Technical appendix to accompany

Stock-based Compensation and CEO (Dis)Incentives

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Appendix A. Proofs

Proof of Proposition 1. The capital evolution equation is given by

$$\frac{dK_t}{dt} = I_t - \delta K_t. \tag{A1}$$

From (2), the target level of capital, J_t, is given by $J_t = e^{Gt}$ for $t < \tau^*$ and $J_t = e^{G\tau^* + g(t-\tau^*)}$ for $t \geq \tau^*$. Imposing $K_t = J_t$ for every t and using (48) the optimal investment policy is given by (8). From (7), the dividend stream is (9). **Q.E.D.**

Proof of Proposition 2. For $t \geq \tau^*$, the $P_{fi,t}^{after}$ stems from integration of future dividends. For $t < \tau^*$, the expectation in $P_{fi,t}^{before}$ can be computed by integration by parts.**Q.E.D**

Proof of Corollary 1: $P_{fi,t}^{before}\left(\lambda^H\right) > P_{fi,t}^{before}\left(\lambda^L\right)$ iff $A_{\lambda^H}^{fi} > A_{\lambda^L}^{fi}$. Substituting, this relation holds iff $z - r - \delta > 0$, which is always satisfied. **Q.E.D.**

Proof of Corollary 2: The first part immediately follows the fact that a manager/owner values the firm as the present value of future dividends discounted at β. The second part follows the utility of the manager at τ^* and $t < \tau^*$. First, after τ^*, there is no benefit from exerting effort. Thus the manager/owner's utility is:

$$U_{Div,\tau^*} = \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)} D_t^g dt = \frac{(z-g-\delta)}{(\beta-g)} e^{G\tau^*}$$

Before τ^*, the utility of the manager for given effort e is

$$\begin{aligned} U_{Div,t}(e) &= E\left[\int_t^{\tau^*} e^{-\beta(u-t)} D_u^G \left(1-c(e)\right) du + e^{-\beta(\tau^*-t)} U_{Div,\tau^*}\right] \\ &= e^{Gt} \frac{(z-G-\delta)}{\beta+\lambda(e)-G}\left[1-c(e)+\lambda\left(e\right)\frac{(z-g-\delta)}{(z-G-\delta)(\beta-g)}\right] \end{aligned}$$

Given H^{Div} in (16), the condition $U_{Div,t}\left(e^H\right) > U_{Div,t}\left(e^L\right)$ translates into (38). **Q.E.D.**

Proof of Proposition 3: As in Corollary 2, from τ^* onward the manager will not exercise high effort, resulting in a utility level at τ^* given by

$$U_{Stock,\tau^*} = \int_{\tau^*}^{\infty} e^{-\beta(s-t)} \left(\eta P_{fi,s}^{after}\right) ds = \frac{\eta\left(z-g-\delta\right)}{(r-g)\left(\beta-g\right)} e^{G\tau^*}.$$

Thus, for $t < \tau^*$ we have

$$\begin{aligned} U_{Stock,t}\left(e\right) = E&\left[\int_t^{\tau^*} e^{-\beta(s-t)}\left(\eta P_{fi,s}^{before}\right)\left(1-c(e)\right) ds + e^{-\beta(\tau^*-t)} U_{Stock,\tau^*}\right] \\ &= \frac{\eta e^{Gt}}{\beta+\lambda(e)-G}\left[A_\lambda^{fi}\left(1-c(e)\right)+\lambda(e)\left(\frac{z-g-\delta}{(r-g)\left(\beta-g\right)}\right)\right]. \end{aligned}$$

Let e^H be the optimal strategy in equilibrium. The price function is then $P_{fi,t}^{before}$ with $A_{\lambda^H}^{fi}$. We then obtain the condition $U_{Stock,t}\left(e^H\right) > U_{Stock,t}\left(e^L\right)$ iff (19) holds. The Nash equilibrium follows. Similarly, if e^L is the optimal strategy in equilibrium, then the price function is $P_{fi,t}^{before}$ with $A_{\lambda^L}^{fi}$. Thus, $U_{Stock,t}\left(e^H\right) < U_{Stock,t}\left(e^L\right)$ iff (19) does not hold. **Q.E.D.**

Proof of Lemma 1. Conditional on the decision to conceal g, the manager must provide a dividend stream D_t^G, as any deviation make her lose her job. Since she cannot affect the stock price, after τ^* her utility only depends on the length of her tenure. Since we normalize the manager's outside options to zero, her optimal choice is to maximize T^{**}. **Q.E.D.**

Proof of Proposition 4: (1) The manager must mimic D_t^G for as long as possible. From (7) this target determines the investments I_t (in point (2) of the proposition) and thus the evolution of capital $\frac{dK_t}{dt} = I_t - \delta K_t$ for given initial condition $\widehat{K}_{\tau^*}$. From the monotonicity properties of differential equations in their initial value and the definition of T^{**} as the time at which $K_{T^{**}} = \underline{K}_{T^{**}} = \xi J_{T^{**}}$, T^{**} must be increasing with $\widehat{K}_{\tau^*}$. The claim follows from Lemma 1.

(3) At time τ^* we have $K_{\tau^*} = J_{\tau^*} = e^{G\tau^*}$. Thus, inserting I_t from point (2) of the proposition in the capital evolution equation, we have

$$\frac{dK_t}{dt}|_{\tau^*} = ze^{G\tau^*} - \delta e^{G\tau^*} - (z - G - \delta)\, e^{G\tau^*} = Ge^{G\tau^*}$$

This implies that $dK_t/dt > dJ_t/dt$ after the switch, and thus $K_{\tau^*+dt} > J_{\tau^*+dt}$. The trajectory of capital at τ^* is then above J_t. By continuity, there is a period $[0, t_1]$ in which $K_t > J_t$. During this period, the ODE for capital accumulation becomes:

$$\frac{dK_t}{dt} = ze^{G\tau^* + g(t-\tau^*)} - \delta K_t - (z - G - \delta)\, e^{Gt}$$

Given the initial condition $K_{\tau^*} = J_{\tau^*} = e^{G\tau^*}$, the ODE solution implies the excess capital:

$$K_t - J_t = e^{G\tau^*}\left[\left(\frac{z-\delta-g}{\delta+g}\right)\left(e^{g(t-\tau^*)} - e^{-\delta(t-\tau^*)}\right) - \frac{z-G-\delta}{\delta+G}(e^{G(t-\tau^*)} + e^{-\delta(t-\tau^*)})\right]$$

As t increases, $K_t - J_t \to -\infty$. Since $K_{\tau^*+dt} - J_{\tau^*+dt} > 0$, there must be a t_1 at which $K_{t_1} - J_{t_1} = 0$. Since $t_1 > \tau^*$, we can define $h^* \equiv t_1 - \tau^*$. Substituting in $K_{t_1} - J_{t_1} = 0$, h^* must satisfy:

$$0 = \left(\frac{z-g-\delta}{\delta+g}\right) e^{-Gh^*}\left(e^{gh^*} - e^{-\delta h^*}\right) + \left(e^{-(\delta+G)h^*} - 1\right)\left[\frac{z-G-\delta}{\delta+G}\right] \tag{A2}$$

For $t > t_1$, $K_t < J_t$, and thus the ODE switches to

$$\frac{dK_t}{dt} = (z-\delta)\, K_t - (z - G - \delta)\, e^{Gt}$$

Given the initial condition $K_{t_1} = J_{t_1}$, the ODE solution yields

$$K_t - J_t = e^{G\tau^*} e^{G(t-t_1+h^*)}\left[1 + \left(e^{-(G-g)h^*} - 1\right) e^{(z-G-\delta)(t-t_1)} - e^{-(G-g)(t-t_1+h^*)}\right]$$

which again diverges to $-\infty$ as $t \to \infty$. From the condition $K_{T^{**}} - J_{T^{**}}\xi = 0$, and defining $h^{**} \equiv T^{**} - \tau^*$, we obtain the equation defining h^{**}:

$$0 \quad = 1 + \left(e^{-(G-g)h^*} - 1\right) e^{(z-G-\delta)(h^{**}-h^*)} - e^{-(G-g)h^{**}}\xi \tag{A3}$$

Q.E.D.

