Learning about the Neighborhood: A Model of Housing Cycles

by Michael Sockin and Wei Xiong

Discussion

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What Does This Paper Do?

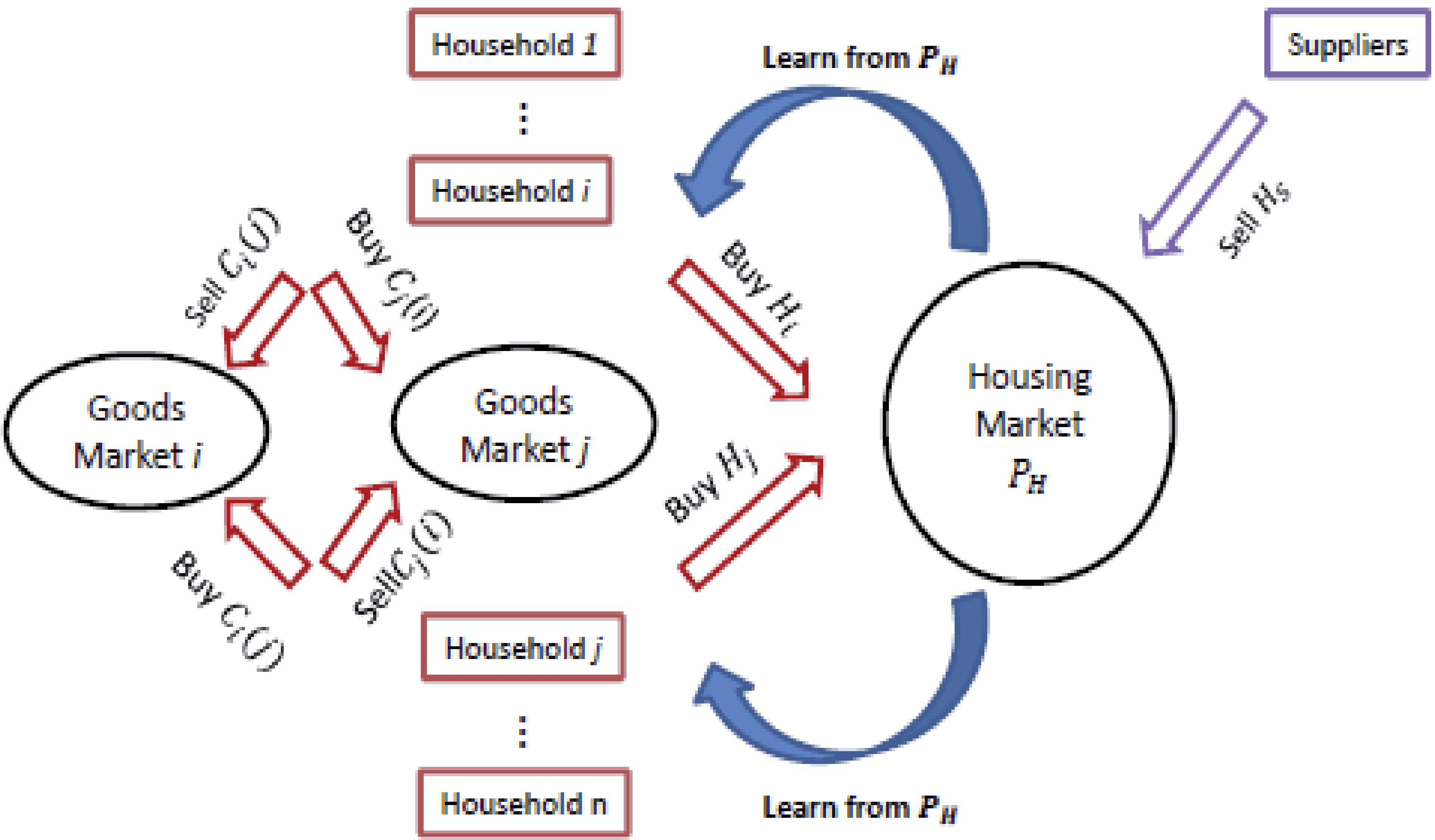


Figure 1: Structure of the Static Model

What Does This Paper Do?

