Option-Implied Currency Risk Premia

by Jakub W. Jurek and Zhikai Xu

Discussion

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What Does This Paper Do?

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$$k^{\mathbb{F}^i}_{s^{ji}_{t+1}}[u] = \left(s^{ji}_t - \alpha^j_t + \alpha^i_t\right)\cdot u + k^{\mathbb{F}^i}_{L^g_{t+1}}\left[\left(\xi^i_t - \xi^j_t\right)\cdot u\right] + k^{\mathbb{F}^i}_{L^i_{t+1}}[u] + k^{\mathbb{F}^i}_{L^j_{t+1}}[-u] \quad (10)$$

$$k^{\mathbb{F}^i}_{L^g_{t+1}}[u] = \ln E^{\mathbb{F}^i}_t\left[\exp\left(u\cdot L^g_{t+1}\right)\right] = \ln E^{\mathbb{P}}_t\left[\exp\left(y^i_{t,t+1} + \left(m^i_{t+1} - m^i_t\right) + u\cdot L^g_{t+1}\right)\right]$$

$$= \left(k_{\tilde{L}^g_{t+1}}\left[u - \xi^i_t\right] - k_{\tilde{L}^g_{t+1}}\left[-\xi^i_t\right]\right)\cdot Z_t \quad (11a)$$

$$k^{\mathbb{F}^i}_{L^i_{t+1}}[u] = \left(k_{\tilde{L}^i_{t+1}}[u-1] - k_{\tilde{L}^i_{t+1}}[-1]\right)\cdot Y^i_t \quad (11b)$$

$$k^{\mathbb{F}^i}_{L^j_{t+1}}[u] = k_{\tilde{L}^j_{t+1}}[u]\cdot Y^j_t \quad (11c)$$

$$k_{s^{ji}_{t+1}}[u] = \left(s^{ji}_t - \alpha^j_t + \alpha^i_t\right)\cdot u + k_{\tilde{L}^g_{t+1}}\left[\left(\xi^i_t - \xi^j_t\right)\cdot u\right]\cdot Z_t + k_{\tilde{L}^i_{t+1}}[u]\cdot Y^i_t + k_{\tilde{L}^j_{t+1}}[-u]\cdot Y^j_t \quad (12)$$

