1.13)	(2.75)	(3.29)	(6.16)	(1.38)	(2.67)	(2.34)	(3.45)
	R2	0.04	0.07	0.10	0.11	0.03	0.10	0.13	0.15	0.02	0.07	0.08	0.10
	N	186	186	186	186	186	186	186	186	186	186	186	186

Panel B: Multivariate Regressions

		IS and IV					NIS and IV					IS and NIS			
		Horizon (Months)					Horizon (Months)					Horizon (Months)			
K/S		3	6	9	12		3	6	9	12		3	6	9	12
0.900	IS	-0.27	0.12	1.13	2.41	NIS	0.11	0.18	-0.04	-0.23	IS	0.06	0.36	0.98	1.71
	t	(-0.86)	(0.28)	(4.16)	(2.37)	t	(0.65)	(0.86)	(-0.20)	(3.22)	t	(0.28)	(1.07)	(2.68)	(12.57)
	IV	0.31	0.24	-0.12	-0.63	IV	0.10	0.23	0.44	0.58	NIS	0.28	0.25	-0.09	-0.55
	t	(1.81)	(1.46)	(-1.50)	(-1.12)	t	(1.05)	(1.67)	(2.14)	(4.89)	t	(1.14)	(0.85)	(-0.52)	(-2.26)
	R2	0.16	0.20	0.21	0.24	R2	0.14	0.20	0.18	0.16	R2	0.12	0.19	0.21	0.24
	N	186	186	186	186	N	186	186	186	186	N	186	186	186	186
0.950	IS	-0.16	0.68	1.30	2.27	NIS	0.22	0.31	0.19	0.21	IS	0.06	0.38	0.80	1.34
	t	(-0.46)	(1.77)	(1.48)	(1.91)	t	(0.81)	(0.76)	(0.57)	(0.68)	t	(0.26)	(1.13)	(3.48)	(-5.39)
	IV	0.27	-0.11	-0.28	-0.56	IV	0.07	0.19	0.33	0.44	NIS	0.29	0.22	0.02	-0.13
	t	(1.29)	(-0.34)	(-0.46)	(-0.77)	t	(0.51)	(0.72)	(1.14)	(1.60)	t	(0.84)	(0.65)	(0.12)	(-0.60)
	R2	0.08	0.13	0.15	0.19	R2	0.08	0.12	0.13	0.14	R2	0.08	0.13	0.15	0.18
	N	186	186	186	186	N	186	186	186	186	N	186	186	186	186
0.975	IS	0.43	0.55	1.99	3.39	NIS	0.23	0.10	0.14	0.32	IS	0.13	0.46	0.97	1.48
	t	(4.09)	(1.59)	(2.02)	(2.09)	t	(0.80)	(0.35)	(0.48)	(0.78)	t	(0.73)	(1.67)	(3.89)	(-5.46)
	IV	-0.22	-0.03	-0.67	-1.14	IV	0.08	0.27	0.37	0.36	NIS	0.16	0.06	-0.15	-0.23
	t	(-1.77)	(-0.11)	(-1.02)	(-1.25)	t	(0.45)	(1.22)	(1.44)	(1.07)	t	(0.65)	(0.23)	(-0.62)	(-1.20)
	R2	0.08	0.11	0.16	0.22	R2	0.07	0.10	0.11	0.12	R2	0.08	0.11	0.14	0.18
	N	186	186	186	186	N	186	186	186	186	N	186	186	186	186
1.000	IS	0.03	0.65	1.01	1.82	NIS	0.04	0.28	0.21	0.45	IS	0.13	0.41	0.92	1.20
	t	(0.22)	(2.39)	(2.11)	(1.95)	t	(0.17)	(0.94)	(0.60)	(0.98)	t	(0.80)	(1.94)	(2.43)	(5.75)
	IV	0.11	-0.15	-0.09	-0.37	IV	0.12	0.19	0.34	0.30	NIS	0.05	0.12	-0.08	-0.02
	t	(0.66)	(-0.47)	(-0.23)	(-0.59)	t	(0.80)	(0.78)	(1.20)	(0.80)	t	(0.26)	(0.55)	(-0.25)	(-0.06)
	R2	0.03	0.10	0.12	0.16	R2	0.03	0.07	0.10	0.11	R2	0.03	0.10	0.12	0.15
	N	186	186	186	186	N	186	186	186	186	N	186	186	186	186

Table D5: SVJ Parameter Estimates: Robustness

This table reports parameter estimates for the SVJ model. BCJ 2005 refers to the parameters from Broadie, Chernov, and Johannes (2009) with sample between 1987 and 2005. CFLL 2012 refers to the parameters from Chambers, Foy, Liebner, and Lu (2014) with sample between 1987 and 2012. HL 2015 refers to the parameters from Hu and Liu (2019) with sample between 1998 and 2015. The risk adjustment for volatility risk is $\kappa^Q = \kappa_v + \xi_v \sigma_v$ with $\xi_v = \gamma\rho$. The risk adjustment for jump risk is $\lambda^Q = \lambda^P \exp\left(-\mu_J \gamma + \frac{1}{2}\gamma^2 \sigma_J^2\right)$ and $\mu_J^Q = \mu_J^P - \gamma\sigma_J^2$. Risk aversion $\gamma = 10$.

