Heterogeneity and Asset Prices: A Different Approach

by Nicolae Garleanu and Stavros Panageas

Discussion

Pietro Veronesi

The University of Chicago Booth School of Business

Main Contribution and Outline of Discussion

- Main contribution of the paper:
  1. *Develop a macro-asset pricing framework that links volatile asset prices and high risk premiums to non-volatile, but persistent movements in the cross-sectional income and consumption distributions.*
  2. *Propose a novel empirical approach to infer low frequency, time-series movements in the marginal agent's consumption [...] by utilizing [..] cross-sectional information.*
  3. *Calibration and success.*

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- Outline of discussion

  1. Locally riskless consumption and volatile asset prices
  2. Limited Risk Sharing
  3. Comments

Locally Riskless Consumption and Stochastic Prices. Power Utility

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- Then:

Interest Rate: $$r(g_t) = \rho + \gamma g_t$$

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\text{Return process:} \qquad \frac{dP_t + D_t dt}{P_t} &= r(g_t)dt + \sigma_P(g_t) dB_t
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- $\Longrightarrow$ Volatile prices with locally deterministic consumption
  - Because the riskless rate is time varying

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- Same economy: $dD_t/D_t = g_t\, dt \quad$ with $\quad dg_t = k(\overline{g} - g_t)dt + \sigma_g dB_t$

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- EZ utility with RRA γ and EIS ψ:

$$f(C_t, V_t) = \frac{\rho\,(1-\gamma)}{1-1/\psi} V_t \left(\left(\frac{C_t}{((1-\gamma)V_t)^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}} - 1 \right)$$

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- Volatile prices and a risk premium with locally deterministic consumption
  - Because the riskless rate is time varying and agents care about future utility

Locally Riskless Consumption and Stochastic Prices

- More generally, one could choose a generic process for economic growth

$$\frac{dD_t}{D_t} = g_t dt$$

with

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- From this, one could reverse engineer (if he/she is very good!)

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- Examples:
  - Affine models
  - Affine-Quadratic Models
  - Gabaix Linearity Generating Models

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- Agents do have different wealth over time, because their total endowment is different and this difference persists
- But with complete markets, all agents' consumption plans have identical marginal rates of substitution
- $\Longrightarrow$ Aggregation gives the result.
- $\Longrightarrow$ With complete markets, income distribution has no impact on equilibrium

Share Processes and Incomplete Markets

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- Therefore, the change in aggregate consumption:

$$dC_t = \underbrace{-\lambda\, C_t\, dt}_{\text{Death}} + \underbrace{\lambda \int_{-\infty}^{t} e^{-\lambda(t-s)} dC_{t,s}}_{\text{Survivors}} + \underbrace{\lambda\, C_{t,t}\, dt}_{\text{New agents}}$$

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- And $C_{t,t}$ depend on t new endowments:

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- This is pretty cool.

