Low Risk Anomalies?

by Schneider, Wagners, and Zechner

Discussion

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Main Contribution and Outline of Discussion

- Main contribution of the paper:
  - Proposes a skew-based explanation of several low-risk anomalies
    - Use approximate stochastic discount factor that loads on "skewness"
    - Use Merton (1974) model to justify several implications for levered equity
      - Levered equity returns are negatively skewed
      - Levered equity has higher market beta
      - Levered equity returns have *less* co-skewness with aggregate return $\Longrightarrow$ risk premia less than implied by CAPM
  - Test the model's implications in the data
    - Use ex-ante option-implied skewness as proxy for co-skewness
    - Explain several low-risk strategies: (i) Bet-against-beta; (ii) high idiosyncratic risk; (iii) distress anomalies are implied by investors' preference for low skewness

- Outline of discussion
  1. Review of Merton (1974) model and its implications
  2. Comments

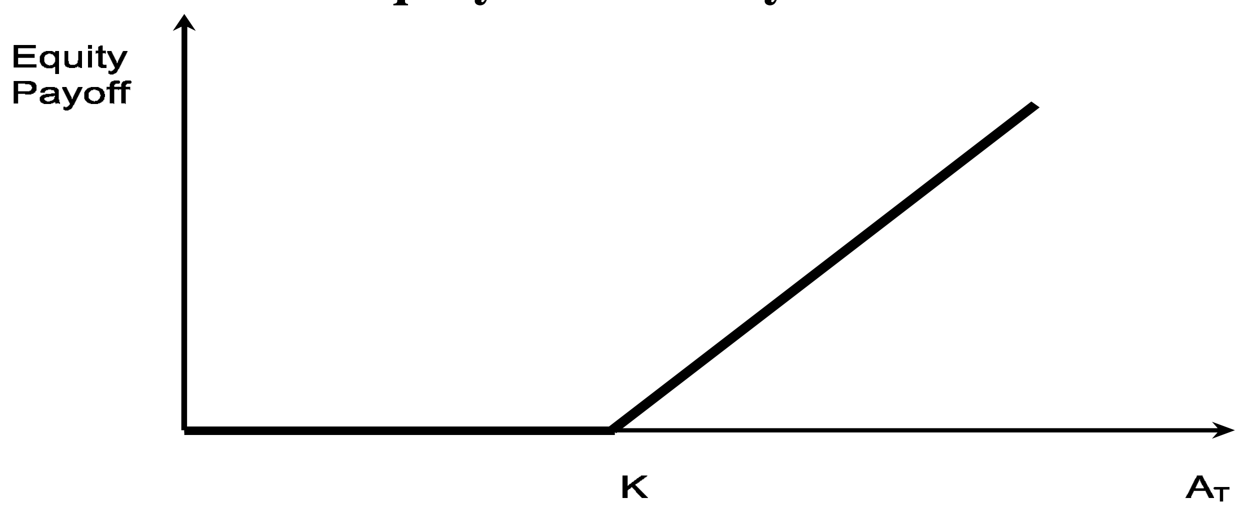
Merton (1974) model

- Firm i's assets are lognormally distributed

$$A_{i,T} = A_{i,0} \times e^{(\mu_A - 1/2\sigma_A^2)T + \sigma_A \sqrt{T} \epsilon_{i,T}}$$

- Firm issues zero coupon bond with face value K.