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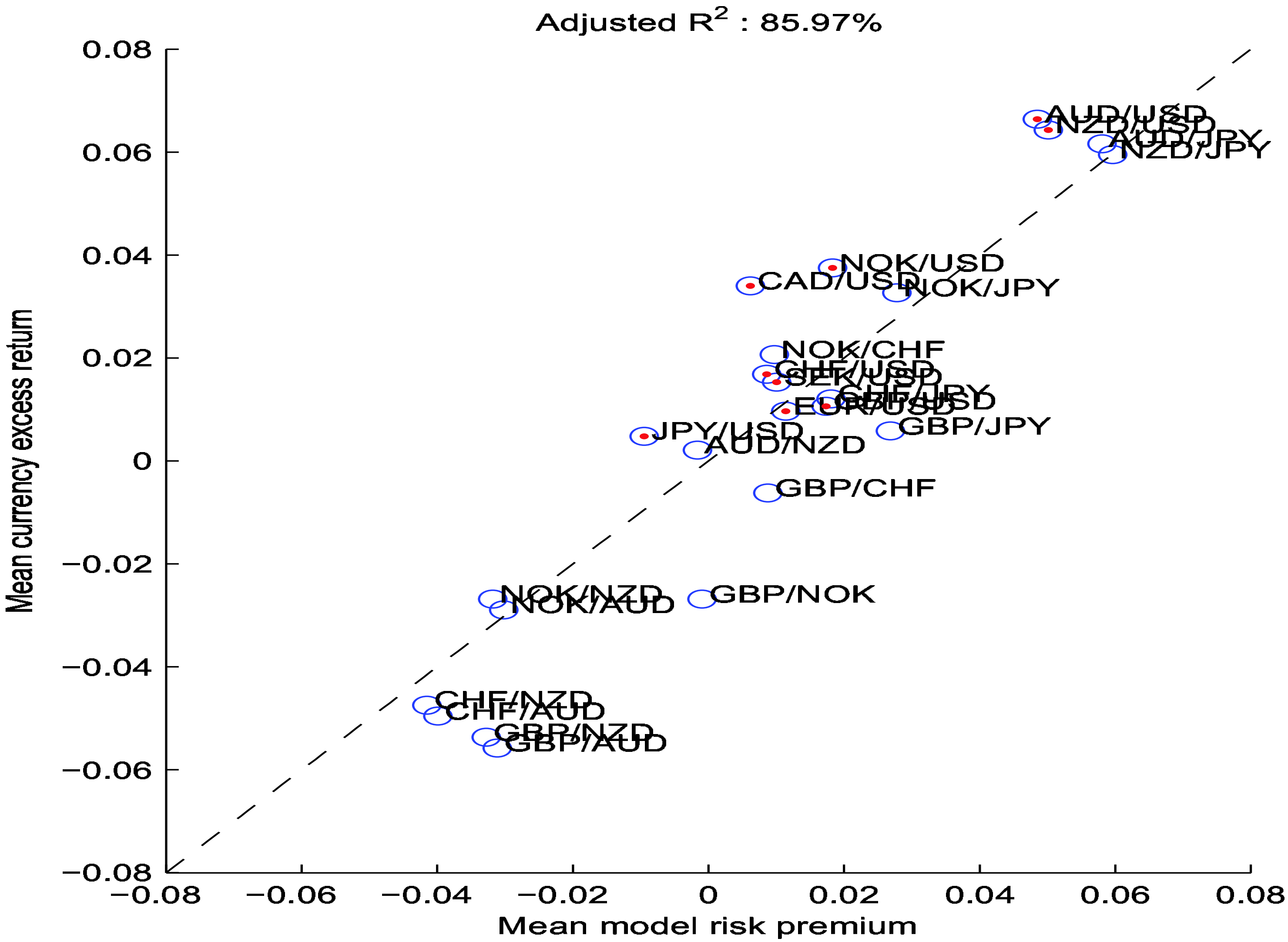
$$+\left(k_{\tilde{L}^i_{t+1}}[u-1] - k_{\tilde{L}^i_{t+1}}[-1]\right)\cdot Y^i_t + k_{\tilde{L}^j_{t+1}}[-u]\cdot Y^j_t \quad (13)$$

$$\lambda^{ji}_{HML,t} = \left(k_{\tilde{L}^g_{t+1}}[\xi^i_t - \xi^j_t] + k_{\tilde{L}^g_t}[-\xi^i_t] - k_{\tilde{L}^g_{t+1}}[-\xi^j_t]\right)\cdot Z_t \quad (20)$$

$$\lambda^i_{refFX,t} = \left(k_{\tilde{L}^i_{t+1}}[1] + k_{\tilde{L}^i_{t+1}}[-1]\right)\cdot Y^i_t \quad (21)$$

$$\lambda^{jk}_t = \left(k_{\tilde{L}^g_{t+1}}[\xi^i_t - \xi^j_t] - k_{\tilde{L}^g_{t+1}}[\xi^i_t - \xi^k_t] + k_{\tilde{L}^g_{t+1}}[-\xi^k_t] - k_{\tilde{L}^g_{t+1}}[-\xi^j_t]\right)\cdot Z_t \quad (22)$$

What Does This Paper Do?



What Does This Paper Do?

- Proposes a relatively general model of option pricing in the FX market with non-Gaussian global and local shocks
- Proposes a methodology to use cross-sectional data on FX options to estimate options' implied currency risk premia
- Implements such methodology using a panel of currency options .
- Finds evidence of global risk factor in currency risk premia, similar to the HML_{FX} factor identified by Lustig, Roussanov, Verdelhan (2011, 2013).
- Uses ex-ante identification to carry out a number of empirical tests, variance decomposition etc.

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(1) Ambitious paper

- Obtaining risk premia from cross-section of options is kind of the holy grail of this literature.
- The methodology is relatively general (cumulants and the like), but it seems it can only be used in the case of currency options.

(2) Paper would benefit from a deeper discussion of the methodology to compute risk premia from options

- The empirical results support the fact that the methodology indeed manages to estimate currency risk premia in the cross-section.
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Outline of discussion:

(A) Review basic no arbitrage relations in currency markets.

(B) Use simple "baby models" to discuss why it is very hard to estimate risk premia from options.

(C) Go back to cross-section.