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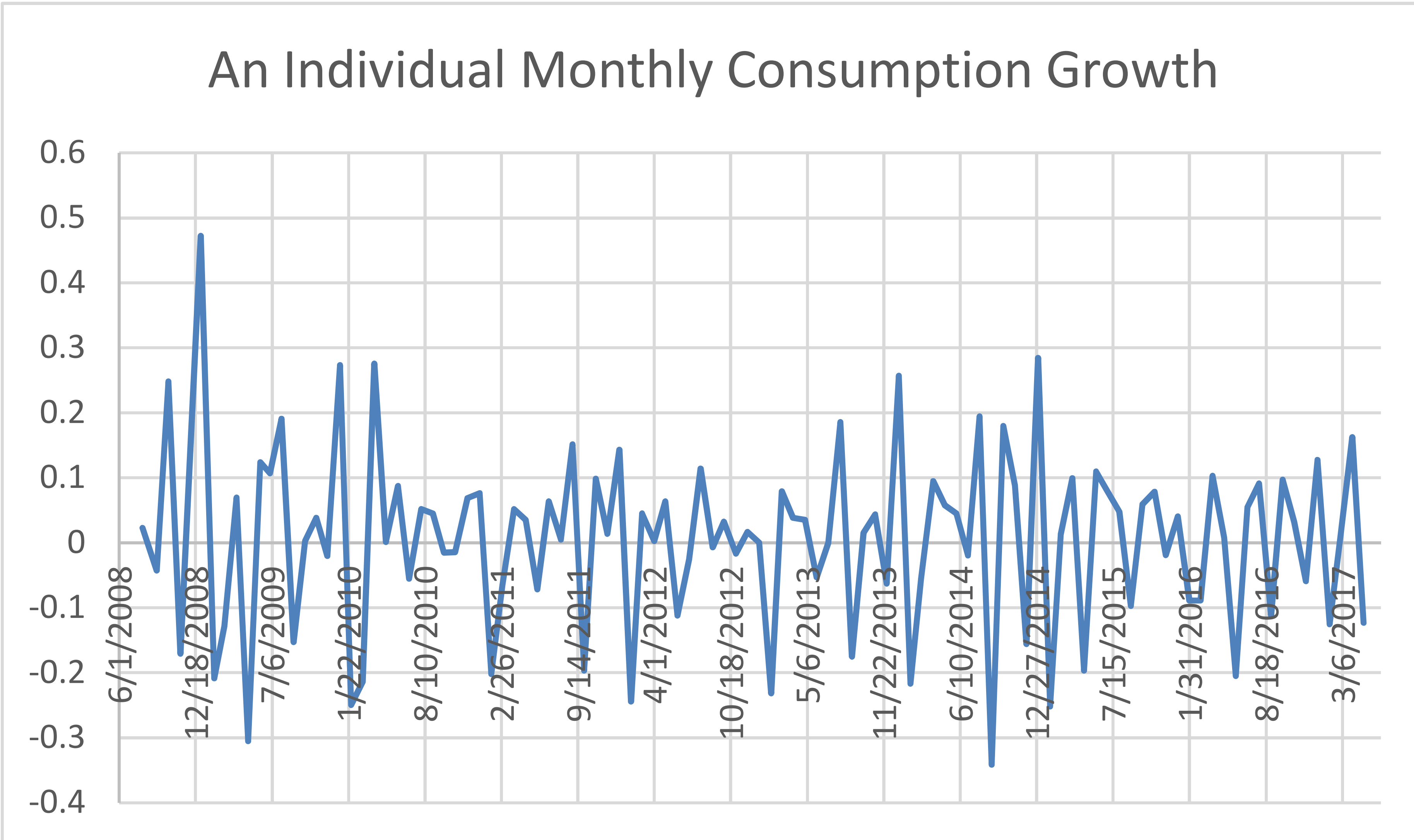
  - Evidence:
    - Survey of 1 agent (me): Smooth income growth.
      Sample 2010 - 2017: Quarterly consumption volatility = 11%


An Individual Monthly Consumption Growth
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
6/1/2008
12/18/2008
7/6/2009
1/22/2010
8/10/2010
2/26/2011
9/14/2011
4/1/2012
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    - Survey of Consumer Expenditures:

Household Quarterly Consumption Growth Moments. (1980 - 2005)

	Individual Growth Rate (%)				Individual Volatility (%)		
	Mean	Median	Std. Dev.		Mean	Median	Std. Dev.
Arithmetic	6.04	-0.63	40.13	Total	36.53	27.10	42.35
Logarithmic	-0.59	-0.66	35.78	Systematic	8.94	6.61	10.42

(Source: Santos and Veronesi "Habits and Leverage", 2017)

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- Interest rate dynamics is critical in the model
  - Relation between cohort effects and interest rates?
  - (Too) tight relation between interest rates and prices?
    - Question: Why is high interest rate *positively* related to high P/D ratio? (Figure 8)
  - Term structure implications?

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  - Relation between cohort effects and interest rates?
  - (Too) tight relation between interest rates and prices?
    - Question: Why is high interest rate *positively* related to high P/D ratio? (Figure 8)
  - Term structure implications?
- Interesting read for sure. Hopefully, the first version (this is the preliminary one) will contain more intuition behind the results to clarify the main forces.