			P-Measure Parameters										Q-Measure Parameters			
		Risk Adjustment	r	δ	μ_S	λ^P	μ_J^P	σ_J^P	κ^P	θ^P	σ_v	ρ	λ^Q	μ_J^Q	σ_J^Q	ξ_v
1	BCJ 2005	No Adjustment	4.50%	2.00%	5.41%	0.91	-3.25%	6.00%	5.330	0.0182	0.140	-0.520	0.91	-3.25%	6.00%	0.00
2	BCJ 2005	Volatility Risk	4.50%	2.00%	5.41%	0.91	-3.25%	6.00%	5.330	0.0182	0.140	-0.520	0.91	-3.25%	6.00%	-5.20
3	BCJ 2005	Jump Risk	4.50%	2.00%	5.41%	0.91	-3.25%	6.00%	5.330	0.0182	0.140	-0.520	1.51	-6.85%	6.00%	0.00
4	BCJ 2005	Volatility and Jump Risk	4.50%	2.00%	5.41%	0.91	-3.25%	6.00%	5.330	0.0182	0.140	-0.520	1.51	-6.85%	6.00%	-5.20
5	CFLL 2012	No Adjustment	3.70%	2.00%	5.83%	1.29	-1.22%	8.94%	2.192	0.0171	0.258	-0.760	1.29	-1.22%	8.94%	0.00
6	CFLL 2012	Volatility Risk	3.70%	2.00%	5.83%	1.29	-1.22%	8.94%	2.192	0.0171	0.258	-0.760	1.29	-1.22%	8.94%	-7.60
7	CFLL 2012	Jump Risk	3.70%	2.00%	5.83%	1.29	-1.22%	8.94%	2.192	0.0171	0.258	-0.760	2.17	-9.21%	8.94%	0.00
8	CFLL 2012	Volatility and Jump Risk	3.70%	2.00%	5.83%	1.29	-1.22%	8.94%	2.192	0.0171	0.258	-0.760	2.17	-9.21%	8.94%	-7.60
9	HL 2015	No Adjustment	2.01%	1.74%	5.06%	0.97	-2.09%	6.77%	5.986	0.0358	0.542	-0.801	0.97	-2.09%	6.77%	0.00
10	HL 2015	Volatility Risk	2.01%	1.74%	5.06%	0.97	-2.09%	6.77%	5.986	0.0358	0.542	-0.801	0.97	-2.09%	6.77%	-8.02
11	HL 2015	Jump Risk	2.01%	1.74%	5.06%	0.97	-2.09%	6.77%	5.986	0.0358	0.542	-0.801	1.50	-6.67%	6.77%	0.00
12	HL 2015	Volatility and Jump Risk	2.01%	1.74%	5.06%	0.97	-2.09%	6.77%	5.986	0.0358	0.542	-0.801	1.50	-6.67%	6.77%	-8.02
13	BCJ 2005	Estimation Risk	4.50%	2.00%	5.41%	0.91	-3.25%	6.00%	5.330	0.0182	0.140	-0.520	1.25	-4.96%	6.99%	0.00
14	CFLL 2012	Estimation Risk	3.70%	2.00%	5.83%	1.29	-1.22%	8.94%	2.192	0.0171	0.258	-0.760	1.64	-2.80%	10.04%	0.00

Figure D1: Expected Excess Returns in SVJ: Robustness

This figure plots one-month expected excess returns by option-implied quantities from Monte Carlo simulations of the SVJ model with no risk adjustments (upper left), volatility risk adjustments (upper right), jump risk adjustments (bottom left), and volatility and jump risk adjustments (bottom right). BCJ 2005 refers to the parameters from Broadie, Chernov, and Johannes (2009) with sample between 1987 and 2005. The corresponding parameter estimates are in Table D5.