Proof of Proposition 5: Let $t > h^{**}$. If a cash shortfall has not been observed by t, then a shift cannot have occurred before $t - h^{**}$. Bayes formula implies that time $T^{**} = \tau^* + h^{**}$ conditional on not observing a cash shortfall by time t has the following conditional distribution

$$F_{T^{**}}(t'|T^{**} > t) = Pr\left(\tau^* < t' - h^{**}|\tau^* > t - h^{**}\right) = \frac{e^{-\lambda(t-h^{**})} - e^{-\lambda(t'-h^{**})}}{e^{-\lambda(t-h^{**})}} = 1 - e^{-\lambda(t'-t)}$$

That is, T^{**} has the exponential distribution $f(T^{**}|\text{no cash shortfall by } t) = \lambda e^{-\lambda(T^{**}-t)}$. The value of $P_{ai,t}$ for $t > h^{**}$ in (22) then follows from the pricing formula (21) and integration by parts.

Let $t < h^{**}$, then the conditional distribution of T^{**} is zero in the range $[t, h^{**}]$, as even a shift at 0 would only be revealed at h^{**}. The density is then $f(T^{**}) = \lambda e^{-\lambda(T^{**}-h^{**})} 1_{(T^{**}>h^{**})}$. Using this density to compute the expectation, we find

$$P_{ai,t} = (z - G - \delta)e^{Gt}\frac{1 - e^{-(r-G)(h^{**}-t)}}{(r - G)} + e^{rt}e^{(G-r)h^{**}}A_\lambda^{ai} \tag{A4}$$

Q.E.D

Proof of Proposition 6: Let $\tau^* > h^{**}$. There are two equilibria to consider: a reveal equilibrium and a conceal equilibrium. In both equilibria, if the manager reveals at τ^*, then her utility depends $P_{fi,t}^{after}$ in equation (11). In contrast, the price path is different under the conceal strategy, depending on the equilibrium: In a conceal equilibrium, investors' expect the manager to conceal and thus her utility depends on $P_{ai,t}$ in (22) until T^{**}. In a reveal equilibrium, then investors expect the manager to reveal and thus her utility under the conceal strategy depends on $P_{fi,t}^{before}$ in (11). We then obtain the utility functions in (24), (25), and (26). A reveal equilibrium is obtained if $U_{Stock,\tau^*}^{reveal} > U_{Stock,\tau^*}^{conceal,fi}$ and a conceal equilibrium obtains if $U_{Stock,\tau^*}^{conceal,ai} > U_{Stock,\tau^*}^{reveal}$. The conditions in the claim are obtained by simple substitution.

Finally, if $\tau^* < h^{**}$, then $U_{Stock,\tau^*}^{conceal,ai}$ depends on the price in (51). We can write

$$U_{Stock,\tau^*}^{conceal,ai} = e^{\beta\tau^*}\int_{\tau^*}^{h^{**}} e^{-\beta t}\eta_p P_{ai,t}dt + e^{\beta\tau^*}\int_{h^{**}}^{T^{**}} e^{-\beta t}\eta_p P_{ai,t}dt.$$

Using the two formulas for $P_{ai,t}$ before and after h^{**}, and taking the integral, tedious calculations show:

$$U^{conceal,ai}_{Stock,\tau^*} = \eta_p \frac{e^{\beta\tau^*}(z-G-\delta)}{r-G}\left(\frac{e^{-(\beta-G)\tau^*}-e^{-(\beta-G)h^{**}}}{\beta-G} - e^{-(r-G)h^{**}}\frac{e^{-(\beta-r)\tau^*}-e^{-(\beta-r)h^{**}}}{\beta-r}\right)$$
$$+\eta_p e^{\beta\tau^*+Gh^{**}}A^{ai}_{\lambda}\left[e^{-rh^{**}}\frac{e^{-(\beta-r)\tau^*}-e^{-(\beta-r)h^{**}}}{\beta-r} + e^{-\beta h^{**}}\frac{1-e^{-(\beta-G)\tau^*}}{\beta-G}\right]$$

Note that at $\tau^* = h^{**}$ we have

$$U^{conceal}_{\tau^*} = e^{\beta\tau^{**}}\eta_p A^{ai}_{\lambda} e^{-(\beta-G)h^{**}}\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G} = \eta_p A^{ai}_{\lambda} e^{Gh^{**}}\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G},$$

which equals the one above for $\tau^* \geq h^{**}$, so that it converges there. **Q.E.D.**

Proof of Proposition 7: Let $t \geq h^{**}$. In a conceal equilibrium with high effort, $P_{ai,t}$ in (22) with $A^{ai}_{\lambda^H}$ determines the wage $w_t = \eta P_{ai,t}$. The expected utility under effort e is:

$$\begin{aligned} U_{Stock,t}(e) &= E\left[\int_t^{\tau^*} e^{-\beta(s-t)}w_s(1-c(e))ds + e^{-\beta(\tau^*-t)}U^{conceal,ai}_{Stock,\tau^*}\right] \\ &= e^{Gt}\eta A^{ai}_{\lambda^H}\frac{\left[1-c(e)+\lambda(e)H^{Stock}\right]}{\beta+\lambda(e)-G} \end{aligned}$$

where $\lambda(e)$ and $c(e)$ are the intensity and the cost of effort under effort choice e. The condition in the Proposition follows from the maximization condition $U_{Stock,t}\left(e^H\right) > U_{Stock,t}\left(e^L\right)$. Finally, given e^H chosen by the manager, then indeed λ^H applies in equilibrium, a conceal equilibrium obtains at τ^* and thus the price function is $P_{ai,t}$ in (22), concluding the proof.

