Technical Appendix

to accompany

Political Uncertainty and Risk Premia

Ľuboš Pástor
University of Chicago,
CEPR, and *NBER*

Pietro Veronesi
University of Chicago,
CEPR, and *NBER*

December 9, 2012

This online Technical Appendix provides proofs and additional theoretical results in support of the analysis presented in the paper. The contents are as follows:

1. Proofs

Proof of Lemma 1: The same argument leading to equation (IA.8) in the Internet Appendix of Pastor and Veronesi (2012) implies that conditional on policy n, $n = 0, 1, .., N$, being chosen at time τ, aggregate capital is given by

$$B_T = B_\tau e^{(\mu + g^n - \frac{1}{2}\sigma^2)(T-\tau) + \sigma(Z_T - Z_\tau)}$$

Thus, exploiting $W_T = B_T$ we have

$$E_\tau \left[\frac{W_T^{1-\gamma}}{1-\gamma} | \text{policy } n \right] = \frac{B_\tau^{1-\gamma}}{1-\gamma} e^{(1-\gamma)(T-\tau)\mu_g^n + \frac{1}{2}(1-\gamma)^2 (T-\tau)^2 \sigma_{g,n}^2 + (\mu - \frac{\gamma}{2}\sigma^2)(T-\tau)(1-\gamma)}$$

It follows immediately that

$$E_\tau \left[\frac{W_T^{1-\gamma}}{1-\gamma} | \text{policy } n \right] > E_\tau \left[\frac{W_T^{1-\gamma}}{1-\gamma} | \text{policy } m \right]$$

if and only if

$$\widetilde{\mu}^n = \mu_g^n + \frac{1}{2}(1-\gamma)(T-\tau)\sigma_{g,n}^2 > \mu_g^m + \frac{1}{2}(1-\gamma)(T-\tau)\sigma_{g,m}^2 = \widetilde{\mu}^m$$

Q.E.D.

Proof of Proposition 1. The government chooses policy $n \in \{0, 1, ..., N\}$ if and only if for all $m \neq n$, $m = 0, 1, ..., N$,

$$E_\tau \left[\frac{C^n W_T^{1-\gamma}}{1-\gamma} | \text{policy } n \right] \quad > \quad E_\tau \left[\frac{C^m W_T^{1-\gamma}}{1-\gamma} | \text{policy } m \right]$$

where recall that $C^0 = 1$. The same calculations as in Lemma 1 lead to the inequality

$$\mu_g^n - \frac{\sigma_{g,n}^2}{2}(T-\tau)(\gamma-1) - \frac{c^n}{(\gamma-1)(T-\tau)} > \mu_g^m - \frac{\sigma_{g,m}^2}{2}(T-\tau)(\gamma-1) - \frac{c^m}{(\gamma-1)(T-\tau)} \tag{B1}$$

The claim follows from the definitions of $\widetilde{\mu}^n$ and $\tilde{c}^n$ in equations (15) and (23). Q.E.D.

Proof of Corollary 1. Immediate from Proposition 1 and equations (16) and (17).

Proof of Corollary 2. As of time t, we have for each $n = 1, ..., N$

$$c^n \sim N\left(\widehat{c}_t^n, \widehat{\sigma}_{c,t}^2\right) \tag{B2}$$

Recall from Proposition 1 that policy $n \in \{1, ..., N\}$ is chosen if and only if

$$\begin{aligned} \widetilde{\mu}^n - \widetilde{c}^n &> \widetilde{\mu}^m - \widetilde{c}^m \qquad m \neq n,\ m = 1, ..., N && \text{(B3)} \\ \widetilde{\mu}^n - \widetilde{c}^n &> x_\tau\ , && \text{(B4)} \end{aligned}$$

where we define

$$x_\tau \equiv \widetilde{\mu}^0 = \widehat{g}_\tau - \frac{\widehat{\sigma}_\tau^2}{2}(T-\tau)(\gamma - 1)\ . \tag{B5}$$

Therefore, the conditional probability at t that policy n is chosen at τ is given by

$$\begin{aligned} p_t^n &= \Pr\begin{pmatrix} \widetilde{\mu}^n - \widetilde{c}^n > \widetilde{\mu}^m - \widetilde{c}^m \text{ for } m \neq n \\ \widetilde{\mu}^n - \widetilde{c}^n > x_\tau \end{pmatrix} \\ &= \int_{-\infty}^{\infty} \Pr\begin{pmatrix} \widetilde{c}^n - \widetilde{\mu}^n + \widetilde{\mu}^m < \widetilde{c}^m \text{ for } m \neq n & |\widetilde{c}^n \\ \widetilde{\mu}^n - \widetilde{c}^n > x_\tau & \end{pmatrix} \phi_{\widetilde{c}^n}\left(\widetilde{c}^n\right) d\widetilde{c}^n \\ &= \int_{-\infty}^{\infty} \Pi_{m\neq n} \Pr\left(\widetilde{c}^n - \widetilde{\mu}^n + \widetilde{\mu}^m < \widetilde{c}^m | \widetilde{c}^n\right) \Pr\left(\widetilde{\mu}^n - \widetilde{c}^n > x_\tau | \widetilde{c}^n\right) \phi_{\widetilde{c}^n}\left(\widetilde{c}^n\right) d\widetilde{c}^n \\ &= \int_{-\infty}^{\infty} \Pi_{m\neq n}\left(1 - \Phi_{\widetilde{c}^m}\left(\widetilde{c}^n - \widetilde{\mu}^n + \widetilde{\mu}^m\right)\right) \Phi_x\left(\widetilde{\mu}^n - \widetilde{c}^n | \widehat{g}_t\right) \phi_{\widetilde{c}^n}\left(\widetilde{c}^n\right) d\widetilde{c}^n \end{aligned}$$

where we used the fact that $\widetilde{c}^m$'s are independent of each other as well as of x_τ. Moreover, from the definition of $x_\tau = \widehat{g}_\tau - \frac{\widehat{\sigma}_\tau^2}{2}(T-\tau)(\gamma-1)$ (see equation (16)) we have $x_\tau | \widehat{g}_t \sim N\left(\widehat{g}_t - \frac{\widehat{\sigma}_\tau^2}{2}(T-\tau)(\gamma-1), \widehat{\sigma}_t^2 - \widehat{\sigma}_\tau^2\right)$.

We note two properties:

1. As $\widehat{g}_t \longrightarrow \infty$, then $p_t^n \longrightarrow 0$ for all $n \in \{1, \ldots, N\}$, as $\Phi_x\left(\widetilde{\mu}^n - \widetilde{c}^n | \widehat{g}_t\right) \longrightarrow 0$.
2. As $t \to \tau$ we have

$$\Phi_x\left(\widetilde{\mu}^n - \widetilde{c}^n | \widehat{g}_t\right) = \int^{\widetilde{\mu}^n - \widetilde{c}^n} \phi_x\left(x | \widehat{g}_t\right) dx \to 1_{\{x_\tau < \widetilde{\mu}^n - \widetilde{c}^n\}} \tag{B6}$$

so that

$$\begin{aligned} p_t^n &= \int_{-\infty}^{\infty} \Pi_{m\neq n}\left(1 - \Phi_{\widetilde{c}^m}\left(\widetilde{c}^n + \widetilde{\mu}^m - \widetilde{\mu}^n\right)\right) \Phi_x\left(\widetilde{\mu}^n - \widetilde{c}^n | \widehat{g}_t\right) \phi_{\widetilde{c}^n}\left(\widetilde{c}^n\right) d\widetilde{c}^n \\ &\to \int_{-\infty}^{\widetilde{\mu}^n - x_\tau} \Pi_{m\neq n}\left(1 - \Phi_{\widetilde{c}^m}\left(\widetilde{c}^n + \widetilde{\mu}^m - \widetilde{\mu}^n\right)\right) \phi_{\widetilde{c}^n}\left(\widetilde{c}^n\right) d\widetilde{c}^n \\ &= p_\tau^n \end{aligned}$$

Q.E.D.

Proof of Lemma A1. Using the same arguments as to obtain equation (IA.20) in the Internet Appendix of Pastor and Veronesi (2012), after the announcement of policy n at time $\tau+$, the state price density is given by

$$E_{\tau+}\left[\pi_T|\text{policy } n\right] = \pi_{\tau+}^n = \lambda^{-1} B_{\tau+}^{-\gamma} e^{-\gamma\mu_g^n(T-\tau)} e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\sigma_{g,n}^2} \tag{B7}$$

Therefore, using also $B_\tau = B_{\tau+}$, the state price density at τ is

$$\begin{aligned}
\pi_\tau &= \sum_{n=0}^{N} p_\tau^n \pi_{\tau+}^n \\
&= \lambda^{-1} \sum_{n=0}^{N} p_\tau^n B_\tau^{-\gamma} e^{-\gamma\mu_g^n(T-\tau)} e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\sigma_{g,n}^2} \\
&= \lambda^{-1} B_\tau^{-\gamma} e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)} \left(\sum_{n=0}^{N} p_\tau^n e^{-\gamma\mu_g^n(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\sigma_{g,n}^2}\right)
\end{aligned}$$

Using the definition $\mu_g^0 = \widehat{g}_\tau$ in equation (18) and the condition

$$p_\tau^0 = 1 - \sum_{n=1}^{N} p_\tau^n \tag{B8}$$

we can rewrite the state price density at τ as

$$\begin{aligned}
\pi_\tau &= \lambda^{-1} B_\tau^{-\gamma} e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)-\gamma\widehat{g}_\tau(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\widehat{\sigma}_\tau^2} \times \\
&\quad \times \left(1+\sum_{n=1}^{N} p_\tau^n \left(e^{-\gamma\left(\mu_g^n-\widehat{g}_\tau\right)(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\left(\sigma_{g,n}^2-\widehat{\sigma}_\tau^2\right)} - 1\right)\right)
\end{aligned}$$

Similarly, after the announcement of policy n, $n = 0, 1, ..., N$, at time $\tau+$, we have

$$E_{\tau+}\left[B_T^{-\gamma} B_T^i|\text{policy } n\right] = N_{\tau+}^{i,n} = B_{\tau+}^{-\gamma} B_{\tau+}^i \times e^{(1-\gamma)\mu_g^n(T-\tau)} e^{\left((1-\gamma)\mu+\frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\sigma_{g,n}^2} \tag{B9}$$

Therefore, we have

$$\begin{aligned}
E_\tau\left[B_T^{-\gamma} B_T^i\right] &= \sum_{n=0}^{N} p_\tau^n N_{\tau+}^{i,n} \\
&= B_\tau^{-\gamma} B_\tau^i e^{\left((1-\gamma)\mu+\frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau)} \times \left(\sum_{n=0}^{N} p_\tau^n \times e^{(1-\gamma)\mu_g^n(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\sigma_{g,n}^2}\right) \\
&= B_\tau^{-\gamma} B_\tau^i e^{\left((1-\gamma)\mu+\frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau)+(1-\gamma)\widehat{g}_\tau(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\widehat{\sigma}_\tau^2} \times \\
&\quad \times \left(1+\sum_{n=1}^{N} p_\tau^n \times \left(e^{(1-\gamma)\left(\mu_g^n-\widehat{g}_\tau\right)(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\left(\sigma_{g,n}^2-\widehat{\sigma}_\tau^2\right)} - 1\right)\right)
\end{aligned}$$

The claim follows from taking the ratio $M_\tau^i = \frac{E_\tau\left[\pi_T B_T^i\right]}{\pi_\tau} = \frac{E_\tau\left[\lambda^{-1} B_T^{-\gamma} B_T^i\right]}{\pi_\tau}$. Q.E.D.

Proof of Lemma A2. From (B7) and (B9) we obtain that if policy n, $n = 0, 1, ..., N$, is selected at $\tau+$, then

$$M_{\tau+}^i = \frac{E_{\tau+}\left[B_T^{-\gamma} B_T^i|\text{policy } n\right]}{E_{\tau+}\left[B_T^{-\gamma}|\text{policy } n\right]} = B_{\tau+}^i e^{\left(\mu-\gamma\sigma^2+\mu_g^n\right)(T-\tau)+\frac{1-2\gamma}{2}(T-\tau)^2\sigma_{g,n}^2} \tag{B10}$$

Q.E.D.

Proof of Proposition 2: To prove this proposition, we need three lemmas:

Lemma B1 . Δb_τ and $\widehat{g}_\tau$ are perfectly correlated, and we can write

$$\begin{aligned} \Delta b_\tau &= E_t\left[\Delta b_\tau\right] + \left(\widehat{g}_\tau - E_t\left[\widehat{g}_\tau\right]\right)\left[\sigma^2/\widehat{\sigma}_t^2 + (\tau - t)\right] \\ &= E_t\left[\Delta b_\tau\right] + \left(x_\tau - E_t\left[x_\tau\right]\right)\left[\sigma^2/\widehat{\sigma}_t^2 + (\tau - t)\right] \end{aligned}$$

Proof of Lemma B1: From Lemma A5 in Pastor and Veronesi (2012), we have that $b_\tau = \log(B_\tau)$ and $\widehat{g}_\tau$ have the conditional joint distribution

$$\begin{pmatrix} b_\tau - b_t \\ \widehat{g}_\tau \end{pmatrix} \sim N\left(\begin{array}{c} E_t\left[\Delta b_\tau\right] \\ E_t\left[\widehat{g}_\tau\right] \end{array}; \begin{pmatrix} V_b, C_{g,b} \\ C_{g,b}, V_{\widehat{g}} \end{pmatrix}\right) \tag{B11}$$

where

$$\begin{aligned} E_t\left[\Delta b_\tau\right] &= \left(\mu + \widehat{g}_t - \frac{1}{2}\sigma^2\right)(\tau - t) \\ E_t\left[\widehat{g}_\tau\right] &= \widehat{g}_t \\ V_b &= (\tau - t)^2\,\widehat{\sigma}_t^2 + \sigma^2\,(\tau - t) \\ V_{\widehat{g}} &= \widehat{\sigma}_t^2 - \widehat{\sigma}_\tau^2 \\ C_{\widehat{g},b} &= \widehat{\sigma}_t^2\,(\tau - t) \end{aligned}$$

We now see that $b_\tau - b_t$ and $\widehat{g}_\tau$ are perfectly correlated. In fact,

$$Corr = \frac{C_{\widehat{g},b}}{\sqrt{V_b V_{\widehat{g}}}} = \frac{\widehat{\sigma}_t^2\,(\tau - t)}{\sqrt{\left((\tau - t)^2\,\widehat{\sigma}_t^2 + \sigma^2\,(\tau - t)\right)\left(\widehat{\sigma}_t^2 - \widehat{\sigma}_\tau^2\right)}} \tag{B12}$$