- Proposes a model of housing choice based on **rational** learning with differentially informed agents about the quality of a location.
- Location quality affect agents' labor productivity, and hence amount of consumption goods produced and traded
- Housing is treated a "good" that is complementary to other consumption goods, and it is produced by special builders.
- House price aggregate information about location quality
  - Higher house price signal higher quality $\Longrightarrow$ increase housing demand (in a partial sense)
- Solve for equilibrium prices and obtain several results about housing demand, supply elasticity, learning, etc.
- Dynamic extension
  - Generate short-term momentum and long-term reversal
  - U-shape relation about price variation and housing supply

Assessment and Outline of Discussion

- Interesting and somewhat plausible mechanism
  - House prices convey information about neighborhood quality
- There are some issues about interpretation and the source of the results that need to be clarified
- Paper is still a major challenge to read and even more to understand (it is indeed Preliminary). Advise to wait for first clean version.
- My discussion:

(A) Static asymmetric information models

(B) The mechanism in a simplified version of model

(C) Further comments on the paper.

Static Asymmetric Information Model - I

- Consider a model a la' Hellwig (1980).
- Risky asset with payoff $A \sim N(0, v_A)$, and random supply $H_S = \xi \sim N(0, v_\xi)$.
- Each agent i observes signal $s_i = A + \epsilon_i$, chooses H_i in risky asset and B_i in riskless bond to maximize the expected utility from final wealth $W_i = H_i A + B_i R$:

$$\max_{H_i, B_i} E\left[-e^{-\eta W_i} | s_i, P_H\right] \text{ subject to } W_0 = P_H H_i + B_i$$

- Conjecture linear equilibrium:

$$P_H = a + b\ A + c\ \xi$$

- Vector $(A, P_H, s_i)'$ is jointly normal

$$\Longrightarrow A|_{s_i, P_H} \sim N\left(\mu_A(s_i, P_H), \overline{v}_A\right) \text{ where } \mu_A(s_i, P_H) = h_1\left(P_H - a\right) + h_2 s_i$$

(h_1, h_2 and $\overline{v}_A$ depend on parameters.)

- Expected utility is

$$E\left[-e^{-\eta H_i A + B_i} | s_i, P_H\right] = -e^{-\eta H_i \mu_A + B_i + \frac{1}{2}\eta^2 H_i^2 \overline{v}_A}$$

Static Asymmetric Information Model - II

- Maximizing this utility is equivalent to max the exponent, obtaining

$$H_i = \frac{\mu_A(s_i, P_H) - P_H}{\eta \overline{v}_A} = \frac{h_1(P_H - a) + h_2 s_i - P_H}{\eta \overline{v}_A}$$

- Integrate on both sides and impose market clearing $\int H_i di = H_S = \xi$

$$\int H_i di = \frac{-h_1 a + (h_1 - 1)P_H + h_2 \int s_i di}{\eta \overline{v}_A} \Longrightarrow \xi = \frac{-h_1 a + (h_1 - 1)P_H + h_2 A}{\eta \overline{v}_A}$$

- Solve for P_H:

$$P_H = \underbrace{-\frac{h_1 a}{1 - h_1}}_{a} + \underbrace{\frac{h_2}{1 - h_1}}_{b} A \underbrace{-\frac{\eta \overline{v}_A}{1 - h_1}}_{c} \xi$$

- From Bayes formula, $h_2 > 0$ and $sign(h_1) = sign(b)$.

$\Longrightarrow b > 0$ and thus $1 - h_1 > 0$.

  $* \Longrightarrow$ information effect $(= h_1)$ always weaker than price effect $(= 1)$.

  $* \; P_H \uparrow \Longrightarrow E[A|P_H] \uparrow$ but makes asset more costly. Latter effect dominates.