A similar proof holds for $t < h^{**}$. In this case, we obtain

$$\begin{aligned} U_{Stock,t}(e) &= \int_t^{\infty} e^{-(\beta+\lambda(e))(u-t)}\left(\eta P_{ai,u}(1-c(e))\right) + \lambda(e)\, e^{-(\beta+\lambda(e))(u-t)}U^{conceal,ai}_{Stock,u}du \\ &= \int_t^{h^{**}} e^{-(\beta+\lambda(e))(u-t)}\left[\left(\eta P_{ai,u}(1-c(e))\right) + \lambda(e)\,U^{conceal,ai}_{Stock,u}\right]du \\ &\quad + e^{-(\beta+\lambda(e))(h^{**}-t)}U_{Stock,h^{**}}(e) \end{aligned}$$

where $U_{Stock,h^{**}}(e)$ is the utility computed in the first part for $t = h^{**}$. We solve this expression numerically, and check $U_t(e^H) > U_t(e^L)$. **Q.E.D.**

Proof of Proposition 8: (a) If $t \geq k$, then $\tau^* \geq k$. Thus, $w_t = \eta P_{t-k}$ implies the utility from revealing and concealing under the reveal equilibrium are

$$
\begin{aligned}
U^{\text{Reveal}}_{\text{Delayed},\tau^*} &= \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)} w_t dt = \int_{\tau^*}^{\tau^*+k} e^{-\beta(t-\tau^*)} \eta P^{before}_{t-k} dt + \int_{\tau^*+k}^{\infty} e^{-\beta(t-\tau^*)} \eta P^{after}_{t-k} dt \\
&= \eta e^{G(\tau^*-k)} \left(A^{fi}_{\lambda} \frac{1-e^{-(\beta-G)k}}{\beta-G} + \left(\frac{z-g-\delta}{(r-g)(\beta-g)} \right) e^{-(\beta-G)k} \right) \\
U^{\text{Conceal}}_{\text{Delayed},\tau^*} &= \int_{\tau^*}^{T^{**}} e^{-\beta(t-\tau^*)} w_t dt = \int_{\tau^*}^{\tau^*+h^{**}} e^{-\beta(t-\tau^*)} \eta P^{before}_{t-k} dt \\
&= \eta e^{G(\tau^*-k)} A^{fi}_{\lambda} \frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}
\end{aligned}
$$

Thus, algebra shows that $U^{\text{Reveal}}_{\text{Delayed},\tau^*} > U^{\text{Conceal}}_{\text{Delayed},\tau^*}$ iff

$$
A^{fi}_{\lambda} \frac{1-e^{-(\beta-G)(h^{**}-k)}}{\beta-G} < \left(\frac{z-g-\delta}{(r-g)(\beta-g)} \right)
$$

The claim follows from the definition of $\underline{k}$ in (31).

(b) Moving to $k < t < \tau^*$, we have for $i = H, L$, the ex-ante utility is

$$
\begin{aligned}
U^i_t &= E\left[\int_t^{\tau^*} e^{-\beta(s-t)} \eta P^{before}_{s-k} \left(1-c^i\right) ds + e^{-\beta(\tau^*-t)} U^{\text{Reveal}}_{Delayed,\tau^*} \right] \\
&= \frac{A^{fi}_{\lambda} \left(1-c^i\right) \eta e^{Gt} e^{-Gk}}{\beta+\lambda^i-G} + \eta \frac{\lambda^i e^{Gt}}{\beta+\lambda^i-G} \left[A^{fi}_{\lambda} e^{-Gk} \frac{1-e^{(\beta-G)k}}{\beta-G} + \left(\frac{z-g-\delta}{(r-g)(\beta-g)} \right) e^{-\beta k} \right]
\end{aligned}
$$

Using the definition of $\underline{k}$ in (31), and substituting for the last term in the parenthesis, tedious algebra shows that $U^H_t > U^L_t$ iff

$$
c^H \left[\frac{\beta+\lambda^L-G}{\beta+\lambda^H-G} \right] < \left(\lambda^L-\lambda^H\right) e^{-(\beta-G)(h^{**}-\underline{k}+k)}
$$

Choosing k as small as possible, we obtain condition (32). **Q.E.D.**

Proof of Proposition 9. We start with the reveal equilibrium. The proof follows from comparing the utility functions (33) and (34). Consider the first term in (33). For $t \in [\tau^*, \tau^*+k]$ we have

$$
P^{after}_{fi,t} - P^{before}_{fi,t-k} = \left(\frac{z-g-\delta}{r-g} \right) e^{G\tau^* + g(t-\tau^*)} - A^{fi}_{\lambda} e^{G(t-k)}
$$

Exploiting the definition of k^*_{fi}, algebra shows that if $k < k^*_{fi}$ then $P^{after}_{fi,t} - P^{before}_{fi,t-k} < 0$ for all $t \in [\tau^*, \tau^*+k]$. It follows that the first term in (33) is always zero for $k < k^*_{fi}$. Turning to the second term of (33), for $t > \tau^*+k$, we have

$$
\begin{aligned}
\int_{\tau^*+k}^{\infty} e^{-\beta(t-\tau^*)} \eta \max\left(P^{after}_{fi,t} - P^{after}_{fi,t-k}, 0\right) dt &= \eta \left(\frac{z-g-\delta}{r-g} \right) e^{(\beta+G-g)\tau^*} \int_{\tau^*+k}^{\infty} e^{-\beta t} \left(e^{gt} - e^{g(t-k)} \right) dt \\
&= \eta \left(\frac{z-g-\delta}{r-g} \right) e^{G\tau^*} \left(1-e^{-gk}\right) \frac{e^{-(\beta-g)k}}{\beta-g}
\end{aligned}
$$

Consider now the utility under a conceal strategy (34), which is a deviation under the reveal equilibrium. If the CEO conceals, his utility is

$$
\begin{aligned}
U_{Option,\tau^*}^{\text{Conceal},fi} &= \int_{\tau^*}^{\tau^*+h^{**}} e^{-\beta(t-\tau^*)}\eta \max\left(P_{fi,t}^{before} - P_{fi,t-k}^{before}, 0\right) dt \\
&= \eta A_\lambda^{fi}\left(1-e^{-Gk}\right) e^{G\tau^*} \frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}
\end{aligned}
$$

The second equality stems from noticing that $P_{fi,t}^{before} - P_{fi,t-k}^{before} = A_\lambda^{fi}\left(e^{Gt} - e^{G(t-k)}\right) = A_\lambda^{fi} e^{Gt}\left(1-e^{-Gk}\right) > 0$, which implies that the utility (34) is the same as with stock-based compensation, but with the adjustment $\left(1-e^{-Gk}\right)$. Finally, for a reveal Nash equilibrium to be supported we must have $U_{Stock,\tau^*}^{\text{Reveal},fi} > U_{Stock,\tau^*}^{\text{Conceal},fi}$, which yields

$$
\left(\frac{z-g-\delta}{r-g}\right)\left(1-e^{-gk}\right)\frac{e^{-(\beta-g)k}}{\beta-g} > A_\lambda^{fi}\left(1-e^{-Gk}\right)\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}
$$

Using the definition of k_{fi}^* some algebra shows that this condition is equivalent to

$$
\frac{\left(e^{gk}-1\right)}{\left(1-e^{-Gk}\right)}\left(\frac{\beta-G}{\beta-g}\right) > e^{Gk_{fi}^*} e^{\beta k}\left(1-e^{-(\beta-G)h^{**}}\right)
$$

For any k, as $g \to 0$, the left-hand-side converges to 0, while the right-hand-side is independent of g. The claim follows.