Equity holders Payoff at T



- Levered equity is

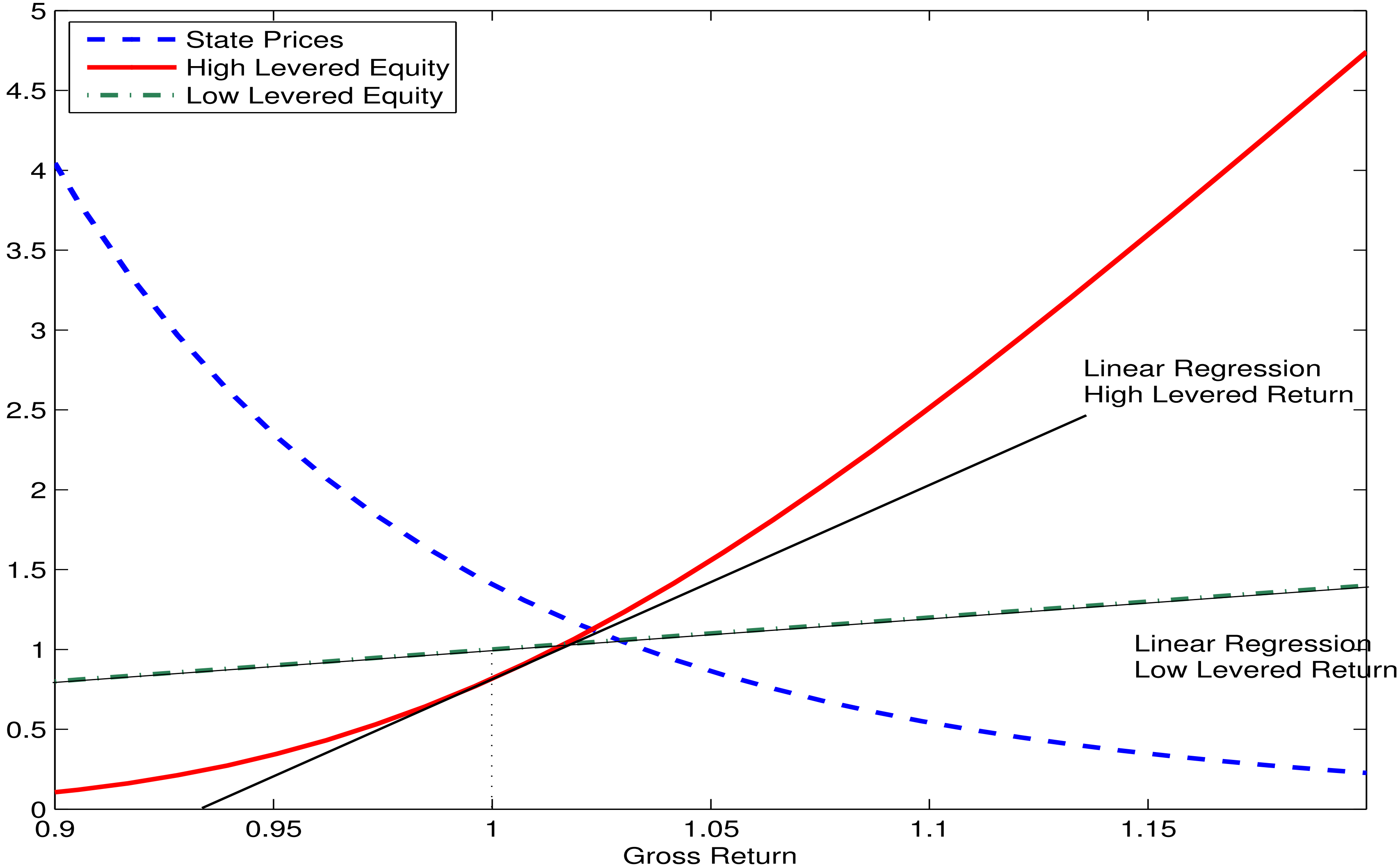
$$S_t = \text{Call Option}$$

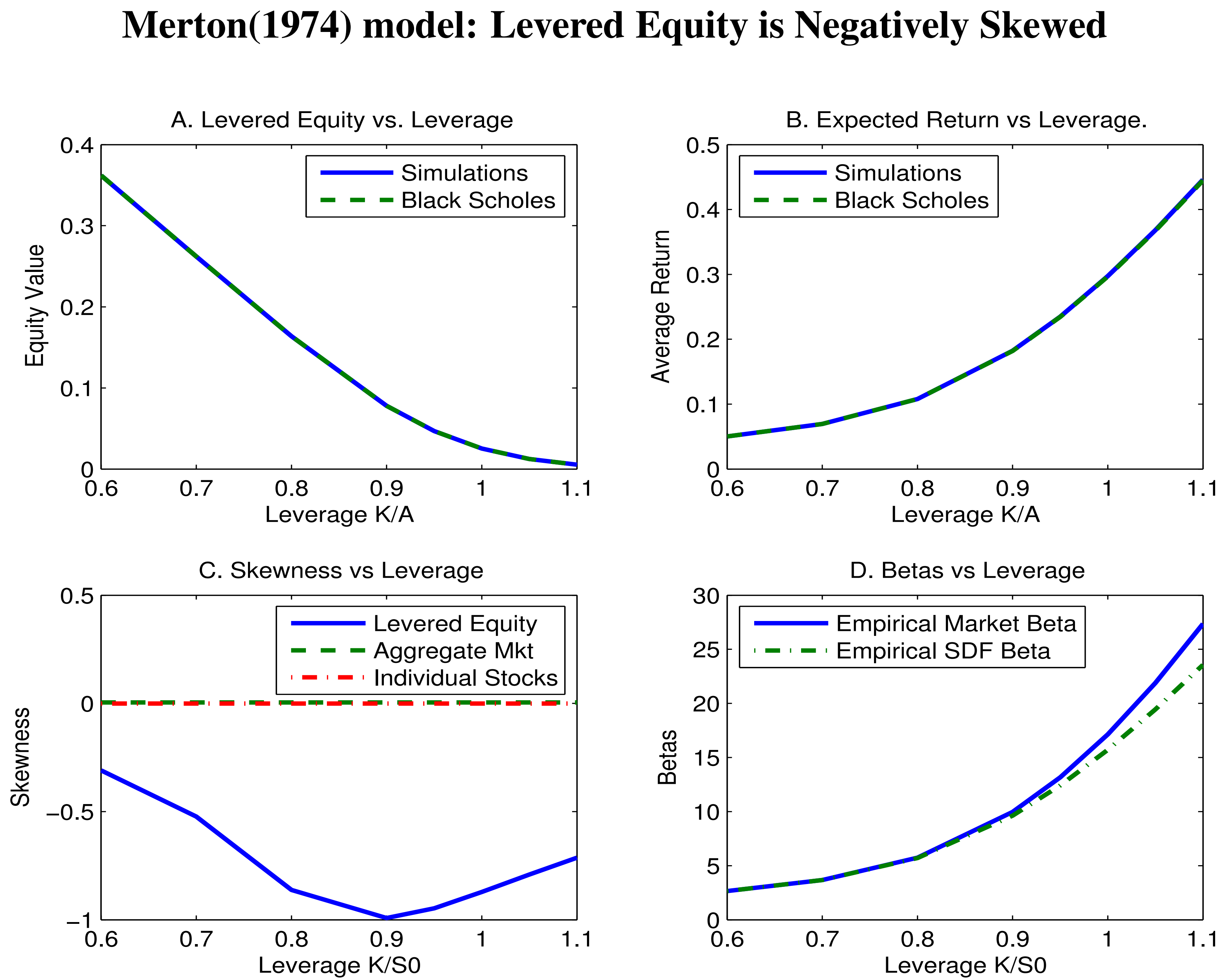
- or, equivalently

$$S_t = A_t + \text{Put Option} - \text{Bonds}$$

Merton(1974) model: Levered Equity and Implicit Put Protection

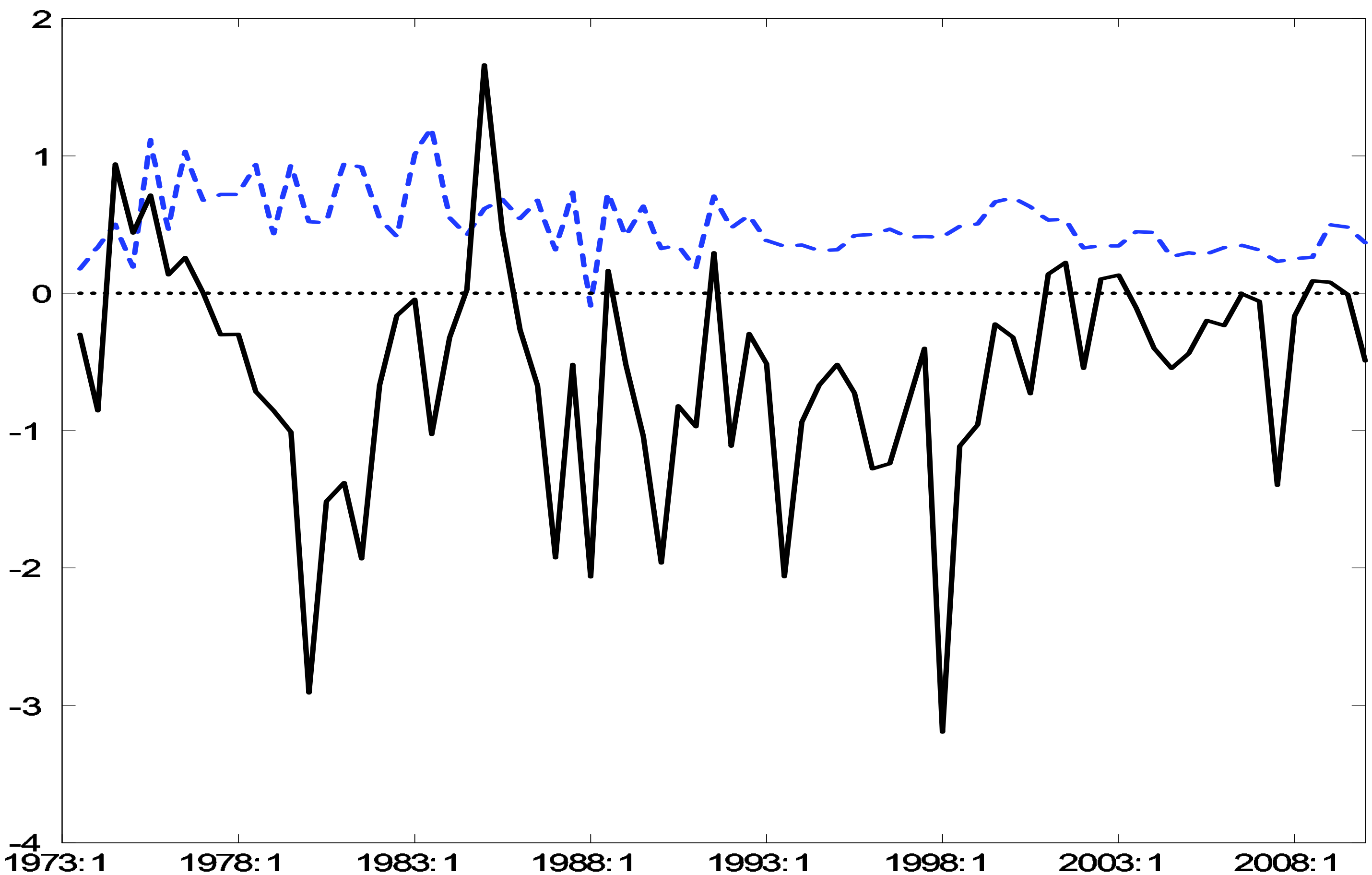
- Implicit put protection (limited liability) is valuable if aversion to skewness




Merton(1974) model: Levered Equity is Negatively Skewed
A. Levered Equity vs. Leverage
Simulations
Black Scholes
Equity Value
Leverage K/A
B. Expected Return vs Leverage.
Simulations
Black Scholes
Average Return
Leverage K/A
C. Skewness vs Leverage
Levered Equity
Aggregate Mkt
Individual Stocks
Skewness
Leverage K/S0
D. Betas vs Leverage
Empirical Market Beta
Empirical SDF Beta
Betas
Leverage K/S0

Data: Individual Stocks' Equity Returns are *Positively* Skewed

- Aggregate stock returns are negatively skewed.
- Individual stock returns are *positively* skewed, on average.



(Source: Rui Albuquerque, Skewness in Stock Returns: Reconciling the Evidence on Firm versus Aggregate Returns," RFS, 2012)

Data: Individual Stocks' Equity Returns are *Positively* Skewed

Table. Skewness and Leverage

Annual portfolio sort on leverage. The sample is individual stocks that are or used to be in the S&P500 index sampled at daily frequency. The sample is 1964 to 2014 (COMPUSTAT Sample).

	Lev	Mean	Std	Skew	exKurt
1	0.01	0.16	0.33	0.20	4.20
2	0.06	0.15	0.31	0.19	4.47
3	0.16	0.14	0.31	0.23	4.58
4	0.31	0.14	0.32	0.29	4.94
5	0.59	0.14	0.38	0.38	5.22

- Merton (1974) intuition hinges on
  1. Underlying firms' assets are log-normal
  2. Leverage is exogenous

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  1. Underlying firms' assets are log-normal
  2. Leverage is exogenous
- But this paper is about *co-skewness*.

Table 5

Skewness by firm size decile and by firm R^3 decile. Reported for each decile are mean firm size, R^3, risk-neutral skewness, and realized return skewness at daily, monthly, and quarterly horizons.

