

# Information Acquisition in Financial Markets: a Correction\*

Gadi Barlevy  
Federal Reserve Bank of Chicago  
230 South LaSalle  
Chicago, IL 60604

Pietro Veronesi  
Graduate School of Business  
University of Chicago  
Chicago, IL 60637

January 17, 2008

## Abstract

This note provides a proper example for the channel of strategic complementarities we proposed in our 2000 paper "Information Acquisition in Financial Markets". As pointed out in Chamley (2007), our earlier example contained an error, which, once corrected, reveals that our example actually exhibits strategic substitutability in information acquisition.

## JEL Codes: G14, D82, D84

In our 2000 paper "Information Acquisition in Financial Markets" we argued that contrary to the conventional wisdom set forth in Grossman and Stiglitz (1980), it was theoretically possible that as more traders in financial markets acquire information, equilibrium prices would change in such a way that it became more difficult for remaining agents to infer the fundamentals from prices. We presented an example we thought demonstrated this claim. However, as was subsequently pointed out to us by Christophe Chamley, the expression we used for the value of information in that paper (expression 3.5) was incorrect. As demonstrated by Chamley (2007), using the correct expression for the value of learning reveals that learning is in fact a strategic substitute in our example.

This leaves open the question of whether there is an example consistent with our original conjecture. This note provides a proper example of the mechanism we previously attempted to model.<sup>1</sup> The example involves changing the way fundamentals and noise are *jointly distributed*, in contrast with our previous paper in which we tried to change the *functional form* for the distribution of each variable. While this amounts to a different technical assumption, allowing fundamentals and noise to be correlated captures precisely what we attempted to model in our earlier paper, a stochastic environment in which more agents learning can make prices harder, not easier, to read.

---

\*We are grateful to Christophe Chamley for discovering our error, and to Christian Hellwig for his comments.

<sup>1</sup>Since our paper was published, there have been other papers that demonstrated information acquisition in financial markets can exhibit strategic complementarity, e.g. Veldkamp (2006), Chamley (2006), and Ganguli and Yang (2006). However, these papers rely on different mechanisms than the one we conjectured in our 2000 paper.

The intuition for our result is as follows. Consider an environment in which as fundamentals change, additional factors that do not impact on fundamentals tend to change as well, in a way that has an offsetting effect on the price of the asset. For example, firm managers might take advantage of periods in which fundamentals are high to issue more shares, perhaps because prices tend to be higher on average at these times or because good fundamentals tend to be associated with new investment opportunities that necessitate new funds. This increase in supply may counteract the effect of more favorable fundamentals on the price of the asset. As another example, if good fundamentals are correlated with higher future income for some agents, they will liquidate some of their assets to consume more today. The resulting sales pressure will counteract the positive news about fundamentals. In such situations, as more agents acquire information, prices will be pushed up (down) when fundamentals are high (low). However, this change may no longer serve to make prices more informative. Due to the offsetting factors, such a shift will cause the distribution of prices for different fundamentals to converge towards the same values, making it harder to infer fundamentals from prices. This is precisely what we sought to demonstrate in our previous paper.

We begin our discussion with a simple example based on the setup in our 2000 paper. This example illustrates why allowing noise to be correlated with fundamentals can produce strategic complementarities in information acquisition. We then argue that correlation can be a source of complementarities quite generally, including in the original specification of Grossman and Stiglitz (1980).

## 1 The $2 \times 2$ Case

To construct a relatively simple example of complementarities, we turn to the framework of our 2000 paper. Briefly, agents must choose to allocate their wealth between money and an asset that pays a random amount  $\tilde{\theta}$  per share. There is a unit mass of risk-neutral agents who can observe  $\tilde{\theta}$  if they pay a cost  $c > 0$ . The demand of informed traders is denoted  $x^I(\tilde{\theta}, P)$ , and of uninformed is denoted  $x^U(P)$ . Traders can spend at most their initial endowment, equal to one unit of money, and cannot sell assets short. Demand for the asset by noise traders is  $\frac{w}{P} - \tilde{x}$  for some  $w > 0$ , where  $\tilde{x}$  is random.