State Price Densities and Exchange Rates

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$$M_t^{\pounds} V_t^{\pounds} = E_t \left[M_T^{\pounds} V_T^{\pounds} \right] \tag{1}$$

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- Other relations may also work.

Baby Model I

- Consider the following two SDFs for US and UK:

$$
\begin{aligned}
\frac{dM^{\$}}{M^{\$}} &= -r_{\$}dt + \sigma_{\$,g}dW_g + \sigma_{\$,\$}dW_{\$} \\
\frac{dM^{£}}{M^{£}} &= -r_{£}dt + \sigma_{£,g}dW_g + \sigma_{£,£}dW_{£}
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- Let the exchange rate follow the process

$$\frac{dS_t}{S_t} = (\mu_S - r_{£})\,dt + \sigma_{S,g}dW_g + \sigma_{S,\$}dW_{\$} + \sigma_{S,£}dW_{£}$$

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- Using Ito's Lemma, this gives the PDE

$$r_{\$}V^{\$} = \frac{\partial V^{\$}}{\partial t} + \frac{\partial V^{\$}}{\partial S}(\mu_S - r_{£})\,S + \frac{1}{2}\frac{\partial^2 V^{\$}}{\partial S^2}S^2\boldsymbol{\sigma}_S\boldsymbol{\sigma}'_S + \frac{\partial V^{\$}}{\partial S}S\boldsymbol{\sigma}_S\boldsymbol{\sigma}'_{M,\$}$$

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- This PDE must hold also for $V = S$ (with the addition of the "dividend yield" $r_£$), which gives the risk premium for a US investor to purchase British pounds

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- No reference to expected return μ_S nor to the pricing kernel parameters $\boldsymbol{\sigma}_{M,\$}$.

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- Note that all shocks are "Gaussian" $\Longrightarrow$ related to Jurek and Xu finding that Gaussian shocks do not allow one to estimate risk premia from options.

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- Let's just add a BIG common shock to pricing kernels:

$$\frac{dM^{\$}}{M^{\$}} = -\left[r_{\$} + \lambda_g E_t\left(J^{\$}_{M,g} - 1\right)\right] dt + \boldsymbol{\sigma}_{M,\$} d\boldsymbol{W} + \left(J^{\$}_{M,g} - 1\right) dQ_g$$
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- Consider again the same steps. Now S_t must depend on dQ_g as well (at least):

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- Ito's Lemma in this case gives

$$r_{\$} V^{\$} = \frac{\partial V^{\$}}{\partial t} + \frac{\partial V^{\$}}{\partial S}\left(\mu_S - r_{£}\right) S + \frac{1}{2}\frac{\partial^2 V^{\$}}{\partial S^2} S^2 \boldsymbol{\sigma}_S \boldsymbol{\sigma}'_S + \frac{\partial V^{\$}}{\partial S} S \boldsymbol{\sigma}_S \boldsymbol{\sigma}'_{M,\$}$$

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SAME COMPONENT AS PREVIOUS CASE

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- Like before, a dollar security $V^{\$}$ must satisfy $E_t\left[d\left(V^{\$} M^{\$}\right)\right] = 0$.
- Ito's Lemma in this case gives

$$\begin{aligned}r_{\$} V^{\$} &= \frac{\partial V^{\$}}{\partial t} + \frac{\partial V^{\$}}{\partial S}\left(\mu_S - r_{£}\right) S + \frac{1}{2}\frac{\partial^2 V^{\$}}{\partial S^2} S^2 \boldsymbol{\sigma}_S \boldsymbol{\sigma}'_S + \frac{\partial V^{\$}}{\partial S} S \boldsymbol{\sigma}_S \boldsymbol{\sigma}'_{M,\$} \\ &\quad - \frac{\partial V^{\$}}{\partial S} S \lambda_g E\left[J_{S,g} - 1\right] + \lambda_g E\left[\left(V^{\$}\left(J_{S,g} S\right) - V^{\$}(S)\right) J^{\$}_{M,g}\right]\end{aligned}$$

JUMP-RELATED COMPONENT OF PDE

Baby Model II

- This equation must hold for $V^{\$} = S$ (adding the "dividend yield" $r_{£}$), obtaining:

$$\mu_S - r_{\$} = \underbrace{-\boldsymbol{\sigma}_S \boldsymbol{\sigma}'_{M,\$}}_{-\text{Diffusive}Cov\left(dS/S,\, dM^{\$}/M^{\$}\right)} \underbrace{-\lambda_g E_t\left[(J_{S,g}-1)\left(J^{\$}_{M,g}-1\right)\right]}_{-\text{Jump}Cov\left(dS/S,\, dM^{\$}/M^{\$}\right)}$$