Panel A: Skewness by size decile

Decile	*Logsize*	R^3	*Daily*	*Monthly*	*Quarterly*	*Risk-neutral**
1	15.2958	0.0027	0.0791	0.1149	0.0004	0.2922
2	16.4537	0.0042	0.1640	0.1569	0.0284	−0.0247
3	17.1150	0.0066	0.1938	0.1593	0.0283	−0.0847
4	17.6665	0.0105	0.2217	0.1397	0.0271	−0.1518
5	18.1857	0.0172	0.2137	0.1076	0.0174	−0.1575
6	18.7247	0.0254	0.1978	0.0682	0.0031	−0.1530
7	19.2952	0.0367	0.1693	0.0224	−0.0218	−0.1877
8	19.9304	0.0490	0.1534	−0.0121	−0.0289	−0.1874
9	20.7692	0.0667	0.1211	−0.0357	−0.0398	−0.1995
10	22.4310	0.1187	0.0478	−0.0630	−0.0514	−0.2602

Panel B: Skewness by R^3 decile

Decile	R^3	*Logsize*	*Daily*	*Monthly*	*Quarterly*	*Risk-neutral**
1	−0.0041	17.0304	0.1229	0.1215	0.0205	−0.1380
2	0.0006	16.9429	0.1406	0.1382	0.0196	−0.2052
3	0.0022	17.2797	0.1720	0.1287	0.0215	−0.1593
4	0.0049	17.6481	0.1703	0.1095	0.0120	−0.1979
5	0.0093	18.0858	0.1794	0.0877	0.0034	−0.1748
6	0.0164	18.5832	0.1813	0.0642	−0.0050	−0.1966
7	0.0271	19.0573	0.1750	0.0387	−0.0095	−0.1858
8	0.0438	19.5741	0.1607	0.0147	−0.0227	−0.1894
9	0.0734	20.2110	0.1547	−0.0157	−0.0371	−0.2057
10	0.1640	21.4555	0.1050	−0.0292	−0.0397	−0.2484

(Source: Engle and Mistry, "Priced risk and asymmetric volatility in the cross section of skewness," *Journal of Econometrics,* 2014)

Positive Skewess

Negative Risk Neutral Skewness
More negative for bigger firms

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Higher Co-Skewness ==> Higher Risk Neutral Skewness ?

A Simple Model of Co-Skewness – 1

- We want:
  - Aggregate negative skewness
  - Positive average skewness
- Aggregate Factor (Market):

$$F_T = F_0 \times e^{(\mu - 1/2\sigma^2)T + \sigma_F \sqrt{T}\epsilon_T} \times (1 - \delta_F J_{F,T})$$

  - where $J_T = 1$ with probability $P(T) = e^{-\lambda T}$, and $\delta_F > 0$

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- Individual firm's assets at T:

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- With a large number of firms, aggregate wealth at T is

$$W_T = \int A_{i,T} di = F_T$$

A Simple Model of Co-Skewness. – 2

- Pricing Kernel (= marginal CRRA utility at T – assume zero risk free rate)

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- If $\delta_F = \delta_A = 0 \Longrightarrow$ Black-Scholes model.
- If $0 < \delta_F < \delta_A \Longrightarrow$ (i) $\log(F_T)$ is neg. skewed; (ii) $\log(A_{i,T})$ is pos. skewed.

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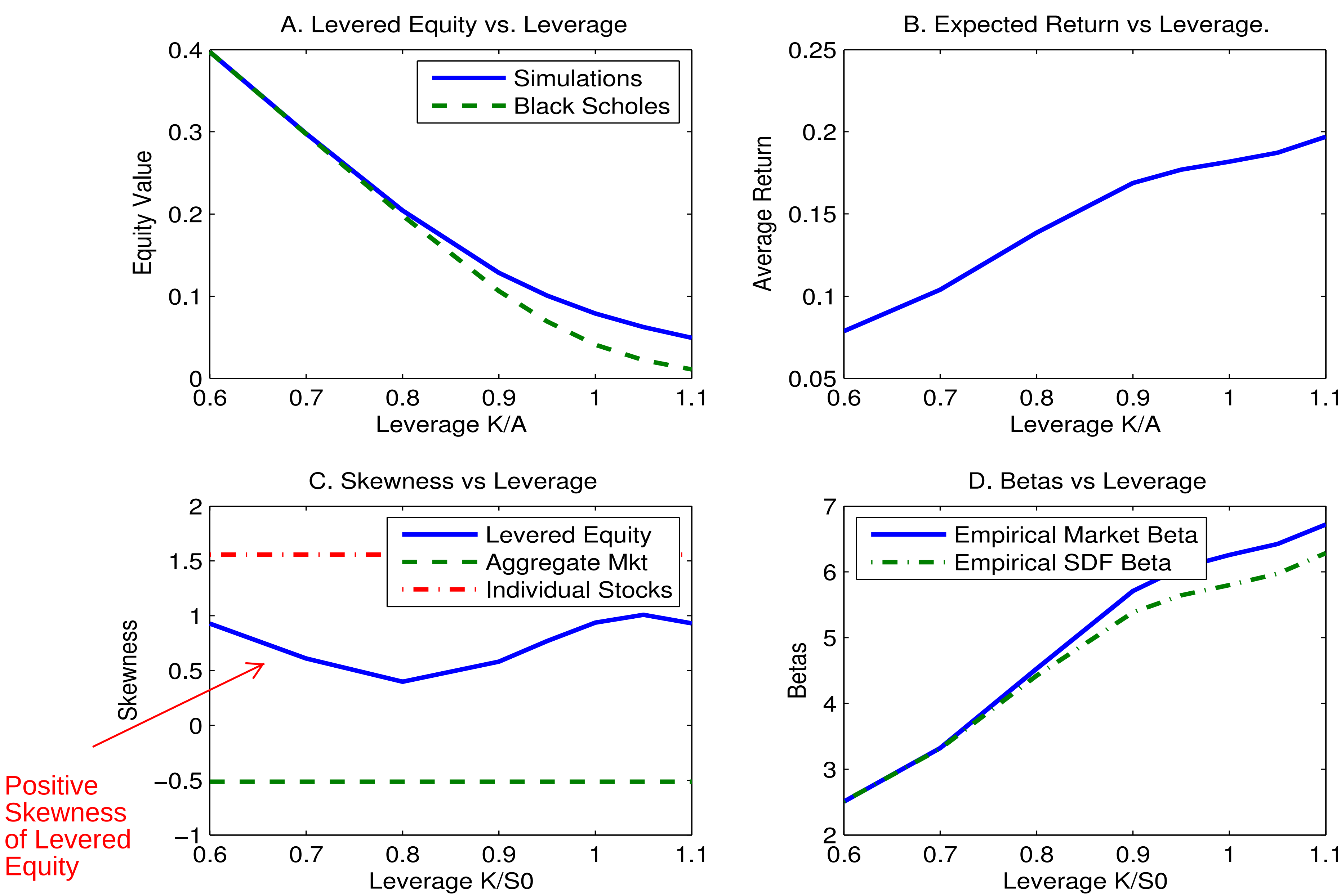
- If $0 < \delta_F < \delta_A \Longrightarrow$ (i) $\log(F_T)$ is neg. skewed; (ii) $\log(A_{i,T})$ is pos. skewed.
- Questions:
  - Can we find parameters so that levered equity S_t is also positively skewed?
  - What is the expected return of levered equity? How does it depend on (i) market beta; (ii) SDF beta?