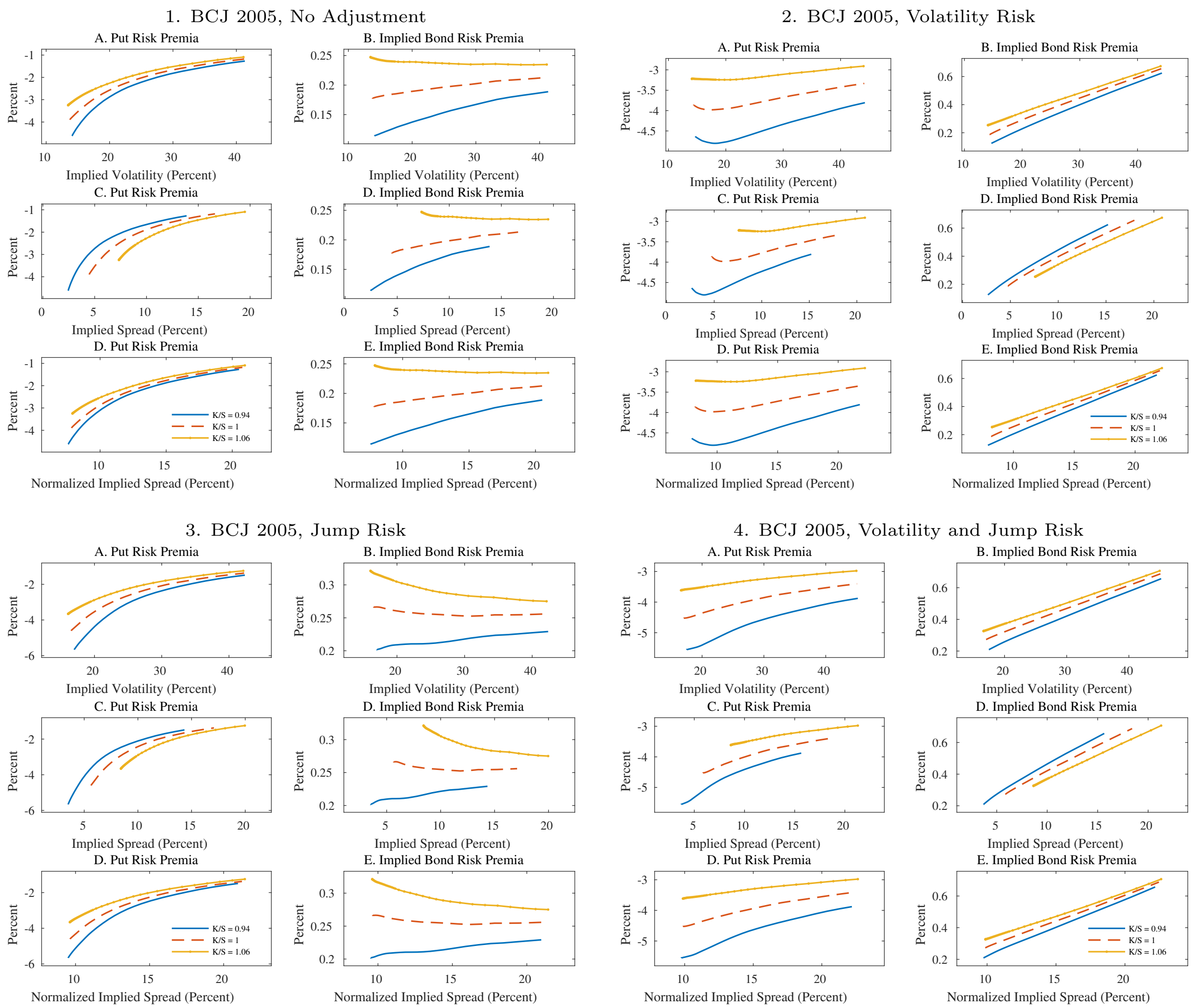

Figure D1 (Continued): Expected Excess Returns in SVJ: Robustness

This figure plots one-month expected excess returns by option-implied quantities from Monte Carlo simulations of the SVJ model with no risk adjustments (upper left), volatility risk adjustments (upper right), jump risk adjustments (bottom left), and volatility and jump risk adjustments (bottom right). CFLL 2012 refers to the parameters from Chambers, Foy, Liebner, and Lu (2014) with sample between 1987 and 2012. The corresponding parameter estimates are in Table D5.