Using the fact that

$$\widehat{\sigma}_\tau^2 = \frac{1}{\frac{1}{\widehat{\sigma}_t^2} + \frac{1}{\sigma^2}\,(\tau - t)} = \frac{\widehat{\sigma}_t^2\sigma^2}{\sigma^2 + \widehat{\sigma}_t^2\,(\tau - t)} \tag{B13}$$

we find

$$\begin{aligned} Corr &= \frac{\widehat{\sigma}_t^2\,(\tau - t)}{\sqrt{\left((\tau - t)^2\,\widehat{\sigma}_t^2 + \sigma^2\,(\tau - t)\right)\left(\widehat{\sigma}_t^2 - \frac{\widehat{\sigma}_t^2\sigma^2}{\sigma^2+\widehat{\sigma}_t^2(\tau-t)}\right)}} \\ &= \frac{\widehat{\sigma}_t^2\,(\tau - t)}{\sqrt{(\tau - t)\left(\widehat{\sigma}_t^2\left(\sigma^2 + \widehat{\sigma}_t^2\,(\tau - t)\right) - \widehat{\sigma}_t^2\sigma^2\right)}} \\ &= \frac{\widehat{\sigma}_t^2\,(\tau - t)}{\sqrt{(\tau - t)\left(\widehat{\sigma}_t^2\right)^2(\tau - t)}} = 1 \end{aligned}$$

It follows that we can write

$$\Delta b_\tau = E_t[\Delta b_\tau] + \{\widehat{g}_\tau - E_t[\widehat{g}_\tau]\}\frac{C_{b,\widehat{g}}}{V_{\widehat{g}}} = E_t[\Delta b_\tau] + \{\widehat{g}_\tau - E_t[\widehat{g}_\tau]\}\sqrt{\frac{V_b}{V_{\widehat{g}}}}$$

$$= E_t[\Delta b_\tau] + \{\widehat{g}_\tau - E_t[\widehat{g}_\tau]\}\frac{\widehat{\sigma}_t^2(\tau - t)}{\widehat{\sigma}_t^2 - \widehat{\sigma}_\tau^2} = E_t[\Delta b_\tau] + \{\widehat{g}_\tau - E_t[\widehat{g}_\tau]\}\left[\sigma^2/\widehat{\sigma}_t^2 + (\tau - t)\right]$$

where we also used the equality

$$\widehat{\sigma}_t^2 - \widehat{\sigma}_\tau^2 = \widehat{\sigma}_t^2 - \frac{\widehat{\sigma}_t^2\sigma^2}{\sigma^2 + \widehat{\sigma}_t^2(\tau - t)} = \frac{\left(\widehat{\sigma}_t^2\right)^2(\tau - t)}{\sigma^2 + \widehat{\sigma}_t^2(\tau - t)}$$

From the definition of x_τ, it also follows that $x_\tau - E_t[x_\tau] = \widehat{g}_\tau - E_t[\widehat{g}_\tau]$. Q.E.D.

Lemma B2: The conditional distribution of $\Delta b_\tau = b_\tau - b_t = \log(B_\tau/B_t)$ conditional on time-t information and policy n being chosen at time τ is

$$f(\Delta b_\tau|S_t, n \text{ at } \tau) \tag{B14}$$

$$= \frac{\phi_{\Delta b_\tau}(\Delta b_\tau)}{p_t^n}\int_{-\infty}^{\widetilde{\mu}^n - E_t[x_\tau] - (\Delta b_\tau - E_t[\Delta b_\tau])\frac{\widehat{\sigma}_t^2}{(\tau - t)\widehat{\sigma}_t^2 + \sigma^2}} \Pi_{m\neq n}\left(1 - \Phi_{\widetilde{c}^m}\left(\widetilde{c}^n - \widetilde{\mu}^n + \widetilde{\mu}^m\right)\right)\phi_{\widetilde{c}^n}(\widetilde{c}^n)\, d\widetilde{c}^n \tag{B15}$$

where $\phi_{\Delta b_\tau}(\Delta b_\tau)$ is the normal density with mean $E_t[\Delta b_\tau] = \left(\mu + \widehat{g}_t - \frac{1}{2}\sigma^2\right)(\tau - t)$ and variance $V_b = (\tau - t)^2\widehat{\sigma}_t^2 + \sigma^2(\tau - t)$. In addition, $E_t[x_\tau] = \widehat{g}_t - \frac{\widehat{\sigma}_\tau^2}{2}(T - \tau)(\gamma - 1)$.

Note that $f(\Delta b_\tau|S_t, \kappa \text{ at } \tau)$ does not depend on the current value of log capital, b_t, hence the conditional dependence only on S_t and time t.

Proof of Lemma B2. The conditional CDF is

$$F_{\Delta b_\tau}\left(\Delta b|S_t, \begin{array}{c} x_\tau < \widetilde{\mu}^n - \widetilde{c}^n \\ \widetilde{c}^n - \widetilde{\mu}^n + \widetilde{\mu}^m < \widetilde{c}^m \text{ for } m \neq n \end{array}\right) = \frac{\Pr\left(\Delta b_\tau < \Delta b, \begin{array}{c} x_\tau < \widetilde{\mu}^n - \widetilde{c}^n \\ \widetilde{c}^n - \widetilde{\mu}^n + \widetilde{\mu}^m < \widetilde{c}^m \text{ for } m \neq n \end{array} |S_t\right)}{\Pr\left(\begin{array}{c} x_\tau < \widetilde{\mu}^n - \widetilde{c}^n \\ \widetilde{c}^n - \widetilde{\mu}^n + \widetilde{\mu}^m < \widetilde{c}^m \text{ for } m \neq n \end{array} |S_t\right)} \tag{B16}$$

The denominator is just p_t^n from Corollary 2. Consider the numerator. From Lemma B1:

$$\Delta b_\tau - E_t[\Delta b_\tau] = \{x_\tau - E_t[x_\tau]\}\left(\sigma^2/\widehat{\sigma}_t^2 + (\tau - t)\right)$$

which implies

$$x_\tau = E_t[x_\tau] + \{\Delta b_\tau - E_t[\Delta b_\tau]\}\frac{\widehat{\sigma}_t^2}{(\sigma^2 + \widehat{\sigma}_t^2(\tau - t))}$$

Thus, the joint distribution can be written as

$$\Pr\left(\Delta b_\tau < \Delta b, \begin{array}{c} x_\tau < \widetilde{\mu}^n - \widetilde{c}^n \\ \widetilde{c}^n - \widetilde{\mu}^n + \widetilde{\mu}^m < \widetilde{c}^m \end{array} |S_t\right)$$

$$= \Pr\left(\Delta b_\tau < \Delta b, \begin{array}{c} E_t[x_\tau] + \{\Delta b_\tau - E_t[\Delta b_\tau]\} \frac{\widehat{\sigma}_t^2}{\left(\sigma^2 + \widehat{\sigma}_t^2(\tau - t)\right)} < \widetilde{\mu}^n - \widetilde{c}^n \\ \widetilde{c}^n - \widetilde{\mu}^n + \widetilde{\mu}^m < \widetilde{c}^m \end{array} |S_t\right)$$

$$= \int_{-\infty}^{\infty} \Pr\left(\Delta b_\tau < \Delta b, \begin{array}{c} \widetilde{c}^n < \widetilde{\mu}^n - E_t[x_\tau] - \{\Delta b_\tau - E_t[\Delta b_\tau]\} \frac{\widehat{\sigma}_t^2}{\left(\sigma^2 + \widehat{\sigma}_t^2(\tau - t)\right)} \\ \widetilde{c}^n - \widetilde{\mu}^n + \widetilde{\mu}^m < \widetilde{c}^m \end{array} |\widetilde{c}^n, S_t\right) \phi_{\widetilde{c}^n}(\widetilde{c}^n)\, d\widetilde{c}^n$$

$$= \int_{-\infty}^{\Delta b} \int_{-\infty}^{\widetilde{\mu}^n - E_t[x_\tau] - \{\Delta b_\tau - E_t[\Delta b_\tau]\} \frac{\widehat{\sigma}_t^2}{\left(\sigma^2 + \widehat{\sigma}_t^2(\tau - t)\right)}} \Pi_{m \neq n} \left[1 - \Phi_{\widetilde{c}^m}\left(\widetilde{c}^n - \widetilde{\mu}^n + \widetilde{\mu}^m\right)\right] \phi_{\widetilde{c}^n}(\widetilde{c}^n)\, d\widetilde{c}^n \phi_{\Delta b_\tau}(\Delta b_\tau) d\Delta b_\tau$$

where we exploited the independence across $\widetilde{c}^m$ and with respect to Δb_τ. Substituting into (B16) and taking the first derivative with respect to Δb, we obtain the density (B15). Q.E.D.

Lemma B3: The distribution of $\widehat{g}_\tau$ conditional on time-t information and no new policy being chosen at time τ is

$$f(\widehat{g}_\tau | \text{no policy change at } \tau) = \frac{\phi_{\widehat{g}_\tau}(\widehat{g}_\tau | \widehat{g}_t)}{p_t^0} \Pi_{n=1}^N \left(1 - \Phi_{\widetilde{c}^n}\left(\widetilde{\mu}^n - \widehat{g}_\tau + \frac{\widehat{\sigma}_\tau^2}{2}(T - \tau)(\gamma - 1)\right)\right) \tag{B17}$$

where $\phi_{\widehat{g}_\tau}(\widehat{g}_\tau | \widehat{g}_t)$ is the conditional normal density of $\widehat{g}_\tau$, namely, $\mathrm{N}(\widehat{g}_t, \widehat{\sigma}_t^2 - \widehat{\sigma}_\tau^2)$.

Proof of Lemma B3: The conditional CDF is given by

$$\begin{aligned}
& F_{\widehat{g}_\tau}(g | \text{no policy change at } \tau) \\
& = F_{\widehat{g}_\tau}(g | x_\tau > \widetilde{\mu}^n - \widetilde{c}^n \text{ for all } n) \\
& = \frac{\Pr\left(\widehat{g}_\tau < g \ \&\ \widehat{g}_\tau > \widetilde{\mu}^n - \widetilde{c}^n + \frac{\widehat{\sigma}_\tau^2}{2}(T - \tau)(\gamma - 1) \text{ for all } n\right)}{\Pr(x_\tau > \widetilde{\mu}^n - \widetilde{c}^n \text{ for all } n)} \\
& = \frac{\int_{-\infty}^{\infty} \Pr\left(\widehat{g}_\tau < g \ \&\ \widehat{g}_\tau > \widetilde{\mu}^n - \widetilde{c}^n + \frac{\widehat{\sigma}_\tau^2}{2}(T - \tau)(\gamma - 1) \text{ for all } n | \widehat{g}_\tau\right) \phi_{\widehat{g}_\tau}(\widehat{g}_\tau | \widehat{g}_t)\, d\widehat{g}_\tau}{p_t^0} \\
& = \frac{\int_{-\infty}^{\infty} 1_{\{\widehat{g}_\tau < g\}} \Pi_{n=1}^N \left(1 - \Phi_{\widetilde{c}^n}\left(\widetilde{\mu}^n - \widehat{g}_\tau + \frac{\widehat{\sigma}_\tau^2}{2}(T - \tau)(\gamma - 1)\right)\right) \phi_{\widehat{g}_\tau}(\widehat{g}_\tau | \widehat{g}_t)\, d\widehat{g}_\tau}{p_t^0} \\
& = \frac{\int_{-\infty}^{g} \Pi_{n=1}^n \left(1 - \Phi_{\widetilde{c}^n}\left(\widetilde{\mu}^n - \widehat{g}_\tau + \frac{\widehat{\sigma}_\tau^2}{2}(T - \tau)(\gamma - 1)\right)\right) \phi_{\widehat{g}_\tau}(\widehat{g}_\tau | \widehat{g}_t)\, d\widehat{g}_\tau}{p_t^0}
\end{aligned}$$

Taking the first derivative with respect to g, we obtain the density (B17). Q.E.D.

Proof of Proposition 2: We know that

$$\pi_t = E_t[\pi_{\tau+}] = \sum_{n=0}^{N} E_t[\pi_{\tau+} | n \text{ at } \tau]\, p_t^n \tag{B18}$$

Note that for $n=1,...,N$

$$
\begin{aligned}
E_t\left[\pi_{\tau+}|n \text{ at } \tau\right] &= E_t\left[\lambda^{-1}B_{\tau+}^{-\gamma}e^{-\gamma\mu_g^n(T-\tau)}e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\sigma_{g,n}^2}|n \text{ at } \tau\right] \\
&= \lambda^{-1}e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)-\gamma\mu_g^n(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\sigma_{g,n}^2}e^{-\gamma b_t}E_t\left[e^{-\gamma(b_\tau-b_t)}|n \text{ at } \tau\right] \\
&= \lambda^{-1}B_t^{-\gamma}e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)}\times \\
&\quad \times e^{-\gamma\mu_g^n(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\sigma_{g,n}^2}\left[\int_{-\infty}^{\infty}e^{-\gamma\Delta b_\tau}f\left(\Delta b_\tau|S_t,n \text{ at } \tau\right)d\Delta b_\tau\right]
\end{aligned}
$$

Similarly, for $n=0$ we have

$$
\begin{aligned}
E_t\left[\pi_{\tau+}|0 \text{ at } \tau\right] &= E\left[\lambda^{-1}B_{\tau+}^{-\gamma}e^{-\gamma\widehat{g}_\tau(T-\tau)}e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\widehat{\sigma}_\tau^2}|0 \text{ at } \tau\right] \\
&= \lambda^{-1}e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\widehat{\sigma}_\tau^2}e^{-\gamma b_t}E_t\left[e^{-\gamma\Delta b_\tau-\gamma\widehat{g}_\tau(T-\tau)}|0 \text{ at } \tau\right] \\
&= \lambda^{-1}e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\widehat{\sigma}_\tau^2}e^{-\gamma b_t}\times \\
&\quad \times E_t\left[e^{-\gamma\left(E_t[\Delta b_\tau]+\{\widehat{g}_\tau-E_t[\widehat{g}_\tau]\}\sqrt{\frac{V_b}{V_{\widehat{g}}}}\right)-\gamma\widehat{g}_\tau(T-\tau)}|0 \text{ at } \tau\right] \\
&= \lambda^{-1}B_t^{-\gamma}e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)}\times e^{-\gamma\widehat{g}_t(T-\tau)+\frac{\gamma^2}{2}(T-\tau)^2\widehat{\sigma}_\tau^2} \\
&\quad \times\int_{-\infty}^{\infty}e^{-\gamma\left(E_t[\Delta b_\tau]+\{\widehat{g}_\tau-E_t[\widehat{g}_\tau]\}\sqrt{\frac{V_{b_\tau}}{V_{\widehat{g}_\tau}}}\right)-\gamma(T-\tau)(\widehat{g}_\tau-\widehat{g}_t)}f\left(\widehat{g}_\tau|0 \text{ at } \tau\right)d\widehat{g}_\tau
\end{aligned}
$$

The result follows from comparing the terms in equations (28) and (A1) with the ones above, and defining in this proposition $\mu_g^0=\widehat{g}_t$ and $\sigma_{g,0}^2=\widehat{\sigma}_\tau^2$. Q.E.D.