(Diffusive Risk Premium) (Jump Risk Premium)

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$$0 = \text{BSMPDE} - \frac{\partial V^{\$}}{\partial S} S \lambda_g E\left[(J_{S,g}-1)\, J^{\$}_{M,g}\right] + \lambda_g E\left[\left(V^{\$}(J_{S,g}S) - V^{\$}(S)\right) J^{\$}_{M,g}\right]$$

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1. Diffusive risk premium again disappears from pricing equation $\Longrightarrow$ options cannot inform about that component.

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2. Options though depend on jump component of pricing kernel $\Longrightarrow$ options may reveal some information about it.

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- Note:

  1. Diffusive risk premium again disappears from pricing equation $\Longrightarrow$ options cannot inform about that component.
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- But what exactly can be identified? Let's expand the jump risk premium

Baby Model II

$$\text{Jump Risk Premium} = -\lambda_g E\left[(J_{S,g}-1)\, J_{M,g}^{\$}\right] \qquad + \qquad \lambda_g E\left[J_{S,g}-1\right]$$

Baby Model II

$$\text{Jump Risk Premium} = -\underbrace{\lambda_g E\left[(J_{S,g}-1)\,J_{M,g}^{\$}\right]}_{\substack{\text{Identifiable from Options} \\ \text{(Risk Neutral Dist.)}}} + \underbrace{\lambda_g E\left[J_{S,g}-1\right]}_{\substack{\text{Not Identifiable from Options} \\ \text{(Physical Dist.)}}}$$

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- But let's do a bit more work. From $S_t = M_t^{£}\left(M_t^{\$}\right)^{-1} \Longrightarrow J_{S,g} = J^{£}_{M,g}\left(J^{\$}_{M,g}\right)^{-1}$.

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$$\text{Jump Risk Premium} = -\lambda_g\left[E\left(J_{M,g}^{£}\right) - E\left(J_{M,g}^{\$}\right)\right] + \lambda_g E\left[J_{M,g}^{£}\left(J_{M,g}^{\$}\right)^{-1} - 1\right]$$

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Baby Model II

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- Normalize $E[J_g] = 1$ (wlog) and substitute:

$$\text{Jump Risk Premium} = -\lambda_g\left(\beta^{£} - \beta^{\$}\right) \quad + \quad \frac{\lambda_g}{\beta^{\$}}\left(\beta^{£} - \beta^{\$}\right)$$

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- Not bad.... So: how do we identify λ_g, $\beta^{£}$ and $\beta^{\$}$?

Baby Model II

- The PDE now becomes

$$0 = \mathsf{BSMPDE} \quad - \; \frac{\partial V^{\$}}{\partial S}\, S\, \lambda_g \beta^{\$} \left(\frac{\beta^{£}}{\beta^{\$}} - 1 \right) + \lambda_g \beta^{\$} \left(V^{\$} \left(\frac{\beta^{£}}{\beta^{\$}} S \right) - V^{\$}\,(S) \right)$$

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$\Longrightarrow$ options on $\$/£$ can then identify them.

- Indeed, in this case, the option pricing formula is very simple (Merton (1976))

$$V^{\$}(S) = \sum_{n=0}^{\infty} \frac{e^{-(\lambda_g \beta_{\$})T} [(\lambda_g \beta_{\$})T]^n}{n!} BSM \left(S e^{b(n)T}, K, r_{\$}, r_{£}, \boldsymbol{\sigma}_S \boldsymbol{\sigma}_S', T \right)$$

  – where BSM is the Black, Scholes, Merton formula, and

$$b(n) = -(\lambda_g \beta^{\$}) \left[\left(\frac{\beta^{£}}{\beta^{\$}} \right) - 1 \right] + \frac{n}{T} \log \left(\frac{\beta^{£}}{\beta^{\$}} \right)$$

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- Given a bunch of options data, we can estimate $(\lambda_g \beta^{\$})$ and $\left(\frac{\beta^{£}}{\beta^{\$}} \right)$.