In our 2000 paper, we assumed  $\tilde{x}$  had a continuous distribution. However, the intuition is more transparent if both  $\tilde{\theta}$  and  $\tilde{x}$  take on only two values:  $\tilde{\theta} \in \{\underline{\theta}, \bar{\theta}\}$  where  $\bar{\theta} > \underline{\theta}$ , and  $x \in \{x_0, x_1\}$  where  $x_1 > x_0$ . We return to the case where  $\tilde{x}$  has a continuous distribution in the next section. We refer to the four states of the world as  $\omega_1$  through  $\omega_4$ , with probabilities  $\pi_1$  through  $\pi_4$ , as follows:

	$x_0$	$x_1$
$\bar{\theta}$	$\omega_1$	$\omega_2$
$\underline{\theta}$	$\omega_3$	$\omega_4$

	$x_0$	$x_1$
$\bar{\theta}$	$\pi_1$	$\pi_2$
$\underline{\theta}$	$\pi_3$	$\pi_4$

Let  $z$  denote the fraction of traders who acquire information. Consider the case where  $z = 0$ , so all traders are uninformed. An equilibrium price function assigns a price to each of the four states. Since all traders are uninformed, it seems natural to restrict attention to equilibria in which  $P(\omega_1) = P(\omega_3)$  and  $P(\omega_2) = P(\omega_4)$ , i.e. where the price for a realization of  $\tilde{x}$  is not measurable with respect to  $\tilde{\theta}$ . We thus rule out equilibria in which agents coordinate to buy the asset based on unobservable realizations of  $\tilde{\theta}$ . One can The existence of such an equilibrium follows by construction.

Define  $P_0 \equiv P(\omega_1) = P(\omega_3)$  as the price of the asset when  $\tilde{x} = x_0$  and  $P_1 \equiv P(\omega_2) = P(\omega_4)$  as the

price of the asset when  $\tilde{x} = x_1$ . Since we wish to demonstrate the possibility that prices become less informative once some agents become informed, we need prices to contain some information when  $z = 0$ . Thus, we restrict attention to parameters that ensure  $P_0 \neq P_1$ . Below we derive sufficient conditions for this assumption. The information set of an uninformed agent given this equilibrium price corresponds to the partition

$$\Omega_0 = \{\{\omega_1, \omega_3\}, \{\omega_2, \omega_4\}\}. \quad (1)$$

Next, suppose  $z > 0$ . First, note that in general,  $z > 0$  can only be an equilibrium if  $P(\omega_2) = P(\omega_3)$ . This is because state  $\omega_1$  (respectively,  $\omega_4$ ) involves low (high) supply and high (low) fundamentals, implying  $P(\omega_1)$  will be strictly higher than the price in any other state while  $P(\omega_4)$  will be strictly lower.<sup>2</sup> It follows that if  $P(\omega_2) \neq P(\omega_3)$ , the price would fully reveal  $\omega$ . But then no agent would agree to pay to learn  $\omega$ , so  $z = 0$ . Hence, the only candidate equilibrium in which  $z > 0$  is one in which  $P(\omega_2) = P(\omega_3)$ , and the information of an uninformed agent corresponds to the partition

$$\Omega_z = \{\{\omega_1\}, \{\omega_2, \omega_3\}, \{\omega_4\}\}. \quad (2)$$

Comparing  $\Omega_z$  to  $\Omega_0$  reveals that neither partition refines the other. Hence, there is no sense in which uninformed agents know more about the fundamentals when some traders are informed than when none are. Equilibrium prices do not inherently become more informative as more traders acquire information — they simply convey *different* information. This contrasts with what Grossman and Stiglitz (1980) find in their model, where prices become more informative in a well-defined sense as more agents become informed (specifically, prices are more informative in the Blackwell sense).