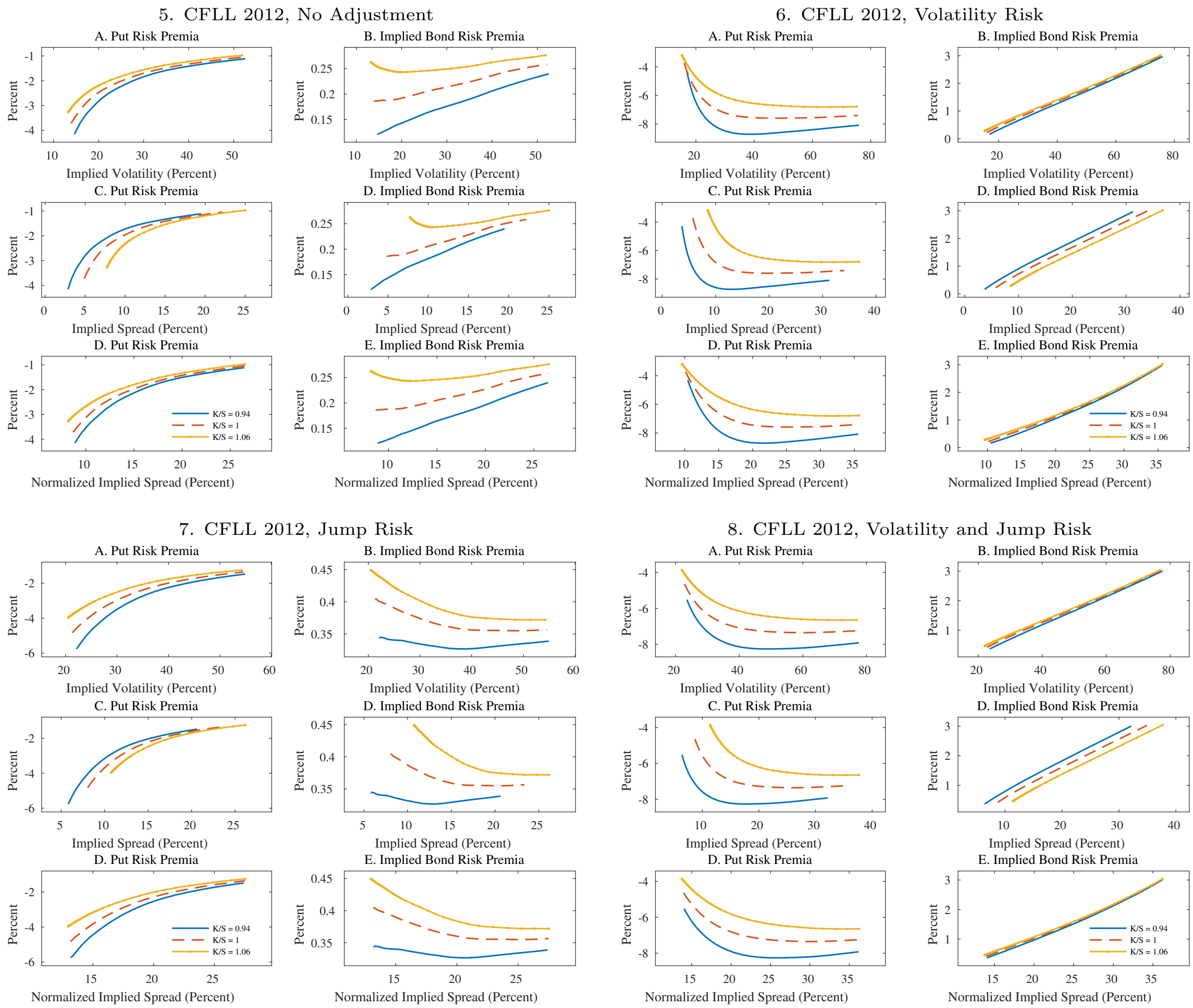

Figure D1 (Continued): Expected Excess Returns in SVJ: Robustness

This figure plots one-month expected excess returns by option-implied quantities from Monte Carlo simulations of the SVJ model with no risk adjustments (upper left), volatility risk adjustments (upper right), jump risk adjustments (bottom left), and volatility and jump risk adjustments (bottom right). HL 2015 refers to the parameters from Hu and Liu (2019) with sample between 1998 and 2015. The corresponding parameter estimates are in Table D5.

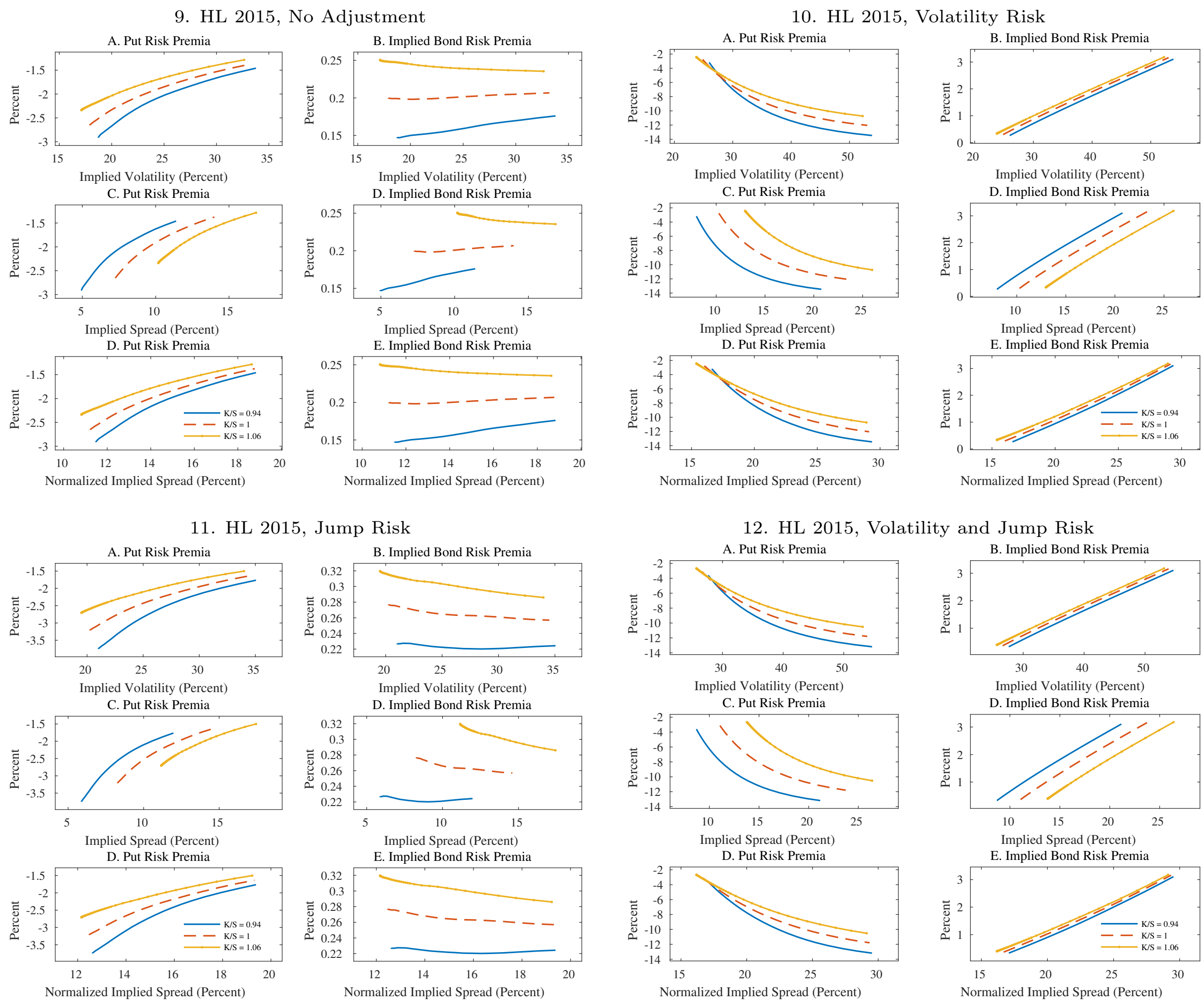

Figure D1 (Continued): Expected Excess Returns in SVJ: Robustness

This figure plots one-month expected excess returns by option-implied quantities from Monte Carlo simulations of the SVJ model with estimation risk adjustments. BCJ 2005 refers to the parameters from Broadie, Chernov, and Johannes (2009) with sample between 1987 and 2005. CFLL 2012 refers to the parameters from Chambers, Foy, Liebner, and Lu (2014) with sample between 1987 and 2012. The corresponding parameter estimates are in Table D5.