Proof of Proposition 3: The result follows from an application of Ito's Lemma to equation (28), and recalling that π_t is a martingale, and thus $E_t[d\pi_t/\pi_t]=0$. Q.E.D.

Proof of Corollary 3: The probability that the old policy will be retained is

$$
\begin{aligned}
p_t^0 &= \Pr\left(\widetilde{\mu}^n-\widetilde{c}^n<\widehat{g}_\tau-\frac{\widehat{\sigma}_\tau^2}{2}(T-\tau)(\gamma-1)\ \text{ for all } n>0\right) \\
&= \int_{-\infty}^{\infty}\Pr\left(\widetilde{\mu}^n-\widetilde{c}^n<\widehat{g}_\tau-\frac{\widehat{\sigma}_\tau^2}{2}(T-\tau)(\gamma-1)\ \text{ for all } n>0|\widehat{g}_\tau\right)\phi\left(\widehat{g}_\tau|\widehat{g}_t\right)d\widehat{g}_\tau \\
&= \int_{-\infty}^{\infty}\prod_{n=1}^{N}\left[1-\Phi_{\widetilde{c}^n}\left(\widetilde{\mu}^n-\widehat{g}_\tau+\frac{\widehat{\sigma}_\tau^2}{2}(T-\tau)(\gamma-1)\right)\right]\phi\left(\widehat{g}_\tau|\widehat{g}_t\right)d\widehat{g}_\tau
\end{aligned}
$$

Thus, for a given $\widehat{c}_t^n$, for $n=1,..,N$, we have that as $\widehat{g}_t\to-\infty$, $p_t^0\to 0$ and $dp_t/d\widehat{g}_t\to 0$. From the proof of Proposition 2, for a sufficiently small $\widehat{g}_t$, the state price density converges to

$$
\pi_t = E_t\left[\pi_{\tau+}\right]\to\sum_{n=1}^{N}p_t^n E\left[\pi_{\tau+}|n \text{ at } \tau+\right]
$$

$$= \lambda^{-1} B_t^{-\gamma} e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)} \sum_{n=1}^{N} p_t^n e^{-\gamma\mu_g^n(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,n}^2} E\left[e^{-\gamma\Delta b_\tau}|S_t, n \text{ at } \tau\right] \quad \text{(B19)}$$

As $\widehat{g}_t$ declines so that $p_t^0 \to 0$, we have $E\left[e^{-\gamma\Delta b_\tau}|S_t, n \text{ at } \tau\right] \to E\left[e^{-\gamma\Delta b_\tau}|S_t\right]$. Indeed, note that in the proof of the distribution of Δb_τ in Lemma B2, we have

$$F\left(\Delta b_\tau|S_t, \text{n } at\ \tau\right) = \Pr\left(\Delta b_\tau < \Delta b_t|S_t, \begin{array}{c} x_\tau < \widetilde{\mu}^n - \widetilde{c}^n \\ \widetilde{\mu}^m - \widetilde{c}^n < \widetilde{\mu}^n - \widetilde{c}^n \text{ for } m \neq n \end{array}\right)$$

The statement $p_t^0 = 0$ implies that the event $x_\tau < \widetilde{\mu}^n - \widetilde{c}^n$ is certain to be realized. Thus:

$$F\left(\Delta b_\tau|S_t, \text{n } at\ \tau\right) = \Pr\left(\Delta b_\tau < \Delta b_t|S_t, \widetilde{\mu}^m - \widetilde{c}^n < \widetilde{\mu}^n - \widetilde{c}^n \text{ for } m \neq n\right) = \Pr\left(\Delta b_\tau < \Delta b_t|S_t\right)$$

the last step due to the independence of $\widetilde{c}^n$, $\widetilde{c}^m$ from Δb_τ process. We then obtain

$$\begin{aligned}
\pi_t &= \lambda^{-1} B_t^{-\gamma} e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)} \sum_{n=1}^{N} p_t^n e^{-\gamma\mu_g^n(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,n}^2} E\left[e^{-\gamma\Delta b_\tau}|S_t, n \text{ at } \tau\right] \\
&= \lambda^{-1} B_t^{-\gamma} e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)} E\left[e^{-\gamma\Delta b_\tau}|S_t\right] \sum_{n=1}^{N} p_t^n e^{-\gamma\mu_g^n(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,n}^2} \\
&= \lambda^{-1} B_t^{-\gamma} e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-\tau)} \\
&\quad \times e^{-\gamma\left(\left(\mu+\widehat{g}_t-\frac{1}{2}\sigma^2\right)(\tau-t)\right)+\frac{1}{2}\gamma^2\left((\tau-t)^2\widehat{\sigma}_t^2+(\tau-t)\sigma^2\right)} \sum_{n=1}^{N} p_t^n e^{-\gamma\mu_g^n(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,n}^2}
\end{aligned}$$

where the last step follows from Lemma B2:

$$\Delta b_\tau|S_t \sim N\left(\left(\mu + \widehat{g}_t - \frac{1}{2}\sigma^2\right)(\tau - t), (\tau - t)^2 \widehat{\sigma}_t^2 + (\tau - t)\sigma^2\right)$$

It follows that for $\widehat{g}_t$ sufficiently small, $\Omega\left(S_t\right)$ converges to

$$\Omega\left(S_t\right) = e^{-\gamma\left(\left(\mu+\widehat{g}_t-\frac{1}{2}\sigma^2\right)(\tau-t)\right)+\frac{1}{2}\gamma^2\left((\tau-t)^2\widehat{\sigma}_t^2+(\tau-t)\sigma^2\right)} \sum_{n=1}^{N} p_t^n e^{-\gamma\mu_g^n(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,n}^2} \quad \text{(B20)}$$

Taking the first derivative with respect to $\widehat{g}_t$ and dividing by $\Omega(S_t)$, we find that as $\widehat{g}_t \to -\infty$,

$$\sigma_{\pi,0} = \frac{1}{\Omega\left(S_t\right)} \frac{\partial\Omega\left(S_t\right)}{\partial\widehat{g}_t} \widehat{\sigma}_t^2 \sigma^{-1} \to -\gamma(\tau - t)\widehat{\sigma}_t^2 \sigma^{-1}$$

proving the first part of the statement of Corollary 3.

To prove the second part of the statement of Corollary 3, from property 1 in the proof of Corollary 2, for a given distribution of $\widetilde{c}^n$, we have $p_t^0 \to 1$ as $\widehat{g}_t \to \infty$. It follows that the state price density converges to one that assigns zero probability to a policy change:

$$\begin{aligned}
\pi_t &\to E_t\left[\pi_{\tau+}|0 \text{ at } n\right] = \lambda^{-1} E_t\left[B_T^{-\gamma}|0 \text{ at } n\right] \\
&= \lambda^{-1} B_t^{-\gamma} e^{-\gamma\widehat{g}_t(T-t)} e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(T-t)+\frac{\gamma^2}{2}(T-t)^2\widehat{\sigma}_t^2} \quad \text{(B21)}
\end{aligned}$$

It follows that in this case, for $\widehat{g}_t$ sufficiently large, $\Omega(S_t)$ converges to

$$\Omega\left(S\right)=e^{\left(-\gamma\mu+\frac{1}{2}\gamma(\gamma+1)\sigma^2\right)(\tau-t)}e^{-\gamma\widehat{g}_t(T-t)+\frac{1}{2}\gamma^2(T-t)^2\widehat{\sigma}_t^2}$$

Taking the first derivative with respect to $\widehat{g}_t$ and dividing by $\Omega(S_t)$, we find that as $\widehat{g}_t\to\infty$,

$$\sigma_{\pi,0}=\frac{1}{\Omega\left(S_t\right)}\frac{\partial\Omega\left(S_t\right)}{\partial\widehat{g}_t}\widehat{\sigma}_t^2\sigma^{-1}\to-\gamma(T-t)\widehat{\sigma}_t^2\sigma^{-1}$$

Q.E.D.

Proof of Corollary 4: From expression (B21), we see that the state price density does not depend on any $\widehat{c}_t^n$. Hence, we have $\frac{1}{\Omega(S_t)}\frac{\partial\Omega(S)}{\partial\widehat{c}^n}=0$. Q.E.D.

Proof of Proposition 4: The proof is identical to the proof of Proposition 2, except that we have to calculate

$$E_t\left[\pi_{\tau+}M_{\tau+}^i\right]=\sum_{n=0}^{N}p_t^nE_t\left[\pi_{\tau+}M_{\tau+}^i|n\text{ at }\tau\right]$$

From (B9), for $n=1,..,N$:

$$\begin{aligned}E_t\left[\pi_{\tau+}M_{\tau+}^i|n\text{ at }\tau\right]&=\lambda^{-1}E_t\left[N_{\tau+}^i|n\text{ at }\tau\right]\\&=\lambda^{-1}E_t\left[B_{\tau+}^{-\gamma}B_{\tau+}^i\times e^{(1-\gamma)\mu_g^n(T-\tau)}e^{\left((1-\gamma)\mu+\frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\sigma_{g,n}^2}|n\text{ at }\tau\right]\\&=\lambda^{-1}e^{(1-\gamma)\mu_g^n(T-\tau)}e^{\left((1-\gamma)\mu+\frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\sigma_{g,n}^2}E_t\left[e^{-\gamma b_\tau+b_\tau^i}|n\text{ at }\tau\right]\end{aligned}$$

Now, recall

$$\frac{B_\tau^i}{B_t^i}=\frac{B_\tau}{B_t}e^{-\frac{1}{2}\sigma_1^2(T-\tau)+\sigma_1\left(Z_\tau^i-Z_t^i\right)}\qquad\text{(B22)}$$

which implies

$$e^{b_\tau^i}=e^{b_t^i+b_\tau-b_t-\frac{1}{2}\sigma_1^2(T-\tau)+\sigma_1\left(Z_\tau^i-Z_t^i\right)}\qquad\text{(B23)}$$

For $n=1,...,N$, we then have:

$$\begin{aligned}E_t\left[\pi_{\tau+}M_{\tau+}^i|n\text{ at }\tau\right]&=\lambda^{-1}B_t^{-\gamma}B_t^ie^{(1-\gamma)\mu_g^n(T-\tau)}e^{\left((1-\gamma)\mu+\frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\sigma_{g,n}^2}\\&\quad E_t\left[e^{(1-\gamma)\Delta b_\tau}|n\text{ at }\tau\right]\\&=\lambda^{-1}B_t^{-\gamma}B_t^ie^{(1-\gamma)\mu_g^n(T-\tau)}e^{\left((1-\gamma)\mu+\frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau)+\frac{(1-\gamma)^2}{2}(T-\tau)^2\sigma_{g,n}^2}\\&\quad\int e^{(1-\gamma)\Delta b_\tau}f\left(\Delta b_\tau|S_t,n\text{ at }\tau\right)d\Delta b_\tau\end{aligned}$$

Similarly, for $n=0$, we have:

$$E_t\left[\pi_{\tau+}M_{\tau+}^i|0\text{ at }\tau\right]=\lambda^{-1}E_t\left[N_{\tau+}^i|0\text{ at }\tau\right]$$

$$= \lambda^{-1} E_t \left[B_\tau^{-\gamma} B_\tau^i \times e^{(1-\gamma)\widehat{g}_\tau (T-\tau)} e^{\left((1-\gamma)\mu + \frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau) + \frac{(1-\gamma)^2}{2}(T-\tau)^2 \widehat{\sigma}_\tau^2} | 0 \text{ at } \tau \right]$$

$$= \lambda^{-1} e^{\left((1-\gamma)\mu + \frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau) + \frac{(1-\gamma)^2}{2}(T-\tau)^2 \widehat{\sigma}_\tau^2} E_t \left[e^{-\gamma b_\tau + b_\tau^i + (1-\gamma)\widehat{g}_\tau (T-\tau)} | 0 \text{ at } \tau \right]$$

$$= \lambda^{-1} B_t^{-\gamma} B_t^i e^{\left((1-\gamma)\mu + \frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau) + \frac{(1-\gamma)^2}{2}(T-\tau)^2 \widehat{\sigma}_\tau^2} E_t \left[e^{(1-\gamma)\Delta b_\tau + (1-\gamma)\widehat{g}_\tau (T-\tau)} | 0 \text{ at } \tau \right]$$

$$= \lambda^{-1} B_t^{-\gamma} B_t^i e^{\left((1-\gamma)\mu + \frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau) + \frac{(1-\gamma)^2}{2}(T-\tau)^2 \widehat{\sigma}_\tau^2}$$

$$\times E_t \left[e^{(1-\gamma)\left(E_t[\Delta b_\tau] + \{\widehat{g}_\tau - E_t[\widehat{g}_\tau]\} \sqrt{\frac{V_b}{V_{\widehat{g}}}} \right) + (1-\gamma)\widehat{g}_\tau (T-\tau)} | 0 \text{ at } \tau \right]$$

$$= \lambda^{-1} B_t^{-\gamma} B_t^i e^{(1-\gamma)\widehat{g}_t (T-\tau) + \left((1-\gamma)\mu + \frac{1}{2}\gamma(\gamma-1)\sigma^2\right)(T-\tau) + \frac{(1-\gamma)^2}{2}(T-\tau)^2 \widehat{\sigma}_\tau^2}$$

$$\times \int e^{(1-\gamma)\left(E_t[\Delta b_\tau] + \{\widehat{g}_\tau - E_t[\widehat{g}_\tau]\} \sqrt{\frac{V_b}{V_{\widehat{g}}}} \right) + (1-\gamma)(\widehat{g}_\tau - \widehat{g}_t)(T-\tau)} f\left(\widehat{g}_\tau | S_t, 0 \text{ at } \tau\right) d\widehat{g}_\tau$$

The result follows from comparing the terms in equations (35), (A1), and (A2) with the ones above, and defining in this proposition $\mu_g^0 = \widehat{g}_t$ and $\sigma_{g,0}^2 = \widehat{\sigma}_\tau^2$. Q.E.D.