$$E[R_i^S] = \underbrace{\beta^{Mkt}}_{\frac{Cov(R_i^S, R^F)}{Var(R^F)}} E[R^F]; \qquad E[R_i^S] = \underbrace{\beta^{SDF}}_{\frac{Cov(R_i^S, R^\pi)}{Cov(R^F, R^\pi)}} E[R^F]$$

Simple Model ($\lambda = 1, \delta_A = .4, \delta_F = .1$)



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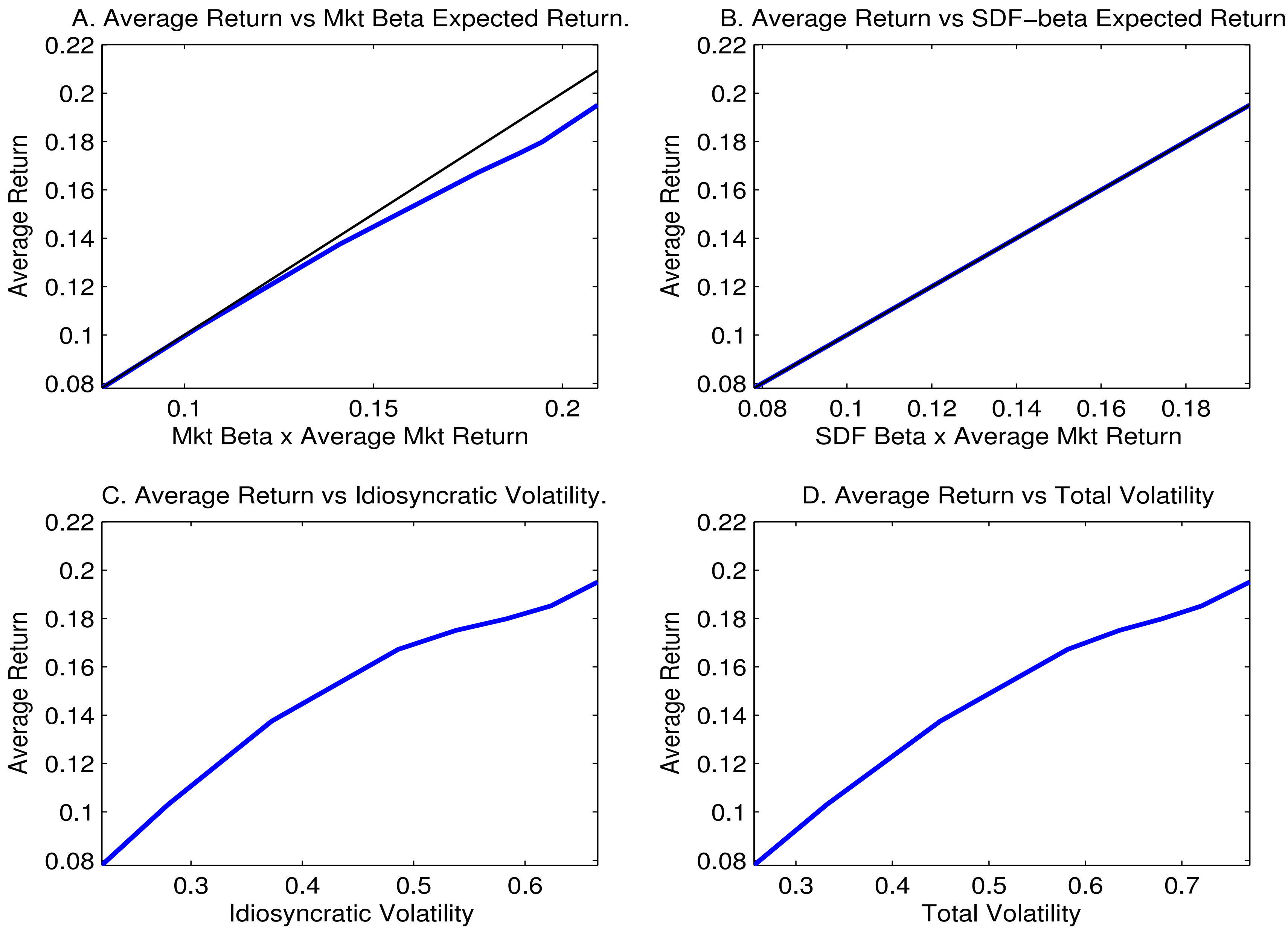