$\Longrightarrow c < 0 \Longrightarrow$ higher supply decrease price

Static Asymmetric Information Model with Power Utility

- Negative exponential utility is not too appealing as

  A. No wealth effect

  B. Relative risk aversion increases with wealth

  B. Equilibrium prices can be negative

- What happens if we use another utility function? Consider power utility:

$$\max_{H_i,B_i} E\left[\frac{(H_iA+B_iR)^{1-\eta}}{1-\eta}|s_i,P_H\right]$$

- A linear or log-linear equilibrium price function does not work in this case.

- Taking FOC of Lagrangean does not help either:

$$E\left[A\left(H_iA+B_i\right)^{-\eta}|s_i,P_H\right]=\lambda_iP_H$$

  – Problem: P_H enters on both sides, and the non-linearity messes up aggregation $\int H_i di = H_S$

  – It can be solved numerically, or using approximation to small payoffs (e.g. Perez (2004, RFS)).

Sockin and Xiong Paper - I

- This paper uses power utility
  - How could it solve the non-linear fixed-point problem discussed earlier?
  - Value of housing depends on complementarity with another consumption good, and there are no riskless bonds to purchase.
- Consider a super stripped down version of model, just to see this mechanism:

$$E\left[\frac{H_i^{1-\eta}}{1-\eta}\,C|s_i,P_H\right] \text{ subject to } W_0 = P_H H_i$$

- Assume:
  - $C = e^A$ is an exogenous random amount of good provided by a 3rd party or nature, e.g. schools, infrastructure, earthquakes
  - Supply of asset H is random: $H_S = e^{\xi}$.
- FOC is far simpler:

$$H_i^{-\eta} E\left[e^A|P_H,s_i\right] = P_H$$

- Assume $p_H = log(P_H)$ is linear:

$$p_h = a + bA + c\xi$$

Sockin and Xiong Paper - II

- Then the *same* learning result above implies $A \sim N(\mu_A(p_H, s_i), \overline{\upsilon}_A)$ and hence

$$E\left[e^A | P_H, s_i\right] = e^{\mu_A(p_H,s_i)+\frac{1}{2}\overline{\upsilon}_A}$$

- It is clear it can be solved, as we can write:

$$H_i^{-\eta} e^{h_1(p_H-a)+h_2 s_i+\frac{1}{2}\overline{\upsilon}_A} = e^{p_H}; \quad \Longrightarrow \quad H_i = e^{\frac{1}{\eta}h_2 s_i} e^{-\frac{1}{\eta}(1-h_1)p_H - \frac{ah_1}{\eta} + \frac{1}{2\eta}\overline{\upsilon}_A}$$

- Integrate both sides

$$\int H_i di = \int e^{\frac{1}{\eta}h_2 s_i} di \; e^{-\frac{1}{\eta}(1-h_1)p_H - \frac{ah_1}{\eta} + \frac{1}{2\eta}\overline{\upsilon}_A} \Longrightarrow e^{\xi} = e^{\frac{1}{\eta}h_2 A + \frac{1}{\eta^2}h_2^2 \upsilon_s} e^{-\frac{1}{\eta}(1-h_1)p_H - \frac{ah_1}{\eta} + \frac{1}{2\eta}\overline{\upsilon}_A}$$

- and solve for the price

$$p_H = \underbrace{\frac{\left(\frac{h_2^2}{2\eta}\upsilon_s - h_1 a + \frac{1}{2}\overline{\upsilon}_A\right)}{(1-h_1)}}_{a} + \underbrace{\frac{h_2}{(1-h_1)}}_{b} A - \underbrace{\frac{\eta}{(1-h_1)}}_{c} \xi$$

  - As before, $h_1 < 1$ and information effect is weaker than price effect.

$\Longrightarrow$ Price p_H is increasing in A and decreasing in supply ξ.

$\Longrightarrow$ Demand H_i is increasing in s_i and decreasing in p_H.