Proof of Proposition 5. The claim follows from an application of Ito's Lemma to the price M_t^i in Proposition 4, and the equilibrium restriction $\mu_M^i = -Cov_t \left(\frac{dM_t^i}{M_t^i}, \frac{d\pi_t}{\pi_t} \right)$. Q.E.D.

Proof of Corollary 5. The proof is identical to that of Corollary 3, except that we analyze the limiting behavior of $E_t \left[\pi_{\tau+} M_{\tau+}^i \right]$. Comparing the proofs of Proposition 2 and Proposition 5, we see that the same arguments discussed in Corollary 3 apply for $E_t \left[\pi_{\tau+} M_{\tau+}^i \right]$. In particular, as $\widehat{g}_t \to -\infty$, we have that for $\widehat{g}_t$ sufficiently small, $H(S_t)$ converges to

$$H(S_t) = e^{(1-\gamma)\left(\left(\mu + \widehat{g}_t - \frac{1}{2}\sigma^2\right)(\tau - t)\right) + \frac{1}{2}(1-\gamma)^2\left((\tau-t)^2 \widehat{\sigma}_t^2 + (\tau - t)\sigma^2\right)} \sum_{n=1}^{N} p_t^n e^{(1-\gamma)\mu_g^n (T-\tau) + \frac{1}{2}(1-\gamma)^2 (T-\tau)^2 \sigma_{g,n}^2} \tag{B24}$$

Hence,

$$\frac{1}{H(S_t)} \frac{\partial H(S_t)}{\partial \widehat{g}_t} \to (1-\gamma)(\tau - t)$$

Therefore, as $\widehat{g}_t \to -\infty$, we have

$$\sigma_{M,0} = \left(\frac{1}{H(S_t)} \frac{\partial H(S_t)}{\partial \widehat{g}_t} - \frac{1}{\Omega(S_t)} \frac{\partial \Omega(S_t)}{\partial \widehat{g}_t} \right) \widehat{\sigma}_t^2 \sigma^{-1} \to (\tau - t) \widehat{\sigma}_t^2 \sigma^{-1}$$

Similarly, as $\widehat{g}_t \to \infty$, we have that for $\widehat{g}_t$ sufficiently large, $H(S_t)$ converges to

$$H(S_t) = e^{\left((1-\gamma)\mu + \frac{1}{2}(1-\gamma)\gamma\sigma^2\right)(\tau - t)} e^{(1-\gamma)\widehat{g}_t (T-t) + \frac{1}{2}(1-\gamma)^2 (T-t)^2 \widehat{\sigma}_t^2} \tag{B25}$$

which implies

$$\frac{1}{H(S_t)} \frac{\partial H(S_t)}{\partial \widehat{g}_t} \to (1-\gamma)(T - t)$$

Hence, as $\widehat{g}_t \to \infty$, we obtain

$$\sigma_{M,0} = \left(\frac{1}{H(S_t)}\frac{\partial H(S_t)}{\partial \widehat{g}_t} - \frac{1}{\Omega(S_t)}\frac{\partial \Omega(S_t)}{\partial \widehat{g}_t}\right)\widehat{\sigma}_t^2\sigma^{-1} \to (T-t)\widehat{\sigma}_t^2\sigma^{-1}$$

Q.E.D.

Proof of Corollary 6. The proof is identical to that of Corollary 4. In particular, from expressions (B21) and (B25), neither $\Omega(S_t)$ nor $H(S_t)$ depend on any $\widehat{c}_t^n$. Thus, both $\partial\Omega(S_t)/\partial\widehat{c}_t^n \to 0$ and $\partial H(S)/\partial\widehat{c}_t^n \to 0$, which implies $\sigma_{M,n} \to 0$ for all n. Q.E.D.

Proof of Proposition 6. From Lemmas A1 and A2, the gross announcement return from announcing policy n is

$$\begin{aligned} 1 + R^n\left(\widehat{g}_\tau\right) &= e^{\left(\mu^n - \widehat{g}_\tau\right)(T-\tau) + \frac{1-2\gamma}{2}(T-\tau)^2\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right)} \times \\ & \times \frac{\left(1 + \sum_{n=1}^N p_\tau^n\left(e^{-\gamma\left(\mu_g^n - \widehat{g}_\tau\right)(T-\tau) + \frac{\gamma^2}{2}(T-\tau)^2\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right)} - 1\right)\right)}{\left(1 + \sum_{n=1}^N p_\tau^n\left(e^{(1-\gamma)\left(\mu_g^n - \widehat{g}_\tau\right)(T-\tau) + \frac{(1-\gamma)^2}{2}(T-\tau)^2\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right)} - 1\right)\right)} \end{aligned}$$

Similarly, recalling the notation $\mu_g^0 = \widehat{g}_\tau$ and $\sigma_{g,0} = \widehat{\sigma}_\tau$, from Lemma A1 and A2 the gross announcement return from announcing no policy change is

$$1 + R^0\left(\widehat{g}_\tau\right) = \frac{\left(1 + \sum_{n=1}^N p_\tau^n\left(e^{-\gamma\left(\mu_g^n - \widehat{g}_\tau\right)(T-\tau) + \frac{\gamma^2}{2}(T-\tau)^2\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right)} - 1\right)\right)}{\left(1 + \sum_{n=1}^n p_\tau^n\left(e^{(1-\gamma)\left(\mu_g^n - \widehat{g}_\tau\right)(T-\tau) + \frac{(1-\gamma)^2}{2}(T-\tau)^2\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right)} - 1\right)\right)} \tag{B26}$$

Therefore, we can write more compactly, for $n = 1, ..., N$,

$$1 + R^n\left(\widehat{g}_\tau\right) = e^{\left(\mu_g^n - \widehat{g}_\tau\right)(T-\tau) + \frac{1-2\gamma}{2}(T-\tau)^2\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right)} \times \left(1 + R^0\left(\widehat{g}\right)\right) \tag{B27}$$

From equations (15) and (16) in the paper, we have

$$\mu_g^n - \widehat{g}_t = \left(\widetilde{\mu}^n - \widetilde{\mu}^0\right) + \frac{\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right)}{2}(T-\tau)(\gamma - 1)$$

For the exponent in equation (B27), we therefore obtain

$$\begin{aligned} & \left(\mu_g^n - \widehat{g}_t\right)(T-\tau) + \frac{1-2\gamma}{2}(T-\tau)^2\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right) \\ = \; & \left(\widetilde{\mu}^n - \widetilde{\mu}^0\right)(T-\tau) + \frac{\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right)}{2}(T-\tau)^2(\gamma-1) + \frac{1-2\gamma}{2}(T-\tau)^2\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right) \\ = \; & \left(\widetilde{\mu}^n - \widetilde{\mu}^0\right)(T-\tau) - \frac{\gamma}{2}(T-\tau)^2\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right) \end{aligned}$$

The claim of Proposition 6 then follows immediately. Q.E.D.

Proof of Corollary 7. Immediate from Proposition 6. Q.E.D.

Proof of Corollary 8. Immediate from Corollary 7. Q.E.D.

Proof of Proposition 7. The expression for the jump risk premium follows immediately from

$$J(S_\tau) = \sum_{n=0}^{N} p_\tau^n R^n(x_\tau)$$

where $R^n(x_\tau)$ are given in Proposition 6. We now see that

$$J(S_\tau) = -Cov_\tau\left(\frac{M_{\tau+}^i}{M_\tau^i} - 1, \frac{\pi_{\tau+}}{\pi_\tau} - 1\right) = -\left\{E_\tau\left[J_M J_\pi\right] - E_\tau\left[J_M\right] E_\tau\left[J_\pi\right]\right\}$$

where, if policy n is chosen, we denote $J_M^n = \frac{M_{\tau+}^n}{M_\tau}$ and $J_\pi^n = \frac{\pi_{\tau+}^n}{\pi_\tau}$. Recall from Proposition 6 that

$$\begin{aligned} J_M^n &= 1 + R^n(x_\tau) \\ &= e^{(\widetilde{\mu}^n - x_\tau)(T-\tau) - \frac{\gamma}{2}(T-\tau)^2\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right)} \times \frac{\left(1 + \sum_{\kappa=1}^N p_\tau^\kappa\left(e^{-\gamma(T-\tau)(\widetilde{\mu}^\kappa - x_\tau) + \frac{\gamma}{2}(T-\tau)^2\left(\sigma_{g,\kappa}^2 - \widehat{\sigma}_\tau^2\right)} - 1\right)\right)}{\left(1 + \sum_{\kappa=1}^N p_\tau^\kappa\left(e^{(1-\gamma)(T-\tau)(\widetilde{\mu}^\kappa - x_\tau)} - 1\right)\right)} \end{aligned}$$

We can compute a similar expression now for the stochastic discount factor. From the expressions for $\pi_{\tau+}^n$ and π_τ in the proof of Lemma A1, it follows that for $n = 1, .., N$

$$J_\pi^n(x_\tau) = \frac{\pi_{\tau+}^n}{\pi_\tau} = e^{-\gamma(\widetilde{\mu}^n - x_\tau)(T-\tau) + \frac{\gamma}{2}\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right)(T-\tau)^2} J_\pi^0(x_\tau)$$

where

$$J_\pi^0(x_\tau) = \frac{\pi_{\tau+}^0}{\pi_\tau} = \frac{1}{\left(1 + \sum_{\kappa=1}^N p_\tau^\kappa\left(e^{-\gamma\left(\widetilde{\mu}^k - x_\tau\right)(T-\tau) + \frac{\gamma}{2}(T-\tau)^2\left(\sigma_{g,k}^2 - \widehat{\sigma}_\tau^2\right)} - 1\right)\right)}$$

This implies that

$$\begin{aligned} J_\pi^n(x_\tau) J_M^n(x_\tau) &= \frac{e^{(1-\gamma)(\widetilde{\mu}^n - x_\tau)(T-\tau)}}{\left(1 + \sum_{\kappa=1}^N p_\tau^\kappa\left(e^{(1-\gamma)(T-\tau)(\widetilde{\mu}^\kappa - x_\tau)} - 1\right)\right)} \qquad \text{for } n = 1, ..., N \\ J_\pi^0(x_\tau) J_M^0(x_\tau) &= \frac{1}{\left(1 + \sum_{\kappa=1}^N p_\tau^\kappa\left(e^{(1-\gamma)(T-\tau)(\widetilde{\mu}^\kappa - x_\tau)} - 1\right)\right)} \end{aligned}$$

It follows that

$$E_\tau\left[J_\pi(x_\tau) J_M(x_\tau)\right] = \sum_{\kappa=1}^N p_\tau^\kappa \left\{\frac{e^{(1-\gamma)(\widetilde{\mu}^\kappa - x_\tau)(T-\tau)}}{\left(1 + \sum_{\kappa=1}^N p_\tau^\kappa\left(e^{(1-\gamma)(T-\tau)(\widetilde{\mu}^\kappa - x_\tau)} - 1\right)\right)}\right\}$$

$$
\begin{aligned}
&+\left(1-\sum_{\kappa=1}^{N} p_\tau^\kappa\right) \frac{1}{\left(1+\sum_{\kappa=1}^{N} p_\tau^\kappa \left(e^{(1-\gamma)(T-\tau)(\widetilde{\mu}^\kappa - x_\tau)}-1\right)\right)} \\
=& \frac{1}{\left(1+\sum_{\kappa=1}^{N} p_\tau^\kappa \left(e^{(1-\gamma)(T-\tau)(\widetilde{\mu}^\kappa - x_\tau)}-1\right)\right)} \left(\sum_{\kappa=1}^{N} p_\tau^\kappa e^{(1-\gamma)(\widetilde{\mu}^\kappa - x_\tau)(T-\tau)} + 1 - \sum_{\kappa=1}^{N} p_\tau^\kappa\right) \\
=& 1
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
E_\tau\left[J_\pi\left(x_\tau\right)\right] &= \sum_{\kappa=1}^{N} p_\tau^\kappa \left\{ \frac{e^{-\gamma \widetilde{\mu}^n (T-\tau)} e^{\frac{\gamma}{2}\sigma_{g,n}^2 (T-\tau)^2}}{e^{-\gamma x_\tau (T-\tau)+\frac{\gamma}{2}\widehat{\sigma}_\tau^2 (T-\tau)^2}} J_\pi^0\left(x_\tau\right)\right\} + \left(1-\sum_{\kappa=1}^{N} p_\tau^\kappa\right) J_\pi^0\left(x_\tau\right) \\
&= \left[\sum_{\kappa=1}^{N} p_\tau^\kappa e^{-\gamma(\widetilde{\mu}^n - x_\tau)(T-\tau)} e^{\frac{\gamma}{2}\left(\sigma_{g,n}^2 - \widehat{\sigma}_\tau^2\right)(T-\tau)^2} + 1 - \sum_{\kappa=1}^{N} p_\tau^\kappa\right] J_\pi^0\left(x_\tau\right) \\
&= 1
\end{aligned}
$$

Thus, we finally obtain

$$
\begin{aligned}
J(x_\tau) &= -Cov_\tau\left(J_M, J_\pi\right) = -\left\{E_\tau\left[J_M J_\pi\right] - E_\tau\left[J_M\right] E_\tau\left[J_\pi\right]\right\} \\
&= E_\tau\left[J_M\right] - 1
\end{aligned}
$$

Q.E.D.

2. Political Risk Premium: Additional Results

In this section we collect some additional results on the political risk premium in the limiting case of $\widehat{g}_t \to -\infty$. Recall that in this limiting case, the probability of retaining the old policy converges to zero (i.e., $p_t^0 \to 0$).

Proposition B1: As $\widehat{g}_t \to -\infty$, then, for every $m = 1, .., N$

$$\frac{1}{\Omega(S_t)}\frac{\partial\Omega(S_t)}{\partial\widehat{c}^m} = \sum_{n=1}^{N} w_{1,t}^n \left(\frac{1}{p_t^n}\frac{\partial p_t^n}{\partial\widehat{c}^m}\right); \qquad \frac{1}{H(S_t)}\frac{\partial H(S_t)}{\partial\widehat{c}^m} = \sum_{n=1}^{N} w_{2,t}^n \left(\frac{1}{p_t^n}\frac{\partial p_t^n}{\partial\widehat{c}^m}\right);$$

where the weights, which sum to one, are given by

$$\begin{aligned} w_{1,t}^n &= \frac{p_t^n e^{-\gamma\mu_g^n(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,n}^2}}{\sum_{n=1}^N p_t^n e^{-\gamma\mu_g^n(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,n}^2}} \\ w_{2,t}^n &= \frac{p_t^n e^{(1-\gamma)\mu_g^n(T-\tau)+\frac{1}{2}(1-\gamma)^2(T-\tau)^2\sigma_{g,n}^2}}{\sum_{n=1}^N p_t^n e^{(1-\gamma)\mu_g^n(T-\tau)+\frac{1}{2}(1-\gamma)^2(T-\tau)^2\sigma_{g,n}^2}} \end{aligned}$$

Proof. Follows immediately from equations (B20) and (B24). Q.E.D.