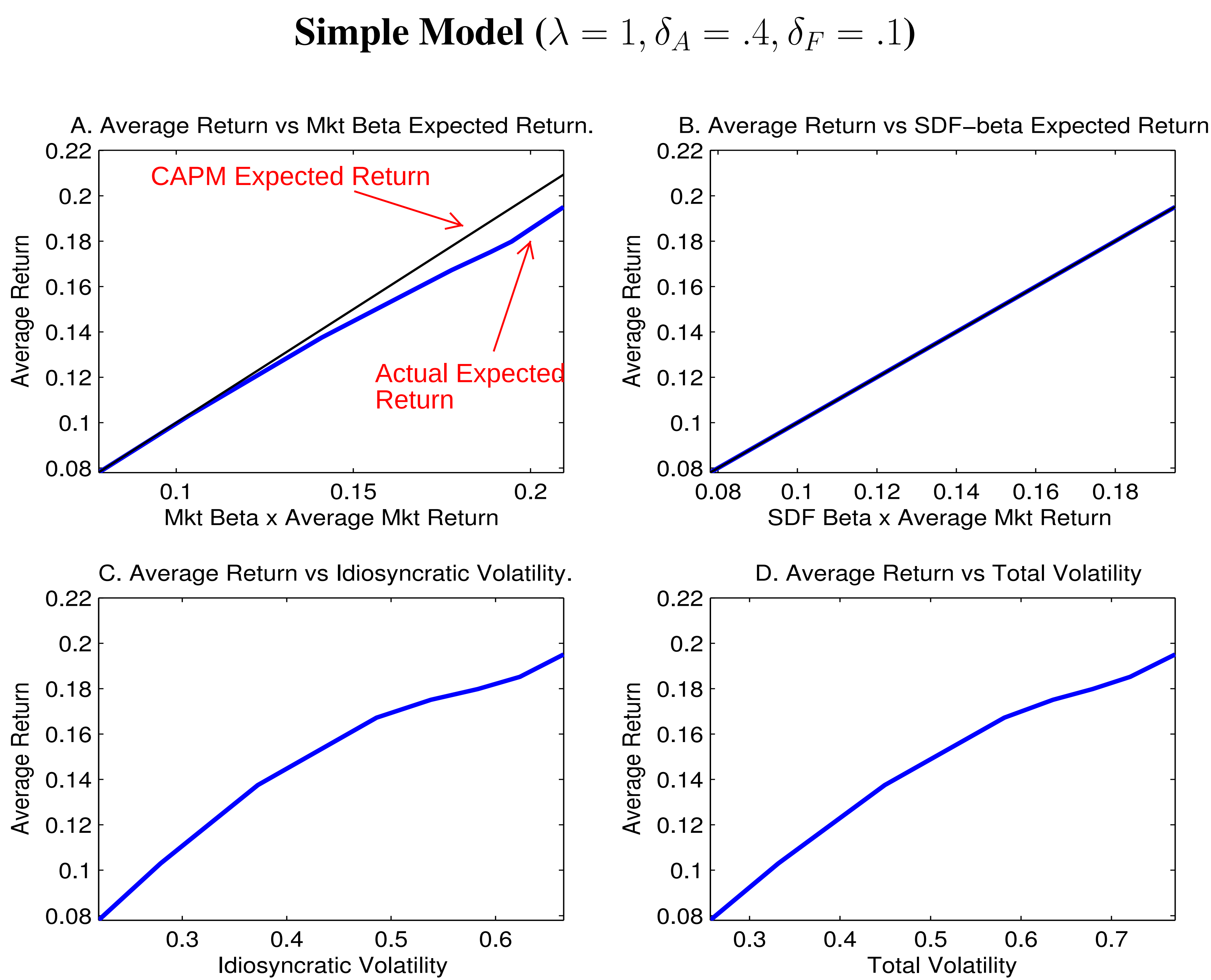
Simple Model ($\lambda = 1, \delta_A = .4, \delta_F = .1$)
A. Average Return vs Mkt Beta Expected Return.
CAPM Expected Return
Actual Expected Return
Average Return
Mkt Beta x Average Mkt Return
B. Average Return vs SDF−beta Expected Return
Average Return
SDF Beta x Average Mkt Return
C. Average Return vs Idiosyncratic Volatility.
Average Return
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D. Average Return vs Total Volatility
Average Return
Total Volatility

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- Higher leverage
  - $\Longrightarrow$ Higher market beta *and* SDF beta
  - $\Longrightarrow \beta^{Mkt} > \beta^{SDF}$

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- Strategy: Bet against beta
  1. Pick a high market beta (H) and a low market beta (L) stock
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- By construction: $R_p = w_L R_L - w_H R_H$ has zero market beta.

$$E[R^p] = \left(\underbrace{\frac{\beta_L^{SDF}}{\beta_L^{Mkt}}}_{\approx 1} - \underbrace{\frac{\beta_H^{SDF}}{\beta_H^{Mkt}}}_{< 1} \right) E[R^{Mkt}] > 0$$

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- How about idiosyncratic volatility and return?