A similar proof holds for the conceal equilibrium. The two utilities at τ^* are

$$
\begin{aligned}
U_{Option,\tau^*}^{\text{Reveal,ai}} &= \int_{\tau^*}^{\tau^*+k} e^{-\beta(t-\tau^*)}\eta_{t-k} \max\left(P_t^{after} - P_{ai,t-k}, 0\right) dt \\
&\quad + \int_{\tau^*+k}^{\infty} e^{-\beta(t-\tau^*)}\eta_{t-k} \max\left(P_t^{after} - P_{t-k}^{after}, 0\right) dt \qquad \text{(A5)}\\
U_{Option,\tau^*}^{\text{Conceal,ai}} &= \int_{\tau^*}^{\tau^*+h^{**}} e^{-\beta(t-\tau^*)}\eta_{t-k} \max\left(P_{ai,t} - P_{ai,t-k}, 0\right) dt \qquad \text{(A6)}
\end{aligned}
$$

In particular, the formula under reveal strategy (a deviation in this equilibrium) is the same as in the previous part, but with A_λ^{ai} in place of A_λ^{fi} in the definition of k_{ai}^*. The utility under the (equilibrium) conceal strategy instead is

$$
U_{Option,\tau^*}^{\text{Conceal},ai} = \int_{\tau^*}^{\tau^*+h^{**}} e^{-\beta(t-\tau^*)}\eta P_{ai,t} - \int_{\tau^*}^{\tau^*+h^{**}} e^{-\beta(t-\tau^*)}\eta P_{ai,t-k} dt
$$

Thus, for $\tau^* > h^{**} + k$, the formula is the same as with stock-based compensation, but with the adjustment $\left(1-e^{-Gk}\right)$. It then follows that a conceal strategy is optimal if and only if

$$
\frac{\left(e^{gk}-1\right)}{\left(1-e^{-Gk}\right)}\left(\frac{\beta-G}{\beta-g}\right) < e^{Gk_{ai}^*} e^{\beta k}\left(1-e^{-(\beta-G)h^{**}}\right)
$$

which is always satisfied if g is small enough. The claim follows. **Q.E.D.**

Proof of Proposition 10: Under cash-flow based compensation, if the manager decides to reveal (resp. conceal) $\tilde{g}_{\tau^*} = g$, the dividend path is given by D_t^g (resp. D_t^G until T^{**}). The expected utilities are, respectively:

$$U_{Div,\tau^*}^{reveal} = \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)}(\eta_d D_t^g)dt = \eta_d e^{G\tau^*}\left(\frac{z-g-\delta}{\beta-g}\right) \quad \text{(A7)}$$

$$U_{Div,\tau^*}^{conceal} = \int_{\tau^*}^{T^*} e^{-\beta(t-\tau^*)}(\eta_d D_t^G)dt = \eta_d(z-G-\delta)e^{G\tau^*}\left(\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}\right) \quad \text{(A8)}$$

A conceal equilibrium obtains if $U_{Div,\tau^*}^{reveal} < U_{Div,\tau^*}^{conceal}$, otherwise a reveal equilibrium obtains. Condition (37) follows from $U_{Div,\tau^*}^{reveal} > U_{Div,\tau^*}^{conceal}$ by rearranging terms.

Let parameters be such that a reveal equilibrium obtains at τ^*. Then, for $t < \tau^*$, the manager with $w_t = \eta_d D_t$ has the same utility function as the manager/owner in Corollary 2, but scaled by η_d. It follows that the same incentives go trough in this case, and the same result as in Corollary 2 applies. **Q.E.D.**

Proof of Proposition 11. (a) Truthful revelation constraint. The two utilities under reveal and conceal are

$$U_{\tau^*}^{\text{Reveal}} = \int_{\tau^*}^{\infty} e^{-\beta(s-\tau^*)} w_s^a ds = \frac{A_a e^{(B_a+C_a)\tau^*}}{\beta - B_a} \quad \text{(A9)}$$

$$U_{\tau^*}^{\text{Conceal}} = \int_{\tau^*}^{T^{**}} e^{-\beta(s-\tau^*)} w_s^b ds = A_b e^{B_b\tau^*}\frac{1-e^{-(\beta-B_b)h^{**}}}{(\beta-B_b)} \quad \text{(A10)}$$

It follows $U_{\tau^*}^{\text{Reveal}} \geq U_{\tau^*}^{\text{Conceal}}$ for all τ^* iff conditions (43) are satisfied.

(b) High effort constraint. For $i = H, L$ and $t < \tau^*$ we can compute

$$\begin{aligned} U_t^i &= E_t\left[\int_t^{\infty} e^{-\beta(s-t)} w_s\left(1-c_s^i\right)ds|i\right] \\ &= E_t\left[\int_t^{\tau^*} e^{-\beta(s-t)} w_s^b\left(1-c_s^i\right)ds + \int_{\tau^*}^{\infty} e^{-\beta(s-t)} w_s^a ds|i\right] \end{aligned}$$

We obtain

$$U_t^i = \frac{A_b e^{B_b t}\left(1-c^i\right)}{(\beta+\lambda^i-B_b)} + \frac{\lambda^i A_a e^{(C_a+B_a)t}}{(\beta-B_a)\left(\beta+\lambda^i-(C_a+B_a)\right)}$$

Thus, the high effort constraint $U_t^H \geq U_t^L$ translates into

$$\frac{A_b e^{B_b t}\left(1-c^H\right)}{(\beta+\lambda^H-B_b)} + \frac{\lambda^H A_a e^{(C_a+B_a)t}}{(\beta-B_a)(\beta+\lambda^H-B_b)} \geq \frac{A_b e^{B_b t}}{(\beta+\lambda^L-B_b)} + \frac{\lambda^L A_a e^{(C_a+B_a)t}}{(\beta-B_a)(\beta+\lambda^L-B_b)}$$

Tedious algebra shows that this constraint is satisfied for all t iff conditions (44) are satisfied.

(c) Outside option constraint. For all t and τ^* we must have $U_t \geq U_t^O$. For $t < \tau^*$, the earlier constraints obtained above imply U_t^H is the relevant utility, which can be written as

$$U_t^H = \left[\frac{A_b\left(1-c^H\right)}{(\beta+\lambda^H-B_b)} + \frac{\lambda^H A_a}{(\beta-B_a)(\beta+\lambda^H-B_b)}\right]e^{B_b t}$$

where we impose the earlier restriction that $B_b = C_a + B_a$. We must have that for all t, $U_t^H \geq A_O e^{B_O\,t}$, which implies

$$\left[A_b\left(1-c^H\right)+\frac{\lambda^H A_a}{(\beta-B_a)}\right]\frac{e^{B_b t}}{(\beta+\lambda^H-B_b)} \geq A_O e^{B_O\,t}$$

This condition is satisfied for all t if and only if conditions in (45) are satisfied.

Similarly, for $t \geq \tau^*$ the constraints above imply that $U_t = U_t^{\text{Reveal}}$, given by (52). Thus, the outside option condition is

$$\frac{A_a}{\beta-B_a}e^{C_a\tau^*}e^{B_a t} \geq A_O e^{B_O t}$$

which is satisfied for all t and τ^* if and only if conditions (45) are satisfied. **Q.E.D.**

Next proposition obtains the conditions that guarantee a (High Effort/Reveal) equilibrium under the combined compensation package in equation (47).

Proposition A1: *Let $\omega^* \in [0,1]$ be such that*

$$\mathcal{L}_2 > \mathcal{L}_1(\omega^*)\left(\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}\right) \tag{A11}$$

$$\frac{\lambda^L+\beta-G}{\lambda^H+\beta-G} > \frac{\mathcal{L}_1(\omega^*)+\lambda^L\mathcal{L}_2}{(1-c^H)\,\mathcal{L}_1(\omega^*)+\lambda^H\mathcal{L}_2} \tag{A12}$$

where $\mathcal{L}_1(\omega)$ and $\mathcal{L}_2$ are in the Appendix. Then, the combined compensation $w = \omega^*\eta_p P_t + (1-\omega^*)\,\eta_d D_t$ achieves the first best (High Effort/Reveal) equilibrium.