Note that the weights are larger for policies with lower means μ_g^n and higher uncertainties $\sigma_{g,n}$. Since all of the $\widehat{c}_t^m$'s are independent of each other and $\sum_{n=1}^N p_t^n = 1$, we have (i) $\frac{\partial p_t^n}{\partial\widehat{c}^n} < 0$ (i.e., an increase in the perceived political cost of policy n decreases the probability of its adoption); and (ii) $\frac{\partial p_t^n}{\partial\widehat{c}^m} > 0$ for $m \neq n$ (i.e., an increase in the political cost of policy m increases the probability of adopting a different policy n).

We now specialize the above result to the case with only two policies, which we denote by H and L. In that case, we obtain analytical solutions:

Proposition B2. For $N = 2$ and $\widehat{g}_t \to -\infty$, we have

$$\begin{aligned} \frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^H} &= -\frac{\int \phi_{\widetilde{c}^H}\left(\widetilde{c}-\widetilde{\mu}^L+\widetilde{\mu}^H\right)\phi_{\widetilde{c}^L}\left(\widetilde{c}\right)d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_1^{-1}+p_t^H\right)}; \quad \frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^L} = \frac{\int \phi_{\widetilde{c}^L}\left(\widetilde{c}-\widetilde{\mu}^H+\widetilde{\mu}^L\right)\phi_{\widetilde{c}^H}\left(\widetilde{c}\right)d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_1^{-1}+p_t^H\right)} \\ \frac{1}{H}\frac{\partial H}{\partial\widehat{c}^H} &= -\frac{\int \phi_{\widetilde{c}^H}\left(\widetilde{c}-\widetilde{\mu}^L+\widetilde{\mu}^H\right)\phi_{\widetilde{c}^L}\left(\widetilde{c}\right)d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_2^{-1}+p_t^H\right)}; \quad \frac{1}{H}\frac{\partial H}{\partial\widehat{c}^L} = \frac{\int \phi_{\widetilde{c}^L}\left(\widetilde{c}-\widetilde{\mu}^H+\widetilde{\mu}^L\right)\phi_{\widetilde{c}^H}\left(\widetilde{c}\right)d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_2^{-1}+p_t^H\right)} \end{aligned}$$

where $\phi_{\widetilde{c}^i}(.)$ is the normal density with mean $\widehat{c}_t^i/(\gamma-1)(T-\tau)$ and variance $\left(\widehat{\sigma}_{c,t}^2-\widehat{\sigma}_{c,\tau}^2\right)/(\gamma-1)^2(T-\tau)^2$, and

$$\begin{aligned} G_1 &= e^{-\gamma\left(\mu_g^H-\mu_g^L\right)(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\left(\sigma_{g,H}^2-\sigma_{g,L}^2\right)} - 1 \\ G_2 &= e^{(1-\gamma)\left(\mu_g^H-\mu_g^L\right)(T-\tau)+\frac{1}{2}(1-\gamma)^2(T-\tau)^2\left(\sigma_{g,H}^2-\sigma_{g,L}^2\right)} - 1 \end{aligned}$$

Proof. With $N = 2$, we have $p_t^L = 1 - p_t^H$. We prove only the expression for $\frac{1}{\Omega(S_t)}\frac{\partial\Omega(S_t)}{\partial\widehat{c}_t^m}$, $m = H, L$, as the other case with $H(S_t)$ can be proved analogously. Using (B20), we have that for $\widehat{g}_t$ sufficiently small, $\Omega(S_t)$ converges to a quantity that is proportional to

$$\Omega(S_t) \;\propto\; e^{-\gamma\widehat{g}_t(\tau-t)+\frac{1}{2}\gamma^2(\tau-t)^2\widehat{\sigma}_t^2}\left[\left(1-p_t^H\right)e^{-\gamma\mu_g^L(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,L}^2}+p_t^H e^{-\gamma\mu_g^H(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,H}^2}\right],$$

where the constant of proportionality is independent of any state variables. Thus,

$$\begin{aligned}
\frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^H} &= \frac{\frac{\partial p_t^H}{\partial\widehat{c}^H}\left[e^{-\gamma\mu_g^H(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,H}^2}-e^{-\gamma\mu_g^L(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,L}^2}\right]}{\left[\left(1-p_t^H\right)e^{-\gamma\mu_g^L(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,L}^2}+p_t^H e^{-\gamma\mu_g^H(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,H}^2}\right]}\\
&= \frac{\frac{\partial p_t^H}{\partial\widehat{c}^H}\left(e^{-\gamma\left(\mu_g^H-\mu_g^L\right)(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\left(\sigma_{g,H}^2-\sigma_{g,L}^2\right)}-1\right)}{\left[1+p_t^H\left(e^{-\gamma\left(\mu_g^H-\mu_g^L\right)(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\left(\sigma_{g,H}^2-\sigma_{g,L}^2\right)}-1\right)\right]}\\
\frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^L} &= \frac{\frac{\partial p_t^H}{\partial\widehat{c}^L}\left[e^{-\gamma\mu_g^H(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,H}^2}-e^{-\gamma\mu_g^L(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,L}^2}\right]}{\left[\left(1-p_t^H\right)e^{-\gamma\mu_g^L(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,L}^2}+p_t^H e^{-\gamma\mu_g^H(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\sigma_{g,H}^2}\right]}\\
&= \frac{\frac{\partial p_t^H}{\partial\widehat{c}^L}\left(e^{-\gamma\left(\mu_g^H-\mu_g^L\right)(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\left(\sigma_{g,H}^2-\sigma_{g,L}^2\right)}-1\right)}{\left[1+p_t^H\left(e^{-\gamma\left(\mu_g^H-\mu_g^L\right)(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\left(\sigma_{g,H}^2-\sigma_{g,L}^2\right)}-1\right)\right]}
\end{aligned}$$

Using the definition of G_1 we obtain

$$\frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^H}=\frac{\frac{\partial p_t^H}{\partial\widehat{c}^H}G_1}{1+p_t^H G_1}\text{ and }\frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^L}=\frac{\frac{\partial p_t^H}{\partial\widehat{c}^L}G_1}{1+p_t^H G_1}$$

We finally compute the sensitivity of probabilities to expected costs. As $\widehat{g}\to-\infty$, we have $p_t^0\to 0$, and we find that p_t^H and p_t^L converge to

$$\begin{aligned}
p_t^H &= \int_{-\infty}^{\infty}\left(1-\Phi_{\widetilde{c}^L}\left(\widetilde{c}^H-\widetilde{\mu}^H+\widetilde{\mu}^L\right)\right)\phi_{\widetilde{c}^H}\left(\widetilde{c}^H\right)d\widetilde{c}^H\\
p_t^L &= \int_{-\infty}^{\infty}\left(1-\Phi_{\widetilde{c}^H}\left(\widetilde{c}^L-\widetilde{\mu}^L+\widetilde{\mu}^H\right)\right)\phi_{\widetilde{c}^L}\left(\widetilde{c}^L\right)d\widetilde{c}^L
\end{aligned}$$

Thus, recalling

$$\widetilde{c}^i|S_t=\frac{c^i}{(\gamma-1)(T-\tau)}\sim N\left(\frac{\widehat{c}_t^i}{(\gamma-1)(T-\tau)},\frac{\widehat{\sigma}_{c,t}^2-\widehat{\sigma}_{c,\tau}^2}{(\gamma-1)^2(T-\tau)^2}\right)$$

and letting $V(t,\tau)=\frac{\widehat{\sigma}_{c,t}^2-\widehat{\sigma}_{c,\tau}^2}{(\gamma-1)^2(T-\tau)^2}$, we have

$$\begin{aligned}
\Phi_{\widetilde{c}^L}\left(\widetilde{c}^H-\widetilde{\mu}^H+\widetilde{\mu}^L\right) &= \int_{-\infty}^{\widetilde{c}^H-\widetilde{\mu}^H+\widetilde{\mu}^L}\frac{1}{\sqrt{2\pi V(t,\tau)}}e^{-\frac{1}{2V(t,\tau)}\left(\frac{\widehat{c}_t^L}{(\gamma-1)(T-\tau)}-\widetilde{c}\right)^2}d\widetilde{c}\\
&= \int_{-\infty}^{\frac{\left(\widetilde{c}^H-\widetilde{\mu}^H+\widetilde{\mu}^L-\frac{\widehat{c}_t^L}{(\gamma-1)(T-\tau)}\right)}{\sqrt{V(t,\tau)}}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\widetilde{c}^2}d\widetilde{c}
\end{aligned}$$

Thus, we obtain

$$\frac{\partial\Phi_{\widetilde{c}^L}\left(\widetilde{c}^H-\widetilde{\mu}^H+\widetilde{\mu}^L\right)}{\partial\widehat{c}_t^L} = -\frac{1}{(\gamma-1)(T-\tau)}\frac{1}{\sqrt{V(t,\tau)}}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left[\frac{\left(\widetilde{c}^H-\widetilde{\mu}^H+\widetilde{\mu}^L-\frac{\widehat{c}_t^L}{(\gamma-1)(T-\tau)}\right)}{\sqrt{V(t,\tau)}}\right]^2} d\widetilde{c}$$
$$= -\frac{1}{(\gamma-1)(T-\tau)}\phi_{\widetilde{c}^L}\left(\widetilde{c}^H-\widetilde{\mu}^H+\widetilde{\mu}^L\right)$$

which gives

$$\frac{\partial p_t^H}{\partial\widehat{c}^L} = \int_{-\infty}^{\infty}-\frac{\partial\Phi_{\widetilde{c}^L}\left(\widetilde{c}^H-\widetilde{\mu}^H+\widetilde{\mu}^L\right)}{\partial\widehat{c}^L}\phi_{\widetilde{c}^H}\left(\widetilde{c}^H\right)d\widetilde{c}^H$$
$$= \frac{1}{(\gamma-1)(T-\tau)}\int_{-\infty}^{\infty}\phi_{\widetilde{c}^L}\left(\widetilde{c}^H-\widetilde{\mu}^H+\widetilde{\mu}^L\right)\phi_{\widetilde{c}^H}\left(\widetilde{c}^H\right)d\widetilde{c}^H$$

Similarly, using $p_t^H = 1-p_t^L$, we also obtain

$$\frac{\partial p_t^H}{\partial\widehat{c}^H} = -\frac{\partial p_t^L}{\partial\widehat{c}^H} = -\frac{1}{(\gamma-1)(T-\tau)}\int_{-\infty}^{\infty}\phi_{\widetilde{c}^H}\left(\widetilde{c}^L-\widetilde{\mu}^L+\widetilde{\mu}^H\right)\phi_{\widetilde{c}^L}\left(\widetilde{c}^L\right)d\widetilde{c}^L$$

This yields the expressions

$$\frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^H} = -\frac{\int\phi_{\widetilde{c}^H}\left(\widetilde{c}-\widetilde{\mu}^L+\widetilde{\mu}^H\right)\phi_{\widetilde{c}^L}\left(\widetilde{c}\right)d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_1^{-1}+p_t^H\right)}$$
$$\frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^L} = \frac{\int\phi_{\widetilde{c}^L}\left(\widetilde{c}-\widetilde{\mu}^H+\widetilde{\mu}^L\right)\phi_{\widetilde{c}^H}\left(\widetilde{c}\right)d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_1^{-1}+p_t^H\right)}$$

An identical argument, using (B24), proves the claim for $\frac{1}{H}\frac{\partial H}{\partial\widehat{c}^m}$, for $m = H, L$. Q.E.D.

This proposition allows us to obtain sufficient conditions for the signs for the sensitivity of $\Omega(S_t)$ and $H(S_t)$ to $\widehat{c}_t^m$, with $m = H, L$. In particular,

Corollary B1: The following two conditions hold:

$$G_1 > 0 \text{ if and only if } \left(\mu_g^H-\mu_g^L\right) < \frac{1}{2}\gamma(T-\tau)\left(\sigma_{g,H}^2-\sigma_{g,L}^2\right)$$
$$G_2 > 0 \text{ if and only if } \left(\mu_g^H-\mu_g^L\right) < \frac{1}{2}(\gamma-1)(T-\tau)\left(\sigma_{g,H}^2-\sigma_{g,L}^2\right)$$

It follows that if both $G_1 > 0$ and $G_2 > 0$, then

$$\frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^H} < 0;\quad \frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^L} > 0;\quad \frac{1}{H}\frac{\partial H}{\partial\widehat{c}^H} < 0;\quad \frac{1}{H}\frac{\partial H}{\partial\widehat{c}^L} > 0$$

Proof. We have

$$G_1 = e^{-\gamma\left(\mu_g^H-\mu_g^L\right)(T-\tau)+\frac{1}{2}\gamma^2(T-\tau)^2\left(\sigma_{g,H}^2-\sigma_{g,L}^2\right)}-1 > 0$$

iff

$$\left(\mu_g^H - \mu_g^L\right) < \frac{1}{2}\gamma\left(T-\tau\right)\left(\sigma_{g,H}^2 - \sigma_{g,L}^2\right)$$

Similarly,

$$G_2 = e^{(1-\gamma)\left(\mu_g^H-\mu_g^L\right)(T-\tau)+\frac{1}{2}(1-\gamma)^2(T-\tau)^2\left(\sigma_{g,H}^2-\sigma_{g,L}^2\right)} - 1 > 0$$

iff

$$\left(\mu_g^H - \mu_g^L\right) < \frac{1}{2}\left(\gamma-1\right)\left(T-\tau\right)\left(\sigma_{g,H}^2 - \sigma_{g,L}^2\right)$$

The remaining part follows from Proposition B2. Q.E.D.

The conditions in this corollary are intuitive. Consider the case in which $\sigma_{g,H}^2 > \sigma_{g,L}^2$, that is, H is a riskier policy than L. In that case, the conditions of this corollary state that if the risky policy H does not have a sufficiently higher expected impact than policy L to offset its higher risk (i.e., if $\mu_g^H - \mu_g^L$ is not large enough), then an increase in the political cost of policy H is good news, as it decreases marginal utility ($\frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^H} < 0$) and increases expected utility (which is proportional to $\frac{1}{1-\gamma}\frac{1}{H}\frac{\partial H}{\partial\widehat{c}^H} > 0$, since $\gamma > 1$). The opposite sign holds for an increase in the cost of the less risky policy L.