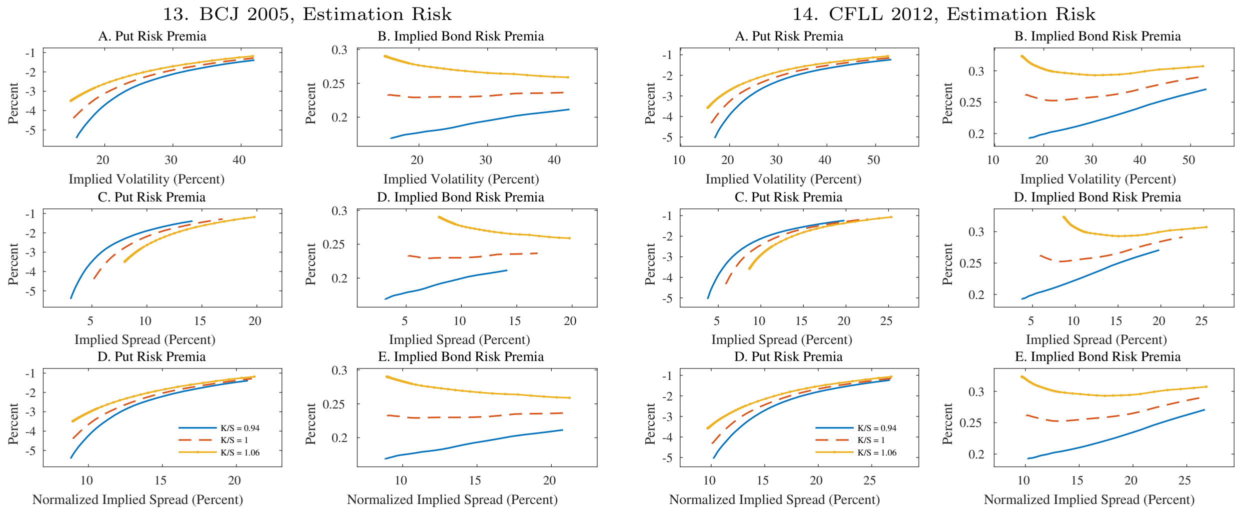

Table D6: Target Predictability Moments in SVSIJ

This table reports target moments from predictive regressions of excess returns on implied spreads in the calibration of the SVSIJ model. The first column is the moneyness bin. The second column is the time-to-maturity bin. The third column is the table with the corresponding regression in the data. The fourth column is the R^2 in the data. The fifth column is the constraint on the R^2 in the calibration. The sixth column is the R^2 in the calibration. OTM is moneyness K/S $\leqslant 0.95$. The corresponding parameter estimates are in Table 8.

		Data		Model	
Moneyness	Maturity (Days)	Table	R^2	Constraint	R^2
Panel A: Put Option Excess Returns					
OTM	$\leqslant 122$	2C	0.00	$R^2 < 0.0075$	0.0016
OTM	$\geqslant 152$	2C	0.00	$R^2 < 0.0075$	0.0012
0.90	91	6A	-0.00		0.0034
0.90	182	6A	-0.00		0.0049
0.90	273	6A	0.00		0.0019
0.90	365	6A	0.00		0.0015
Panel B: Implied Bond Excess Returns					
OTM	$\leqslant 122$	2C	0.05	$R^2 > 0.0491$	0.0789
				$R^2 < 0.0791$	
OTM	$\geqslant 152$	2C	0.03	$R^2 > 0.0237$	0.0237
				$R^2 < 0.0537$	
0.90	91	5A	0.13		0.0930
0.90	182	5A	0.19		0.0669
0.90	273	5A	0.19		0.0318
0.90	365	5A	0.19		0.0101

Table D7: Estimation Properties and Option Price Fit in SVSIJ

This table reports properties (Panel A) and pricing errors (Panel B) in the estimation of the SVSIJ model. The IV absolute error ratio is IV absolute error relative to the implied volatility in the data. OTM is moneyness K/F = 0.95, ATM is K/F = 1.00, and ITM is K/F = 1.05. The corresponding parameter estimates are in Table 8.

Panel A: Estimation Properties		
Log-Likelihood		10786.33
Average Volatility	$\sqrt{\theta_v}$	13.86%
Average Intensity	θ_λ	0.3412
Diffusive Volatility Premium	$\kappa_v^P - \kappa_v^Q$	0.9228
Diffusive Intensity Premium	$\kappa_\lambda^P - \kappa_\lambda^Q$	0.7290
Jump Risk Premium	$\mu_J^P - \mu_J^Q$	2.58%

Panel B: Option Price Fit					
		IV Absolute Error (Percent)		IV Absolute Error Ratio (Percent)	
		$T = 91$ days	$T = 273$ days	$T = 91$ days	$T = 273$ days
OTM	Mean	2.78	2.20	13.17	10.56
	Median	2.15	1.63	11.16	8.33
ATM	Mean	2.97	2.21	16.75	11.77
	Median	2.37	1.51	14.52	8.66
ITM	Mean	3.20	2.35	20.93	13.98
	Median	2.38	1.67	17.44	10.69

Figure D2: Fitted Option Prices in SVSIJ

This figure plots the actual (blue) and fitted (orange) put option price-to-strike ratio by moneyness and time-to-maturity. Model-implied prices are from the SVSIJ model. OTM is moneyness K/F = 0.95, ATM is K/F = 1.00, and ITM is K/F = 1.05.