Condition (A11) guarantees that "Reveal" is optimal at time τ^*, conditional the full information pricing function (11) with $\lambda = \lambda^H$. The second condition (A12) guarantees that "High Effort" is optimal at $t < \tau^*$, conditional on reveal being optimal at time τ^*.

Proof of Proposition A1: First, we need to compute the condition that guarantees a reveal strategy at time τ^*. The equilibrium price function to use in this calculation is $P_{fi,t}^{before}(\lambda^H)$ if conceal (i.e. the manager deviates), and $P_{fi,t}^{after}$ if it reveals. We obtain

$$U_{comb,\tau^*}^{reveal} = \left(\omega\eta\frac{1}{r-g}+(1-\omega)\,\eta_d\right)e^{G\tau^*}\left(\frac{z-g-\delta}{\beta-g}\right)$$

$$U_{comb,\tau^*}^{conceal} = \left(\omega\eta A_\lambda^{fi}+(1-\omega)\,\eta_d\,(z-G-\delta)\right)e^{G\tau^*}\left(\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}\right)$$

Let $\eta_d = \eta/(r-g)$. At τ^*, (A11) follows from $U_{comb,\tau^*}^{reveal} > U_{comb,\tau^*}^{conceal}$, where

$$\mathcal{L}_2 = \frac{z-g-\delta}{(\beta-g)(r-g)}$$

Before τ^*, the expected utility under the combined package depends on both a dividend component and a stock component. For given effort e, the dividend-based component is

$$U_{Div,t}(e) = E\left[\int_t^{\tau^*} e^{-\beta(u-t)}\left(w_u\left(1-c\left(e_u\right)\right)\right)du + e^{-\beta(\tau^*-t)}U^{reveal}_{Div,\tau^*}\right]$$
$$= \frac{e^{Gt}}{\beta+\lambda\left(e\right)-G}\eta_d\left(\left(z-G-\delta\right)\left(1-c\left(e_u\right)\right)+\lambda\left(e_u\right)\left(\frac{z-g-\delta}{\beta-g}\right)\right)$$

The stock-based component, conditional on a reveal equilibrium and thus price $P^{before}_{fi,t}(\lambda^H)$:

$$U_{Stock,t}(e) = E\left[\int_t^{\tau^*} e^{-\beta(u-t)}\left(w_u\left(1-c(e)\right)\right)du + e^{-\beta(\tau^*-t)}U^{reveal}_{Stock,\tau^*}\right]$$
$$= \frac{e^{Gt}}{\beta+\lambda-G}\eta\left(A^{fi}_{\lambda^H}\left(1-c(e)\right)+\frac{\lambda}{r-g}\left(\frac{z-g-\delta}{\beta-g}\right)\right)$$

Thus, the total combined utility before τ^* is $U_{Comb,t}\left(e\right) = \omega U_{Stock,t} + \left(1-\omega\right)U_{Div,t}$. Tedious computations show that (A12) follows from $U_{Comb}\left(e^H\right) > U_{Comb}\left(e^L\right)$, where

$$\mathcal{L}_1\left(\omega\right) = \omega A^{fi}_{\lambda^H} + \left(1-\omega\right)\left(\frac{z-G-\delta}{r-g}\right)$$

and $A^{fi}_{\lambda^H}$ is in (12). Finally, given the behavior of the manager (High Effort/Reveal), the price function is $P^{before}_{fi,t}$ for $t<\tau^*$ and $P^{after}_{fi,t}$ $t\geq\tau^*$. **Q.E.D.**

Corollary A1: *Let λ be the equilibrium intensity. Then:*

1. *Let $C^{fi} = \left(\frac{z-g-\delta}{r-g}\right)$. Under perfect information the average market decline at τ^* is*

$$E_0\left[\frac{P_{\tau^*}-P_{\tau^*-}}{P_{\tau^*-}}\right] = \frac{\frac{(r-G)}{(z-G-\delta)}C^{fi}-1}{\frac{\lambda}{(z-G-\delta)}C^{fi}+1} \tag{A13}$$

2. *Under asymmetric information the average market decline at T^{**} is*

$$E_0\left[\frac{P_{T^*}-P_{T^*-}}{P_{T^*-}}\right] = \frac{\frac{(r-G)}{(z-G-\delta)}C^{ai}-1}{\frac{\lambda}{(z-G-\delta)}C^{ai}+1} \tag{A14}$$

where $C^{ai} = e^{-(G-g)h^{**}}C^{fi} - \left(1-\xi\right)e^{-(G-g)h^{**}} < C^{fi}$

Proof of Corollary A1: (a) In the case of perfect information, we want to compute:

$$E\left[\frac{P^{after}_{\tau^*}-P^{before}_{\tau^*}}{P^{before}_{\tau^*}}\right] = \int_0^\infty \frac{P^{after}_{\tau^*}-P^{before}_{\tau^*}}{P^{before}_{\tau^*}}f\left(\tau^*\right)d\tau^* = \frac{\left(\frac{z-g-\delta}{r-g}\right)-A^{fi}_\lambda}{A^{fi}_\lambda}\int_0^\infty \frac{e^{G\tau^*}}{e^{G\tau^*}}f\left(\tau^*\right)d\tau^*$$
$$= \frac{\left(\frac{z-g-\delta}{r-g}\right)-A^{fi}_\lambda}{A^{fi}_\lambda}$$

Use the definition of A^{fi}_{λ} in (12) to find (A13) after some tedious agebraic steps.

(b) Under asymmetric information:

$$E\left[\frac{P_{T^*}-P_{T^*-}}{P_{T^*-}}\right] = \int_0^\infty \frac{P^L_{ai,T^*}-P_{ai,T^*}}{P_{ai,T^*}} f(T^*)\, dT^*$$

$$= \int_{h^{**}}^\infty \frac{e^{(G-g)(T^*-h^{**})+gT^*}\left(\frac{z-r-\delta}{r-g}+\xi\right)-e^{GT^*}A^{ai}_{\lambda}}{e^{GT^*}A^{ai}_{\lambda}} f(T^*)\, dT^*$$

$$= \frac{\left(\frac{z-r-\delta}{r-g}+\xi\right)e^{-(G-g)h^{**}}-A^{ai}_{\lambda}}{A^{ai}_{\lambda}}$$

where we used $T^{**}=\tau^*-h^{**}$, and the fac that $\int_{h^{**}}^\infty f(T^{**})\, dT^{**}=1$, as the density $f(T^{**})$ gives zero weight to $T^{**}<h^{**}$. Using now the definition of A^{ai}_{λ} in (23), tedious algebra shows (A14). **Q.E.D.**

Appendix B. Additional Material

In this appendix we report some additional numerical results that were not included in the body of the paper. Namely, we show first that for most plausible parameter values, pure stock-based compensation leads to a conceal equilibrium. In addition, we also consider the potential costs to the firm to provide incentives through stocks or dividends. Finally, we characterize the optimal weight on stock in the combined compensation package.