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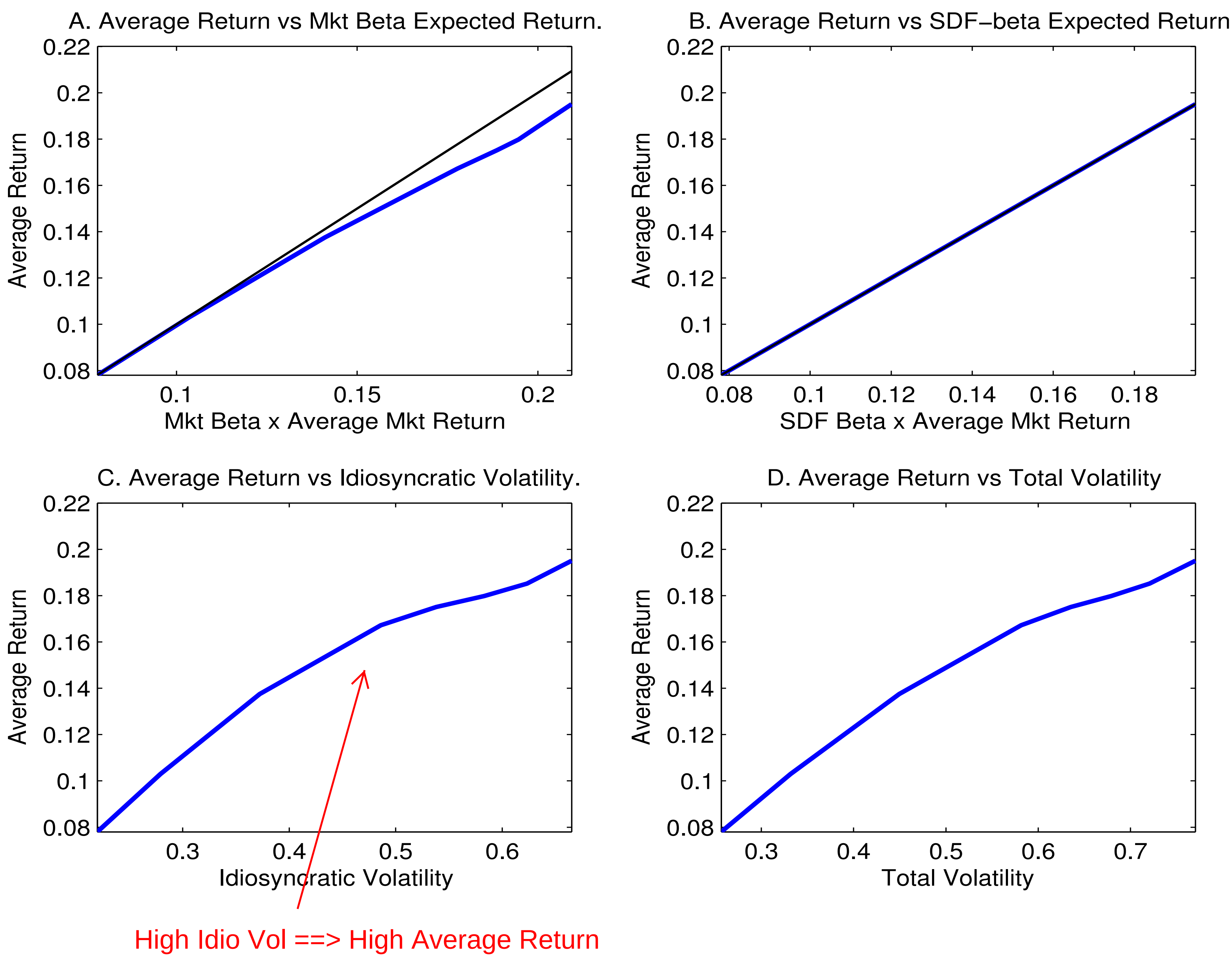
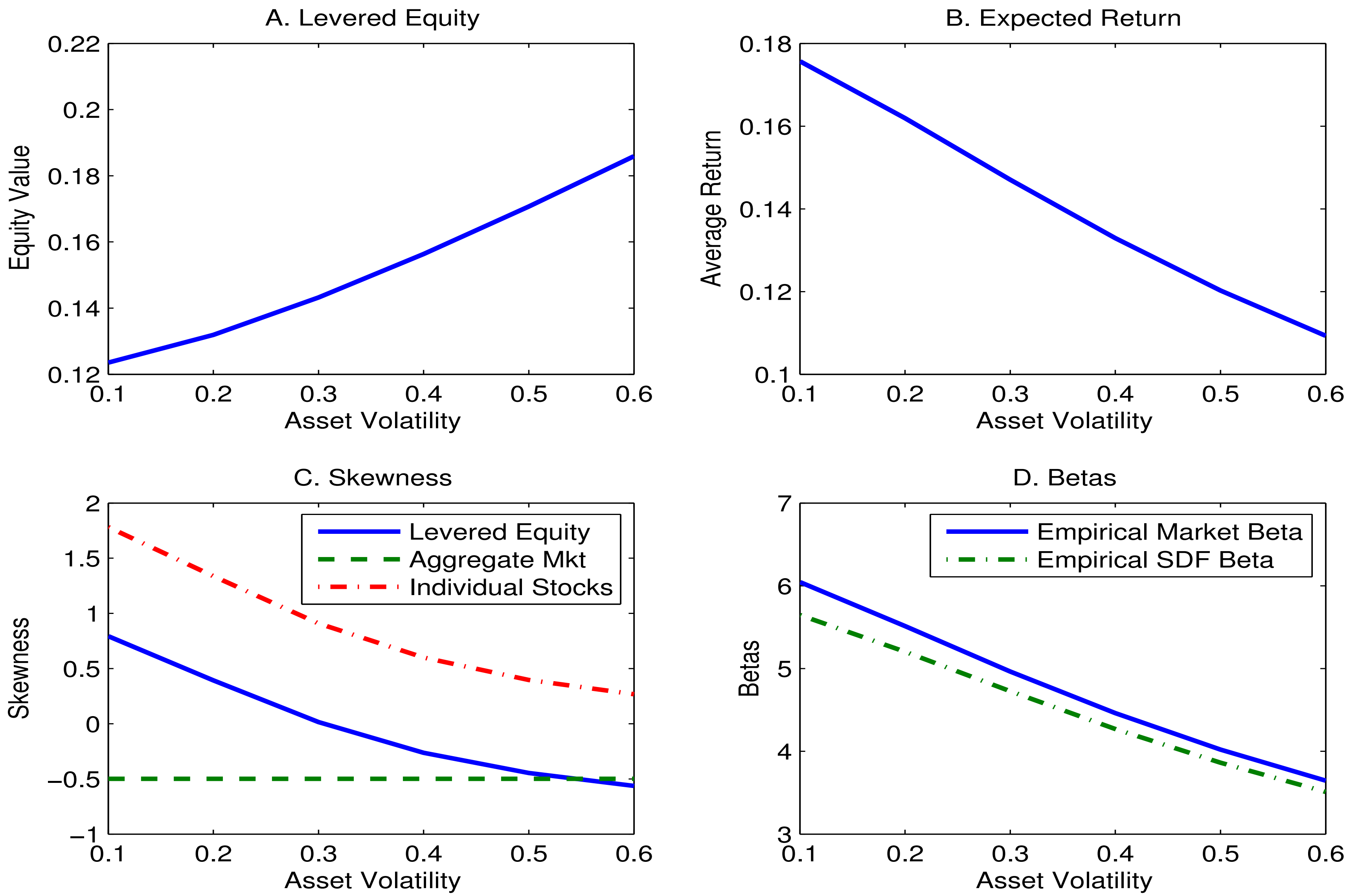