Since the fraction of informed agents  $z$  changes the informational content of prices, we would expect complementarities to occur if the information conveyed by prices when  $z > 0$  is less helpful for making investment decisions than the information conveyed by prices when  $z = 0$ . An extreme example of this occurs when  $\pi_1 = \pi_4 = 0$ , i.e.  $\tilde{x}$  and  $\tilde{\theta}$  are perfectly correlated. In this case, the true state of the world  $\omega$  must be either  $\omega_2$  or  $\omega_3$ . Since  $\Omega_z$  does not distinguish between these two states, equilibrium prices when  $z > 0$  convey no additional information beyond what uninformed traders already know. By contrast, the equilibrium price when  $z = 0$  does distinguish the two states so long as  $P_0 \neq P_1$ . The agent would have no need for additional information when  $z = 0$ , but would value information on  $\tilde{\theta}$  when  $z > 0$ . Learning is thus a strategic complement. The same logic applies if  $x$  and  $\theta$  are imperfectly but positively correlated. Intuitively, the fact that agents can perfectly deduce  $\tilde{x}$  when  $z = 0$  implies that prices convey some information about  $\tilde{\theta}$  given the two are correlated. But since  $\tilde{x}$  and  $\tilde{\theta}$  are *positively* correlated, a high (low) realization of the supply shock  $\tilde{x}$  is likely to be accompanied by high (low) demand from informed agents. Having more informed agents respond to  $\tilde{\theta}$  thus jams the signal about  $\tilde{x}$  that would otherwise be conveyed by the price of the asset.

In sum, if the information content of prices is given by (1) and (2) and  $\tilde{x}$  and  $\tilde{\theta}$  are positively correlated, the value of learning will be higher for some  $z > 0$  than for  $z = 0$ . We now derive sufficient conditions for there to exist an equilibrium price function consistent with (1) and (2), and then show that these conditions are compatible with  $\tilde{x}$  and  $\tilde{\theta}$  being positively correlated.

---

<sup>2</sup>Prices will be extreme in states  $\omega_1$  and  $\omega_4$  if agents are not indifferent between the asset and money in these states. This can be ensured by choosing parameters appropriately.

Consider first the case where  $z = 0$ . We wish to ensure there is a unique equilibrium price with  $P_0 \neq P_1$ . We specifically look for an equilibrium in which uninformed agents prefer the asset when supply is high, i.e.  $\tilde{x} = x_1$ , and money when supply is low, i.e.  $\tilde{x} = x_0$ . Market clearing implies

$$x^I(\tilde{\theta}, P) + x^U(P) + \frac{w}{P} - \tilde{x} = 1. \quad (3)$$

Since  $x^I(\tilde{\theta}, P) + x^U(P) \geq 0$ , we can rearrange (3) to conclude that the price must be at least  $\frac{w}{x_0+1}$  when  $\tilde{x} = x_0$ . Uninformed agents would strictly prefer money to the asset upon learning  $\tilde{x} = x_0$  if the conditional expectation  $E[\tilde{\theta} \mid \tilde{x} = x_0]$  were less than the price when  $\tilde{x} = x_0$ . Hence, if

$$E[\tilde{\theta} \mid \tilde{x} = x_0] = \frac{\pi_1 \bar{\theta} + \pi_3 \underline{\theta}}{\pi_1 + \pi_3} < \frac{w}{x_0 + 1}, \quad (4)$$

uninformed traders would prefer money if they learned  $\tilde{x} = x_0$ . In that case, (3) implies that  $P_0 = \frac{w}{x_0+1}$ . Similarly, since  $x^I(\tilde{\theta}, P) + x^U(P) \leq \frac{1}{P}$ , the price when  $\tilde{x} = x_1$  is at most  $\frac{w+1}{x_1+1}$ . If the equilibrium price reveals  $\tilde{x} = x_1$ , we can be assured that uninformed agents would prefer the asset if

$$E[\tilde{\theta} \mid \tilde{x} = x_1] = \frac{\pi_2 \bar{\theta} + \pi_4 \underline{\theta}}{\pi_2 + \pi_4} > \frac{w + 1}{x_1 + 1} \quad (5)$$