B.1. Stock-Based Compensation and Conceal Equilibrium at $t=\tau^*$

Given the effort choice e and thus λ, when is it optimal to conceal the change in the growth rate of investment opportunities? Figure BI plots the areas in which pure strategy Nash conceal or reveal equilibria obtain under stock-based compensation. The base numerical values of the parameters are in Table I. In all panels, the x-axis reports the initial high growth rate, G, ranging between 0 and 14%, while the y-axis represents a different variable in each panel: in the top panel it is low growth rate g; in the middle panel, it is the return on investment z, which ranges between 12% and 25%, and in the bottom panel, it is the expected time of maturity of the firm $E[\tau^*]=1/\lambda$, that ranges between 3 and 25 years. As the top panel indicates, under stock-based compensation even a 5% difference between G and g is sufficient to induce a conceal Nash equilibrium, in which the manager chooses the conceal strategy and investors rationally anticipate this behavior. There is no pure strategy reveal equilibrium for any combination of G and g. The intuition is as follows: if investors think that the manager follows a reveal strategy, the pricing function would reflect this belief and it is then given by the perfect information pricing formula (11). However, given these high prices, it is optimal for the manager to deviate and conceal the shift in investment opportunities. If in contrast shareholders think that the manager conceals, the pricing function is given by (22), and induces the manager to reveal.

The middle panel of Figure BI plots the areas of conceal and reveal equilibrium under stock-based compensation in the (z, G) space, where z is the return on capital. We see that the conceal strategy choice is, once again, a pervasive equilibrium outcome. In contrast with the top panel, there is a small region in which a reveal Nash equilibrium obtains under stock-based compensation. This is the area in the top-left corner, in which G is small and the return on capital z is extremely high. The intuition is that if growth is low, and return on capital high, there is little gain from concealing the change in investment opportunities (G is low anyway) and the cost of future repercussion is high, as the higher profitability of investments implies higher future prices, and thus a higher utility of the manager.

Finally, the bottom panel reports the conceal and reveal strategy areas under stock-based compensation in the space $(E[\tau^*], G)$. The outcome is once again the same: the conceal equilibrium prevails for most parameters, and especially for high growth G and high expected maturity time τ^* (or low λ).

We mentioned in Section IV.E., after Proposition 10, that a cashflow-based, bonus type of compensation always induces truth revelation at τ^*. Indeed, for all parameter combinations shown in the three panels of Figure BI, the manager compensated with a cashflow based, bonus type of contract always reveals his information. These examples point to a broad dichotomy: the stock-based compensation induces concealing strategy, while the cashflow-based, bonus compensation yields truth revelation. Unfortunately, the latter compensation typically does not induce the manager to exert high effort, as we show next.

B.2. Maximum and minimum weight to stock in CEO package

An important question pertains to the weight to give to stock in a combined compensation to ensure the manager does not retain important information about the company (recall that this discussion applies also to the deferred compensation, see Section IV.B.). Figure BII shows the range of the weight ω in the compensation package that can induce the first best outcome as a function of G. That is, those ω's that satisfy both conditions (**??**) and (**??**). The three panels assumes that the return on investment is 16% (top), 18% (middle) and 20% (bottom). In each panel, the upper line indicates the ω at which the manager is indifferent between conceal and reveal strategies under the high effort choice. For values of ω below that the manager prefers to reveal, which is what long term shareholders would like him to do. The lower line represents the level at which the manager is indifferent between choosing high and low effort, when he is in a reveal equilibrium. For ω above that the manager prefers to exert high effort. Thus the area between the two lines represents all possible weights ω on stocks in the combined compensation package that support the first best.

Notice that when the lower line reaches zero, the cashflow-based compensation alone is enough to induce high effort (recall the top-right area in Figure V). This does not mean that a little stock-based component would necessarily ruin the incentives to reveal, but aggressive stock compensation (or high proportion of ownership) will. Indeed, across the three panels, we see that the maximal weight on stock-based compensation never reaches 40%. That is, it is never optimal to provide more than 40% of the total compensation to CEOs in stocks, however delivered. Still, across panels we see that the lower line is in general above zero, especially if the return on capital z is low, implying that a zero weight to stocks in the CEO compensation package is also suboptimal. Moreover, the first best equilibrium obtains for a larger set of parameters than the conceal equilibrium. Indeed, returning to Figure V, while the stock-based compensation only induced high effort for a sufficiently high return on investment z and growth rate G, and no pure strategy equilibrium exists for lower values of both, the combined compensation package achieves a (High Effort/Reveal) equilibrium for all of the parameter combinations depicted in the figure.[24]

B.3. Cost of CEO's Compensation

We approximate the total costs paid to the CEO under the various cases by computing:

$$V_0 = E\left[\int_0^{\infty} e^{-rt} w_t dt\right] \tag{B15}$$

where w_t is the CEO compensation. We obtain closed form formulas for the three types of compensation, stock-based, cashflow-based, and combined, separately below. The key insight is that we can choose the key parameters to make the present value of the total compensation for the manager practically identical under every model. Given V_0 for each case, we can compute the quantity $P_0 - V_0$, that is, the firm value net of payments to the CEO.

More specifically, we start by setting $\eta_d = 5\%$ as our benchmark value under the dividend-based compensation in the (Low Effort/Reveal) equilibrium. In this case, the CEO's compensation equals 5% of firm value (see (B16) in the appendix). Denote this compensation cost $V_{Div}^{reveal,L}$. Next, we compute the value of η to make the compensation cost under stock-based compensation

[24]This is not a generic statement though, as first best cannot always be achieved for all parameter combinations. For instance, a higher cost of effort reduces the area (z, G) in which first best holds, although the area is still larger than the conceal equilibrium.

(High Effort/Conceal) equilibrium, $V_{Stock}^{Conceal,H}$, equal to the dividend-based (Low Effort/Reveal) equilibrium compensation cost $V_{Div}^{reveal,L}$. That is, such that $V_{Stock}^{Conceal,H} = V_{Div}^{Reveal,L}$. Finally, given these two values for η_d and η, we compute the cost for the combined compensation, denoted $V_{Comb}^{Reveal,H}$. In this latter case, we need to choose the weight ω from the range of possible values (see Figure BII). We choose the minimum ω that induces the CEO to exert costly effort, as in this case, when G and z are high, the combined compensation boils down to a dividend-based compensation with high effort (see bottom panel of Figure BII.)

Given the compensation costs, we then compute the net firm values in the three cases, namely, $\left[P_{fi,\lambda^L,0}^{before,fi}(\lambda^L) - V_{Div}^{reveal,H}\right]$ for dividend-based compensation, $\left[P_{ai,0} - V_{Stock}^{Conceal,H}\right]$ for stock-based compensation, and $\left[P_{fi,0}^{before} - V_{Comb}^{Reveal,H}\right]$ for the combined compensation. These quantities are plotted in Figure BIII as functions of the high growth rate G and for three values of return on investments z. In all panels, the solid line corresponds to the combined compensation case, the dotted line to the stock-based (High Effort/Conceal) equilibrium, and the dashed line to the dividend-based (Low Effort/Reveal) equilibrium. The figure makes apparent two facts: First, inducing high effort increases the net firm value, especially for high growth companies. This is true for both the stock-based compensation, which has the conceal strategy behavior as a side effect, and the combined compensation. Second, the combined compensation equilibrium leads to a higher net firm value compared to both the other equilibria.