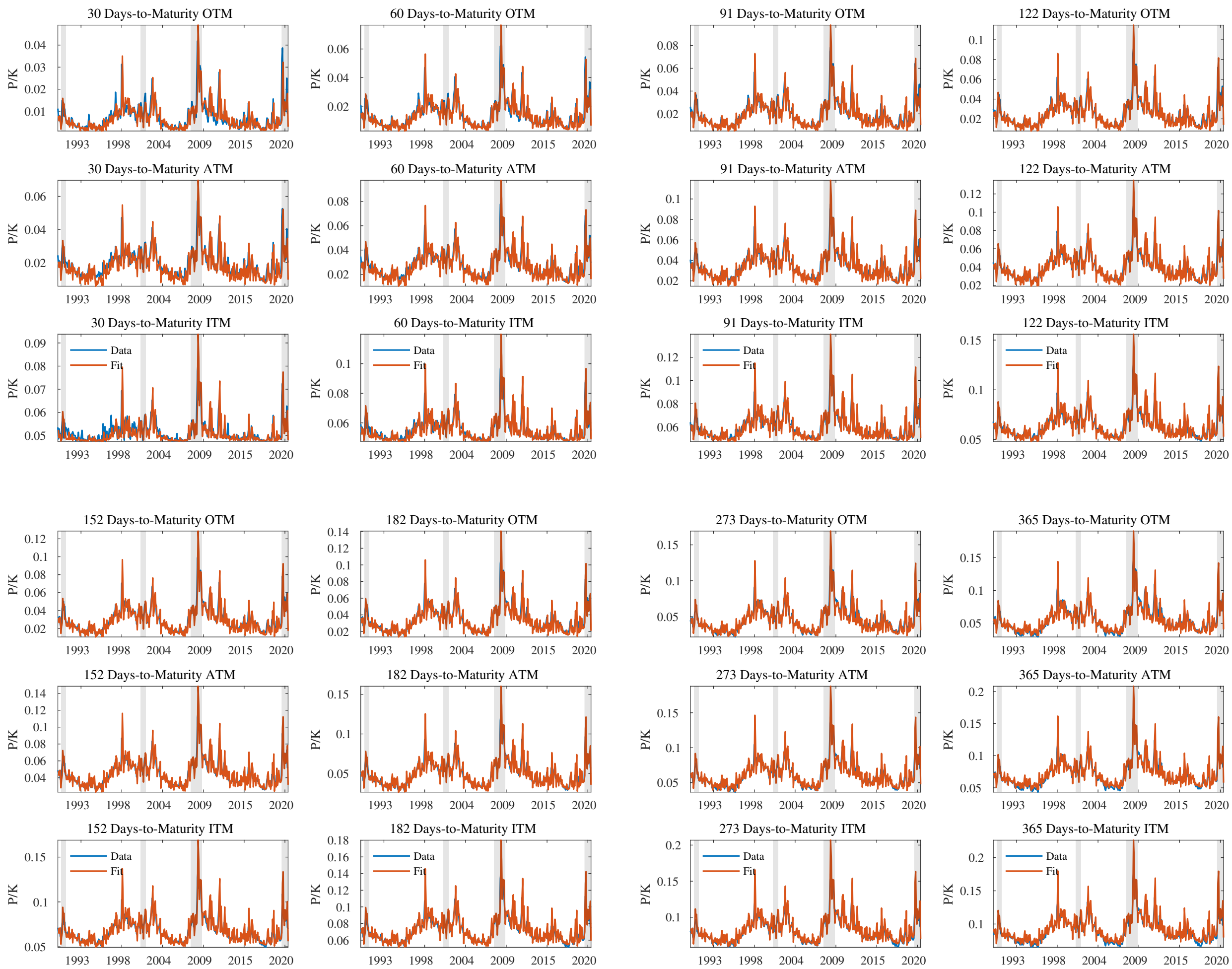

Figure D3: Fitted Option-Implied Spreads in SVSIJ

This figure plots actual (blue) and fitted (orange) option-implied spreads. Panel A plots the average at-the-money implied spread across maturities. Panel B plots the average at-the-money normalized implied spread across maturities. Model-implied spreads are from the SVSIJ model. Grey bands are NBER recessions. The sample is monthly from January 1990 to December 2020.