Sockin and Xiong Paper - III

- The barebone model is too "bare". Sockin and Xiong add:
  - Labor / production at time $t = 2$ (after housing choice made at $t = 1$) with complementarity in consumption
    - Critical is a common productivity shock A;
    - Critical that there is no asymmetric information at $t = 2$ as consumption depend on realized productivity A_j of everybody else.
  - Suppliers of housing:
    - From their profit maximization: $H_S = P_H^k e^{k\xi}$
    - k = supply elasticity.
    - Questions here:
      1. Household budget constraint: $P_H H_i + \int P_j C_j(i) dj = P_i e^{A_i} \ell_i + w_i$
         w_i = wage of the builder = $P_H H_i$. This suggests builder i sell his home to himself (?). Shouldn't we have $w_i = P_H H_S$?
      2. Builders belong to household (see budget constraint), but they have superior information (observe ξ), and their utility and labor costs does not enter the household utility?

Sockin and Xiong Paper - IV

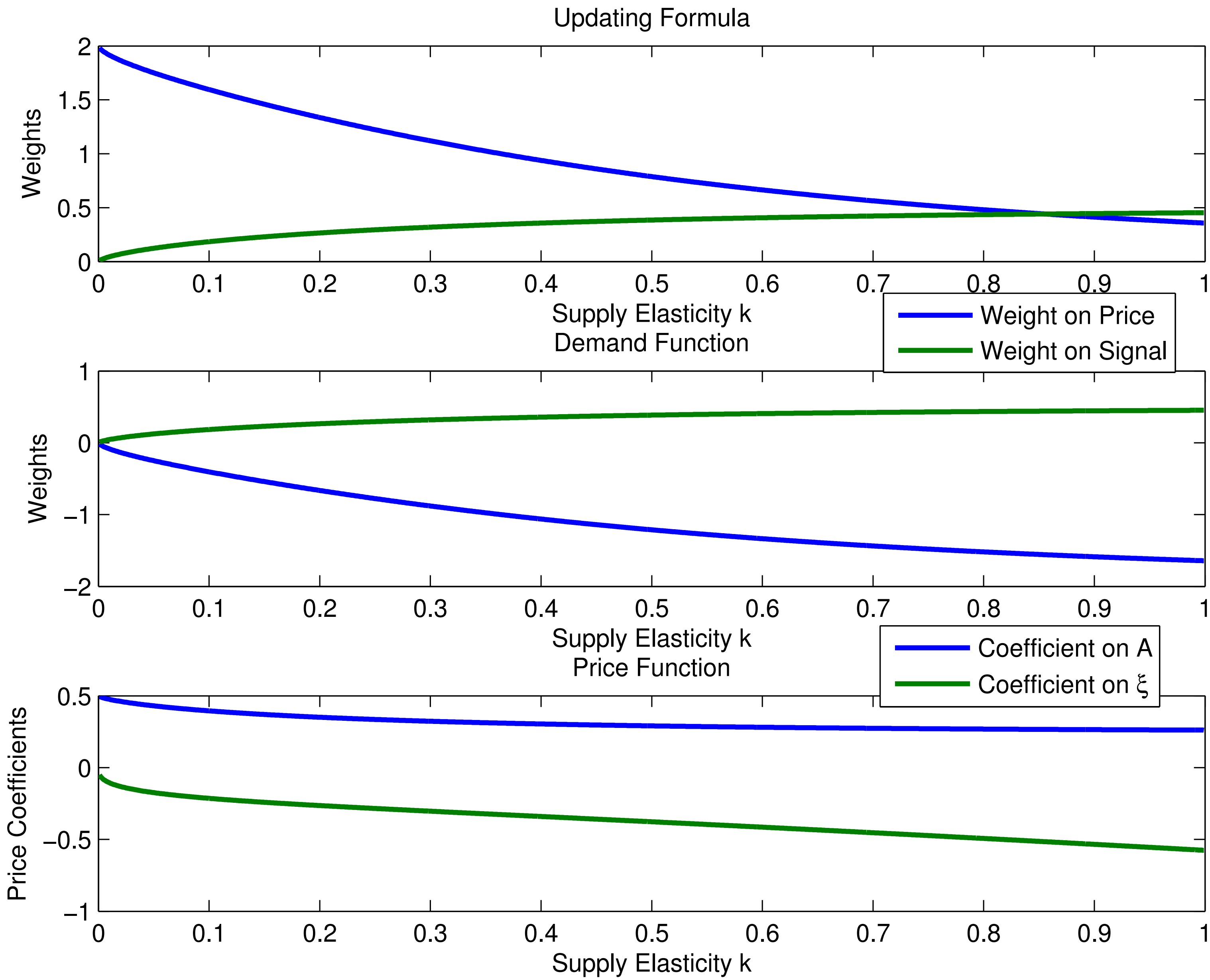
- To see (some) of the impact of these ingredients, let's add to the simple model:
  1. Utility with different weights
  $$E\left[\frac{H_i^{1-\eta}}{1-\eta}C^{\eta_c}|s_i, P_H\right] \text{ subject to } W_0 = P_H H_i$$
  2. Price-dependent supply (obtained from optimality of builders)
  $$H_S = P_H^k e^{k\xi}$$
- The same calculations above give:

$$H_i = e^{\left(\frac{\eta_c}{\eta}h_2\right)s_i + \left(\frac{1}{\eta}(\eta_c h_1 - 1)\right)p_H + \left(\frac{1}{2}\frac{\eta_c^2}{\eta}\overline{v}_A - \frac{\eta_c}{\eta}h_1 a\right)}$$

$$p_H = \underbrace{\frac{\frac{1}{2}\left(\frac{\eta_c^2}{\eta}\right)h_2^2 v_s - \eta_c h_1 a + \frac{1}{2}\eta_c^2\overline{v}_A}{(1+\eta k - \eta_c h_1)}}_{a} + \underbrace{\frac{\eta_c h_2}{(1+\eta k - \eta_c h_1)}}_{b} A - \underbrace{\frac{\eta k}{(1+\eta k - \eta_c h_1)}}_{c} \xi$$

- Note that again $b > 0$ and $c < 0$: Higher A or lower ξ results in higher price
- Now however, demand H_i *may* be *increasing* in price if $\eta_c h_1 - 1 > 0$
  - "Signal" effect may be stronger than "cost" effect.

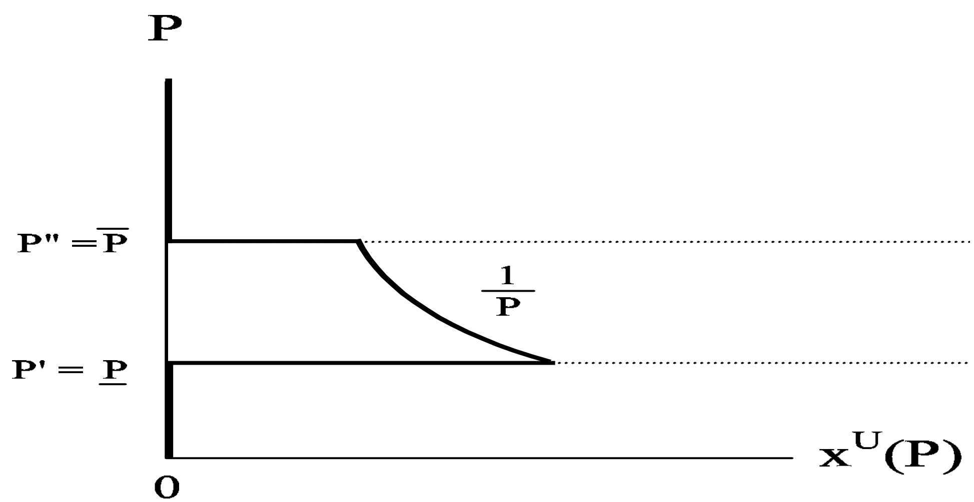
Example: The Impact of Supply Elasticity



Global versus Local Information Effects

- These models however tend to have "global signal effects"
  - Demand is either always decreasing or always increasing in the price P_H.
  - There are cases in which 'backward-bending" demand is reasonable.
    - Wine, Stocks, Houses (?)
- Barlevy and Veronesi (2003) show that in model with (A) informed and uninformed traders, and (B) only two regimes (good or bad)

$\Longrightarrow$ Strong **local** information effects $\Longrightarrow$ backward-bending demand function (and stock market crash).