Formulas for Cost of CEO Compensation: We report here the expected discounted value of the compensation costs. The derivations are left to the technical appendix. Consider first the pure dividend-based compensation under full revelation and low effort. In this case, $w_t = \eta_d D_t$ and therefore

$$V_{Div}^{Reveal,L} = E\left[\int_0^\infty e^{-rt}\eta_d D_t dt\right] = \eta_d A_{\lambda^L}^{fi} \tag{B16}$$

where $A_{\lambda^L}^{fi}$ is given in (12). Similarly, the present value of all payments to the CEO under the high-effort, stock-based compensation and conceal equilibrium, requires $w_t = \eta P_{ai,t}$, where $P_{ai,t}$ is given by (22). We obtain

$$V_{Stock}^{Conceal,H} = \eta\left(z - G - \delta\right)\left(\frac{1 - e^{-(r-G)h^{**}}}{r - G}\right)^2 + \eta A_{\lambda^H}^{ai} e^{-(r-G)h^{**}}\left(h^{**} + \frac{z - G - \delta}{r + \lambda - G}\right) \tag{B17}$$

where A_{λ}^{ai} is given in (23). Finally, the total costs under the full revelation / high effort equilibrium obtained from the combined compensation is given by:

$$V_{Comb}^{Reveal,H} = \omega\eta\left(A_{\lambda^H}^{fi} + \lambda^H \frac{z - g - \delta}{(r - g)^2}\right)\frac{1}{(r - G + \lambda^H)} + (1 - \omega)\,\eta_d A_{\lambda^H}^{fi} \tag{B18}$$

where $A_{\lambda^H}^{fi}$ is given in (12).

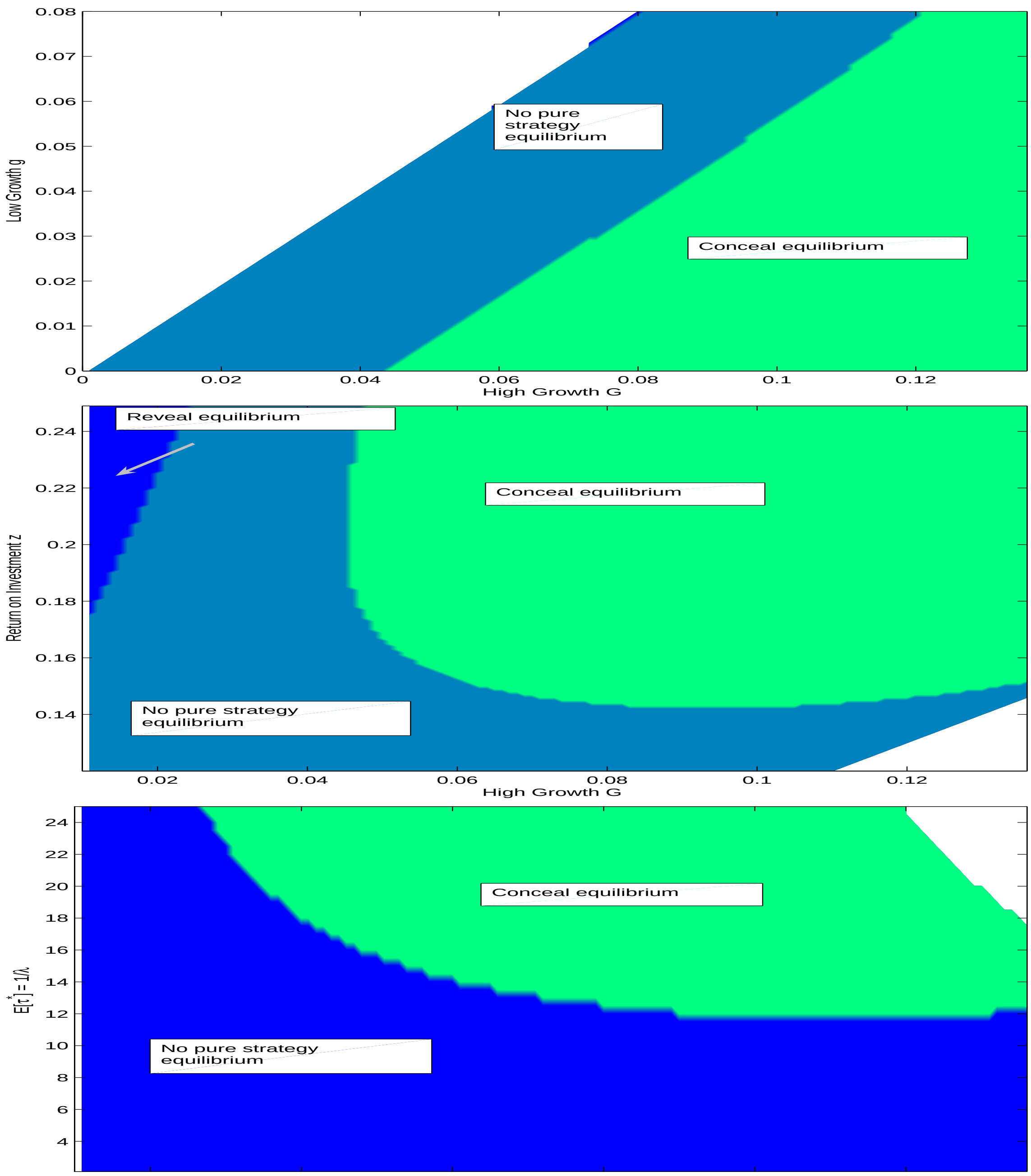

Figure BI: Conceal equilibrium under stock compensation The figure reports the conceal and reveal equilibria areas under stock compensation. In all figures, the $x-$axis reports the initial high growth G. In the top panel, the $y-$axis is the low growth g, in the middle panel, the $y-$axis is the return on capital z; and in the bottom panel, the $y-$axis is given by the expected time to maturity $E[\tau^*] = 1/\lambda$. The base parameters are in Table I.