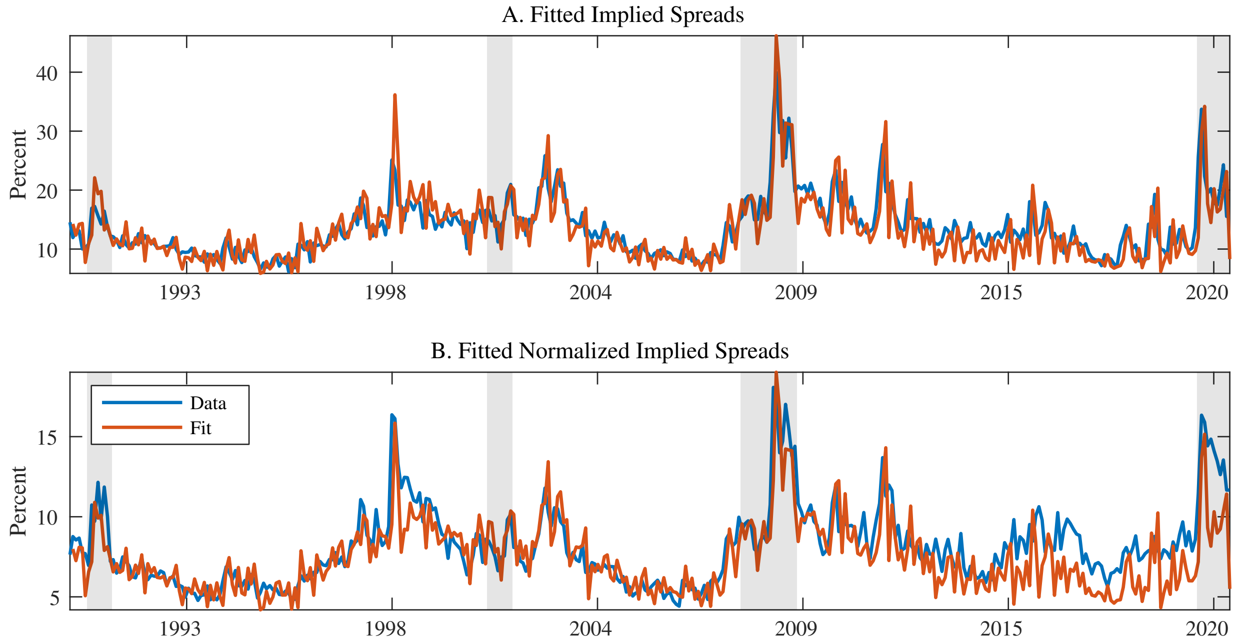

Figure D4: Option-Implied Spreads by Risk-Neutral Volatility/Intensity

Panel A plots the posterior mean of risk-neutral volatility (blue) and risk-neutral intensity (orange) in the estimation of the SVSIJ model. Panel B plots the average at-the-money implied spread (blue) and the average at-the-money normalized implied spread (orange) across maturities. Panel C plots the average convexity of normalized implied spreads across maturities. Grey bands are NBER recessions. The sample is monthly from January 1990 to December 2020.

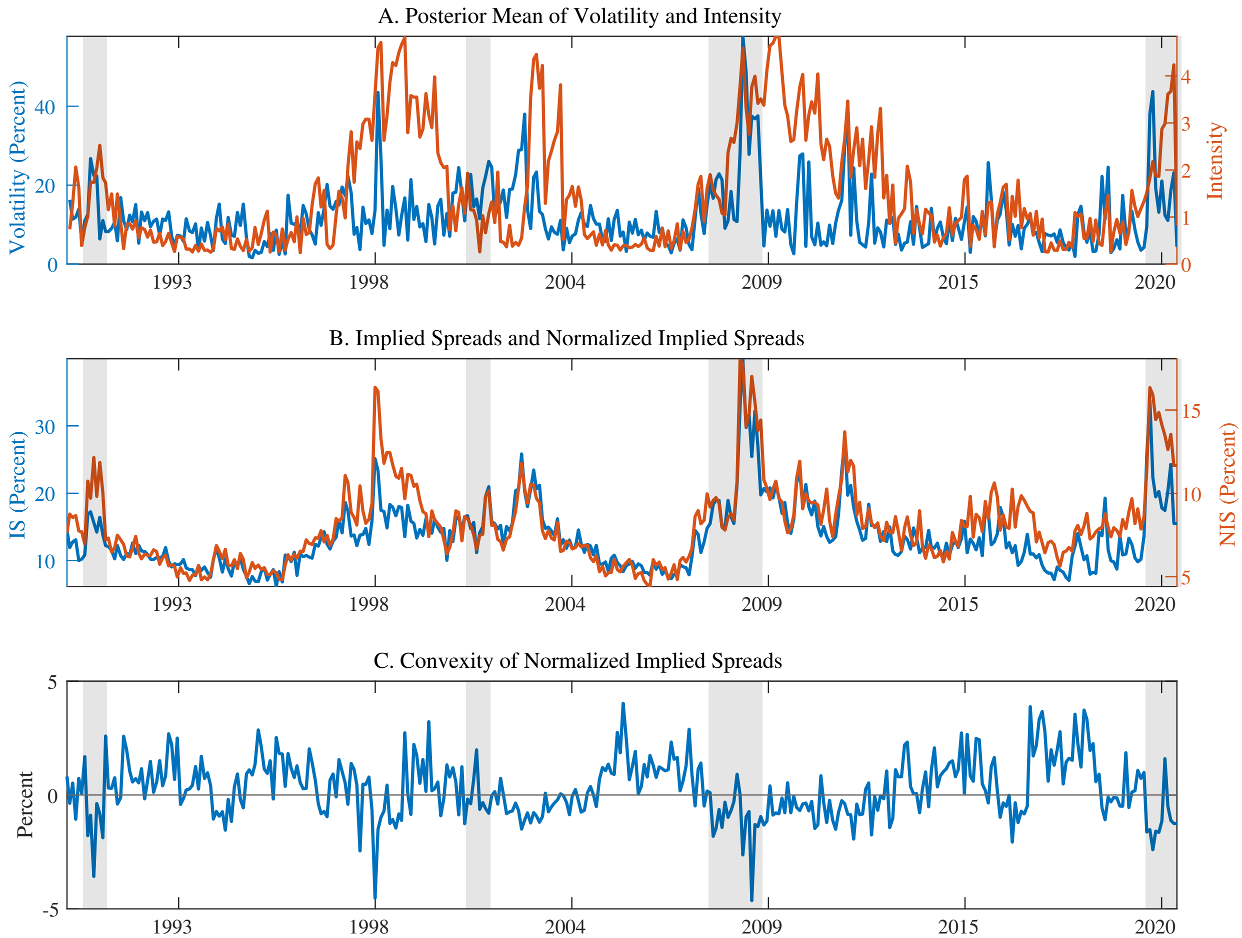

Figure D5: Expected Excess Put Option Returns in SVSIJ

This figure reports average put option returns in the data (blue) and expected excess put option returns in the SVSIJ model (orange) by moneyness and time-to-maturity. Model-implied expected returns are from Monte Carlo simulations of the SVSIJ model. OTM is moneyness K/S = 0.90, ATM is K/S = 1.00, and ITM is K/S = 1.10.