Demand of the Uninformed

Dynamic Model

- Much of the implications discussed in the paper are about dynamics.
  - Overlapping generations with segmentation: Each cohort trade only with itself.
  - $\Longrightarrow$ Repeat of the same model above over and over.
  - Link between times only through dynamics of common factors and learning:

$$A_t = \rho^A A_{t-1} + Z_t^A; \qquad \xi_t = \rho^\xi \xi_{t-1} + Z_t^\xi$$

  - Information structure: A_t and ξ_t revealed with 2 periods lag. (Why 2 periods and not 1?)

- Suppose only one period lag (at t we know A_{t-1} and ξ_{t-1}), then learnings is identical, but price function at t must include A_{t-1} and ξ_{t-1}, as they are known and determines agents expectations. Same calculations as above give:

$$p_H(t) = a + bA_t + c\xi_t + dA_{t-1} + e\xi_{t-1}$$

  (coefficients are much more involved even in this simple case)

- Demand now depends on the current price "dynamics"

Dynamic Model and Momentum - I

- The price of the house now depend on lagged state-variables, which are persistent.
- Sockin and Xiong show that the persistence of state variables is critical to generate short-term momentum (short?) and long-term reversal.
- To generate momentum and long-term reversal, one normally needs time varying risk premia that depend on two factors with different frequencies (see e.g. Albuquerque and Miao (JET, 2014)).
- In this setting, it is not clear what is the source of the variation in risk premia (conditional on public information).
- Indeed, given the lack of trading across cohorts, the house prices at every t has a different marginal buyer, as there is no wealth transfers over time.
- To some extent, the prices $p_H(t)$ are really prices of "different securities" as they pertain to different "neighborhoods" (cohorts).

Dynamic Model and Momentum - II

- Indeed, the interpretation of the results is a bit tricky here.
- We could consider the identical model in a "spatial" setting, still with

$$A_t = \rho_A A_{t-1} + Z_t^A; \quad \xi_t = \rho_\xi \xi_{t-1} + Z_t^\xi$$

- but now t denotes a different "**t**own", rather than **t**ime.
- Agents in town t learn about aggregate local productivity by observing the adjacent town $t-1$ (or $t-2$).
- All the results about pricing would be identical, but it is pretty clear that $p_H(t)$ are prices of a different securities,

$\Longrightarrow$ $p_H(t) - p_H(t-1)$ is not a return, but just price difference across **different** adjacent towns.

Dynamic Model and Supply Elasticity

- With the same caveat, it is interesting the relation between supply elasticity and price variation.
- In the paper, supply elasticity is about external supply shock.
  - Very low elasticity $\Longrightarrow$ price do not respond to "supply noise" $\Longrightarrow$ no asymmetric information (as price fully revealing)
  - Extremely high elasticity $\Longrightarrow$ house price too noisy for non-fundamental reason $\Longrightarrow$ no learning from prices
- This is an interesting channel.
- Do I believe it explain difference between New York and Las Vegas?

Other Minor Comments

- The good "house" in the model could be interpreted in any way as well
  - No lump sum investment, no durable good, no tradeoff purchase / rent
- Assumption of closed neighbors
  - It is not too clear what it means. Are these neighbors, or cities, or states?
- Large number of parameters and moving parts
  - Maybe want to simplify the model a bit?
- What is the numeraire in the model?
  - In these models with utility from final wealth, the bond is normally the numeraire. But there is no bond here, and all consumption goods / houses have non-unit prices.

Conclusion

- Very rich model, and perhaps not everything is necessary, especially to convey intuition of results. But authors endogenize almost everything, which is interesing.
- Need some more thinking about interpretation.
- However, the mechanism per se' seems plausible: Learning dynamics may generate some short-term momentum and long-term reversals, as others have shown already in the asset pricing literature.
- It would be interesting to push the "information story" further, generating local backward bending demand curves, which may have dramatic price effects.