Baby Model II

- The above shows that from $\$/\pounds$ options we can "observe:"

$$\boxed{\widehat{d}_{1,\$/\pounds} = \lambda_g \beta^{\$};} \quad \boxed{\widehat{d}_{2,\$/\pounds} = \beta^{\pounds}/\beta^{\$}}$$

- 2 equations in 3 unknowns. Not enough.

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- Consider now one more option on $\$/€$:

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- 3 equations in 4 unknowns. Still not enough.

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- Consider the cross rate $\pounds/€$:

$$\boxed{\widehat{d}_{1,\pounds/€} = \lambda_g \beta^{\pounds};} \quad \boxed{\widehat{d}_{2,\pounds/€} = \beta^{€}/\beta^{\pounds}}$$

- 4 equations in 4 unknowns.

  [middle equations do not add anything to the ones at the bottom and the top]

Baby Model II

- So, we have 4 equations in 4 unknowns. Can we actually identify the JRP?

$$JRP = \lambda_g \left(\beta^{£} - \beta^{\$}\right) \left(1 - \beta^{\$}\right) \left(\beta^{\$}\right)^{-1}$$

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- Unfortunately not.
- In fact, the equations are not independent. Recall they are:

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- For instance, if $\widehat{\beta}^i = x\beta^i$, for $i = £, \$, €$, and $\widehat{\lambda}_g = (1/x)\lambda_g$, then

$$\widehat{\lambda}_g \widehat{\beta}^{\$} = \lambda_g \beta^{\$}; \quad \widehat{\beta}^{£}/\widehat{\beta}^{\$} = \beta^{£}/\beta^{\$}; \quad \widehat{\lambda}_g \widehat{\beta}^{£} = \lambda_g \beta^{£}; \quad \widehat{\beta}^{€}/\widehat{\beta}^{£} = \beta^{€}/\beta^{£}$$

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- But the JRP is different: If e.g. $\beta^{£} > \beta^{\$}$, then

$$\begin{aligned} JRP &= \widehat{\lambda}_g \left(\widehat{\beta}^{£} - \widehat{\beta}^{\$}\right)\left(1 - \widehat{\beta}^{\$}\right)\left(\widehat{\beta}^{\$}\right)^{-1} \\ &= \lambda_g \left(\beta^{£} - \beta^{\$}\right)\left(1 - x\beta^{\$}\right)\left(x\beta^{\$}\right)^{-1} \in \left[-\lambda_g \left(\beta^{£} - \beta^{\$}\right), \infty\right] \end{aligned}$$

Baby Model III

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- Consider a standard Heston's stochastic volatility model:

$$\frac{dM^{\$}}{M^{\$}} = -r_{\$}dt + \beta_{\$}\sqrt{v}dW_g + \sigma_{\$,\$}dW_{\$}$$
$$\frac{dM^{£}}{M^{£}} = -r_{£}dt + \beta_{£}\sqrt{v}dW_g + \sigma_{£,£}dW_{£}$$

- with $$dv = k(\theta - v)dt + s_v\sqrt{v}dW_v.$$

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- with $$dv = k(\theta - v)dt + s_v\sqrt{v}dW_v.$$
- From no arbitrage, this time we obtain

$$\frac{dS}{S} = (\mu_S(v) - r_{£})dt + (\beta_{£} - \beta_{\$})\sqrt{v}dW_g + \sigma_{£,£}dW_{£} - \sigma_{\$,\$}dW_{\$}$$

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- The risk premium, to identify, is

$$\mu_S(v) - r_{\$} = -Cov\left(\frac{dM^{\$}}{M^{\$}}, \frac{dS}{S}\right) = -(\beta_{£} - \beta_{\$})\beta_{\$}v_t + \sigma^2_{\$,\$}$$

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- Do options allow us to identify $\beta_{\$}$, $\beta_{£}$, v_t and $\sigma^2_{\$,\$}$?