When $\mu_g^H - \mu_g^L$ is high enough to violate both conditions, then $G_1 < 0$ and $G_2 < 0$. In that case, the signs of the partial derivatives are ambiguous because they depend on the signs of $G_1^{-1} + p_t^H$ and $G_2^{-1} + p_t^H$. If the probability of the risky policy is small, then an increase in the cost of the risky policy actually increases marginal utility and decreases expected utility, due to the even smaller chance of adopting a high growth policy.

We continue with a discussion of the iso-utility case, in which we obtain even clearer predictions for the signs of the partial derivatives.

Corollary B2: In the iso-utility case ($\widetilde{\mu}^L = \widetilde{\mu}^H$), the limiting expressions when $\widehat{g} \to -\infty$ simplify to

$$\begin{aligned}
\frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^H} &= -\frac{\int \phi_{\widetilde{c}^H}\left(\widetilde{c}\right)\phi_{\widetilde{c}^L}\left(\widetilde{c}\right)d\widetilde{c}}{\left(\gamma-1\right)\left(T-\tau\right)\left(G_1^{-1}+p_t^H\right)} = -\frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}^L} \\
\frac{1}{H}\frac{\partial H}{\partial\widehat{c}^H} &= \frac{1}{H}\frac{\partial H}{\partial\widehat{c}^L} = 0
\end{aligned}$$

where using $\left(\mu_g^H - \mu_g^L\right) = \frac{1}{2}\left(\sigma_{g,H}^2 - \sigma_{g,L}^2\right)\left(T-\tau\right)\left(\gamma-1\right)$ we have the simpler expression for G_1 given by

$$G_1 = e^{\frac{\gamma}{2}\left(\sigma_{g,H}^2-\sigma_{g,L}^2\right)(T-\tau)^2} - 1 > 0$$

In this case, $G_1 > 0$, and therefore an increase in the political cost of the high-risk policy H is always good news as it decreases the marginal utility of the representative agent. An increase in the cost of policy H of course does not change expected utility, as both policies yield the same utility.

We conclude by deriving a closed-form expression for the political risk premium in the iso-utility case when $\widehat{g}_t \to -\infty$. That is equation (A5) in the paper.

Corollary B3: If $N = 2$, $\widetilde{\mu}^L = \widetilde{\mu}^H$, and $\widehat{g}_t \to -\infty$, then

$$\sigma_{\pi,H} = \frac{1}{\Omega}\frac{\partial \Omega}{\partial \widehat{c}^H}\widehat{\sigma}^2_{c,t}h^{-1} \to -\frac{\int \phi_{\widetilde{c}^H}(\widetilde{c})\,\phi_{\widetilde{c}^L}(\widetilde{c})\,d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_1^{-1}+p_t^H\right)}\widehat{\sigma}^2_{c,t}h^{-1}$$

$$\sigma_{\pi,L} = \frac{1}{\Omega}\frac{\partial \Omega}{\partial \widehat{c}^L}\widehat{\sigma}^2_{c,t}h^{-1} \to \frac{\int \phi_{\widetilde{c}^H}(\widetilde{c})\,\phi_{\widetilde{c}^L}(\widetilde{c})\,d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_1^{-1}+p_t^H\right)}\widehat{\sigma}^2_{c,t}h^{-1}$$

and similarly

$$\sigma_{M,H} = \left(\frac{1}{H}\frac{\partial H}{\partial \widehat{c}^H} - \frac{1}{\Omega}\frac{\partial \Omega}{\partial \widehat{c}^H}\right)\widehat{\sigma}^2_{c,t}h^{-1} \to \frac{\int \phi_{\widetilde{c}^H}(\widetilde{c})\,\phi_{\widetilde{c}^L}(\widetilde{c})\,d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_1^{-1}+p_t^H\right)}\widehat{\sigma}^2_{c,t}h^{-1}$$

$$\sigma_{M,L} = \left(\frac{1}{H}\frac{\partial H}{\partial \widehat{c}^L} - \frac{1}{\Omega}\frac{\partial \Omega}{\partial \widehat{c}^L}\right)\widehat{\sigma}^2_{c,t}h^{-1} \to -\frac{\int \phi_{\widetilde{c}^H}(\widetilde{c})\,\phi_{\widetilde{c}^L}(\widetilde{c})\,d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_1^{-1}+p_t^H\right)}\widehat{\sigma}^2_{c,t}h^{-1}$$

Thus

$$\begin{aligned}\text{Political Risk Premium} &= -\sigma_{\pi,H}\sigma_{M,H} - \sigma_{\pi,L}\sigma_{M,L} \\ &\to 2\left(\frac{\int \phi_{\widetilde{c}^H}(\widetilde{c})\,\phi_{\widetilde{c}^L}(\widetilde{c})\,d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_1^{-1}+p_t^H\right)}\right)^2 \widehat{\sigma}^4_{c,t}h^{-2}\end{aligned}$$

This is equation (A5) in the paper.

Given our parameters $\gamma = 5$, $(T-\tau) = 10$, $\sigma^2_{g,H} = .03$, $\sigma^2_{g,L} = .01$, equal priors $c_H, c_L \sim N\left(-\frac{1}{2}\sigma_c^2, \sigma_c^2\right)$ with $\sigma_c = .1$, so that $\widehat{\sigma}^2_{c,t} = \sigma^2_c = .1^2$ at time $t_0 = \tau - 1$, and $p_H = 0.5$, we obtain $G_1 = e^{\frac{\gamma}{2}\left(\sigma^2_{g,H}-\sigma^2_{g,L}\right)(T-\tau)^2} - 1 = .2214$, and $\int \phi_{\widetilde{c}}(\widetilde{c})^2\,d\widetilde{c}$ =126.1566 [where $\widetilde{c} \sim N\left(\frac{-\frac{1}{2}\sigma_c^2}{(\gamma-1)(T-\tau)}, \frac{\sigma_c^2-\widehat{\sigma}^2_{c,\tau}}{(\gamma-1)^2(T-\tau)^2}\right)$ with $\sigma_{c,\tau} = 0.0447$]. Thus,

$$\left(\frac{\int \phi_{\widetilde{c}^H}(\widetilde{c})\,\phi_{\widetilde{c}^L}(\widetilde{c})\,d\widetilde{c}}{(\gamma-1)(T-\tau)\left(G_1^{-1}+p_t^H\right)}\right) = 0.6287$$

and hence

$$\text{Political Risk Premium} = 2 \times (0.6287)^2 \times .1^4/.05^2 = 3.16\%$$

This is indeed the value that we see in Figure 3 in the paper.

3. Model Extension: Different Signal Precisions

This extension, described in Section 7.1 in the paper, allows the precisions of political signals to vary across policies. In the two-policy case, we have

$$ds_t^H = c^H\, dt + h^H\, dZ_{c,t}^H \tag{B28}$$

$$ds_t^L = c^L\, dt + h^L\, dZ_{c,t}^L\ , \tag{B29}$$

with $h^H \neq h^L$. The figure described in the paper is shown below. The political risk premium has two components, one driven by learning about c^H (shown in yellow at the top) and the other driven by learning about c^L (shown in red, second from the top).

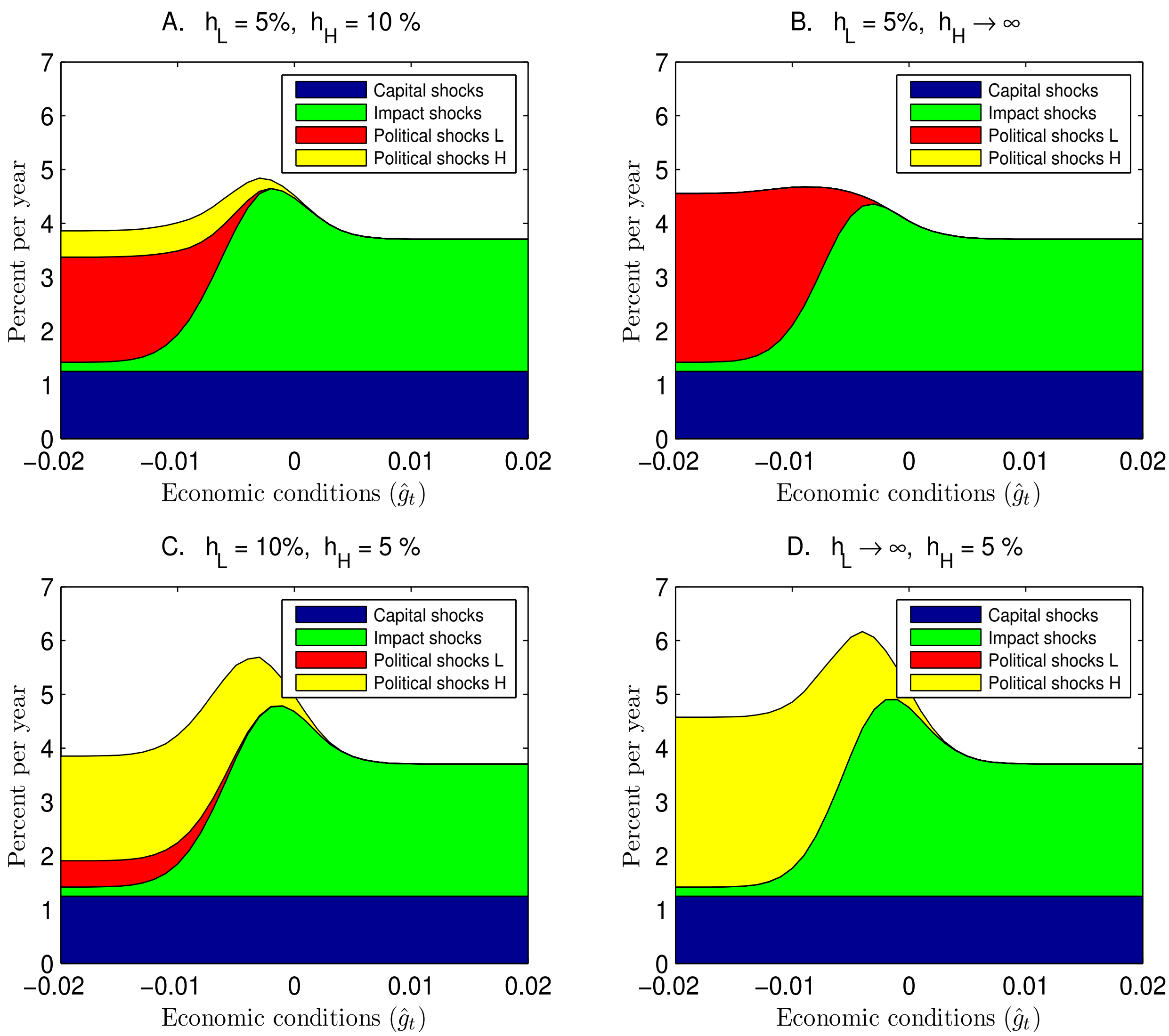

Figure B1. The equity risk premium and its components: Different precisions of political signals.

4. Model Extension: Policy-Unrelated Business Cycles

This extension, described in Section 7.2 in the paper, introduces an additional persistent source of variation in profitability that is unrelated to government policy. In this appendix, we first fill in the details of the calculations that are omitted from the paper. We then show the plots that summarize the main results.

We modify the profitability process from the paper's equation (1) as follows:

$$d\Pi_t^i = (\mu_t + g_t)\, dt + \sigma dZ_t + \sigma_1 dZ_t^i \ ,$$

where μ_t follows the mean-reverting process

$$d\mu_t = \beta\left(\overline{\mu} - \mu_t\right) dt + \sigma_\mu dZ_{\mu,t} \ .$$

Agents do not observe μ_t, but they learn about it by observing $d\Pi_t^i$ and an additional signal:

$$dS_t = \mu_t dt + \sigma_S dZ_{S,t} \ . \tag{B30}$$

Above, β, $\overline{\mu}$, σ_μ, and σ_S are known constants, and $dZ_{\mu,t}$ and $dZ_{S,t}$ are Brownian motions uncorrelated with all others.

4.1. Learning

The set of unknown quantities includes political costs, μ_t, and g_t. Learning about political costs proceeds in the same way as in the paper. Below, we characterize learning about μ_t and g_t. Recall that $g_t = g^0$ is constant until time τ.

We assume that the prior distribution for μ_t and g^0 at time 0 is jointly normal:

$$\begin{pmatrix} \mu_0 \\ g^0 \end{pmatrix} \sim N\left(\begin{pmatrix} \widehat{\mu}_0 \\ \widehat{g}_0 \end{pmatrix}, \begin{pmatrix} \widehat{\sigma}^2_{\mu,0} & \widehat{\sigma}_{\mu g,0} \\ \widehat{\sigma}_{\mu g,0} & \widehat{\sigma}^2_{g,0} \end{pmatrix}\right) \ .$$

Standard arguments for Bayesian learning in continuous time yield the following:

Proposition B3: The posterior distribution at any time $t \leq \tau$ is given by

$$\begin{pmatrix} \mu_t \\ g^0 \end{pmatrix} \sim N\left(\begin{pmatrix} \widehat{\mu}_t \\ \widehat{g}_t \end{pmatrix}, \begin{pmatrix} \widehat{\sigma}^2_{\mu,t} & \widehat{\sigma}_{\mu g,t} \\ \widehat{\sigma}_{\mu g,t} & \widehat{\sigma}^2_{g,t} \end{pmatrix}\right) \ ,$$

where the posterior means follow

$$d\widehat{\mu}_t = \beta\left(\overline{\mu} - \widehat{\mu}_t\right) dt + \sigma^{-1}\left(\widehat{\sigma}^2_{\mu t} + \widehat{\sigma}_{\mu g,t}\right) d\widehat{Z}_{1,t} + \sigma_S^{-1}\widehat{\sigma}^2_{\mu t} d\widehat{Z}_{2,t} \tag{B31}$$

$$d\widehat{g}_t = \sigma^{-1}\left(\widehat{\sigma}_{\mu g,t} + \widehat{\sigma}^2_{g t}\right) d\widehat{Z}_{1,t} + \sigma_S^{-1}\widehat{\sigma}_{\mu g,t} d\widehat{Z}_{2,t} \tag{B32}$$

and the posterior variances and covariances follow

$$
\begin{aligned}
\frac{d\widehat{\sigma}_{\mu,t}^2}{dt} &= -2\beta\widehat{\sigma}_{\mu,t}^2 + \sigma_\mu^2 - \left(\sigma^{-2}\left(\widehat{\sigma}_{\mu t}^2 + \widehat{\sigma}_{\mu g,t}\right)^2 + \sigma_S^{-2}\left(\widehat{\sigma}_{\mu,t}^2\right)^2\right) \\
\frac{d\widehat{\sigma}_{\mu g,t}}{dt} &= -\beta\widehat{\sigma}_{\mu g,t} - \left(\sigma^{-2}\left(\widehat{\sigma}_{\mu t}^2 + \widehat{\sigma}_{\mu g,t}\right)\left(\widehat{\sigma}_{\mu g,t} + \widehat{\sigma}_{gt}^2\right) + \sigma_S^{-2}\widehat{\sigma}_{\mu,t}^2\widehat{\sigma}_{\mu g,t}\right) \\
\frac{d\widehat{\sigma}_{gt}^2}{dt} &= -\left(\sigma^{-2}\left(\widehat{\sigma}_{\mu g,t} + \widehat{\sigma}_{gt}^2\right)^2 + \sigma_S^{-2}\widehat{\sigma}_{\mu g,t}^2\right) \ .
\end{aligned}
$$