It then follows from (3) that  $P_1$  must equal  $\frac{w+1}{x_1+1}$  in equilibrium. As long as

$$\frac{w}{x_0 + 1} \neq \frac{w + 1}{x_1 + 1} \quad (6)$$

we are assured that when  $z = 0$ , there is a unique equilibrium with  $P_0 \neq P_1$ . To ensure there is no other equilibrium in which  $P_0 = P_1$ , we further require

$$E[\tilde{\theta}] = (\pi_1 + \pi_2) \bar{\theta} + (\pi_3 + \pi_4) \underline{\theta} > \frac{w + 1}{x_0 + 1} \quad (7)$$

Under (7), the unconditional expectation of  $\tilde{\theta}$  exceeds the highest possible market clearing price. Hence, if there were an equilibrium in which  $P_0 = P_1$ , uninformed traders would prefer to buy the asset in that equilibrium. But if this is their demand, market clearing would imply  $P_0 \neq P_1$ , a contradiction. In sum, conditions (4) through (7) together imply that when  $z = 0$ , there is a unique equilibrium price that conveys information according to (1).

Next, we provide conditions which imply that there exists some  $z > 0$  such for which  $P(\omega_2) = P(\omega_3)$  is an equilibrium. We look for an equilibrium in which uninformed agents buy the asset at this common price. Suppose

$$E[\tilde{\theta} \mid \omega \in \{\omega_2, \omega_3\}] = \frac{\pi_2 \bar{\theta} + \pi_3 \underline{\theta}}{\pi_2 + \pi_3} > \frac{w + 1}{x_0 + 1} \quad (8)$$

This condition says that the market clearing price if all agents were to buy the asset in state  $\omega_3$  is less than the expected value of the asset. Hence, uninformed agents would wish to buy the asset in this state. If we sharpen (6) to require that there exists a particular  $z^* \in (0, 1)$  such that

$$\frac{w + 1 - z^*}{x_0 + 1} = \frac{w + 1}{x_1 + 1} \quad (9)$$

we can ensure there exists an equilibrium in which  $P(\omega_2) = P(\omega_3)$  when  $z = z^*$ , with the common price given by the expression in (9).

To verify that conditions (4) through (9) are compatible with  $\tilde{x}$  and  $\tilde{\theta}$  being positively correlated, consider the case where  $\pi_1 = \pi_4 = 0$ . In this case, conditions (4) through (9) reduce to the following:

$$\begin{aligned} \text{(i)} \quad \underline{\theta} &< \frac{w+1}{x_1+1} = \frac{w+1-z^*}{x_0+1} \quad \text{for some } z^* \in (0, 1) \\ \text{(ii)} \quad \bar{\theta} &> \frac{w}{x_0+1} \end{aligned} \tag{10}$$

It is easy to find parameters satisfying these conditions. As an example, let  $w = 10$ ,  $x_0 = 0.5$  and  $x_1 = 0.51$ , so  $z^* = 0.1$ . These values imply  $\frac{w+1}{x_1+1} = 7.27$  and  $\frac{w}{x_0+1} = 6.67$ . Now, set  $\underline{\theta} = 5$  and  $\bar{\theta} = 10$  to satisfy (10). If  $\pi_1 = \pi_4 = 0$ , the unique equilibrium when  $z = 0$  involves  $P(\omega_2) = \frac{w+1}{x_1+1} = 7.27$  and  $P(\omega_3) = \frac{w}{x_0+1} = 6.67$ ; uninformed traders hold the asset in state  $\omega_2$  and money in state  $\omega_3$ . Since agents are acting optimally, there is no value to learning  $\tilde{\theta}$ . The only other potential equilibrium is when  $z = z^*$ , in which case  $P(\omega_2) = \frac{w+1}{x_1+1} = 7.27 = \frac{w+1-z^*}{x_0+1} = P(\omega_3)$ ; at this price, uninformed traders prefer the asset. Since uninformed traders end up buying an overvalued asset in state  $\omega_3$ , they would benefit from learning  $\tilde{\theta}$ . In particular, their expected gain would equal