Baby Model III

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$$
\begin{aligned}
r_{\$}V^{\$} \;=\; & \frac{\partial V^{\$}}{\partial t} + \frac{\partial V^{\$}}{\partial S} S \left(r_{\$} - r_{\pounds} \right) + \frac{1}{2} \frac{\partial^2 V^{\$}}{\partial S^2} S^2 \left(\left(\beta_{\pounds} - \beta_{\$} \right)^2 v + \sigma^2_{\pounds,\pounds} + \sigma^2_{\$,\$} \right) \\
& + \frac{\partial V}{\partial v} \left(k(\theta - v) \right) + \frac{1}{2} \frac{\partial^2 V}{\partial v^2} s^2 v
\end{aligned}
$$

Baby Model III

- The PDE this time is

$$r_\$ V^\$ = \frac{\partial V^\$}{\partial t} + \frac{\partial V^\$}{\partial S} S \left(r_\$ - r_£\right) + \frac{1}{2}\frac{\partial^2 V^\$}{\partial S^2} S^2 \left(\left(\beta_£ - \beta_\$\right)^2 v + \sigma^2_{£,£} + \sigma^2_{\$,\$} \right)$$
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- $\Longrightarrow$ At most, from options, we can identify

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\$/€ \text{ options} &\Longrightarrow \widehat{d}_{\$/€} = \beta_{€} - \beta_{\$}; \\
€/£ \text{ options} &\Longrightarrow \widehat{d}_{€/£} = \beta_{£} - \beta_{€};
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  - Note on discrete horizon $[t, t+\tau]$, returns are not Gaussian.

Summary from Baby Models

- Two simple "baby models" with non-Gaussian shocks do **not** allow for the identification of currency risk premium only from the cross-section of options.
- The main problem is that even though one can obtain a number of equations equal to the number of parameters under the physical measure, there are cross-currencies restrictions that decrease the number of independent equations.
- A similar problem seems to occur in Jurek and Xu.
  - Indeed, numerous identifying equations rely on differences in loadings.

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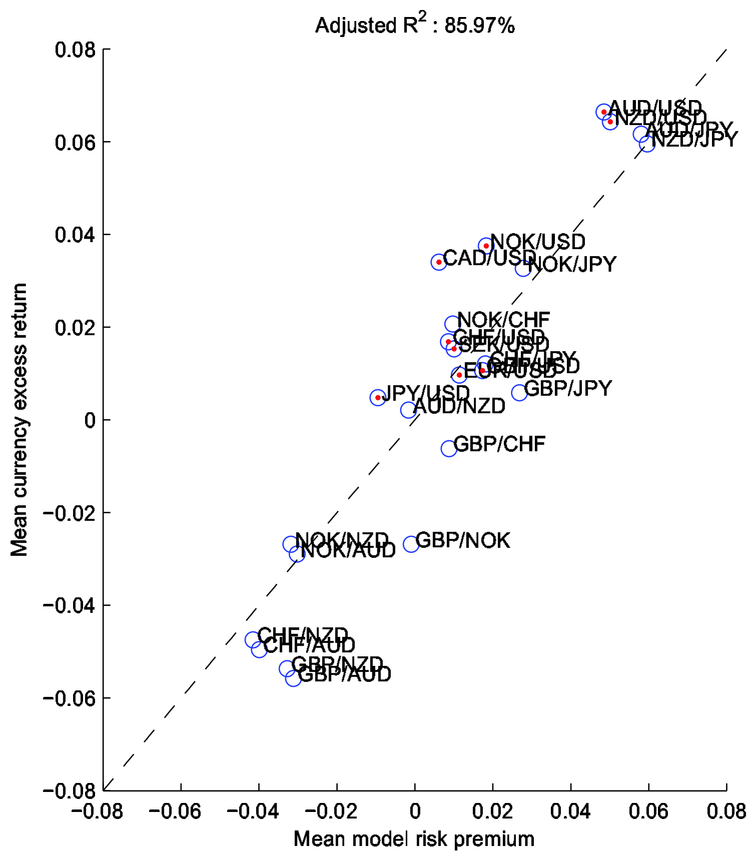
- A similar problem seems to occur in Jurek and Xu.
  - Indeed, numerous identifying equations rely on differences in loadings.
- But then, what is the source of the empirical success of the model?