Above, the new Brownian motions $\left(d\widehat{Z}_{1,t}, d\widehat{Z}_{2,t}\right)$ reflect expectation errors:

$$
\begin{aligned}
d\widehat{Z}_{1,t} &= \sigma^{-1}\left[\frac{dB_t}{B_t} - E_t\left[\frac{dB_t}{B_t}\right]\right] \\
d\widehat{Z}_{2,t} &= \sigma_S^{-1}\left[dS_t - E_t\left[dS_t\right]\right] \ .
\end{aligned}
$$

Note that both new Brownian motions affect both posterior means in equations (B31) and (B32). In the special limiting case $\sigma_S \to \infty$, the second Brownian motion drops out because the signal dS_t is then infinitely imprecise. In that special case, equations (B31) and (B32) simplify to

$$
\begin{aligned}
d\widehat{\mu}_t &= \beta\left(\overline{\mu} - \widehat{\mu}_t\right)dt + \sigma^{-1}\left(\widehat{\sigma}_{\mu t}^2 + \widehat{\sigma}_{\mu g,t}\right)d\widehat{Z}_{1,t} \\
d\widehat{g}_t &= \sigma^{-1}\left(\widehat{\sigma}_{\mu g,t} + \widehat{\sigma}_{gt}^2\right)d\widehat{Z}_{1,t} \ ,
\end{aligned}
$$

and the posterior means become instantaneously perfectly correlated. In particular, news about aggregate profitability ($d\widehat{Z}_{1,t}$) affects both $\widehat{\mu}_t$ and $\widehat{g}_t$. The magnitudes of these effects depend on the degree of uncertainty about $\widehat{\mu}_t$ and $\widehat{g}_t$. If $\widehat{\sigma}_{\mu,t}^2$ is small relative to $\widehat{\sigma}_{gt}^2$, the update on $\widehat{g}_t$ is larger than the update on $\widehat{\mu}_t$, and vice versa. In addition, $\widehat{\mu}_t$ is affected by mean reversion. A higher value of β implies stronger mean reversion and, consequently, a relatively weaker influence of $d\widehat{Z}_{1,t}$. Furthermore, a higher β implies lower uncertainty about $\widehat{\mu}_t$.

4.2. The Government's Policy Decision

We show below that the government's decision rule in this extended model is very similar to the decison rule in the basic model in the paper. For each $n = 0, ..., N$, we redefine the utility score of policy n at time τ as follows:

$$
\widetilde{\mu}^n = \mu_g^n - \frac{\widehat{\sigma}_{g,n}^2}{2}\left(\gamma - 1\right)\left(T - \tau\right) - \left(\gamma - 1\right)Q_1\left(\tau, T\right)\widehat{\sigma}_{\mu g,n} \ ,
$$

where

$$
Q_1\left(t, T\right) = \frac{1 - e^{-\beta(T-t)}}{\beta} \ .
$$

This utility score is the same as the utility score in the paper, except for one additional term, $(\gamma-1)\,Q_1(\tau,T)\,\widehat{\sigma}_{\mu g,n}$. This term involves $\widehat{\sigma}_{\mu g,n}$, the covariance between μ_τ and g^n as of time τ. For $n=0$, this covariance is given by the posterior covariance $\widehat{\sigma}_{\mu g,\tau}$ because some learning about g^0 takes place before time τ. For $n>0$, this is a prior covariance.

Proposition B4: The government chooses policy n at time τ if and only if the following condition holds for all policies $m \neq n,\ m \in \{0,1,\ldots,N\}$:

$$\widetilde{\mu}^n - \widetilde{c}^n > \widetilde{\mu}^m - \widetilde{c}^m \ ,$$

where

$$\widetilde{c}^n = \frac{c^n}{(\gamma-1)\,(T-\tau)} \qquad n = 0,1,\ldots,N \ .$$

The government's decision rule is thus identical to the rule in Proposition 1 in the paper, except for the slightly redefined utility score.

Corollary B4: A policy change occurs at time τ if and only if

$$\widehat{g}_\tau < \max_{n\in\{1,..,N\}} \{\widetilde{\mu}^n - \widetilde{c}^n\} + \frac{\widehat{\sigma}_\tau^2}{2}\,(\gamma-1)\,(T-\tau) + (\gamma-1)\,Q_1(\tau,T)\,\widehat{\sigma}_{\mu g,\tau} \ .$$

This rule is the same as in Corollary 1 the paper, except for the extra term at the end, which reflects the persistent variation in μ_t introduced here.

4.3. Stock Prices

First, we establish the pricing results immediately after the policy decision at time τ.

Proposition B5: The stochastic discount factor at time $\tau+$ conditional on policy n being chosen is given by

$$\begin{aligned}
\pi^n_{\tau+} &= E_{\tau+}\left[B_T^{-\gamma}\middle|\ \text{n at } \tau\right] \\
&= B_\tau^{-\gamma}\exp\Bigg\{\frac{1}{2}\gamma\,(1+\gamma)\,\sigma^2\,(T-\tau) - \gamma\overline{\mu}\,(T-\tau) + \sigma_\mu^2\frac{\gamma^2}{2}Q\,(\tau;T) - \gamma Q_1\,(\tau;T)\,(\widehat{\mu}_t - \overline{\mu}) \\
&\qquad -\gamma\mu_g^n\,(T-\tau) + \frac{1}{2}\gamma^2 Q_1\,(\tau,T)^2\,\widehat{\sigma}^2_{\mu,\tau} + \frac{1}{2}\gamma^2\,(T-\tau)^2\,\widehat{\sigma}^2_{g,n} + \gamma^2 Q_1\,(\tau,T)\,(T-\tau)\,\widehat{\sigma}_{\mu g,n}\Bigg\} \ ,
\end{aligned}$$

where $Q_1(t,T)$ is given above and $Q(t,T)$ is equal to

$$Q\,(t;T) = \left[(T-t) + \frac{1-e^{-2\beta(T-t)}}{2\beta} - 2Q_1\,(t;T)\right] \ .$$

Proposition B6: The M/B at time $\tau+$ conditional on policy n being chosen is

$$\begin{aligned}
\left(\frac{M}{B}\right)^n_{\tau+} &= \exp\Bigg\{-\gamma\sigma^2\,(T-t) + \overline{\mu}\,(T-\tau) + \sigma_\mu^2\frac{1}{2}\,(1-2\gamma)\,Q\,(t;T) + Q_1\,(\tau;T)\,(\widehat{\mu}_t - \overline{\mu}) \\
&\qquad +\mu_g^n\,(T-\tau) + \frac{1}{2}\,(1-2\gamma)\,Q_1\,(\tau,T)^2\,\widehat{\sigma}^2_{\mu,\tau} + \frac{1}{2}\,(1-2\gamma)\,(T-\tau)^2\,\widehat{\sigma}^2_{g,n} \\
&\qquad + (1-2\gamma)\,Q_1\,(\tau,T)\,(T-\tau)\,\widehat{\sigma}_{\mu g,n}\Bigg\} \ .
\end{aligned}$$

Next, we provide the pricing results before time τ, which are the focus of this paper.

Proposition B7: The stochastic discount factor at time $t < \tau$ is given by

$$\pi_t = B_t^{-\gamma} \exp\left\{\frac{1}{2}\gamma\left(1+\gamma\right)\sigma^2\left(T-\tau\right) - \gamma\overline{\mu}\left(T-\tau\right) + \sigma_\mu^2\frac{\gamma^2}{2}Q\left(\tau;T\right) + \frac{1}{2}\gamma^2 Q_1\left(\tau,T\right)^2\widehat{\sigma}_{\mu,\tau}^2\right\}\Omega\left(S_t,t\right),$$

where

$$\Omega\left(S_t,t\right) = E\left[e^{-\gamma\Delta b_\tau - \gamma Q_1(\tau;T)(\widehat{\mu}_\tau-\overline{\mu}) - \gamma\mu_g^n(T-\tau) + \frac{1}{2}\gamma^2(T-\tau)^2\widehat{\sigma}_{g,n}^2 + \gamma^2 Q_1(\tau,T)(T-\tau)\widehat{\sigma}_{\mu g,n}}|S_t\right] ,$$

$\Delta b_\tau = b_\tau - b_t$, and the state variables are $S_t = [\widehat{\mu}_t, \widehat{g}_t, \widehat{c}_{1,t}, .., \widehat{c}_{N,t}, t]$.

The dynamics of the stochastic discount factor are as follows:

$$\frac{d\pi_t}{\pi_t} = \sigma_{\pi,1,t}d\widehat{Z}_{1,t} + \sigma_{\pi,2,t}d\widehat{Z}_{2,t} + \sum_{n=1}^N \sigma_{\pi,c,n,t}d\widehat{Z}_{c,t}^n .$$

where

$$\begin{aligned}
\sigma_{\pi,1,t} &= \left[-\gamma\sigma + \frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{\mu}_t}\sigma^{-1}\left(\widehat{\sigma}_{\mu t}^2 + \widehat{\sigma}_{\mu g,t}\right) + \frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{g}_t}\sigma^{-1}\left(\widehat{\sigma}_{\mu g,t} + \widehat{\sigma}_{gt}^2\right)\right] \\
\sigma_{\pi,2,t} &= \left[\frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{\mu}_t}\sigma_S^{-1}\widehat{\sigma}_{\mu t}^2 + \frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{g}_t}\sigma_S^{-1}\widehat{\sigma}_{\mu g,t}\right] \\
\sigma_{\pi,c,n,t} &= \frac{1}{\Omega}\frac{\partial\Omega}{\partial\widehat{c}_t^n}\widehat{\sigma}_{c,n,t}^2 h^{-1}
\end{aligned}$$

The first variable, $\sigma_{\pi,1,t}$, represents the price of risk associated with the economic shocks from our basic model, namely, shocks associated with surprises in aggregate profitability. The third variable, $\sigma_{\pi,c,n,t}$, is the price of risk associated with the political shocks from our basic model. The second variable, $\sigma_{\pi,2,t}$, is new—it captures the additional economic shocks associated with the signal dS_t.

Proposition B8: The M/B at time $t < \tau$ is given by

$$\frac{M_t}{B_t} = \exp\left\{-\gamma\sigma^2\left(T-\tau\right) + \overline{\mu}\left(T-\tau\right) + \sigma_\mu^2\frac{1}{2}\left(1-2\gamma\right)Q\left(\tau;T\right) + \frac{1}{2}\left(1-2\gamma\right)Q_1\left(\tau,T\right)^2\widehat{\sigma}_{\mu,\tau}^2\right\}\frac{H\left(S_t\right)}{\Omega\left(S_t\right)},$$

where

$$H\left(S_t\right) = E\left[e^{(1-\gamma)\Delta b_\tau + (1-\gamma)Q_1(\tau;T)(\widehat{\mu}_\tau-\overline{\mu}) + (1-\gamma)\mu_g^n(T-\tau) + \frac{1}{2}(1-\gamma)^2(T-\tau)^2\widehat{\sigma}_{g,n}^2 + (1-\gamma)^2 Q_1(\tau,T)(T-\tau)\widehat{\sigma}_{\mu g,n}}|S_t\right] .$$

The stock return process is given by

$$\frac{dM_t}{M_t} = \mu_{M,t}dt + \sigma_{M1,t}d\widehat{Z}_{1t} + \sigma_{M2,t}d\widehat{Z}_{2t} + \sum \sigma_{M,c,n,t}d\widehat{Z}_{c,t}^n ,$$

where

$$\sigma_{M1,t} = \sigma + \left(\frac{1}{H}\frac{\partial H}{\partial \widehat{\mu}_t} - \frac{1}{\Omega}\frac{\partial \Omega}{\partial \widehat{\mu}_t}\right)\sigma^{-1}\left(\widehat{\sigma}^2_{\mu t} + \widehat{\sigma}_{\mu g,t}\right) + \left(\frac{1}{H}\frac{\partial H}{\partial \widehat{g}_t} - \frac{1}{\Omega}\frac{\partial \Omega}{\partial \widehat{g}_t}\right)\sigma^{-1}\left(\widehat{\sigma}_{\mu g,t} + \widehat{\sigma}^2_{gt}\right)$$

$$\sigma_{M2,t} = \left(\frac{1}{H}\frac{\partial H}{\partial \widehat{\mu}_t} - \frac{1}{\Omega}\frac{\partial \Omega}{\partial \widehat{\mu}_t}\right)\sigma_S^{-1}\widehat{\sigma}^2_{\mu t} + \left(\frac{1}{H}\frac{\partial H}{\partial \widehat{g}_t} - \frac{1}{\Omega}\frac{\partial \Omega}{\partial \widehat{g}_t}\right)\sigma_S^{-1}\widehat{\sigma}_{\mu g,t}$$

$$\sigma_{M,c,n,t} = \left(\frac{1}{H}\frac{\partial H}{\partial \widehat{c}^n_t} - \frac{1}{\Omega}\frac{\partial \Omega}{\partial \widehat{c}^n_t}\right)\widehat{\sigma}^2_{c,n,t}h^{-1} .$$

The risk premium has the same expression as in the paper, with one additional term:

$$\mu_{M,t} = \sigma_{M1,t}\sigma_{\pi,1,t} + \sigma_{M2,t}\sigma_{\pi,2,t} + \sum_n \sigma_{M,c,n,t}\sigma_{\pi,c,n,t} .$$

Given the large number of state variables, we compute $\Omega(S_t)$ and $H(S_t)$ by Monte Carlo integration. This method is efficient because we know the joint distribution of the relevant stochastic variables, namely, $\Delta b_\tau, \widehat{\mu}_\tau$ and $\widehat{g}_\tau$. (The variable $\widehat{g}_\tau$ appears in the relevant expectations because $\mu^0_g = \widehat{g}_\tau$.) This joint distribution as of time $t < \tau$ is given by

$$\begin{pmatrix} \Delta b_\tau \\ \widehat{\mu}_\tau \\ \widehat{g}_\tau \end{pmatrix} \sim N\left(\begin{pmatrix} (\widehat{\mu}_t - \overline{\mu})\,Q_1(t,\tau) + (\overline{\mu} + \widehat{g}_t)(\tau - t) \\ \overline{\mu} + (\widehat{\mu}_t - \overline{\mu})\,e^{-\beta(\tau - t)} \\ \widehat{g}_t \end{pmatrix}, \int \Sigma(s)\,\Sigma(s)'\,ds\right) ,$$

where

$$\Sigma(s) = \begin{pmatrix} \sigma & 0 \\ \sigma^{-1}\left(\widehat{\sigma}^2_{\mu s} + \widehat{\sigma}_{\mu g,s}\right) & \sigma_S^{-1}\widehat{\sigma}^2_{\mu s} \\ \sigma^{-1}\left(\widehat{\sigma}_{\mu g,s} + \widehat{\sigma}^2_{g,s}\right) & \sigma_S^{-1}\widehat{\sigma}_{\mu g,s} \end{pmatrix} .$$

We can compute $\int \Sigma(s)\,\Sigma(s)'\,ds$ numerically, construct a large number of draws of $\Delta b_\tau, \widehat{\mu}_\tau$ and $\widehat{g}_\tau$ from their joint distribution, and calculate the relevant expectations by averaging across those draws. We also simulate the political costs $\widehat{c}_{n,\tau}$ by drawing them from their normal distributions.