$$\pi_3 (1 - \underline{\theta}/P(\omega_3)) \tag{11}$$

which is strictly positive as long as  $\pi_3 > 0$ . Since  $z^* \in (0, 1)$ , agents must be indifferent between learning  $\tilde{\theta}$  and not learning. Hence, for  $z^*$  to be an equilibrium, the cost of information  $c$  must exactly equal the value of information in (11). Under this assumption, the complementarity in learning allows multiple equilibria: either no agents are informed or a fraction  $z^*$  of agents are informed. Note that conditions (4) through (9) would continue to hold for these parameters even if we slightly increased  $\pi_1$  and  $\pi_4$  and slightly decreased  $\pi_2$  and  $\pi_3$ , so the value of information can be higher when  $z = z^*$  than when  $z = 0$  even when  $\tilde{x}$  and  $\tilde{\theta}$  are imperfectly positively correlated.

## 2 Correlation as a Source of Complementarities

The previous section derived sufficient conditions in the  $2 \times 2$  case for the value of information to be higher when some fraction of agents are informed than when none are. The advantage of the  $2 \times 2$  case is that it yields distinct information sets  $\Omega_z$  for  $z = 0$  and  $z > 0$  (expressions (1) and (2), respectively) that show prices need not convey more information when more traders are informed. However, the discrete nature of this example can make the results seem special and knife-edge, especially since an equilibrium price only exists at two isolated values of  $z$ . We now argue that allowing noisy supply and fundamentals to be positively correlated inherently captures the scenario we had attempted to model in our 2000 paper, namely that when some agents acquire information they can change prices in such a way that makes it more difficult for remaining traders to infer the fundamentals from the price of the asset. We confirm this by demonstrating that correlation between noisy supply and fundamentals will lead to complementarities when  $\tilde{x}$  has a continuous distribution, as well as when both  $\tilde{x}$  and  $\tilde{\theta}$  are continuously distributed as in Grossman and Stiglitz (1980).

In our original paper we questioned whether informed traders would generally cause prices to be more informative as in the Grossman and Stiglitz (1980) model. A key feature of their model is that as more agents became informed, certain prices became more likely to be associated with certain fundamentals, e.g. very high prices would essentially emerge only when fundamentals were favorable.

We conjectured there might be circumstances in which this was not the case. A positive correlation between noisy supply and fundamentals turns out to be just such a circumstance. When the two variables are positively correlated,  $\tilde{x}$  and  $\tilde{\theta}$  are likely to either both be high or both be low. When few traders are informed, prices will largely depend on, and generally vary with,  $\tilde{x}$ . As a result, different realizations of  $\tilde{\theta}$  are likely to be associated with distinct prices, namely the prices associated with the most common realizations of  $\tilde{x}$  for a given  $\tilde{\theta}$ . By contrast, when many traders are informed, prices will depend on both  $\tilde{\theta}$  and  $\tilde{x}$ . Since the two variables have offsetting effects on the price – high values of  $\tilde{\theta}$  increase the price while high values of  $\tilde{x}$  decrease it – different realizations of  $\tilde{\theta}$  will tend to be associated with similar prices. It is then harder to infer  $\tilde{\theta}$  from prices. Nothing in this intuition relies on the idiosyncratic features of the  $2 \times 2$  case.