Summary from Baby Models



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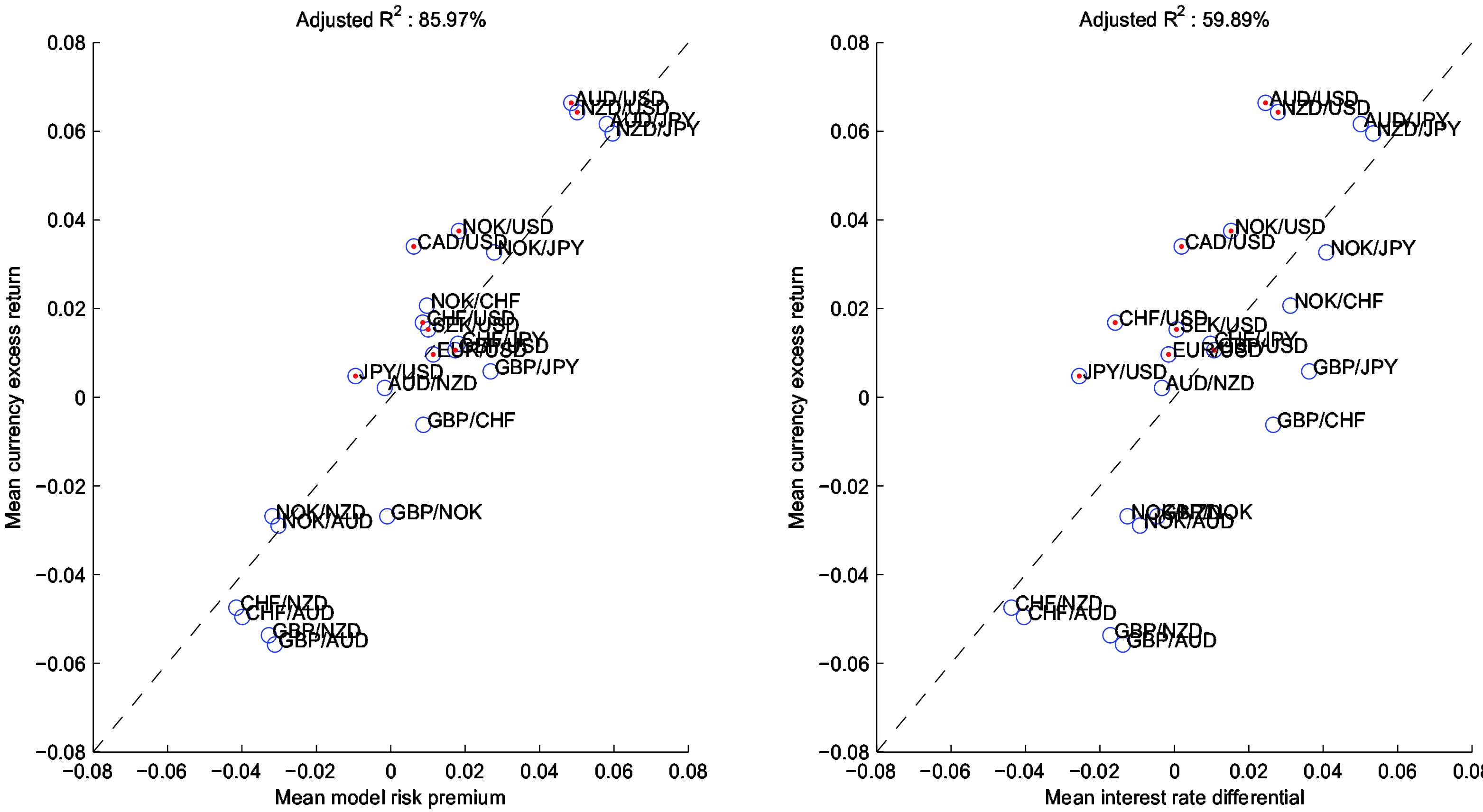

- Ex-ante option-implied currency risk premia line up with *realized* excess returns.
- Options predict currency risk premia better than interest rate differential.

Back to Baby Model II

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- Of course, this holds also for £-JPRs, or €-JPRs etc.
  (but not cross-currencies JPRs, such as $\frac{JPR^{\$,i}}{JPR^{£,i}}$).
- This result is unfortunately not general, but it points at a potentially "easier" identification of the cross-section of risk premia from option.

Conclusions

- Ambitious paper.
- Getting at risk premia from options is tough.
- This paper seems to be getting quite interesting empirical results from the cross-section of options.
- Next step is to clearly understand how and why the estimation methodology works
  - More work there is clearly needed, but it is likely to bring about similar empirical results.
- It would be interesting to explore the difference between estimation of *level* of FX risk premia, and *cross-section* of FX risk premia, which may require different estimation methods, as shown in the simple example.