Bayes Formula

- The following work both for Helwig model and for the simplified case discussed in the text. The only difference is whether we use P or $\log P$. The joint distribution (from the perspective of the investor) of $(A, \log P, s_i)$ is

$$\begin{pmatrix} A \\ \log P \\ s_i \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ a \\ 0 \end{pmatrix}, \begin{pmatrix} v_A & bv_A & v_A \\ bv_A & b^2 v_A + c^2 v_\xi & bv_A \\ v_A & bv_A & v_A + v_s \end{pmatrix}\right)$$

or

$$\begin{pmatrix} A \\ \log P \\ s_i \end{pmatrix} \sim N\left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \begin{pmatrix} \Sigma_{11}, \Sigma_{12} \\ \Sigma_{21}, \Sigma_{22} \end{pmatrix}\right)$$

- Using the properties of conditions normals, we have (let $p_H = \log(P_H)$)

$$(A)\,|_{p_H, s_i} \sim N\left(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}\left(\begin{pmatrix} p_H \\ s_i \end{pmatrix} - \mu_2\right), v_A - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}'\right)$$

where

$$\begin{aligned} \Sigma_{12} &= \begin{pmatrix} bv_A & v_A \end{pmatrix} \\ \Sigma_{22} &= \begin{pmatrix} b^2 v_A + c^2 v_\xi & bv_A \\ bv_A & v_A + v_s \end{pmatrix} \end{aligned}$$

- Therefore, algebra gives (this is tedious, but for completeness):

$$\Sigma_{22}^{-1} = \frac{1}{\left(b^2 v_A + c^2 v_\xi\right) v_s - b^2 v_A^2} \begin{pmatrix} v_s & -b v_A \\ -b v_A & b^2 v_A + c^2 v_\xi \end{pmatrix}$$

- so that

$$\begin{aligned} \Sigma_{12}\Sigma_{22}^{-1} &= \frac{1}{\left(b^2 v_A + c^2 v_\xi\right)\left(v_A + v_s\right) - b^2 v_A^2} \begin{pmatrix} b v_A & v_A \end{pmatrix} \begin{pmatrix} v_A + v_s & -b v_A \\ -b v_A & b^2 v_A + c^2 v_\xi \end{pmatrix} \\ &= \frac{1}{\left(b^2 v_A + c^2 v_\xi\right)\left(v_A + v_s\right) - b^2 v_A^2} \left[b v_A v_A + v_s - b v_A^2, -b^2 v_A^2 + b^2 v_A + c^2 v_\xi v_A\right] \\ &= \frac{1}{b^2 v_A \left(v_A + v_s - v_A\right) + c^2 v_\xi \left(v_A + v_s\right)} \left[b v_A \left(v_A + v_s - v_A\right), c^2 v_\xi v_A\right] \end{aligned}$$

- and

$$\begin{aligned} \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{12}' &= \frac{1}{b^2 v_A \left(v_A + v_s - v_A\right) + c^2 v_\xi v_s} \left[b v_A \left(v_A + v_s - v_A\right), c^2 v_\xi v_A\right] \begin{pmatrix} b v_A \\ v_A \end{pmatrix} \\ &= v_A^2 \frac{b^2 v_s + c^2 v_\xi}{b^2 v_A v_s + c^2 v_\xi \left(v_A + v_s\right)} \end{aligned}$$

- Finally

$$A|_{s_i, \log P} \sim N\left(h_1\left(p_H - a\right) + h_2 s_i, \overline{v}_A\right)$$

- where

$$\begin{aligned}
h_1 &= \frac{b v_A v_s}{v_s b^2 v_A + v_\xi \left(v_A + v_s\right) c^2} \\
h_2 &= \frac{c^2 v_A v_\xi}{v_s v_A b^2 + v_\xi \left(v_A + v_s\right) c^2} \\
\overline{v}_A &= v_A - v_A^2 \frac{b^2 v_s + c^2 v_\xi}{b^2 v_A v_s + c^2 v_\xi \left(v_A + v_s\right)}
\end{aligned}$$

- Note that $sign(h_1) = sign(b)$ and $h_2 > 0$.