Before we confirm that our mechanism can occur in more general settings, it is worth commenting whether it is reasonable to allow noise to be correlated with fundamentals. Grossman and Stiglitz originally assumed the two are independent, and their specification was subsequently adopted by others. However, the motivation for this assumption was convenience rather than to capture some essential feature of the economic environment being modelled. In fact, there are various circumstances in which the non-fundamental forces that noise trading is meant to capture will naturally be correlated with fundamental forces. For example, one justification for noise trading is liquidity: agents sell assets not in response to changes in the fundamentals but because they need immediate cash. Under this interpretation, one might expect that over the business cycle, fundamentals will be low when liquidity demands are high. We even appealed to this scenario in our 2000 paper to motivate our parameter choices. Ironically, the proper example for complementarities we provide here relies on the opposite correlation. But there are circumstances in which a positive correlation is more appropriate than a negative one, e.g. if there is a strong hedging component to the demand for assets, or if financing frictions lead firms to issue more equity during favorable times. Indeed, one way to motivate the correlation in the  $2 \times 2$  example (and the generalization we discuss below) is to assume that a fraction of those who initially own the asset have access to a private technology with a rate of return  $R$ . If more private projects tend to be available in good times, a seemingly plausible assumption, then this fraction will tend to be higher when  $\tilde{\theta}$  is high, and as long as  $R$  is sufficiently large those traders who have such opportunities will sell their stock holdings to invest in the private technology. This implies the supply shock  $\tilde{x}$  will be positively correlated with  $\tilde{\theta}$ .

To demonstrate that the complementarity in the previous section does not hinge on the special features of the  $2 \times 2$  example, consider the case where  $\tilde{x}$  has a continuous distribution. Since  $\tilde{x}$  and  $\tilde{\theta}$  can be correlated, the distribution of  $\tilde{x}$  will generally vary with the realization  $\tilde{\theta}$ . To ensure the two will be positively correlated, we assume  $f(x|\tilde{\theta} = \bar{\theta})$  is increasing in  $x$  and  $f(x|\tilde{\theta} = \underline{\theta})$  is decreasing in  $x$ . Thus, a high value for  $\tilde{\theta}$  tends to be associated with higher realizations of  $\tilde{x}$ . In contrast to the  $2 \times 2$  case, the equilibrium price will now be defined for any  $z \in [0, 1]$ , and as we now show it will be possible for the value of information to increase in  $z$  over a range rather than at two distinct points.

To simplify matters, suppose  $\tilde{\theta}$  equals  $\bar{\theta}$  and  $\underline{\theta}$  with probability  $\frac{1}{2}$  each, so the unconditional expectation of  $\tilde{\theta}$  is  $\frac{\underline{\theta} + \bar{\theta}}{2}$ . We further impose a symmetry assumption on the conditional density for  $x$  given  $\tilde{\theta}$ : Let  $x^*$  denote the value of  $\tilde{x}$  for which the market clearing price equals  $\frac{\underline{\theta} + \bar{\theta}}{2}$  when half of the wealth of non-noise traders is allocated to buying the asset, i.e.  $x^*$  solves

$$\frac{w + 1/2}{x^* + 1} = \frac{\underline{\theta} + \bar{\theta}}{2}.$$

We assume the support of  $x$  is restricted to  $[0, 2x^*]$ , and that

$$f(x|\tilde{\theta} = \bar{\theta}) = f(2x^* - x|\tilde{\theta} = \underline{\theta})$$

This assumption implies the following will be an equilibrium. Uninformed agents hold money if the price of the asset exceeds  $\frac{\theta + \bar{\theta}}{2}$ , hold the asset if the price is below  $\frac{\theta + \bar{\theta}}{2}$ , and are indifferent between the two if the price equals  $\frac{\theta + \bar{\theta}}{2}$ . The price  $P(x, \theta)$  is the unique function consistent with the market clearing condition given this demand, and is given by