We calibrate the model for the two-policy case and the parameter values in Table 1. We choose $\overline{\mu} = 10\%$ for consistency with Table 1. We also choose $\beta = 0.35$ and $\sigma_\mu = 2\%$, which correspond to estimates of the mean-reverting process for aggregate profitability reported in Pástor and Veronesi (2006). We set the prior variance $\widehat{\sigma}^2_{\mu,0} = \sigma_\mu$ and the prior covariance $\widehat{\sigma}_{\mu g,n} = 0$ for all n. We set this prior covariance between μ_τ and g^n equal to zero, for simplicity, because there is no obvious reason to make a different assumption. We vary σ_S from 1% to 5%, 10%, and infinity. We construct two types of plots, which are described in the paper. We show those plots below.

The first plot described in the paper is shown below. The political risk premium is shown in yellow at the top. The new (fourth) component of the risk premium, driven by the signals dS_t from equation (B30), is shown in red, second from the top.

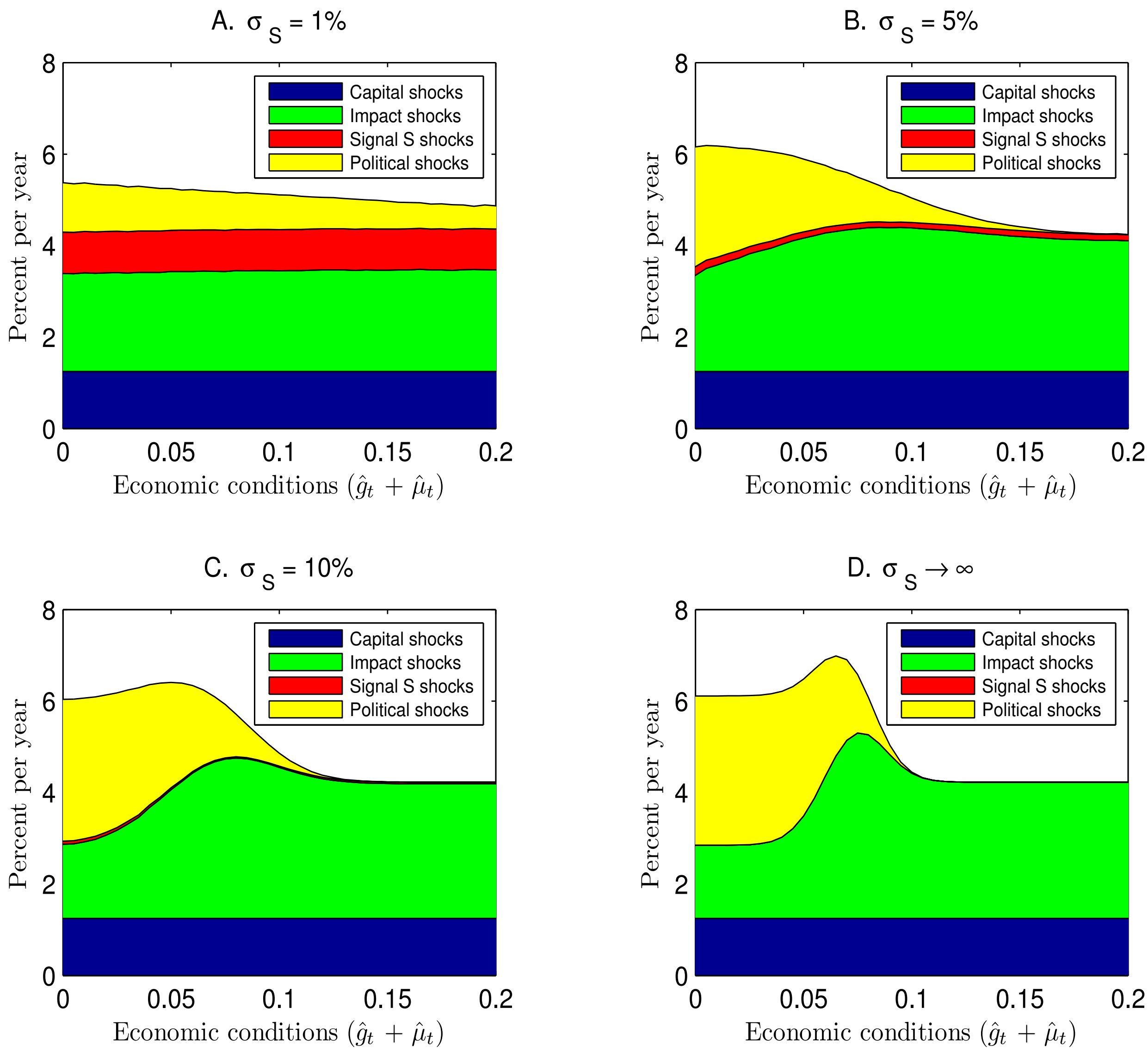

Figure B2. The equity risk premium and its components: Policy-unrelated business cycles.

The second plot described in the paper is shown below. The value of $\sigma_S = 5\%$, and the other parameters are described in the paper.

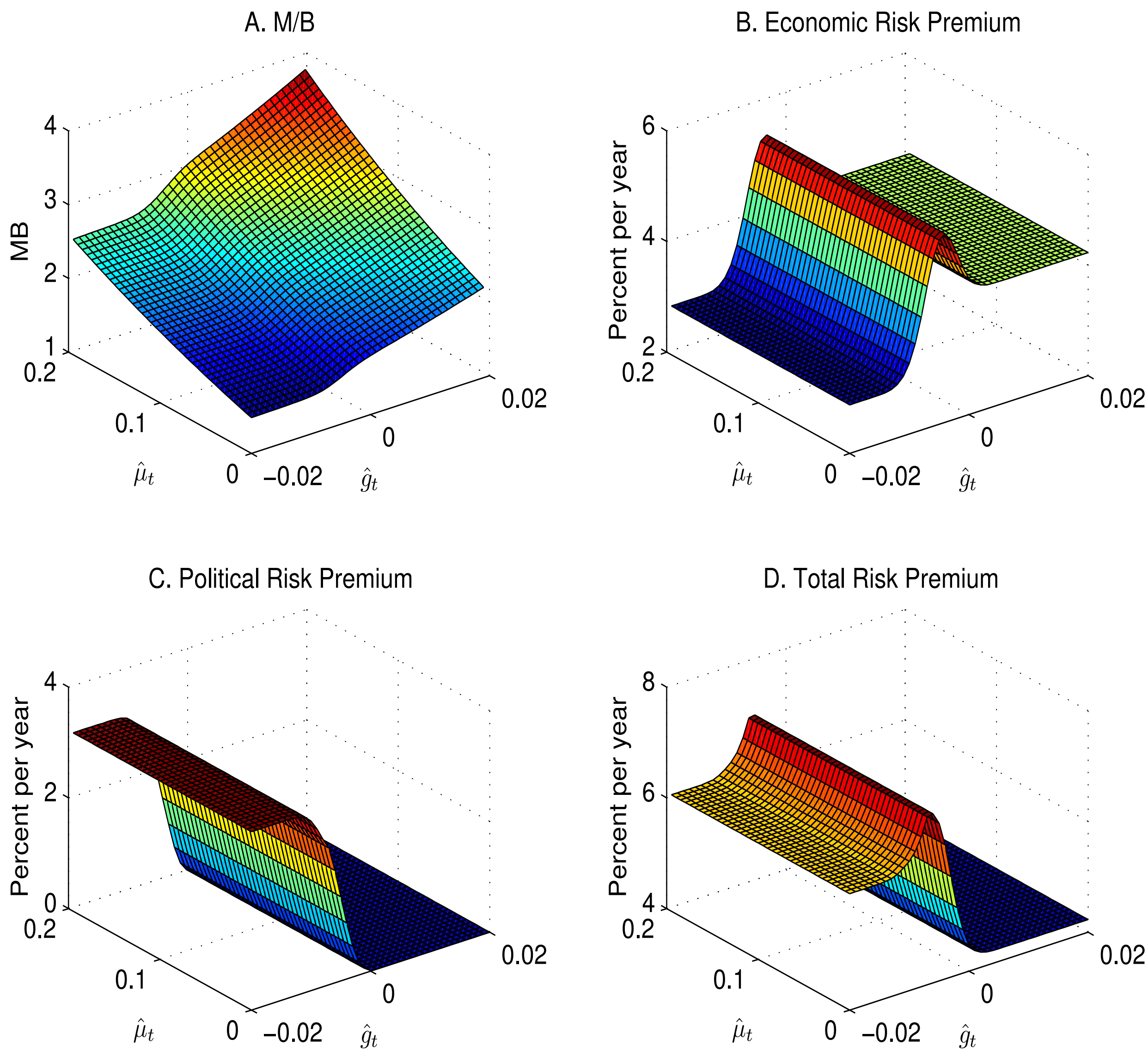

Figure B3. Stock prices and risk premia: Policy-unrelated business cycles. $\sigma_S = 5\%$

Finally, the plot below is analogous to the plot on the previous page, except that $\sigma_S \to \infty$. This plot is very similar to the previous one, which is why we do not elaborate on the small differences induced by σ_S in the paper.

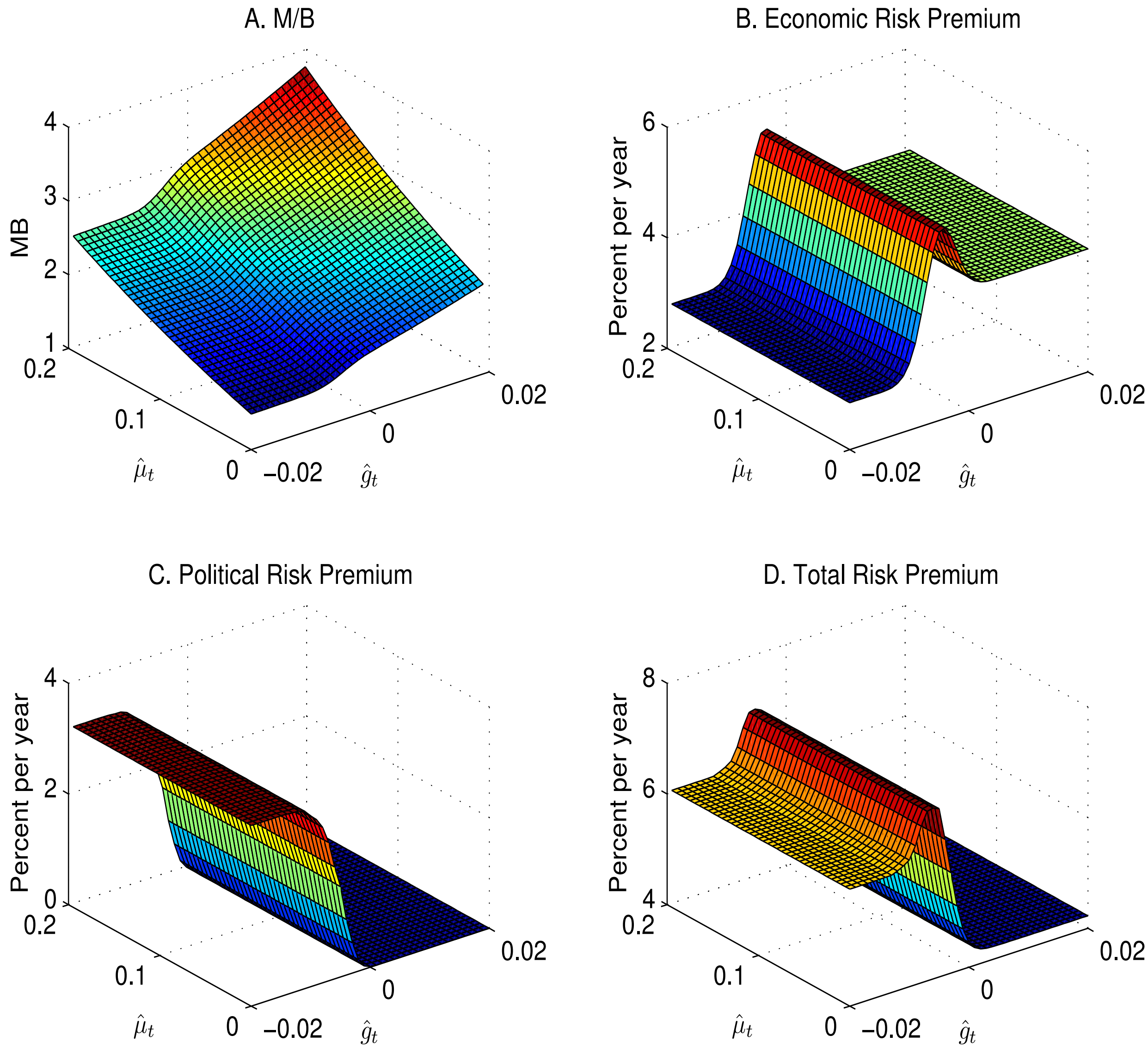

Figure B4. Stock prices and risk premia: Policy-unrelated business cycles. $\sigma_S \to \infty$

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