Simple Model ($\lambda = 1, \delta_A = .4, \delta_F = .1$)

- Now fix leverage $K = 0.9$ and change idiosyncratic asset volatility σ_A.



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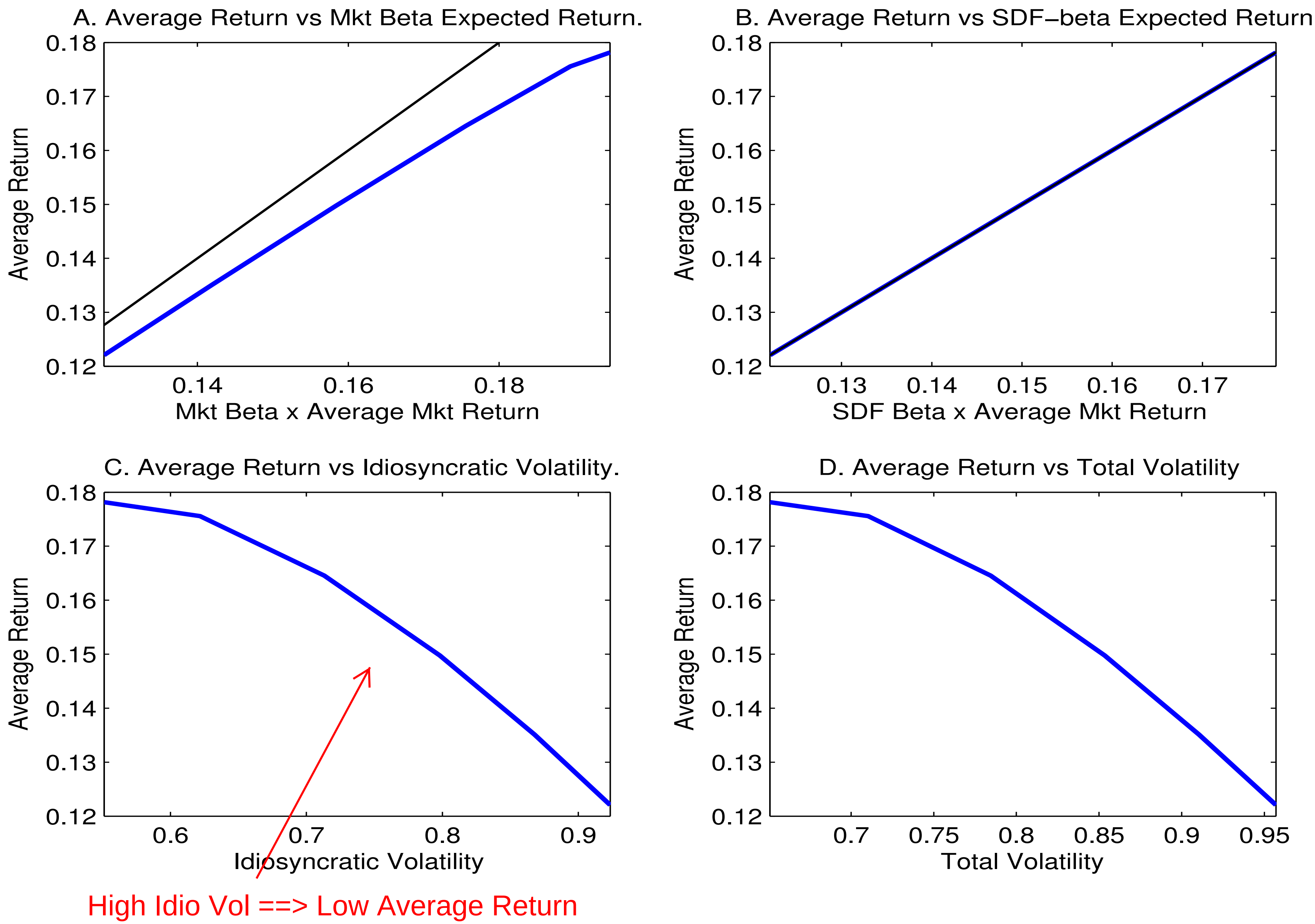

Concluding Remarks

1. Mechanism, paper, and especially empirical results are interesting.
   - Need to fix the "negative skeweness" issue for individual securities
     * Is *ex-ante* skewness still the proper measure of co-skewness in the model?
   - Need to relate it to Engle and Mistry (Journal of Econometrics 2014)
   - Need to relate it to Tim Johnson (JF, 2004)
     * Use a Merton's model to shows that high idio vol $\Longrightarrow$ low risk premia.
   - Note on idio volatility
     * High leverage $\Longrightarrow$ high idio vol *and* high risk premia
     * High asset vol $\Longrightarrow$ high idio vol *and* low risk premia
       $\Longrightarrow$ need to study interaction effects.

2. If you take the mechanism seriously, need to sort on credit risk (under P).
   - How big are the effects for reasonable parameters?

3. Consider other "leverage" mechanisms
   - Operating leverage
   - Labor leverage