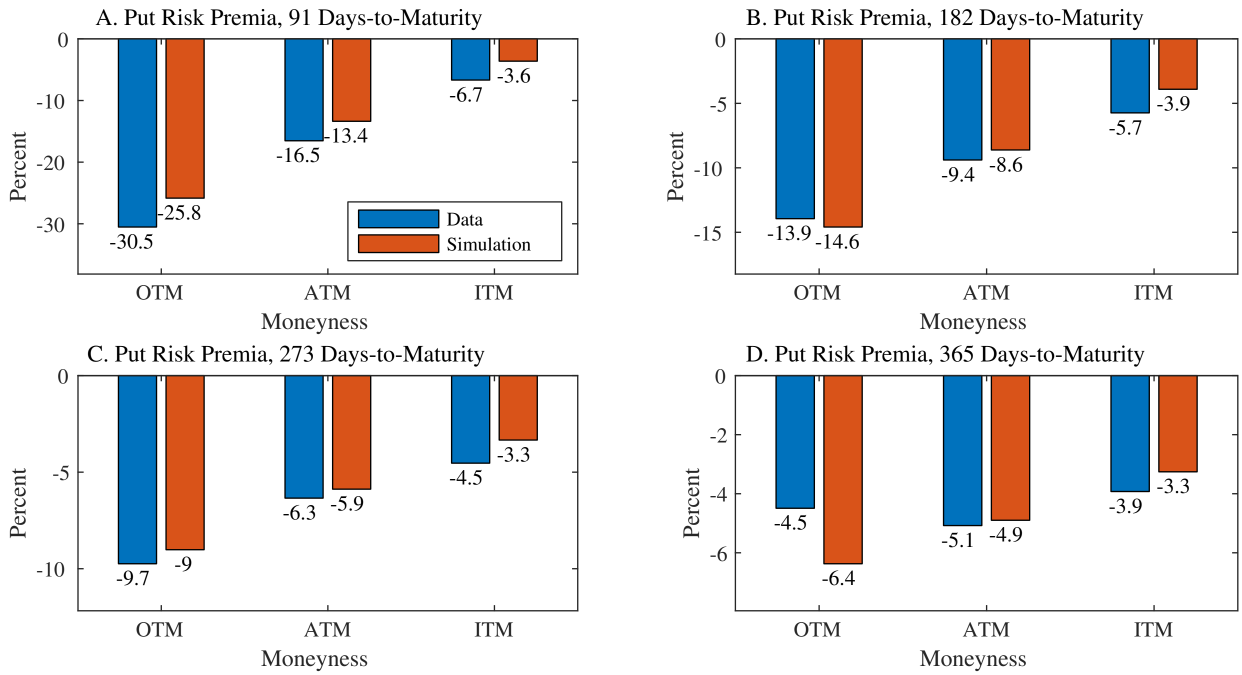

Table D8: One-Month Return Predictability in SVSIJ

This table reports estimates from predictive regressions of one-month put option excess returns and one-month implied bond excess returns on implied spreads (left column) and normalized implied spreads (right column) in simulations of the SVSIJ model. The standalone value is the approximate population value in a single, long sample. The brackets contain the 5th and 95th percentile from 100,000 samples of 371 months. Options are out-of-the-money with 91 (Panel A), 182 (Panel B), 273 (Panel C), and 365 (Panel D) days-to-maturity. Table 2 reports analogous regressions in the data.

	Implied Spread		Normalized Implied Spread	
	Put	Implied Bond	Put	Implied Bond
Panel A: 91 Days-to-Maturity				
b	1.61	0.04	15.30	0.09
	[-2.90, 5.58]	[0.03, 0.06]	[-73.54, 80.53]	[0.05, 0.13]
R2	0.00	0.04	0.00	0.04
	[-0.00, 0.01]	[0.01, 0.08]	[-0.00, 0.01]	[0.01, 0.07]
Panel B: 182 Days-to-Maturity				
b	1.02	0.05	1.25	0.06
	[-2.28, 4.32]	[0.03, 0.07]	[-1.27, 3.83]	[0.04, 0.08]
R2	0.00	0.05	0.00	0.06
	[-0.00, 0.01]	[0.02, 0.09]	[-0.00, 0.01]	[0.02, 0.10]
Panel C: 273 Days-to-Maturity				
b	0.42	0.05	0.36	0.04
	[-1.43, 2.30]	[0.03, 0.08]	[-1.38, 2.09]	[0.02, 0.07]
R2	0.00	0.04	0.00	0.03
	[-0.00, 0.01]	[0.01, 0.07]	[-0.00, 0.01]	[0.01, 0.06]
Panel D: 365 Days-to-Maturity				
b	0.02	0.06	0.02	0.04
	[-0.08, 0.11]	[0.03, 0.10]	[-1.38, 1.39]	[0.02, 0.07]
R2	0.00	0.03	0.00	0.02
	[-0.00, 0.01]	[0.00, 0.06]	[-0.00, 0.01]	[0.00, 0.05]

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