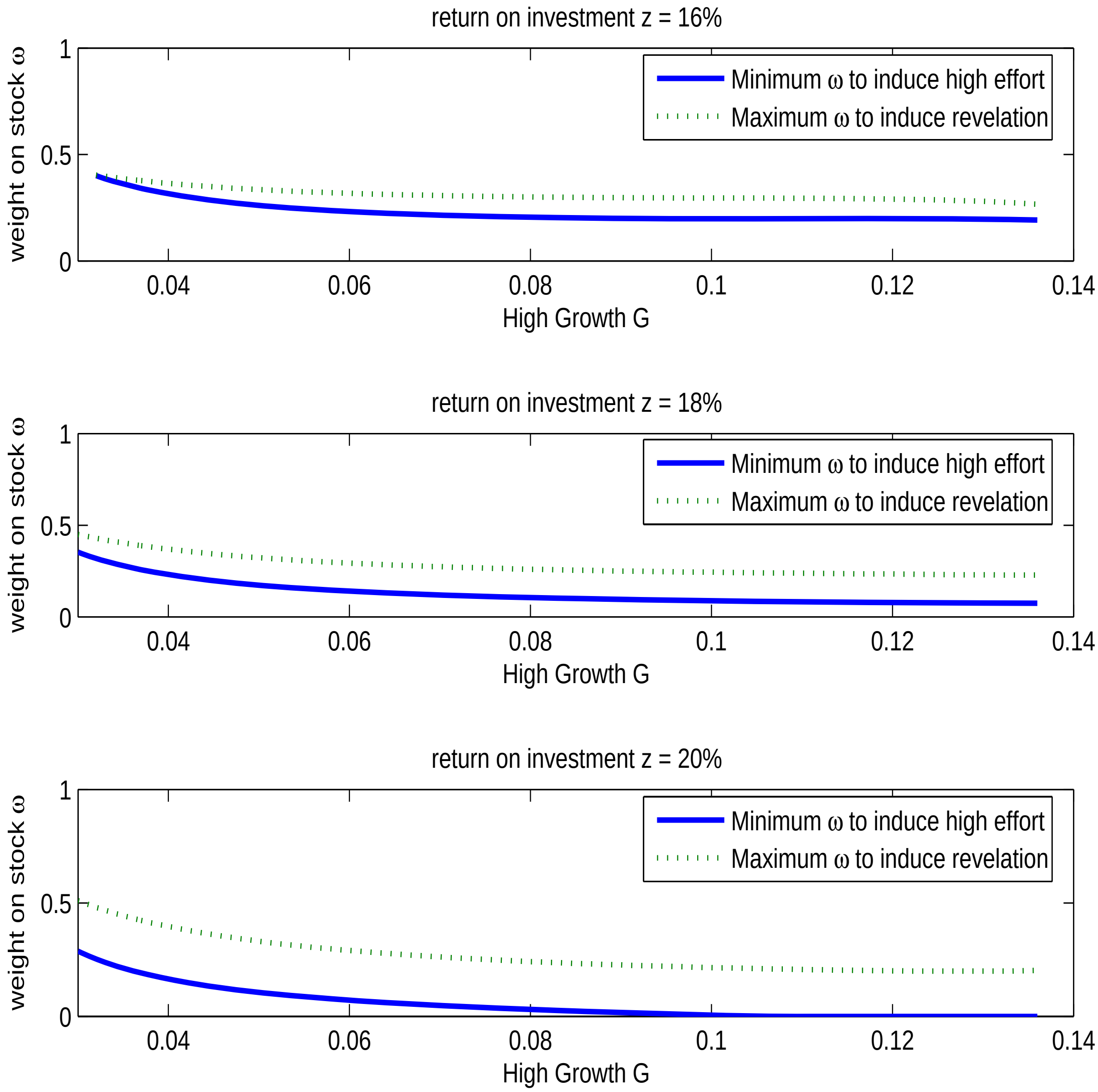

Figure BII: Optimal Weight ω on Stocks in Compensation Package. This figure reports the range of weights on the stock component of the combined compensation package that induces the first best for shareholders, that is, the high effort / reveal equilibrium. In each panel, which only differ for the level of return on capital z, the top line is the maximum ω that still induces the manager to reveal the shift in investment opportunuties, while the bottom line is the minimum ω that induces the manager to exert high effort. The remaining parameters are in Table I.

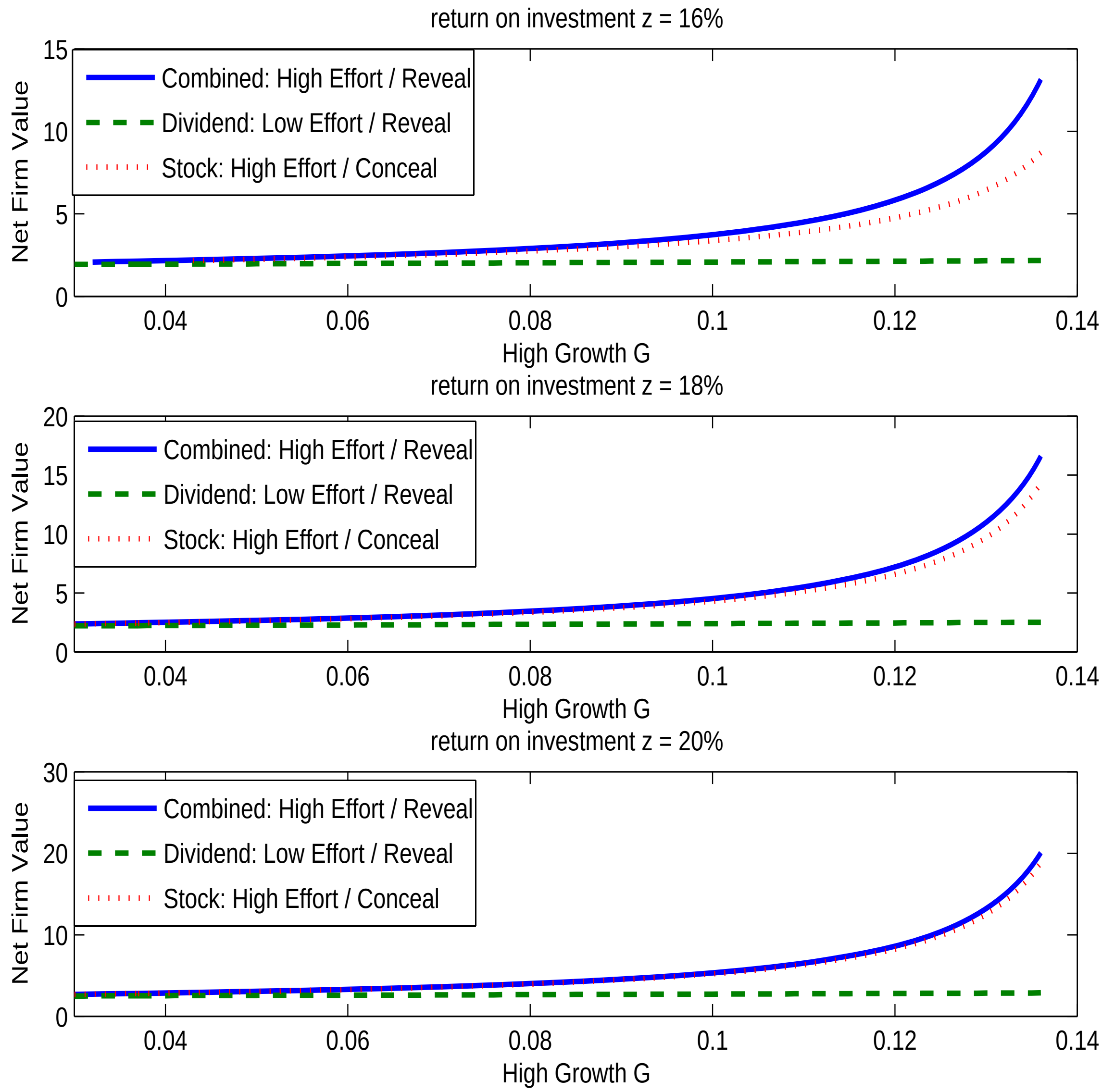

Figure BIII: Firm Value Net of CEO's Incentive Contract Cost. This figure compares the firm value net of the CEO incentive contract costs in the first best equlibrium under the combined compensation package (solid line) to the firm value under (a) dividend compensation when CEO exerts low effort (dashed line), and (b) stock-based compensation when CEO exerts high effort but conceals the worsening of investment opportunities at τ^* (dotted line). Each panel corresponds to a different return on capital z. The combined package in each panel is the one corresponding to the minimum weight ω to stock that still induces the CEO to exerts high effort. $\eta_d = 5\%$ while for each panel η_p is chosen so that the cost to the firm under case (a) and (b) is the same, and thus differens across G and z cases. The remaining parameters are in Table I.