$$P(x, \bar{\theta}) = \begin{cases} \frac{w}{x+1} & \text{if } x+1 \leq \frac{w}{\bar{\theta}} \\ \bar{\theta} & \text{if } x+1 \in \left( \frac{w}{\bar{\theta}}, \frac{w+z}{\bar{\theta}} \right) \\ \frac{w+z}{x+1} & \text{if } x+1 \in \left[ \frac{w+z}{\bar{\theta}}, \frac{2(w+z)}{\bar{\theta} + \bar{\theta}} \right] \\ \frac{\theta + \bar{\theta}}{2} & \text{if } x+1 \in \left( \frac{2(w+z)}{\bar{\theta} + \bar{\theta}}, \frac{2(w+1)}{\bar{\theta} + \bar{\theta}} \right) \\ \frac{w+1}{x+1} & \text{if } x+1 \geq \frac{2(w+1)}{\bar{\theta} + \bar{\theta}} \end{cases} \quad P(x, \underline{\theta}) = \begin{cases} \frac{w}{x+1} & \text{if } x+1 \leq \frac{2w}{\theta + \bar{\theta}} \\ \frac{\theta + \bar{\theta}}{2} & \text{if } x+1 \in \left( \frac{2w}{\theta + \bar{\theta}}, \frac{2(w+1-z)}{\theta + \bar{\theta}} \right) \\ \frac{w+1-z}{x+1} & \text{if } x+1 \in \left[ \frac{2(w+1-z)}{\theta + \bar{\theta}}, \frac{w+1-z}{\underline{\theta}} \right] \\ \underline{\theta} & \text{if } x+1 \in \left( \frac{w+1-z}{\underline{\theta}}, \frac{w+1}{\underline{\theta}} \right) \\ \frac{w+1}{x+1} & \text{if } x+1 \geq \frac{w+1}{\underline{\theta}} \end{cases}$$

An implication of this price function is that for any  $p \neq \frac{\theta + \bar{\theta}}{2}$  between  $\underline{\theta}$  and  $\bar{\theta}$ , the equilibrium price is such that there exists a unique  $x_0(p)$  such that  $P(x_0(p), \underline{\theta}) = p$  and a unique  $x_1(p)$  such that  $P(x_1(p), \bar{\theta}) = p$ . Using the monotonicity of  $f(x|\tilde{\theta} = \theta)$  and the direction of asymmetry between  $x_0(p) - x^*$  and  $x_1(p) - x^*$  for  $p > \frac{\theta + \bar{\theta}}{2}$  and  $p < \frac{\theta + \bar{\theta}}{2}$ , it follows that

$$\Pr(\theta = \bar{\theta} \mid P(\cdot, \cdot) = p) \gtrless \frac{1}{2} \text{ for } p \lesseqgtr \frac{\theta + \bar{\theta}}{2}$$

This confirms that the demand schedule of uninformed is optimal given these prices. To generate complementarities, we need a sufficiently strong positive correlation between  $\theta$  and  $x$ , which amounts to a restriction on the rate at which  $f(x|\tilde{\theta} = \underline{\theta})$  decreases with  $x$ . One example of a function that gives rise to complementarities is the truncated normal, i.e.

$$f(x|\tilde{\theta} = \underline{\theta}) = \frac{e^{-\frac{x^2}{2\sigma^2}}}{\sigma\sqrt{2\pi} [\Phi(2x^*) - \frac{1}{2}]} \text{ for } x \in [0, 2x^*]$$

where  $\Phi(\cdot)$  denotes the CDF of a normal with mean 0 and variance  $\sigma^2$ . The correct expression for the value of information, as described by Chamley (2007), is given by

$$g(z) = \frac{1}{2} \int_{\frac{2w}{\theta + \bar{\theta}} - 1}^{\frac{w+1-z}{\underline{\theta}} - 1} \left[ 1 - \frac{\underline{\theta}}{P(x, \underline{\theta})} \right] f(x|\tilde{\theta} = \underline{\theta}) dx + \frac{1}{2} \int_{\frac{w+z}{\bar{\theta}} - 1}^{\frac{2(w+z)}{\theta + \bar{\theta}} - 1} \left[ \frac{\bar{\theta}}{P(x, \bar{\theta})} - 1 \right] f(x|\tilde{\theta} = \bar{\theta}) dx \quad (12)$$

Figure 1 illustrates  $g(z)$  for the following parameter values:  $\bar{\theta} = 1.0$ ,  $\underline{\theta} = 0.9$ ,  $w = 1.1$ , and  $\sigma = 0.2$ . The value of information is increasing as  $z$  ranges from 0 to about 0.5 and is decreasing in  $z$  thereafter. This reflects the interaction of the two opposing forces. On the one hand, as  $z$  increases, the distribution of prices when  $\theta = \bar{\theta}$  concentrates mass on the same prices as the distribution of prices when  $\theta = \underline{\theta}$ . This force would be present whenever the conditional density function for high (low) values of  $\theta$  concentrated mass on high (low) values of  $x$ , and would tend to make information about fundamentals more valuable. At the same time, the value of using the information to trade optimally shrinks as prices are pushed closer to fundamentals and the payoff on the asset converges to that on money, and this force eventually causes the value of information to decrease with  $z$ .

Nothing in the above argument hinges on the distribution of  $\tilde{\theta}$  being discrete. Indeed, a recent short note by Ganguli and Yang (2007) shows that if noise and fundamentals are positively correlated,

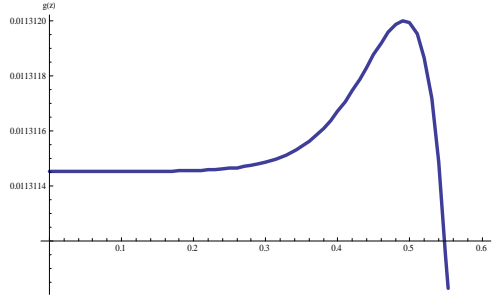


Figure 1: The function  $g(z)$

information acquisition will be a strategic complement in the original Grossman and Stiglitz framework, where both  $\tilde{x}$  and  $\tilde{\theta}$  have continuous distributions. Suppose agents have negative exponential utility  $U(c) = -e^{-\gamma c}$  rather than linear preferences as in our framework. Suppose further that the dividend on the asset is  $\tilde{\theta} + \tilde{\varepsilon}$  where  $\tilde{\theta}$  and  $\tilde{\varepsilon}$  are i.i.d. normal with mean zero and variances  $\sigma_{\tilde{\theta}}^2$  and  $\sigma_{\tilde{\varepsilon}}^2$  respectively. The supply of the asset  $\tilde{x}$  is equal to  $\rho\tilde{\theta} + \tilde{y}$  where  $\tilde{y}$  is independent of all other variables and is itself normally distributed with mean zero and variance  $\sigma_{\tilde{y}}^2$ . Ganguli and Yang show that the value of becoming informed is increasing in  $z$  over the interval  $[0, \rho\gamma\sigma_{\tilde{\varepsilon}}^2)$  and decreasing otherwise. This result reflects the same mechanism, namely that when the two variables are correlated, informed traders can act to make the distribution of prices for different fundamentals more similar rather than more distinct, and this in turn can result in learning being a strategic complement even as prices are pushed closer to their fundamental values.

### 3 References

Barlevy, Gadi and Pietro Veronesi, 2000. “Information Acquisition in Financial Markets” *Review of Economic Studies*, 67(1), p79-90.

Chamley, Christophe, 2006. “Complementarities in Information Acquisition with Short-Term Trades” Mimeo (first version: 2005)

Chamley, Christophe, 2007. “Strategic Substitutability in ‘Information Acquisition in Financial Markets’” forthcoming in *Review of Economic Studies*.

Ganguli, Jayant and Liyan Yang, 2006. “Supply Signals, Complementarities, and Multiplicity in Asset Prices and Information Acquisition” Mimeo, Cornell University.

Ganguli, Jayant and Liyan Yang, 2007. “Strategic Complementarities with Correlated Noise and Fundamentals” Mimeo, Cornell University.

Veldkamp, Laura, 2006. “Media Frenzies in Markets for Financial Information” *American Economic Review*, 96(3), p577-601.